Partial privatization and environmental policies

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Abstract

The paper compares emission tax and emission quota in a mixed duopoly when the partial privatization of a public firm is allowed. Furthermore, we consider the following two cases with regard to the objective of the public firm: (1) the public firm maximizes the weighted average of its profit and welfare and (2) the public firm maximizes the weighted average of its profit and the sum of consumer surplus and producer surplus. We show that emission tax is welfare superior to emission quota regardless of the degree of partial privatization in (1), whereas the former is inferior to the latter when the degree of partial privatization is high in (2).

Key words: environment; mixed duopoly; quota; tax

JEL classification: L33, Q58

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1 Introduction

Environmental policies in a mixed oligopoly have been analyzed in recent years. With regard to the previous studies on environmental policy, Bárcena-Ruiz and Garzón (2006), Beladi and Chao (2006), Wang and Wang (2009), and Wang et al. (2009) investigate the effects of emission tax in a domestic market, while Ohori (2006a, 2006b) considers the same in an international market. With regard to the earlier comparative works on environmental policies, Kato (2006) compares tradable emission permits with non-tradable emission permits, Naito and Ogawa (2009) compare emission standard with emission tax, and Kato (2010) compares emission taxes and emission quotas. The emergence of these works is linked to the phenomenon that the concern for environmental problems has been growing in both the developed and developing countries, where mixed oligopolies are quite common.[1]

This paper investigates welfare comparisons between emission tax and emission quota in a mixed duopoly where the partial privatization of a public firm is allowed. Further, we consider two cases for the objective of the public firm: (1) the public firm maximizes the weighted average of its profit and welfare and (2) the public firm maximizes the weighted average of its profit and the sum of consumer surplus and producer surplus (CSPS). In this paper, we call the public firm in the former case a welfare maximizer and that in the latter case a CSPS maximizer.

We comment on the two objectives of the public firm. The first objective has a theoretical basis. As is often assumed in previous studies on standard mixed oligopoly, the objective of the public firm is to maximize social welfare. This setting is also used in studies on the environmental problems in a mixed oligopoly (Bárcena-Ruiz and Garzón (2006); Cato (2008); and Naito and Ogawa (2009)). In this setting, we can consider the role of the public firm facing environmental problems in the framework of a standard mixed oligopoly: does the existence of a welfare-maximizing public firm enhance welfare given the environmental problem?

[1]See Bárcena-Ruiz and Garzón (2006) and Ohori (2006a).

However, there exist some earlier works where the public firm has another objective—CSPS maximization—such as Beladi and Chao (2006), Ohori (2006b), and Wang and Wang (2009).[2] As is pointed out by Ohori (2006a), several public firms—as they were at that time in many developing countries—showed poor performance with regard to environmental quality. Therefore, the second objective of the public firm seems to be quite natural, and has a practical basis. It might be worthwhile to comparatively analyze welfare under both objectives of the public firm.

The work most related to this paper is Kato (2010). Kato (2010) examines the effects of emission tax and emission quota in a mixed duopoly and shows that whether or not emission tax is welfare superior to emission quota is determined by the values of the parameters of the production and abatement cost functions when the public firm is a pure welfare maximizer, whereas emission quota is always welfare superior to emission tax when the public firm is a pure CSPS maximizer. Kato (2010) also shows that when the public firm is a pure welfare maximizer, full privatization yields the lowest welfare regardless of the cost parameters, whereas it might enhance welfare for some range of the parameters when the public firm is a pure CSPS maximizer.

However, Kato (2010) does not consider the partial privatization of the public firm. As Wang et al. (2009) show that partial privatization enhances welfare under emission tax, there is a possibility that the degree of partial privatization might change the superiority of these environmental policies. Therefore, this paper examines how the degree of partial privatization affects welfare superiority between emission tax and emission quota. Further, this paper also investigates how the difference in the public firm's objective affects the

[2]With regard to Wang and Wang (2009) and Ohori (2006b), the objective of the public firm (the framework) is slightly different from ours. In Wang and Wang (2009), the public firm's objective is to maximize consumer surplus and its profit. In Ohori (2006b), the consumption externality is considered although the objective of the public firm is as in this paper. However, as the public firm does not incorporate environmental damage and tax revenue into its objective in Wang and Wang (2009), and as Ohori (2006b)'s model is the same as ours, we model our work on both Wang and Wang (2009) and Ohori (2006b).

above results.

This paper is organized as follows. The next section describes our model. Sections 3 and 4 derive the equilibrium outcomes under both emission tax and emission quota in the case where the public firm is a welfare maximizer and in the case where the public firm is a CSPS maximizer, respectively. Section 5 compares the equilibrium outcomes of these two cases. Section 6 concludes the main text. The appendixes contain the proofs of the propositions.

2 Model

We follow the model used by Bárcena-Ruiz and Garzón (2006), Naito and Ogawa (2009), and Kato (2010). Consider an industry with two firms—one public (firm 0) and one private (firm 1)—producing a homogeneous good. The inverse demand function of the good is given by $p = \alpha - Q$, where $Q = q_0 + q_1$ denotes the total output; q_i $(i = 0, 1)$, the output of firm i; p, the price of the good; and $\alpha > 0$. Both firms have symmetric production cost functions given by $c_i^p(q_i) = cq_i^2/2$.

Pollution e_i is generated by production. Producing one unit of output generates one unit of pollution. Firms can reduce their pollution by reducing their output or by making abatement effort a_i. Firm i's emission can be represented as $e_i = max\{q_i - a_i, 0\}$. The abatement cost function of firm i is $c_i^a(a_i) = ka_i^2/2$.[3] The profit of firm i is given by

$$\pi_i(q_0, q_1, a_i) = (\alpha - q_0 - q_1)q_i - \frac{cq_i^2}{2} - \frac{ka_i^2}{2}. \tag{1}$$

Welfare is the sum of consumer surplus (CS), producer surplus (PS), and environmental damage, and is given by

$$W(q_0, q_1, a_0, a_1) = \int_0^Q (\alpha - s)ds - \sum_{i=0}^{1} \frac{cq_i^2}{2} - \sum_{i=0}^{1} \frac{ka_i^2}{2} - \frac{(e_0 + e_1)^2}{2}, \tag{2}$$

[3] We follow the assumption that the cost of production and of abatement effort are additively separable. For the causes underlying the usage of this assumption, see Bárcena-Ruiz and Garzón (2006) and Kato (2010).

where the last term represents the environmental damage.

Next, we define the objective function of each firm. The objective function of the private firm is its own profit. With respect to the objective function of the public firm (U or $\bar{U}$), we consider the following two cases:

$$\text{Public firm is a welfare maximizer: } U = \theta W + (1-\theta)\pi_0, \tag{3}$$

$$\text{Public firm is a CSPS maximizer: } \bar{U} = \theta(CS + PS) + (1-\theta)\pi_0, \tag{4}$$

where $\theta \in [0, 1]$. When $\theta = 0$, the public firm is a pure profit maximizer regardless of its objective, and when $\theta = 1$, the public firm is a pure welfare maximizer in the former case and a pure CSPS maximizer in the latter case. θ is understood as the share holding of the public sector, and $1 - \theta$, as that of the private sector.[4]

The decision-making sequence of the government and the firms is as follows. First, in each case, the government chooses the level of regulation, given the degree of partial privatization θ. Then, each firm simultaneously chooses its output and abatement effort.

To simplify analysis, we assume the parameters of the cost functions c and k are equal to 1. Of course, the value of these parameters might affect the results of the paper. However, given this value, we can obtain notable results that are quite different from those in previous studies.

3 Public firm is a welfare maximizer

3.1 Emission tax

Consider a situation in which the government imposes an emission tax. The maximization problem of each firm is given by

$$\max_{q_0, a_0} \quad U^t(q_0, q_1, a_0, a_1), \tag{5}$$

$$\max_{q_1, a_1} \quad \pi_1^t(q_0, q_1, a_1), \tag{6}$$

[4]For a rationalization of this objective function, see Bös (1991).

where we denote $U^t = \theta W + (1-\theta)\pi_0^t$ and π_i^t as $\pi_i(q_i, q_j, a_i) - te_i$ $(j \neq i = 0, 1)$, where t denotes emission tax.

The first-order conditions of the above maximization problem are as follows:

$$\frac{\partial U^t}{\partial q_0} = \alpha - 3q_0 - (1+\theta)q_1 + \theta(a_0 + a_1) - (1-\theta)t = 0, \tag{7}$$

$$\frac{\partial U^t}{\partial a_0} = -(1+\theta)a_0 - \theta a_1 + \theta(q_0 + q_1) + (1-\theta)t = 0, \tag{8}$$

$$\frac{\partial \pi_1^t}{\partial q_1} = \alpha - q_0 - 3q_1 - t = 0, \tag{9}$$

$$\frac{\partial \pi_1^t}{\partial a_1} = -a_1 + t = 0. \tag{10}$$

When $\theta = 1$, the public firm is a pure welfare maximizer, and consequently, the term related to emission tax does not appear in (7) and (8). We note that given the model and the assumption, the concavity of the objective function of each firm in all cases is satisfied. Therefore, the second-order conditions of the maximization problem of each firm are satisfied in all cases. This, however, is not shown in this paper.

Given the above behavior of each firm, the government maximizes welfare with regard to t. We can obtain the following equilibrium outcomes:

$$q_0^T = \frac{3\alpha(40 - 56\theta + 59\theta^2 - 13\theta^3)}{\Delta^T}, \quad q_1^T = \frac{3\alpha(40 - 72\theta + 81\theta^2 - 38\theta^3 + 7\theta^4)}{\Delta^T},$$

$$a_0^T = \frac{\alpha(72 - 100\theta + 107\theta^2 - 34\theta^3 + 9\theta^4)}{\Delta^T}, \quad a_1^T = \frac{\alpha(72 - 124\theta + 143\theta^2 - 61\theta^3 + 6\theta^4)}{\Delta^T},$$

$$e_0^T = \frac{\alpha(48 - 68\theta + 70\theta^2 - 5\theta^3 - 9\theta^4)}{\Delta^T}, \quad e_1^T = \frac{\alpha(4 - 4\theta + 3\theta^2)(12 - 11\theta + 5\theta^2)}{\Delta^T},$$

$$t^T = \frac{\alpha(72 - 124\theta + 143\theta^2 - 61\theta^3 + 6\theta^4)}{\Delta^T}, \quad Q^T = \frac{3\alpha(80 - 128\theta + 140\theta^2 - 51\theta^3 + 7\theta^4)}{\Delta^T},$$

$$A^T = \frac{\alpha(144 - 224\theta + 250\theta^2 - 95\theta^3 + 15\theta^4)}{\Delta^T}, \quad E^T = \frac{2\alpha(4 - \theta)(12 - 17\theta + 17\theta^2 - 3\theta^3)}{\Delta^T},$$

$$W^T = \frac{3\alpha^2(96 - 160\theta + 176\theta^2 - 70\theta^3 + 9\theta^4)}{2\Delta^T},$$

where $\Delta^T = 552 - 940\theta + 1049\theta^2 - 442\theta^3 + 69\theta^4 > 0$. A and E denote the total abatement effort and the total emission, respectively. In the subsequent analysis, the superscripts T and Q denote the equilibrium outcomes under emission tax and emission quota when the public firm is a welfare maximizer, respectively.

3.2 Emission quota

Consider a situation in which the government imposes an emission quota. The maximization problems of firm 0 and firm 1 are given by

$$\max_{q_0,a_0} \quad U(q_0,q_1,a_0,a_1) \ \text{ s.t. } \bar{e} = e_0(q_0,a_0), \tag{11}$$

$$\max_{q_1,a_1} \quad \pi_1(q_0,q_1,a_1) \ \text{ s.t. } \bar{e} = e_1(q_1,a_1). \tag{12}$$

In our subsequent analysis, we restrict our attention to the case where each emission quota is binding because we focus on the comparison of the effectiveness of emission tax and emission quota.

We substitute $a_i = a_i(q_i,\bar{e}) = q_i - \bar{e}$ into each objective function, and then define U^q as $U(q_0,q_1,a_0(q_0,\bar{e}),a_1)$ and π_1^q as $\pi_1(q_0,q_1,a_1(q_1,\bar{e}))$.

Calculating the first-order conditions of the maximization problem of each firm, we find that

$$\frac{\partial U^q}{\partial q_0} = \alpha + \bar{e} - (4-\theta)q_0 - q_1 = 0, \tag{13}$$

$$\frac{\partial \pi_1^q}{\partial q_1} = \alpha + \bar{e} - q_0 - 4q_1 = 0. \tag{14}$$

Given the behavior of each firm, the government maximizes welfare with regard to $\bar{e}$. The equilibrium outcomes are as follows:

$$q_0^Q = \frac{18\alpha(15-4\theta)}{\Delta^Q}, \qquad q_1^Q = \frac{6\alpha(3-\theta)(15-4\theta)}{\Delta^Q},$$

$$a_0^Q = \frac{\alpha(162-18\theta-5\theta^2)}{\Delta^Q}, \qquad a_1^Q = \frac{\alpha(162-108\theta+19\theta^2)}{\Delta^Q},$$

$$\bar{e}^Q = e_i^Q = \frac{\alpha(108-54\theta+5\theta^2)}{\Delta^Q},$$

$$Q^Q = \frac{6\alpha(6-\theta)(15-4\theta)}{\Delta^Q}, \qquad A^Q = \frac{2\alpha(162-63\theta+7\theta^2)}{\Delta^Q},$$

$$E^Q = \frac{2\alpha(108-54\theta+5\theta^2)}{\Delta^Q},$$

$$W^Q = \frac{3\alpha^2(108-54\theta+5\theta^2)}{\Delta^Q},$$

where $\Delta^Q = 1242 - 666\theta + 91\theta^2 > 0$.

4 Public firm is a CSPS maximizer

4.1 Emission tax

Consider a situation in which the government imposes an emission tax. The maximization problem of each firm is given by

$$\max_{q_0,a_0} \quad \bar{U}^t(q_0,a_0,q_1,a_1), \tag{15}$$

$$\max_{q_1,a_1} \quad \pi_1^t(q_0,q_1,a_1). \tag{16}$$

Here, we denote $\bar{U}^t$ as $\theta(CS+PS)+(1-\theta)\pi_0^t$ where $PS=\sum \pi_i^t$.

The first-order conditions of the above maximization problem are as follows:

$$\frac{\partial \bar{U}^t}{\partial q_0} = \alpha-(3-\theta)q_0-q_1-t, \tag{17}$$

$$\frac{\partial \bar{U}^t}{\partial a_0} = -a_0+t=0, \tag{18}$$

$$\frac{\partial \pi_1^t}{\partial q_1} = \alpha-q_0-3q_1-t=0, \tag{19}$$

$$\frac{\partial \pi_1^t}{\partial a_1} = -a_1+t=0. \tag{20}$$

We note that even when $\theta=1$, the term related to the emission tax appears in (17) and (18), which is different from when the public firm is a welfare maximizer (see (7) and (8)).

The government chooses t to maximize welfare, given the firms' behavior. We obtain the following equilibrium outcomes:

$$q_0^{T'}=\frac{6\alpha(20-7\theta)}{\Delta^{T'}}, \qquad q_1^{T'}=\frac{3\alpha(20-7\theta)(2-\theta)}{\Delta^{T'}},$$

$$a_i^{T'}=\frac{2\alpha(36-20\theta+3\theta^2)}{\Delta^{T'}},$$

$$e_0^{T'}=\frac{2\alpha(3+\theta)(8-3\theta)}{\Delta^{T'}}, \qquad e_1^{T'}=\frac{\alpha(48-62\theta+15\theta^2)}{\Delta^{T'}},$$

$$t^{T'}=\frac{2\alpha(36-20\theta+3\theta^2)}{\Delta^{T'}}, \qquad Q^{T'}=\frac{3\alpha(20-7\theta)(4-\theta)}{\Delta^{T'}},$$

$$A^{T'}=\frac{4\alpha(36-20\theta+3\theta^2)}{\Delta^{T'}}, \qquad E^{T'}=\frac{\alpha(96-64\theta+9\theta^2)}{\Delta^{T'}},$$

$$W^{T'}=\frac{3\alpha^2(96-64\theta+9\theta^2)}{2\Delta^{T'}},$$

where $\Delta^{T'} = 552 - 388\theta + 69\theta^2 > 0$.

In the subsequent analysis, we use superscripts T' and Q' to denote the equilibrium outcomes under emission tax and emission quota when the public firm is a CSPS maximizer, respectively.

4.2 Emission quota

Consider a situation in which the government imposes an emission quota. The maximization problem of each firm is given by

$$\max_{q_0, a_0} \quad \bar{U}(q_0, q_1, a_0, a_1) \;\text{ s.t. } \bar{e} = e_0(q_0, a_0), \tag{21}$$

$$\max_{q_1, a_1} \quad \pi_1(q_0, q_1, a_1) \;\text{ s.t. } \bar{e} = e_1(q_0, a_0). \tag{22}$$

We substitute $a_i = a_i(q_i, \bar{e}) = q_i - \bar{e}$ into each objective function, and then define $\bar{U}^q$ as $\bar{U}(q_0, q_1, a_0(q_0, \bar{e}), a_1)$.

Calculating the first-order conditions for the maximization problem of each firm, we get

$$\frac{\partial \bar{U}^q}{\partial q_0} = \alpha + \bar{e} - (4-\theta)q_0 - q_1 = 0, \tag{23}$$

$$\frac{\partial \pi_1^q}{\partial q_1} = \alpha + \bar{e} - q_0 - 4q_1 = 0. \tag{24}$$

The government chooses $\bar{e}$ to maximize W, given the firms' behavior. Calculating the equilibrium outcomes, we find that the equilibrium outcomes are the same as that under emission quota when the public firm is a welfare maximizer, that is, (23) and (24) are the same as (13) and (14), respectively. This is because emission quota is binding on all firms: the emission of each firm is constant. In this case, whether or not environmental damage and tax revenue are included in the public firm's objective does not affect the first-order conditions of the maximization problem of the public firm under emission quota. Therefore, the decision of the public firm is the same regardless of its objective. The equivalence between Q and Q' holds.

To summarize, we find that the behavior of the public firm differs with objective under emission tax, but stays the same under emission quota.

5 Comparison between emission quota and emission tax

Using the results of the aforementioned sections, we compare the equilibrium outcomes under two environmental policies. First, we obtain the following relationships for the equilibrium outcomes in each case.

Proposition 1. *When the public firm is a welfare maximizer,*

$$
\begin{aligned}
&q_1^T \leq q_1^Q \leq q_0^Q \leq q_0^T, \ \textit{for all } \theta \in [0,1],\\
&a_1^Q \leq a_1^T \leq a_0^T \leq a_0^Q, \ \textit{for all } \theta \in [0,1],\\
&e_1^T \leq e_1^Q = e_0^Q \leq e_0^T, \ \textit{for all } \theta \in [0,1],\\
&Q^T \leq Q^Q \quad \textit{if and only if } \theta \in \left[0, \theta^Q\right],\\
&A^Q \leq A^T \quad \textit{if and only if } \theta \in \left[0, \theta^A\right],\\
&E^T \leq E^Q \quad \textit{if and only if } \theta \in \left[0, \theta^E\right],
\end{aligned}
$$

where $\theta^Q \doteqdot 0.22$, $\theta^A \doteqdot 0.69$, $\theta^E \doteqdot 0.57$, *and the strong inequality holds if* $\theta \neq 0$.

When the public firm is a CSPS maximizer,

$$
\begin{aligned}
&q_1^{T'} \leq q_1^{Q'} \leq q_0^{Q'} \leq q_0^{T'}, \ \textit{for all } \theta \in [0,1],\\
&a_1^{Q'} \leq a_1^{T'} = a_0^{T'} \leq a_0^{Q'}, \ \textit{for all } \theta \in [0,1],\\
&e_1^{T'} \leq e_1^{Q'} = e_0^{Q'} \leq e_0^{T'}, \ \textit{for all } \theta \in [0,1],\\
&Q^{Q'} \leq Q^{T'}, \ \textit{for all } \theta \in [0,1],\\
&A^{Q'} \leq A^{T'}, \ \textit{for all } \theta \in [0,1],\\
&E^{Q'} \leq E^{T'} \quad \textit{if and only if } \theta \in \left[0, \bar{\theta}\right],
\end{aligned}
$$

where $\bar{\theta} = (81 - \sqrt{3551})/43 \doteqdot 0.50$ *and the strong inequality holds if* $\theta \neq 0$.

Proof. A simple comparison of the equilibrium outcomes yields the results in Proposition 1. For the proofs of some comparisons, see Appendix A. □

The intuition behind Proposition 1 is as follows. First, we deal with the comparison of the equilibrium outcomes of each firm under the two objectives on q_i, a_i, and e_i. In both cases, when θ increases, the public firm has an incentive to produce more than the private firm because the public firm chooses its output considering consumer surplus. However, an increase in the output of the public firm decreases the output of the private firm and this results in an increase in the inefficiency of product allocation. Taking into consideration this negative effect, it is desirable for the private firm to produce more. Therefore, the government chooses the emission quota level to enhance welfare with the efficiency of production allocation considered. As a result, the differences in the emission and output of both firms are smaller under emission quota than under emission tax. Emission quota requires more (less) abatement effort from the public (private) firm as compared to emission tax. These results are common under both objectives. We note that the public firm produces more when the public firm is a CSPS maximizer than when it is a welfare maximizer because in the latter case, the public firm chooses its output by taking into consideration not only consumer surplus but also environmental damage.

Next, we compare the equilibrium outcomes of Q, A, and E under the two objectives. We first consider the case when the public firm is a welfare maximizer. Suppose that θ is small. Simple calculation shows that emission quota (tax) increases (decreases) in θ as shown in Figure 1. From the results obtained in the previous section, we note that the public firm—when it is a welfare maximizer—behaves as if it is a CSPS maximizer under emission quota. Given this fact, we get that the difference between the increases in the public firm's output under the two objectives is small. On the other hand, the difference between the output levels of the private firm under the two objectives is large. This is because under emission quota, the private firm chooses its output by taking into consideration not only the production cost but also the abatement cost since it can adjust its abatement effort by choosing its output to satisfy its emission constraint. However, under emission tax, the private firm chooses its output by taking into consideration only its production cost and emission tax payment because it chooses its abatement to be equal to the emission tax level. As a result, given an increase in the output of the public firm, the private firm does not decrease its output under emission quota as much as under emission tax. Therefore, $Q^Q > Q^T$. With regard to the total abatement effort, from the aforementioned results, we get that emission tax requires the abatement of both firms to be equal to the emission tax level to a large extent, whereas the emission quota is more relaxed for the private firm and more severe for the public firm. Given this asymmetric effect for the abatement cost, $A^T > A^Q$, and hence, $E^Q > E^T$.

We suppose that θ is large. In this case, the effects of emission tax on both firms are asymmetric: emission tax does not directly affect the behavior of the public firm whereas the emission quota level is the same for both firms. The public firm can expand its output more under emission tax. Therefore, $Q^T > Q^Q$. With respect to the total abatement effort, the public firm voluntary makes the abatement effort under emission tax, whereas it makes a much larger abatement effort under emission quota to meet the severe emission constraint (see Figure 1). This yields that $A^Q > A^T$, and hence, $E^T > E^Q$.

Finally, we consider the case when the public firm is a CSPS maximizer. Under emission tax, the public firm has a strong incentive to produce more when the public firm is a CSPS maximizer than when the public firm is a welfare maximizer because the public firm does not consider environmental damage when it is a CSPS maximizer. This strong incentive to expand output yields $Q^T > Q^Q$. Different from the case when the public firm is a welfare maximizer, emission tax requires the abatement effort of each firm to be perfectly equal to the emission tax level. Under emission tax, simple calculation yields that the emission tax level monotonically increases in θ. Therefore, $A^T > A^Q$. With regard to the total emission, whether or not the emission under emission tax is larger than that under emission quota is not determined uniquely. When θ is small, it is desirable from the welfare viewpoint to increase consumer surplus at the expense of an increase in environmental damage. However, when θ is large, under emission tax, an increase in the total output results in a considerable increase in the inefficiency of production allocation and environmental damage. To reduce these negative effects, the emission tax level is kept sufficiently high. Therefore, $E^Q > E^T$ when θ is large.

Finally, we compare welfare under emission tax with that under emission quota. Then, we obtain the following proposition:

Proposition 2.

$$\textit{When the public firm is a welfare maximizer,} \quad W^Q \leq W^T \quad \textit{for all } \theta \in [0,1].$$
$$\textit{When the public firm is a CSPS maximizer,} \quad W^{Q'} \leq W^{T'} \quad \textit{if and only if } \theta \in \left[0, \bar{\theta}\right].$$

Here, $\bar{\theta} = (81 - \sqrt{3551})/43 \fallingdotseq 0.50$.

Proof. See Appendix B. □

Figures 2 and 3 show welfare comparison when the public firm is a welfare maximizer and a CSPS maximizer, respectively. In Figure 3, we denote $\hat{\theta} \fallingdotseq 0.82$.

The intuition behind Proposition 2 is as follows. First, we consider welfare comparison when the public firm is a welfare maximizer. Note that under emission quota, the public firm chooses its output and abatement effort as if the public firm is a CSPS maximizer. In this case, the public firm is no longer a pure welfare maximizer even if $\theta = 1$, and therefore, the behavior of the public firm might result in a larger distortion in welfare than that under emission tax. In particular, when θ is small, an increase in emission quota yields a large enough increase in environmental damage, and then, $W^T > W^Q$. When θ is large, environmental damage and the inefficiency of the production allocation are larger under emission tax than under emission quota. However, consumer surplus is also larger under emission tax. The latter positive effect dominates the former negative effects because the government can control the behavior of the private firm to a large extent by choosing the emission tax level: the public firm is closer to being a welfare maximizer, and then, emission tax can be regarded as a way to control the private firm.[5] Therefore, regardless of θ, emission tax is welfare superior to emission quota when the public firm is a welfare maximizer.

[5] In this paper, we assume that $c = k = 1$. If we consider alternative values, the results may change.

Next, we consider the case when the public firm is a CSPS maximizer. A simple calculation shows that emission tax increases in $\theta \in [0, 1]$ whereas emission quota increases (decreases) in $\theta \in [0, 9/11]$ ($\theta \in [9/11, 1]$). When θ is small, the positive effects of an increase in consumer surplus and tax revenue dominate the negative effects of an increase in environmental damage and the inefficiency of production allocation. However, when θ is large, the strong incentive for the public firm to produce more results in the negative effect being quite large, and then, the negative effect dominates the positive effect. As a result, emission tax is welfare superior (inferior) to emission quota when θ is small (large) and the public firm is a CSPS maximizer.

With regard to the public firm being a CSPS maximizer, we finally refer to the result in Kato (2010), which does not consider the partial privatization of the public firm, that is, considers only $\theta \in \{0, 1\}$. Kato (2010) shows that $W^{Q'} > W^{T'} > W^P$ in the same setting as in this paper, where the superscript P denotes the equilibrium under emission tax/quota after full privatization ($\theta = 0$). From the results in this paper, if we consider the partial privatization of the public firm, we find that the degree of partial privatization affects the welfare ranking of the environmental policies, and that this effect is dependent on the public firm's objective.

6 Concluding remarks

The paper compares emission tax with emission quota in a mixed duopoly when the partial privatization of a public firm can be allowed. Furthermore, we consider the following two cases with regard to the objective of the public firm: (1) the public firm maximizes the weighted average of its profit and welfare and (2) the public firm maximizes the weighted average of its profit and CSPS. We show that emission tax is welfare superior to emission quota regardless of the degree of partial privatization in the former case, whereas emission tax is welfare inferior to emission quota when the degree of partial privatization is high in the latter case.

We discuss the possible implication of our results. If the public sector, which owns a public firm, emphasizes both the environmental problem and economic development (for example, as in the developed countries), the government should impose emission tax. However, if the public sector shows a marked disdain for the environmental problem (for example, as in the developing countries) and the public firm is slated for gradual privatization, the government should first impose emission quota, and then, impose emission tax after privatization is well under way. When choosing environmental policies, we have to pay attention to the situation of the public firm and the degree of privatization.

Appendix A

Proof of Proposition 1. First, we compare Q^Q and Q^T. We obtain

$$Q^Q - Q^T = \frac{3\phi(\theta)\theta^2\alpha}{\Delta^Q\Delta^T}, \tag{25}$$

where $\phi(\theta) = 148 - 672\theta - 112\theta^2 + 385\theta^3 - 85\theta^4$. We examine whether $\phi(\theta)$ is positive.

$$\begin{aligned}\phi'(\theta) &= -672 - 224\theta + 1155\theta^2 - 340\theta^3 \\ &= -14\theta(4 - 5\theta)^2 - 595(1 - \theta^2) - 77 + 10\theta^3 < 0 \ for\ all\ \theta \in [0, 1].\end{aligned}$$

From the results, we find that $\phi(\theta)$ is monotonically decreasing in $\theta \in [0, 1]$. As $\phi(0) = 148 > 0$ and $\phi(1) = -336 < 0$, there exits some θ^Q for which $Q^Q - Q^T = 0$. After calculations, we have $\theta^Q \fallingdotseq 0.22$.

Second, we compare A^Q and A^T. We obtain

$$A^Q - A^T = \frac{3\rho(\theta)\theta^2\alpha}{\Delta^Q\Delta^T}, \tag{26}$$

where $\rho(\theta) = -2248 + 5444\theta - 3972\theta^2 + 1252\theta^3 - 133\theta^4$. We examine whether $\rho(\theta)$ is positive.

$$\begin{aligned}\rho'(\theta) &= 5444 - 7944\theta + 3753\theta^2 - 532\theta^3 \\ &= 3753(1 - \theta)^2 + 438(1 - \theta) + 532(1 - \theta^3) + 721 > 0 \ for\ all\ \theta \in [0, 1].\end{aligned}$$

From the results, we find that $\rho(\theta)$ is monotonically increasing in $\theta \in [0,1]$. As $\rho(0) = -2248 < 0$ and $\rho(1) = 342 > 0$, there exits some θ^A for which $A^Q - A^T = 0$. After calculations, we have $\theta^A \fallingdotseq 0.69$.

Third, we compare E^Q and E^T. We obtain

$$E^Q - E^T = \frac{6\chi(\theta)\theta^2\alpha}{\Delta^Q\Delta^T}, \tag{27}$$

where $\chi(\theta) = 1198 - 3058\theta + 1930\theta^2 - 433\theta^3 + 24\theta^4$. We examine whether $\chi(\theta)$ is positive.

$$\begin{aligned}\chi'(\theta) &= -3058 + 3860\theta - 1299\theta^2 + 96\theta^3 \\ &= -1299(1-\theta)^2 - 1262(1-\theta) - 96(1-\theta^3) - 401 < 0 \; \textit{for all } \theta \in [0,1].\end{aligned}$$

From the results, we find that $\chi(\theta)$ is monotonically decreasing in $\theta \in [0,1]$. As $\chi(0) = 1198 > 0$ and $\chi(1) = -339 < 0$, there exits some θ^E for which $E^Q - E^T = 0$. After calculations, we have $\theta^E \fallingdotseq 0.57$.

Finally, we compare $E^{Q'}$ and $E^{T'}$. We obtain

$$E^{Q'} - E^{T'} = -\frac{3\nu(\theta)\theta^2\alpha}{\Delta^Q\Delta^{T'}},$$

where $\nu(\theta) = 70 - 162\theta + 43\theta^2$. We denote $\bar{\theta} \in [0,1]$ as the solution of $\nu(\bar{\theta}) = 0$. After calculations, we have $\bar{\theta} = (81 - \sqrt{3551})/43 \fallingdotseq 0.50$. □

Appendix B

Proof of Proposition 2. First, we compare welfare under emission quota and emission tax when the public firm is a welfare maximizer. We obtain

$$W^Q - W^T = -\frac{9[(1-\theta)^2\{10(1-2\theta)^2 + 62\} + 36\theta^2(1-\theta) + 16 - 8\theta^3 + 3\theta^4]\theta^2\alpha}{\Delta^Q\Delta^T} < 0.$$

Next, we compare welfare under emission quota and emission tax when the public firm is a CSPS maximizer. We obtain

$$W^{Q'} - W^{T'} = -\frac{9\nu(\theta)\theta^2\alpha^2}{2\Delta^Q\Delta^{T'}}.$$

The above relationship depends on the sign of $\nu(\theta)$. By quantifying the condition, we obtain Proposition 2. □

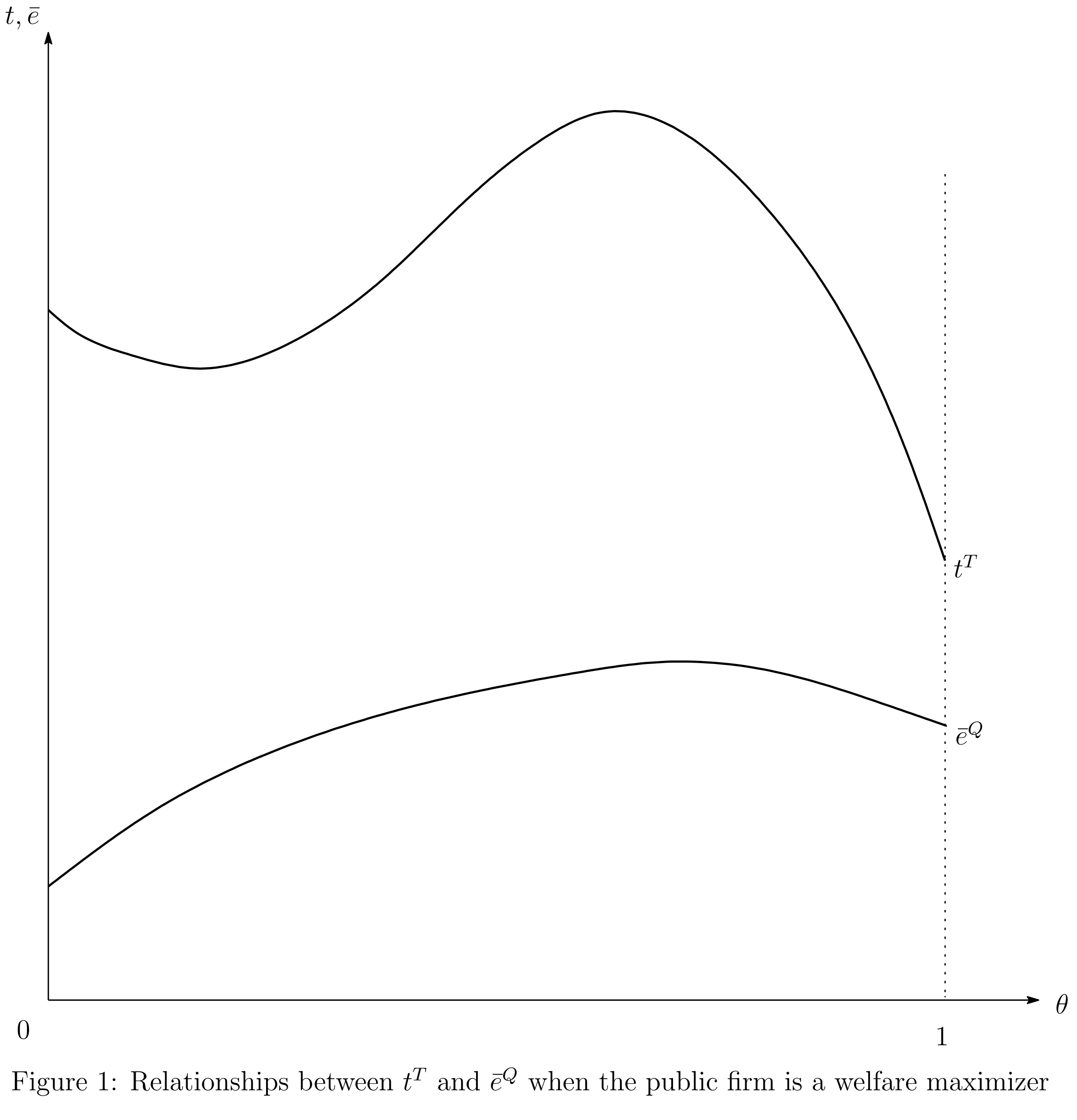

Figure 1: Relationships between t^T and $\bar{e}^Q$ when the public firm is a welfare maximizer

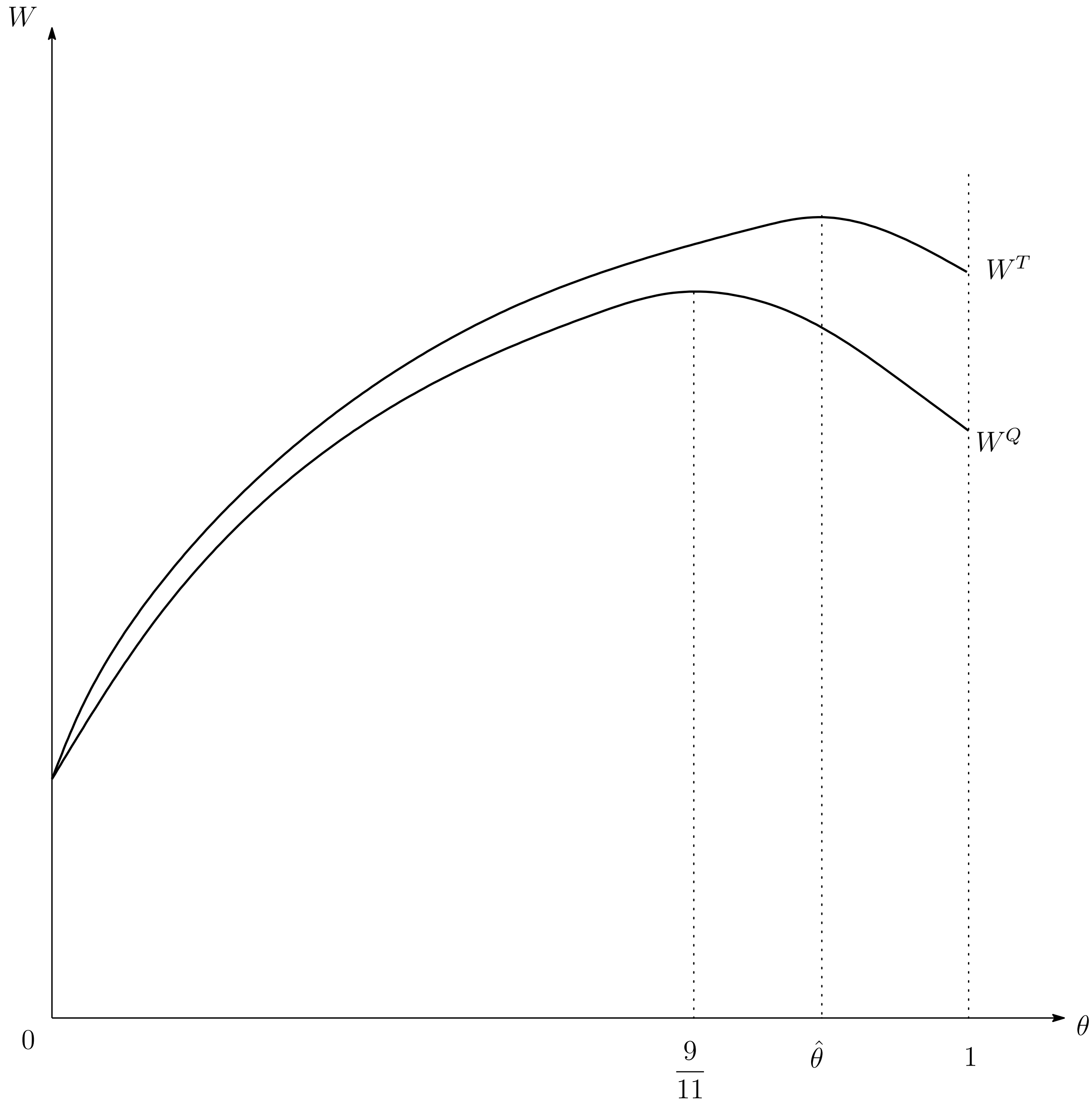

Figure 2: Welfare comparison when the public firm is a welfare maximizer

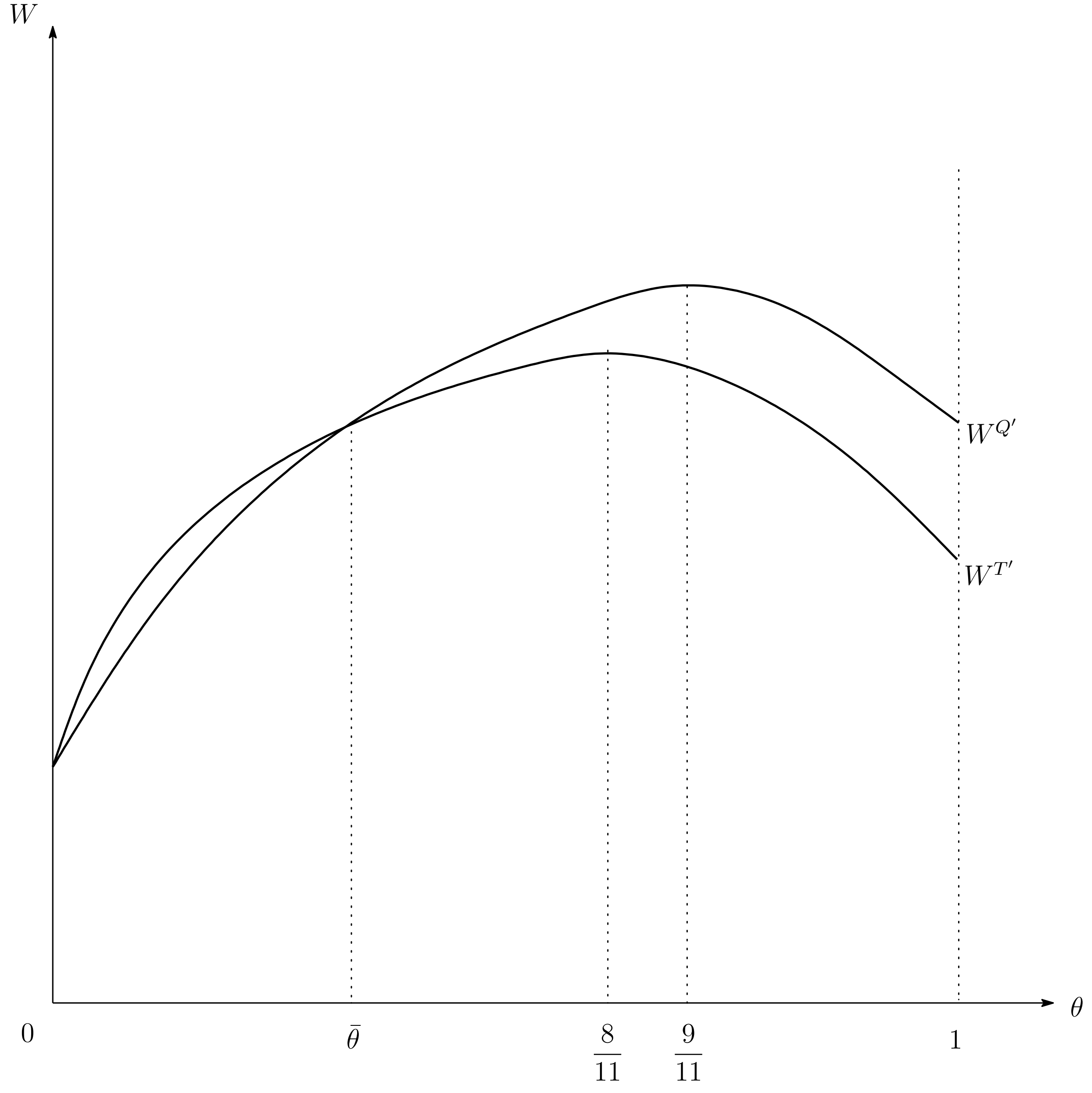

Figure 3: Welfare comparison when the public firm is a CSPS maximizer

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