Asymmetric demographic shocks and institutions: The impact on international capital flows and welfare

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Asymmetric Demographic Shocks and Institutions: The Impact on International Capital Flows and Welfare*

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Abstract

This paper examines the consequences of an asymmetric negative fertility shock on capital formation, saving/investment imbalance, and welfare. The framework of analysis is a Diamond-type overlapping-generations small open economy with capital market imperfection. The capital market imperfection is modelled through a symmetric wedge between foreign investor and domestic investor return on capital. The shock is transmitted to the small open economy depending on whether the wedge is below a given threshold. If the wedge is not too high, capital first flows in the small open economy to exploit the difference in returns on capital. After the shock has occurred, capital is repatriated in order to finance the old age consumption of rest of the world investors. If capital flows internationally, lifetime utility in the small open economy decreases unambiguously for individuals born one period before the shock occurs. Provided that the small open economy is initially below its golden rule, individuals born after the time the shock has occurred experience an increase in their lifetime utility.

JEL classification: F21; F32; H55; J10

Keywords: population aging; capital market imperfection; open economy; capital flows; welfare

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1 Introduction

There exist large differences in the timing and the size of the aging phenomenon across regions of the world. Those differences are transitory (see United Nations [18]). In a two country framework, under standard neoclassical assumptions, a partial equilibrium implication of the life cycle hypothesis is that the bulk of the saving supply triggered by the rapid aging country should flow to the slower aging country, where capital is relatively scarce and labor relatively abundant. This prediction matches with the past decades surge in capital flows to younger/poorer countries following their capital market liberalization. However, those international capital flows appear to be limited compared to what the neoclassical theory would predict, as claimed in Lucas [15]. Indeed, several capital market imperfections are likely to impede demographic differences from fostering international capital flows. In addition, those capital inflows have been unevenly distributed across younger/poorer countries. These stylized facts on capital movements, documented in Prasad and al. [16], suggest a role for the interaction between aging differences and capital market imperfections in explaining both the timing, the magnitude and the distribution of international capital flows across receiving countries. In the present framework, through introducing exogenous differences in fertility rates, I generate differences in capital returns between "source" (older) countries and "recipient" (younger) countries that in turn explain the magnitude and the timing of the flows. In the present paper, I provide an explanation for the uneven distribution of those flows across "recipient" countries through introducing a wedge between domestic and foreign investors return to capture diversion directed toward foreign investors.[1] Diversion can be undertaken by private agents (e.g. thievery, squatting or mafia protection) or public agent (e.g. confiscatory taxation and corruption). Hall and Jones [12] provide empirical evidence of the role of institutions and government policies in explaining cross-country differences in investment, productivity and thus output per worker. Shleifer and Wolfenzon [17] model how agency costs stemming from inefficient corporate governance and law enforcement mechanisms impede foreign capital from flowing to capital-scarce countries. Alfaro et al. [1] provide empirical evidence of the importance of the quality of institutional arrangements in explaining the relative lack of capital flows to developing countries. They argue that the effect of institutions on capital flows is the main channel through which the former affects output per capita.

[1] My argument takes further the main insight of the "push and pull" literature initiated by Calvo et al. [7].

The literature has been relatively silent on the relevance of the interaction between differences in the demographic dynamic and institutions in explaining international capital movements but abundant on the closed economy consequences of aging.[2] A recent literature has been addressing the economic consequences of aging differences in an open economy perspective using large scale simulation models.[3] Among others, Attanasio and Violante [3] and Brooks [5] simulations results point to a significant role of population age structure differences in explaining capital flow from fast aging OECD countries to slower aging emerging markets. Brooks' [5] simulation results also predict a future reversal in the direction of those international capital flows. Indeed, Brooks' [5] predictions suggest that capital will flow from currently younger countries to currently older ones as the former will enter into the fast aging stage of the demographic transition. There is however an important caveat to the literature that is being addressed in the present paper. To the extent of my knowledge, there is no study that analyzes the open economy adjustment to an asymmetric demographic shock in presence of capital market imperfections.[4] Arezki [2], building on Higgins [13], provides empirical evidence of the relevance of the interaction between population age structure differences and institutional quality in explaining current account position, using a panel of up to 115 countries over the period 1970 to 2000.

In this paper, I analyze the consequences of an asymmetric negative fertility shock on capital formation, saving/investment imbalance, and welfare. The framework of analysis is a Diamond-type overlapping-generations small open economy with a wedge between domestic and foreign investors return to capture diversion. The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the consequence of a rest of the world negative fertility shock on capital formation, saving/investment imbalances and welfare. In section 3.1, I find that the rest of the world shock is transmitted to the small open economy, depending on whether the wedge between the domestic investor return and foreign investor return on capital is below a given threshold. In section 3.2, I find that if the wedge is not too high, capital first flows in the small open economy in order to exploit the differences in returns on capital. After the shock has occurred, capital is repatriated

[2] Bosworth and al.[4] provide a useful survey on the financial and macroeconomic consequences of aging.

[3] Geide-Stevenson [10] and Groezen and Leers [11] focus on the open economy consequence of aging using Diamond type overlapping generations models in presence of various pension arrangements .

[4] Kenc and al. [14] developed a simulation model to analyze the consequence of aging in the European Union for Turkey, introducing imperfect capital mobility.

in order to finance old-aged consumption of rest of the world investors. In section 3.3, I find that if capital flows internationally, lifetime utility in the small open economy decreases unambiguously for individuals born one period before the shock occurred. Provided that the small open economy is initially below its golden rule of capital accumulation, individuals born after the shock has occurred experience an increase in their lifetime utility. Section 4 concludes.

2 The Model

The model consists of a small open economy and the rest of the world identical in every respect except in demographic patterns. Each country is represented by competitive output and factor markets, two overlapping generations (OLG), and an identical well-behaved constant returns to scale production function f, a model due to Diamond [8]. Labor is not mobile. Capital is perfectly mobile. The model is entirely standard except that I assume a symmetric wedge between domestic investor and foreign investor return on capital to capture diversion. The wedge is the the result of iceberg costs on capital return repatriation, so that for each unit of capital invested abroad a lump sum amount τ of the return is lost in transit.

All variables associated with the small open economy with respect to the rest of the world are distinguished by the upper script SOE and by the upper script RW respectively on the relevant variables. The variables associated with the rest of the world are also distinguished by an upper bar that indicates their exogenous nature.

2.1 Notation

$c_{1,t}^{i}$ = consumption while young by an individual living at time t in country i ; $i \in \{SOE, RW\}$;

$c_{2,t+1}^{i}$ = consumption while old by an individual at time $t+1$ living in country i; $i \in \{SOE, RW\}$;

s_t^i = aggregate asset owned while young by an individual at time t living in country i; for $i \in \{SOE, RW\}$;

w_t^i = real wage at time t in country i; for $i \in \{SOE, RW\}$;

r_t^i = interest rate on country i individual assets carried from period $t-1$ into period t in country i; for $i \in \{SOE, RW\}$;

k_t^i = capital labor ratio in country i at time t; $i \in \{SOE, RW\}$;

n_t^i = rate of growth of population in country i from period $t-1$ into t; for $i \in \{SOE, RW\}$;

ρ = pure rate of time preference; $\rho > 0$;

τ = wedge between foreign investor and domestic investor return on capital.

2.2 Individuals

Individuals in both regions live two periods: they work in the first period of their lives, and retire in the second. During the first period of their life each individual supplies inelastically one unit of labor. The optimization problem for an individual living in country i is given by equations (1), (2) and (3). The utility of lifetime consumption is maximized subject to an intertemporal budget constraint given by (2). The properties on U^i ensure that the intertemporal budget constraint will hold with equality and that an interior solution will be obtained for $c^i_{1,t}, c^i_{2,t+1}$ for $i \in \{SOE, RW\}$.

$$\underset{c^i_{1,t}, c^i_{2,t+1}}{max} \; U^i(c^i_{1,t}, c^i_{2,t+1}) = u(c^i_{1,t}) + \frac{1}{1+\rho} u(c^i_{2,t+1}) \tag{1}$$

$$c^i_{1,t} + \frac{c^i_{2,t+1}}{1+r^*_{t+1}} \leq w^i_t \tag{2}$$

$$c^i_{1,t}, \; c^i_{2,t+1} \geq 0 \tag{3}$$

For simplicity, I assume that the utility function is time separable and that the subutility function, u, is logarithmic. Thus the optimal saving of a young individual born at time t in country i is given by the following expression:

$$s^i_{1,t} = \frac{w^i_t}{2+\rho} \tag{4}$$

Optimal portfolio return namely r^* is the result of investors behavior analyzed in the following.

2.3 Firms

Firms located in region i maximize profits taking as given domestic factor prices. Equations (5) and (6) state that the capital rental market and labor market in region i are competitive.[5] I assume that capital fully depreciates over a period of time. Thus gross investment equals net investment.

$$r^i_t = f'(k^i_t) - 1 \tag{5}$$

$$w^i_t = f(k^i_t) - k^i_t f'(k^i_t) \tag{6}$$

[5] For $i \in \{SOE, RW\}$.

2.4 Investors

Given the presence of iceberg costs, the effective return associated with foreign investment equals domestic investor return minus a wedge, τ. Investors optimize the return on their portfolio taking as given returns in both locations. Optimization decision for a given investor i at period t is formally given by the following expression:[6]

$$r_t^* = \max(r_t^i,\ r_t^j - \tau) \tag{7}$$

2.5 Asset Allocation Equilibria

As a result of investors portfolio return optimization, the world economy can be at three different asset allocation equilibria:

1. At equilibrium of type 1, individuals located in both regions invest all their assets domestically. At this equilibrium, the rate of return on aggregate assets for an individual living in a given region equals her domestic return.

2. At equilibrium of type 2, small open economy individuals invest all their assets domestically and rest of the world individuals invest their assets in both locations. At this equilibrium, individuals living in the small open economy receive r^{SOE} as a return on their aggregate assets. Individuals living in the rest of the world receive $r^{SOE} - \tau$ as a return on their aggregate assets.

3. At equilibrium of type 3, small open economy individuals invest their assets in both locations and rest of the world individuals invest all their assets domestically. At this equilibrium, the small open economy individuals receive $\overline{r}^{RW} - \tau$ as a return on their aggregate assets. Individuals living in the rest of the world receive $\overline{r}^{RW}$ as a return on their aggregate assets.

For the purpose of realism, I am only interested in equilibria where in both regions individuals invest in the small open economy that corresponds to type 1 and 2 equilibria. At these two equilibria, an individual of a given region i born at time t receives her domestic return r^i as a return on her aggregate asset.[7]

[6]For i and $j \in \{SOE, RW\}$ with $i \neq j$.
[7]For $i \in \{SOE, RW\}$.

2.6 Equilibrium

Let us now collect the equations characterizing the equilibrium in both regions. In the following subsection, I also present the way I model the occurrence of an asymmetric negative fertility shock, whose consequences are analyzed in the following section.

2.6.1 Rest of the World

Whether the world economy is at type 1 or type 2 equilibrium does not affect the rest of the world economy. Indeed, given the small open economy assumption, the rest of the world economy is unaffected by changes affecting the small open economy. The dynamic equilibrium of the rest of the world economy, described in details in appendix A.1, corresponds to the equilibrium of a standard closed economy OLG model.

Further assuming that the production function f is twice differentiable and follows the Inada conditions, I obtain a first order difference equation in k_t^{RW} that describes the evolution of the model from arbitrary initial conditions given by the following expression:[8]

$$k_{t+1}^{RW}\left[(1+n_{t+1}^{RW})\right]-\left[\frac{f(k_t^{RW})-k_t^{RW}f'(k_t^{RW})}{2+\rho}\right]=0 \tag{8}$$

I assume that the rest of the world is subject to a fertility shock. γ^{RW} denotes the size of the shock and h^{RW} denotes the time pattern of the shock.[9] The fertility rate of the rest of the world economy at time t, n_t^{RW} is assumed to be equal to the sum of its steady state value n_{ss}^{RW} (being further normalized to zero) and its deviation from the steady state given by $\gamma^{RW}h_t^{RW}$ as described in expression (9). The variables evaluated at the economy steady state are distinguished by a lower index ss.

$$n_t^{RW}=n_{ss}^{RW}+\gamma^{RW}h_t^{RW} \tag{9}$$

I characterize the steady state of the rest of the world economy through formally giving the expression of its domestic returns at the steady state:

$$\overline{r}_{ss}^{RW}=f'(\overline{k}_{ss}^{RW})-1 \tag{10}$$

with $\overline{k}_{ss}^{RW}$ corresponding to the fix point solution of the difference equation (8).

[8] The Inada formally rewrites $f(0)=0$; $f'\succ 0$; $f'(0)=+\infty$ and $f'(0)=0$.

[9] In the case of a one period shock, h_t^{RW} takes the form:$\{0$ at $t=0$, ..., 0 at $t=t_0-1$, -1 at $t=t_0$, 0 at $t=t_0+1$,...$\}$
with t_0 denoting the time at which the shock occurs.

2.6.2 Small Open Economy

As stated before, I am only interested in equilibria where individuals living in both regions invest in the small open economy. I therefore describe the dynamic equilibrium of the small open economy for type 1 and type 2 equilibria where the small open economy individuals receive r^{SOE} as a return on their aggregate assets invested. I assume that the small open economy is not subject to any fertility shock. Thus its fertility rate at any period t, n_t^{SOE}, is at its steady state value (being further normalized to zero).

At the equilibrium of type 1, the small open economy is as if it were in autarky. The dynamic equilibrium of the small open economy from initial conditions guaranteeing equilibrium of type 1, corresponds to the equilibrium of a standard closed economy OLG model.

At the equilibrium of type 2, rest of the world individuals invest in the small open economy. The small open economy dynamic equilibrium is affected by changes in the rest of the world economy. At this equilibrium the no arbitrage condition between rest of world investors return on domestic investment and their effective return on investment in the small open economy is now binding. Thus the domestic return in the small open economy is pinned down by the rest of the world domestic return. The dynamic equilibrium of capital labor ratio at equilibrium of type 2 in the small open economy is formally determined by the following expression:

$$k_t^{SOE} = f'^{-1}(\overline{r}_t^{RW} + 1 + \tau) \tag{11}$$

At the steady state, the small open economy is identical to rest of the world economy in per capita terms.

3 Consequences of a Rest of the World Fertility Shock

In this section, I analyze the consequences of a rest of the world negative fertility shock on capital formation, saving/investment imbalances and welfare for both regions. I assume that the initial conditions in both regions are such that the world is at type 1 equilibrium. Depending on whether the wedge is below a given threshold, the occurrence of a shock is likely to drive the world economy to an equilibrium of type 2.

3.1 Capital Formation

The consequences of an asymmetric fertility shock on capital formation are analyzed first in the rest of the world economy. Then I explore the transmission of the consequences of such an asymmetric shock to the small open economy.

3.1.1 Rest of the World

I assume that the rest of the world is subject to a transitory negative fertility shock.[10] [11] To study the consequences of such a shock on the rest of the world economy, I differentiate (8) with respect to n_t^{RW} around the economy steady state. I obtain the following expression:

$$\frac{d\bar{k}_{t+1}^{RW}}{d\gamma^{RW}}(1+n_{ss}^{RW}) = \left[\frac{-k_{ss}^{RW} f''(k_{ss}^{RW})}{2+\rho}\right]\frac{d\bar{k}_t^{RW}}{d\gamma^{RW}} - k_{ss}^{RW} h_{t+1}^{RW} \tag{12}$$

To ensure local stability of the system described by equation (12), I assume that $\frac{d\bar{k}_{t+1}^{RW}}{d\bar{k}_t^{RW}} < 1$ that is equivalent to $\left[\frac{-k_{ss}^{RW} f''(k_{ss}^{RW})}{(2+\rho)(1+n_{ss}^{RW})}\right] < 1$.

At time t_0, a decline in the fertility rate mechanically increases the capital labor ratio through a reduction in the capital required to endow new workers. Formally, the short term impact of a negative fertility shock occurring at time t_0 on fertility is given by $\frac{-\bar{k}h_{t_0}}{(1+n_{ss}^{RW})} > 0$. After period t_0, a higher capital labor ratio leads to a higher real wage and saving rate. Indeed, the competitive labor market condition (6) implies that a lower employment level translates into a wage increase, thus increasing individual lifetime resources. Given that consumption at both ages are not inferior goods, consumptions in both periods are an increasing function of labor income. Consumption of the working age individuals increases but by less than their real wages, so that their saving increases, thus increasing the capital labor ratio (and so on and so forth).

Given the transitory nature of the shock and the stability condition imposed on the system, the effect of the shock vanishes over time. A transitory fertility shock has therefore no persistent impact on the rest of the world capital labor ratio. Only a permanent fertility shock has a long run impact on the rest of the world capital labor ratio. Formally, the long run effect of a permanent fertility shock, $\frac{dk^{RW}}{d\gamma^{RW}}$, is given by the fixed point solution to equation (12):[12]

$$\frac{dk^{RW}}{d\gamma^{RW}} = \frac{-k_{ss}^{RW} h^{RW}}{(1+n_{ss}^{RW}) + \frac{k_{ss}^{RW} f''(k_{ss}^{RW})}{2+\rho}} > 0 \tag{13}$$

[10] I only consider a one period shock for expositional purpose. My main results are qualitatively similar in the case of a multi-period shock.

[11] Given the specific form of the utility function, whether the shock is anticipated or not does not affect individuals behavior.

[12] The time pattern of a permanent negative shock occurring at time t_0, h^{RW}, takes the form: $\{0 \text{ at } t=0, ..., 0 \text{ at } t=t_0-1, \ -1 \text{ at } t=t_0, \ -1 \text{ at } t=t_0+1, ...\}$.

The stability condition suffices to ensure that the long run effect of a permanent fertility shock is positive.

The impact of an asymmetric transitory shock on the rest of the world interest rate is given by the following expression:

$$\frac{d\overline{r}_t^{RW}}{d\gamma^{RW}} = f''(k_{ss}^{RW})\frac{d\overline{k}_t^{RW}}{d\gamma^{RW}} \leq 0 \tag{14}$$

I now establish a proposition on the consequences of an asymmetric demographic shock on capital formation in the small open economy.

3.1.2 Small Open Economy

Proposition 1 *A rest of the world negative fertility shock translates into an increase in the small open economy capital labor ratio if the wedge between domestic and foreign investor return on capital is lower than the exogenous short run impact of a negative fertility shock on the rest of the world interest rate.*

Proof. When one starts from steady state in both regions, if τ is higher than the short run difference between the two regions before any potential transmission that formally implies $\tau \geq \left|\frac{d\overline{r}_{t_0}^{RW}}{d\gamma^{RW}}\right|$, the world economy is at the equilibrium of type 1 at time t_0. Given the stability condition imposed on (12), the world economy remains at the equilibrium of type 1 for all $t > t_0$. Thus there is no effect on capital formation in the small open economy that formally implies $\frac{dk_t^{SOE}}{d\gamma^{RW}} = 0$ for all t if $\tau \geq \left|\frac{d\overline{r}_{t_0}^{RW}}{d\gamma^{RW}}\right|$.

When one starts from steady state in both regions, if τ is smaller than the short run difference between the two regions before any potential transmission that formally implies $\tau < \left|\frac{d\overline{r}_{t_0}^{RW}}{d\gamma^{RW}}\right|$, the world economy jumps at the equilibrium of type 2 at time t_0. For all $t > t_0$ for which $\tau < \left|\frac{d\overline{r}_t^{RW}}{d\gamma^{RW}}\right|$, the world economy is at equilibrium of type 2. Given the stability condition on (12), for all $t \geq t^*$, with t^* such that $\tau \geq \left|\frac{d\overline{r}_{t^*}^{RW}}{d\gamma^{RW}}\right|$ the world economy is at equilibrium of type 1. Thus there is a positive impact of the rest of the world asymmetric shock on capital formation in the small open economy that formally implies $\frac{dk_t^{SOE}}{d\gamma^{RW}} > 0$ for all t such that $t_0 \geq t > t^*$. ■

The impact of a rest of the world negative fertility shock on the small open economy capital formation is formally given by the following expression:[13] [14]

[13] Substituting $\frac{d\overline{k}_{t_0}^{RW}}{d\gamma^{RW}}$ in (14), I obtain $\left|\frac{d\overline{r}_{t_0}^{RW}}{d\gamma^{RW}}\right| = \frac{f''(k_{ss}^{RW})k_{ss}^{RW}}{(1+n_{ss}^{RW})}$.

[14] t_0 is such that $\tau < \left|\frac{d\overline{r}_{t_0}^{RW}}{d\gamma^{RW}}\right|$ and t^* is such that $\tau \geq \left|\frac{d\overline{r}_{t^*}^{RW}}{d\gamma^{RW}}\right|$.

$$\frac{dk_t^{SOE}}{d\gamma^{RW}} = \left[\frac{1}{f''(k_{ss}^{RW})}\right]\left(\frac{d\overline{r}_t^{RW}}{d\gamma^{RW}} + \tau\right) > 0 \text{ for } t^* > t \geq t_0 \quad (15)$$

$$= 0 \text{ otherwise} \quad (16)$$

Appendix A.2 formally describes the detail of the derivation that leads to the above expression.

The transmission mechanism is interpreted as follows. If the wedge level is below a certain threshold, a rest of the world negative fertility shock leads to a decrease in the return on capital in that region. Capital flows to the small open economy in order to exploit the difference in returns. The world economy reaches the equilibrium of type 2. Thus the small open economy capital labor ratio increases. As the shock vanishes in the rest of the world, small open economy capital inflows vanish over time too. Capital flows stop when the difference between domestic returns in both regions is lower than the wedge, so that there is no incentive to invest abroad. The world economy returns to the equilibrium of type 1. Thus there is no long term effect of a rest of the world transitory fertility shock on the small open economy capital formation.

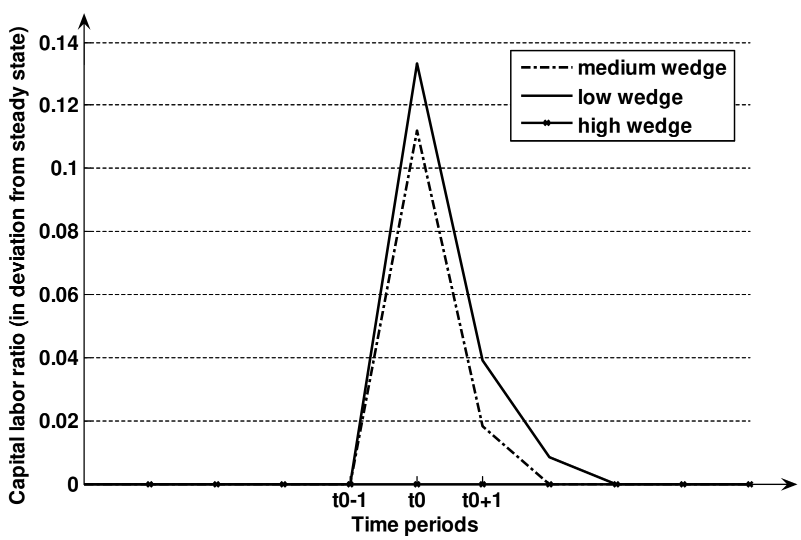

Figure 1: Evolution of Small Open Economy Capital Formation over Time

From (15), it is straightforward to show that given the assumption on diminishing returns, a marginally higher wedge limits the transmission of a rest of the world shock in terms of higher capital labor ratio. Indeed the equilibrium is reached through capital flowing to the small open economy, up to the point where there is no arbitrage between returns in the different locations. Thus a marginally higher wedge reduces the level of capital flows to the small open economy necessary to fulfill the no arbitrage condition between returns. Figure 1 displays the evolution over time of the small open economy capital formation for different degrees of diversion.[15]

In the following subsection, I establish two propositions on the consequences of a rest of the world negative fertility shock on international capital movements.

3.2 Saving/Investment Imbalance

In the following, I analyze the consequences of an asymmetric shock both on the small open economy balance-of-trade surplus (deficit), B^{SOE} and on the current account surplus (deficit), G^{SOE}.

The small open economy balance-of-trade surplus at time t is the excess of **net** domestic product at time t, Y_t^{SOE}, over domestic absorption. Domestic absorption is the sum of aggregate consumption at time t, C_t^{SOE}, and domestic capital formation used in the production at time $t+1$, K_{t+1}^{SOE}:

$$B_t^{SOE} = Y_t^{SOE} - C_t^{SOE} - K_{t+1}^{SOE} \tag{17}$$

with

$$Y_t^{SOE} = F(K_t^{SOE}) - \tau_t(K_t^{SOE} - S_{t-1}^{SOE}) \tag{18}$$

with τ_t given by (37). In per capita terms the net production can be rewritten:

$$y_t^{SOE} = f(k_t^{SOE}) - \tau_t(k_t^{SOE} - \frac{s_{t-1}^{SOE}}{1+n_t^{SOE}}) \tag{19}$$

Indeed, in presence of iceberg costs the relevant measure of domestic production is domestic production net of transit losses.

Formally, the per capita balance-of-trade surplus of the small open economy is given by the following expression:

$$b_t^{SOE} = y_t^{SOE} - c_{1,t}^{SOE} - \frac{c_{2,t}^{SOE}}{(1+n_t^{SOE})} - k_{t+1}^{SOE}(1+n_{t+1}^{SOE}) \tag{20}$$

[15] The figures are based on the following Cobb-Douglas technology, $f(k) = k^{0.33}$. Further more $\rho = 0.5$ and the low, medium and high wedge levels correspond respectively to the following values of τ: $\tau_{low} = \frac{1}{20}\left(\frac{d\overline{r}_{t_0}^{RW}}{d\gamma^{RW}}\right)$, $\tau_{medium} = \frac{1}{5}\left(\frac{d\overline{r}_{t_0}^{RW}}{d\gamma^{RW}}\right)$, $\tau_{high} = \frac{d\overline{r}_{t_0}^{RW}}{d\gamma^{RW}}$.

Under the assumption that the technology is constant returns to scale, after some rearrangements, I obtain:

$$b_t^{SOE} = (1 + r_t^{SOE} - \tau_t)\left[k_t^{SOE} - \frac{s_{t-1}^{SOE}}{(1 + n_t^{SOE})}\right] + s_t^{SOE} - k_{t+1}^{SOE}(1 + n_{t+1}^{SOE}) \tag{21}$$

The current account surplus is the excess of **net national** product over domestic absorption. Net national product equals net domestic product at time t, Y_t^{SOE}, plus net foreign investment income at time t that is formally given by $r_t^{SOE}(S_t^{SOE} - K_t^{SOE})$.[16] The small open economy current account surplus at time t is given by:

$$G_t^{SOE} = Y_t^{SOE} + r_t^{SOE}(S_t^{SOE} - K_t^{SOE}) - C_t^{SOE} - K_{t+1}^{SOE} \tag{22}$$

In per capita terms, the current account surplus reduces to the following expression:

$$g_t^{SOE} = (1 - \tau_t)\left[k_t^{SOE} - \frac{s_{t-1}^{SOE}}{(1 + n_t^{SOE})}\right] + s_t^{SOE} - k_{t+1}^{SOE}(1 + n_{t+1}^{SOE}) \tag{23}$$

At the steady state, the small open economy balance-of-trade and the current account equal zero, as the two regions become identical in every respect (in per capita terms). However, if the wedge is strictly below a certain threshold given by $\left|\frac{d\bar{r}_{t_0}^{SOE}}{d\gamma^{RW}}\right|$, the occurrence of a rest of the world fertility shock is likely to impact upon the small open economy balance-of-trade and current account. To analyze the effect of an asymmetric demographic shock on saving/investment imbalance, I differentiate expressions (21) and (23) with respect to n_t^{RW}. Appendix A.3 presents the details of the linearization of those expressions.

I can now establish the following proposition on the consequences of a rest of the world negative fertility shock on saving/investment imbalance in the small open economy.

Proposition 2 *At the end of time $t_0 - 1$, capital flows into the small open economy, provided that the wedge between domestic and foreign investor return is lower than the exogenous short run impact of a negative*

[16] Net foreign investment income equals interest rate payments on the difference between the small open economy aggregate assets at time t minus domestic capital stock installed in the small open economy at time t. The economy being either at equilibrium of type 1 or 2, the interest rate received on small open economy individuals assets, r^{SOE}, equals the interest rate served on foreign investment in the small open economy (before transit losses occur).

fertility shock on the rest of the world interest rate. If the latter condition holds, at the end of time t_0 capital is repatriated in order to finance the old-aged consumption of rest of the world investors.

Proof. see appendix A.4. ■

At the end of time $t_0 - 1$, capital flows to the small open economy in order to exploit differences in returns, provided that the wedge is below a given threshold. The deviation from steady state of the current account position of the small open economy at time $t_0 - 1$ is given by the following expression:

$$\frac{dg_{t_0-1}^{SOE}}{d\gamma} = -\frac{dk_{t_0}^{SOE}}{d\gamma^{RW}} \leq 0 \tag{24}$$

At time t_0, capital flows out of the small open economy. Indeed, capital is repatriated to the rest of the world in order to finance old-aged investors' consumption. Formally, this is described by the following expression:

$$\frac{dg_{t_0}^{SOE}}{d\gamma} = \left[1 + \frac{-k_{ss}^{SOE} f''\left(k_{ss}^{SOE}\right)}{2+\rho}\right] \frac{dk_{t_0}^{SOE}}{d\gamma^{RW}} - \frac{dk_{t_0+1}^{SOE}}{d\gamma^{RW}} > 0 \tag{25}$$

I establish another proposition as regards the capital movements resulting from a rest of the world asymmetric shock.

Proposition 3 *Provided that the wedge is below a certain threshold, a marginally higher wedge limits the magnitude of the small open economy capital inflows and outflows resulting from of a rest of the world asymmetric fertility shock.*

Proof. For $t = t_0 - 1$, I combine (15) together with (24), and I then differentiate this combination with respect to τ. Using the fact that the rest of the world interest is exogenous, I obtain $\frac{d\left(\frac{dg_{t_0-1}^{SOE}}{d\gamma}\right)}{d\tau} = -\left[\frac{1}{f''\left(\overline{r}_{ss}^{RW}+1\right)}\right] > 0$. A marginally higher wedge improves the small open economy current account position at time $t_0 - 1$.

For $t = t_0$, I combine (25) and (15), and then I differentiate this combination. It is then straightforward to show that $\frac{d\left(\frac{dg_{t_0}^{SOE}}{d\gamma}\right)}{d\tau} < 0$. A marginally higher wedge deteriorates the current account position at time t_0. ■

At the equilibrium, a marginally higher wedge requires that for a given rest of the world interest rate, the return in the small open economy will be higher. The diminishing returns assumption implies that the small open economy capital labor ratio should be lower for a given rest of the world interest rate. Thus less capital should flow to the small open economy to adjust for a rest of the world negative fertility shock. Figure 2 displays the evolution over time of the small open economy current account position for different degrees of diversion.

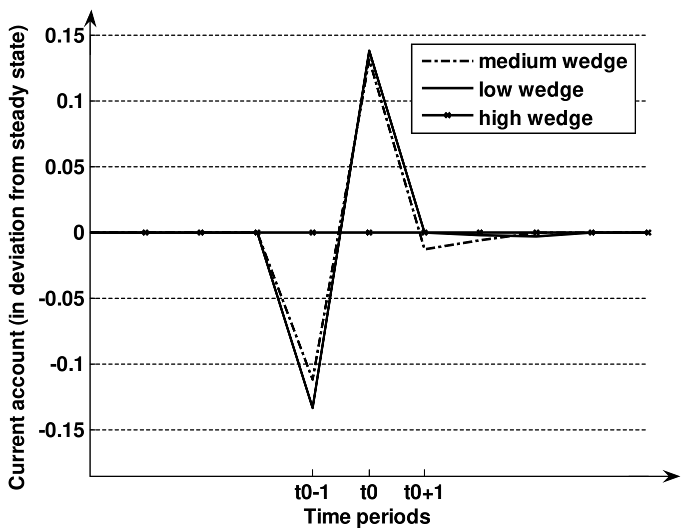

Figure 2: Evolution of Small Open Economy Current Account Position over Time

In the following subsection, I establish two propositions on the consequences of such an asymmetric shock on lifetime utility of individuals living in both regions.

3.3 Welfare Analysis

In order to evaluate the impact of a fertility shock on lifetime utility, I differentiate the lifetime utility function of individuals born at time t and living in region i with respect to n_t^{RW}.[17] The details of the linearization are shown in appendix A.5.

In the following subsection, I establish a proposition on the impact of a rest of the world negative fertility shock on the lifetime utility of rest of the world individuals.

3.3.1 Rest of the World

Proposition 4 *Rest of the world individuals' lifetime utility decreases unambiguously for individuals born one period before the shock occurs.*

[17] For $i \in \{SOE, RW\}$.

Under the assumption that the economy is initially below the golden rule, individuals born after the shock has occurred experience an increase in their lifetime utility.

Proof. see the following. ■

Generations born before period $t_0 - 1$ do not experience any change in their lifetime utility. The generation born at time $t_0 - 1$ experiences an unambiguous decrease in its lifetime utility. Indeed, a negative fertility shock decreases this generation old-aged utility flow through lower interest rate payments. Formally, the deviation from steady state of the lifetime utility of rest of the world individuals born at time $t_0 - 1$ is given by the following expression:

$$\frac{dU_{t0-1}^{RW}}{d\gamma^{RW}} = \frac{1}{c_{2ss}^{RW}} \frac{1}{1+\rho} \frac{dc_{2,t0}^{RW}}{d\gamma} < 0 \tag{26}$$

with

$$\frac{dc_{2,t0}^{RW}}{d\gamma} = \left[\frac{w_{ss}^{RW}}{2+p}\right] \frac{d\overline{r}_{t0}^{RW}}{d\gamma^{RW}} < 0 \tag{27}$$

Using (14), it is straightforward to show that the impact on lifetime utility of generation born at time $t_0 - 1$ in the rest of the world is negative.

Generations born after $t_0 - 1$ experience an increase in their lifetime utility provided that the economy is below the golden rule. For individuals born after the shock has occurred, a negative fertility shock increases unambiguously their young-aged utility flow through an increase in their real wage. However, the effect of such a shock on old-aged utility flows is ambiguous. Indeed, there are two opposite effects resulting from a fertility rate shock on old-aged utility flows. First, a negative fertility shock tends to increase the amount of saving available at retirement age through a real wage increase. Second, a negative fertility shock tends to decrease old-aged wealth through a decrease in interest payments for a given real wage. The overall effect of a negative fertility shock on lifetime utility is positive if the rest of the world economy is initially below the golden rule of capital accumulation. Formally, the condition can be rewritten $n_{ss}^{RW} < r_{ss}^{RW}$. The details of the proof is provided in appendix A.6. Indeed, if the rest of the world economy is initially below the golden rule, the economy is dynamically efficient. A negative fertility shock raising capital labor ratio further increases the economy efficiency.

I can now establish a proposition on the consequence of such an asymmetric shock on lifetime utility of an individual living in the small open economy.

3.3.2 Small Open Economy

Proposition 5 *If capital flows internationally, lifetime utility decreases unambiguously for small open economy individuals born one period before the shock occurs. Provided that the small open economy is initially below its golden rule, small open economy individuals born after the time the shock has occurred experience an increase in their lifetime welfare.*

Proof. I need to distinguish between two cases depending on whether or not capital flows internationally following a rest of the world negative fertility shock. If $\tau \geq \left|\frac{d\bar{r}_{t0}^{RW}}{d\gamma_{t0}^{RW}}\right|$, capital does not flow internationally following an asymmetric fertility shock. The world economy remains at capital equilibrium of type 1. Small open economy lifetime utility is not affected for all generations. If $\tau < \left|\frac{d\bar{r}_{t0}^{RW}}{d\gamma_{t0}^{RW}}\right|$, capital does flow internationally and the welfare of some generations living in the small open economy is affected. For small open economy individuals born before period $t_0 - 1$, there is no impact of an asymmetric shock on their lifetime utility. For individuals born at period $t_0 - 1$, lifetime utility decreases unambiguously for this generation. Provided that the economy is initially below the golden rule, a rest of the world negative fertility shock increases lifetime utility in the small open economy for the generation born after $t_0 - 1$. Formally, for individuals born after period $t_0 - 1$, lifetime utility increases if $n_{ss}^{SOE} < r_{ss}^{SOE}$.The details of the proof is provided in appendix A.6. ■

These results are interpreted as follows. A small open economy aging slower than the rest of the world will experience change in welfare for some of its individuals provided that capital flows internationally. If the latter condition holds, the small open economy generation born one period before the shock occurs will unambiguously face a decrease in its lifetime utility. Generations born after the shock has occurred will experience an increase in welfare, provided that the small open economy is initially below the golden rule of capital accumulation. Figure 3 displays the evolution of small open economy individuals' lifetime utility over their birth periods for two different degrees of diversion.

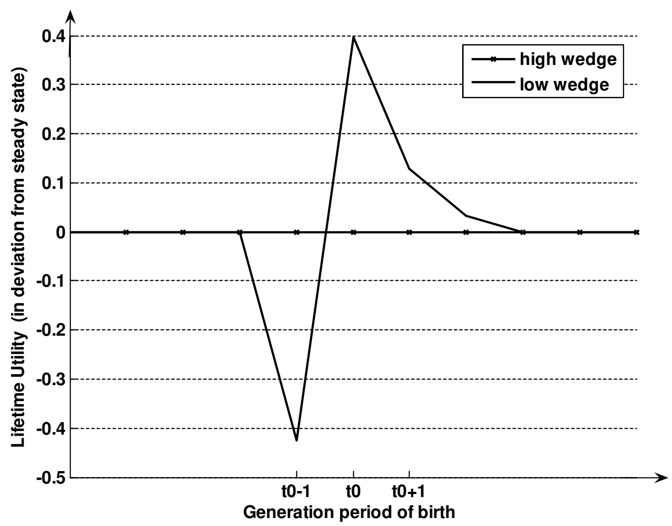

Figure 3: Evolution of Small Open Economy Individuals Lifetime Utility

4 Conclusion

Our objective in this paper has been to analyze the consequences of an asymmetric fertility shock on capital formation, saving/investment imbalance, and welfare. The framework of analysis is a Diamond-type overlapping-generations small open economy with diversion. Diversion is modelled through a symmetric wedge between foreign investor and domestic investor return on capital. A number of results are obtained. A rest of the world negative fertility shock is transmitted to the small open economy, depending on whether the wedge is below a given threshold. If the wedge is not too high, capital first flows into the small open economy to exploit the difference in returns on capital. After the shock has occurred, capital is repatriated in order to finance the old-aged consumption of rest of the world investors. If capital flows internationally, lifetime utility in the small open economy decreases unambiguously for individuals born one period before the shock occurs. Provided that the small open economy is below its golden rule, individuals born after the time the shock has occurred experience an increase in their lifetime utility.

Those results have important implication for policies. Indeed, it suggests that countries with young population and low capital intensity should implement policies to strengthen institutions in order to allow foreign investors in older countries to take advantage of differential in returns. The transitory nature of those demographic differences call for rapid reforms in order for relatively younger countries with low capital intensity not to miss such opportunity. In addition, the consequences of those induced capital flows on welfare suggests that not all generations in those younger countries will benefit. From a political economy perspective, this suggest that government in those younger countries should perhaps compensate the losers in order to build consensus around reforms aiming at attracting foreign investment to exploit temporary differences in returns due do differences in demographic dynamics.

The model can be extended in a number of directions. Within the context of our small open economy Diamond type model with diversion, this framework will analyze the adjustment of an asymmetric shock through labor mobility in the presence of transaction costs.[18] A welfare comparison on the adjustment to an asymmetric demographic shock through capital movement and labor movement can also be conducted.

[18] Galor [9] analyzes the welfare implications of international labor movement in a two-country overlapping generations framework, in presence of time preference rate differences.

A Appendix

A.1 Rest of the World Dynamic Equilibrium

Individuals optimization problem

$$\underset{c_{1,t}^{RW},c_{2,t+1}^{RW}}{max} U(c_{1,t}^{RW}, c_{2,t+1}^{RW}) = u(c_{1,t}^{RW}) + \frac{1}{1+\rho}u(c_{2,t+1}^{RW}) \tag{28}$$

$$c_{1,t}^{RW} + \frac{c_{2,t+1}^{RW}}{1+r_{t+1}^{RW}} \leq w_t^{RW} \tag{29}$$

$$c_{1,t}^{RW}, c_{2,t+1}^{RW} \geq 0 \tag{30}$$

Firms optimization problem

$$r_t^{RW} = f'(k_t^{RW}) - 1 \tag{31}$$

$$w_t^{RW} = f(k_t^{RW}) - k_t^{RW} f'(k_t^{RW}) \tag{32}$$

Capital market equilibrium condition

$$s_{1,t}^{RW} = k_{t+1}^{RW}(1 + n_{t+1}^{RW}) \tag{33}$$

$$k_t^{RW} \geq 0 \tag{34}$$

The first order difference equation in k_t^{RW} that describes the evolution of the model from arbitrary initial conditions is given by the following expression:

$$k_{t+1}^{RW}\left[(1 + n_{t+1}^{RW})\right] - \left[\frac{f(k_t^{RW}) - k_t^{RW} f'(k_t^{RW})}{2+\rho}\right] = 0 \tag{35}$$

A.2 Capital Formation

For t such that $t < t_0$, that is before the shock occurs, a rest of the world negative fertility shock has no impact on the small open economy capital formation.

For t such that $t \geq t_0$, I formally derive the consequences of a rest of the world negative fertility shock on the small open economy interest rate through differentiating the following expression with respect to n_t^{RW} around the world economy steady state.

$$r_t^{SOE} - \tau_t = \overline{r}_t^{RW} \tag{36}$$

with τ_t such that

$$\tau_t = \tau \text{ for all } t \text{ such that } \tau < \left|\frac{d\overline{r}_t^{RW}}{d\gamma^{RW}}\right| \tag{37}$$

$$= 0 \text{ otherwise} \tag{38}$$

Note first that for all t for which $\left|\frac{d\overline{r}_t^{RW}}{d\gamma^{RW}}\right| > \tau$, the rest of the world investor no arbitrage condition, that is formally given by $\overline{r}_t^{RW} = r_t^{SOE} - \tau$, is binding and the world economy is at equilibrium of type 2.

After differentiating (37), the following expression then holds:

$$\frac{d\tau_t}{d\gamma^{RW}} = \tau \text{ for all } t \text{ such that } \tau < \left|\frac{d\overline{r}_t^{RW}}{d\gamma^{RW}}\right|$$

$$= 0 \text{ otherwise} \tag{39}$$

After differentiating expression (36), I formally have that:[19]

$$\frac{dr_t^{SOE}}{d\gamma^{RW}} = \frac{d\overline{r}_t^{RW}}{d\gamma^{RW}} + \tau \text{ for all } t \text{ such that } \tau < \left|\frac{d\overline{r}_t^{RW}}{d\gamma^{RW}}\right| \tag{40}$$

$$= 0 \text{ otherwise} \tag{41}$$

To determine the effect of a fertility shock on the small open economy capital formation, I differentiate the following expression with respect to n_t^{RW} around the small open economy steady state.

$$k_t^{SOE} = f'^{-1}(\overline{r}_t^{RW} + \tau_t + 1) \tag{42}$$

That reduces to the following expression:

$$\frac{dk_t^{SOE}}{d\gamma^{RW}} = \left[\frac{1}{f''(k_{ss}^{RW})}\right]\left(\frac{d\overline{r}_t^{RW}}{d\gamma^{RW}} + \tau\right) > 0 \ \forall t \text{ such that } \tau < \left|\frac{d\overline{r}_t^{RW}}{d\gamma^{RW}}\right|$$

$$= 0 \text{ otherwise} \tag{43}$$

[19]Note that if the condition $\left|\frac{d\overline{r}_t^{RW}}{d\gamma^{RW}}\right| \geqslant \tau$ holds, it is as if the wedge variable, τ_t, is subject to a shock of intensity τ, simultaneous to the fertility shock and that lasts until the above condition holds.

A.3 Current Account Expression Linearization

To determine the effect of a rest of the world shock on the small open economy balance-of-trade, I differentiate the balance-of-trade expression namely equation (21) with respect to n_t^{RW} around the small open economy steady state. Formally, I obtain the following expression:[20]

$$\frac{db_t^{SOE}}{d\gamma^{RW}} = (1 + r_{ss}^{SOE}) \left[\frac{dk_t^{SOE}}{d\gamma^{RW}} - \frac{ds_{t-1}^{SOE}}{d\gamma^{RW}} \right] + \frac{ds_t^{SOE}}{d\gamma^{RW}} - \frac{dk_{t+1}^{SOE}}{d\gamma^{RW}} \tag{44}$$

After some substitutions, I obtain the deviation from steady state equilibrium of the small open economy balance-of-trade.

$$\frac{db_t^{SOE}}{d\gamma} = \left[(1 + r_{ss}^{SOE}) \frac{k_{ss}^{SOE} f''(k_{ss}^{SOE})}{2+\rho} \right] \frac{dk_{t-1}^{SOE}}{d\gamma^{RW}} + \tag{45}$$

$$\left[(1 + r_{ss}^{SOE}) + \frac{-k_{ss}^{SOE} f''(k_{ss}^{SOE})}{2+\rho} \right] \frac{dk_t^{SOE}}{d\gamma^{RW}} - \frac{dk_{t+1}^{SOE}}{d\gamma^{RW}} \tag{46}$$

In order to determine the effect of such an asymmetric shock on the small open economy current account, I differentiate (23) with respect to n_t^{RW} around the small economy steady state. Formally, I obtain the following expression:

$$\frac{dg_t^{SOE}}{d\gamma} = \frac{ds_{1,t}^{SOE}}{d\gamma} - \frac{d(\frac{s_{1,t-1}^{SOE}}{(1+n_t^{SOE})})}{d\gamma} + \frac{dk_t^{SOE}}{d\gamma} - \frac{d(k_{t+1}^{SOE}(1+n_{t+1}^{SOE}))}{d\gamma} \tag{47}$$

After some substitutions, I obtain the small open economy current account deviation from steady state:

$$\frac{dg_t^{SOE}}{d\gamma^{RW}} = \left[\frac{dk_t^{SOE}}{d\gamma^{RW}} - \frac{ds_{t-1}^{SOE}}{d\gamma^{RW}} \right] + \frac{ds_t^{SOE}}{d\gamma^{RW}} - \frac{dk_{t+1}^{SOE}}{d\gamma^{RW}} \tag{48}$$

After some manipulations, I finally get:

$$\frac{dg_t^{SOE}}{d\gamma} = \left[\frac{k_{ss}^{SOE} f''(k_{ss}^{SOE})}{2+\rho} \right] \frac{dk_{t-1}^{SOE}}{d\gamma^{RW}} + \left[1 + \frac{-k_{ss}^{SOE} f''(k_{ss}^{SOE})}{2+\rho} \right] \frac{dk_t^{SOE}}{d\gamma^{RW}} - \frac{dk_{t+1}^{SOE}}{d\gamma^{RW}} \tag{49}$$

[20] Note that at the steady state equilibrium, the world economy is at capital ownership of type 1. Thus $\tau > \left| \frac{d\bar{\tau}_t^{RW}}{d\gamma^{RW}} \right|$, so that $\tau_t = \tau_{ss} = 0$, for all t such that the small open economy balance of trade and current account position equal zero.at steady state.

A.4 Saving/Investment Imbalance

In the following section, I investigate the sign of a rest of the world negative fertility shock on both the balance-of-trade and the current account position, using the differential expressions presented in appendix A.3.

I need to distinguish between two cases, depending on whether or not the transmission of the shock operates.

If $\tau \geq \left|\frac{dr_{t0}^{RW}}{d\gamma_t^{RW}}\right|$, for all time periods the world economy remains at equilibrium of type 1. A rest of the world negative fertility shock has no impact either on the small open economy balance-of-trade and the current account position.

If $\tau < \left|\frac{dr_{t0}^{RW}}{d\gamma_{t0}^{RW}}\right|$, a rest of the world negative fertility shock affects the small open economy balance-of-trade and the current account position. Assuming the world economy is initially at its steady state, before the end of t_0-1, the balance-of-trade and the current account are not affected by a rest of the world shock. At the time period the shock occurs, that is between $t_0 - 1$ and t_0, the small open economy experiences capital inflows. Saving from the rest of the world is flowing into the small open economy in order to finance capital which will be used in the production at time t. Using (45) and (49), the small open economy deviation from steady state of the trade balance and the current account position at the end of time $t_0 - 1$ is given by the following expression:

$$\frac{db_{t_0-1}^{SOE}}{d\gamma} = \frac{dg_{t_0-1}^{SOE}}{d\gamma} = -\frac{dk_{t_0}^{SOE}}{d\gamma^{RW}} < 0 \tag{50}$$

Indeed, in presence of diminishing returns, capital flows from the rest of the world to the small open economy to exploit the difference in returns resulting from the rest of the world fertility shock.

At the end of time t_0, the capital flows out of the small open economy as capital is repatriated to the rest of the world in order to finance old-aged investor consumption that formally rewrites $\frac{dg_t^{SOE}}{d\gamma} > 0$ for $t = t_0$. Indeed, I have formally that:

$$\frac{dg_{t_0}^{SOE}}{d\gamma} = \left[1 + \frac{-k_{ss}^{SOE} f''(k_{ss}^{SOE})}{2+\rho}\right] \frac{dk_{t_0}^{SOE}}{d\gamma^{RW}} - \frac{dk_{t_0+1}^{SOE}}{d\gamma^{RW}} \tag{51}$$

Note that from (15) and (14) I have $\frac{dg_t^{SOE}}{d\gamma} > 0$ if :

$$\left[1 + \frac{-k_{ss}^{SOE} f''(k_{ss}^{SOE})}{2+\rho}\right] \frac{dr_{t_0}^{RW}}{d\gamma^{RW}} - \frac{dr_{t_0+1}^{RW}}{d\gamma^{RW}} < 0 \tag{52}$$

so that it reduces to:

$$\left[1+\frac{-k_{ss}^{SOE}f''(k_{ss}^{SOE})}{2+\rho}\right] > \frac{dk_{t_0+1}^{RW}}{dk_{t_0}^{RW}} \tag{53}$$

From (12), I have that $\frac{dg_{t_0}^{SOE}}{d\gamma} > 0$. At period t_0 capital flows out of the small open economy in order to finance the rest of the world individuals old-aged consumption. It is also straightforward to show that at time t_0, a rest of the world shock has a positive impact on the small open economy balance-of-trade.

For $t \geq t_0 + 1$, given the transitory nature of the shock it can be shown that there are no capital flows from the rest of the world to the small open economy.

Substituting (15) into (49) gives $\frac{dg_t^{SOE}}{d\gamma} = 0$. Indeed, the details of substitution are as follows:

$$\left[\frac{k_{ss}^{SOE}f''(k_{ss}^{SOE})}{2+\rho}\right]\frac{d\overline{r}_{t-1}^{RW}}{d\gamma^{RW}} - \frac{d\overline{r}_{t+1}^{RW}}{d\gamma^{RW}} = -\left[1+\frac{-k_{ss}^{SOE}f''(k_{ss}^{SOE})}{2+\rho}\right]\frac{d\overline{r}_t^{RW}}{d\gamma^{RW}} \tag{54}$$

Using (14) I obtain the following expression:

$$\left[\frac{k_{ss}^{SOE}f''(k_{ss}^{SOE})}{2+\rho}\right]\frac{d\overline{k}_{t-1}^{RW}}{d\gamma^{RW}} - \frac{d\overline{k}_{t+1}^{RW}}{d\gamma^{RW}} = -\left[1+\frac{-k_{ss}^{SOE}f''(k_{ss}^{SOE})}{2+\rho}\right]\frac{d\overline{k}_t^{RW}}{d\gamma^{RW}} \tag{55}$$

Dividing (55) by $\frac{dk_t^{RW}}{d\gamma^{RW}}$, I obtain the following expression:

$$\left[\frac{k_{ss}^{SOE}f''(k_{ss}^{SOE})}{2+\rho}\right]\frac{d\overline{k}_{t-1}^{RW}}{dk_t^{RW}} - \frac{d\overline{k}_{t+1}^{RW}}{d\overline{k}_t^{RW}} = -\left[1+\frac{-k_{ss}^{SOE}f''(k_{ss}^{SOE})}{2+\rho}\right] \tag{56}$$

Recall that from (12), I have for $t > t_0$:

$$\frac{d\overline{k}_{t+1}^{RW}}{d\overline{k}_t^{RW}} = \frac{-k_{ss}^{SOE}f''(k_{ss}^{SOE})}{2+\rho} \tag{57}$$

so that I obtain the following valid identity:

$$-1-\frac{-k_{ss}^{SOE}f''(k_{ss}^{SOE})}{2+\rho} = \left[-1-\frac{-k_{ss}^{SOE}f''(k_{ss}^{SOE})}{2+\rho}\right] \tag{58}$$

Similarly, it is also straightforward to show that for $t \geq t_0$, there is no effect of a rest of the world shock on the small open economy balance-of-trade.

As perfect capital mobility is assumed, the adjustment occurs instantaneously at the end of period $t_0 - 1$ (when capital flows into the small open economy) and at the end of time t_0 (when capital is repatriated in order to finance old-aged consumption of individuals living in the rest of the world).

A.5 Lifetime Utility Function Linearization

Let U_t^i describe the lifetime utility of individuals born at time t living in region i for $i = \{SOE; RW\}$. Formally, U_t^i is given by the following expression:

$$U_t^i = u(c_{1,t}^i) + \frac{u(c_{2,t+1}^i)}{1+\rho} \tag{59}$$

with $c_{1,t}^i$ and $c_{2,t+1}^i$ given by:

$$c_{1,t}^i = w_t^i \left(\frac{1+\rho}{2+p}\right) \tag{60}$$

$$c_{2,t+1}^i = \left(\frac{w_t^i}{2+p}\right)(1+r_{t+1}^i) \tag{61}$$

Differentiating (59) with respect to n_t^{RW} around the economy steady state gives the following expression:

$$\frac{dU_t^i}{d\gamma^{RW}} = u'(c_{1,ss}^i)\frac{dc_{1,t}^i}{d\gamma^{RW}} + u'(c_{2,ss}^i)\frac{1}{1+\rho}\frac{dc_{2,t+1}^i}{d\gamma^{RW}} \tag{62}$$

with the corresponding consumption profile at steady state:

$$c_{1,ss}^i = w_{ss}^i \left[\frac{1+\rho}{2+p}\right] \tag{63}$$

$$c_{2,ss}^i = \left[\frac{w_{ss}^i}{2+p}\right](1+r_{ss}^i) \tag{64}$$

Factors prices deviation from steady state are given by the following expressions:

$$\frac{dw_t^i}{d\gamma^{RW}} = -k_{ss}^i f''(k_{ss}^i)\frac{dk_t^i}{d\gamma^{RW}} \tag{65}$$

$$\frac{dr_t^i}{d\gamma^{RW}} = f''(k_{ss}^i)\frac{dk_t^i}{d\gamma^{RW}} \tag{66}$$

Young and old-aged consumption deviations from steady state are given by:

$$\frac{dc^i_{1,t}}{d\gamma^{RW}} = \frac{dw^i_t}{d\gamma^{RW}}\left[\frac{1+\rho}{2+p}\right] \tag{67}$$

$$\frac{dc^i_{2,t+1}}{d\gamma^{RW}} = \frac{dw^i_t}{d\gamma^{RW}}\frac{1}{2+p}(1+r^i_{ss}) + \left[\frac{w^i_{ss}}{2+p}\right]\frac{dr^i_{t+1}}{d\gamma^{RW}} \tag{68}$$

Given the logarithmic utility assumption, the deviation from the steady state of the lifetime utility of an individual born at time t in region i reduces to the following expression:

$$\frac{dU^i_t}{d\gamma^{RW}} = \frac{1}{c^i_{1,ss}}\frac{dc^i_{1,t}}{d\gamma^{RW}} + \frac{1}{c^i_{2,ss}}\frac{1}{1+\rho}\frac{dc^i_{2,t+1}}{d\gamma^{RW}} \tag{69}$$

A.6 Lifetime Utility

In the following, I prove that a negative fertility shock affects positively the lifetime welfare of an individual living in country i (for $i = \{SOE; RW\}$) provided that the economy i is below its golden rule of capital accumulation.

Combining (67) and (68) with (69), I obtain the following expression:

$$\frac{dU^i_t}{d\gamma^{RW}} = \frac{dw^i_t}{d\gamma^{RW}}\left[\frac{1}{c^i_{1,ss}}\left(\frac{1+\rho}{2+p}\right) + \frac{(1+r^i_{ss})}{c^i_{2,ss}}\frac{1}{1+\rho}\frac{1}{2+p}\right] + \frac{dr^i_{t+1}}{d\gamma^{RW}}\left[\frac{1}{c^i_{2,ss}}\frac{1}{1+\rho}\left(\frac{w^i_{ss}}{2+p}\right)\right] \tag{70}$$

The impact of lifetime utility of an individual born after $t_0 - 1$ is positive if the following condition holds:

$$\frac{dw^i_t}{d\gamma^{RW}}(1+r^i_{ss})\left[\frac{(2+\rho)}{w^i_{ss}}\right] > -\frac{dr^i_{t+1}}{d\gamma^{RW}} \tag{71}$$

using the above expression combined with (12), (65) and (66), I obtain the following condition:

$$\frac{(1+r^i_{ss})}{(1+n^i_{ss})} > \frac{dk^i_{t+1}}{dk^i_t} \tag{72}$$

Given the local stability condition, a sufficient condition for a negative fertility shock to affect positively the lifetime welfare of an individual living in country i is given by

$$r^i_{ss} > n^i_{ss} \tag{73}$$

If the economy is above the golden rule that formally rewrites, $r_{ss}^i < n_{ss}^i$, two cases need to be distinguished. First, if the following inequalities hold, then a negative fertility shock affects positively the lifetime welfare of an individual living in country i born after period t_0.

$$\frac{dk_{t+1}^i}{dk_t^i} < \frac{(1 + r_{ss}^i)}{(1 + n_{ss}^i)} < 1 \tag{74}$$

Second, if the following inequalities hold, then a negative fertility shock affects negatively the lifetime welfare of an individual living in country i born after period t_0.

$$\frac{(1 + r_{ss}^i)}{(1 + n_{ss}^i)} < \frac{dk_{t+1}^i}{dk_t^i} < 1 \tag{75}$$

Thus the economy being above the golden rule is a necessary but not sufficient condition for a negative fertility shock affect negatively the lifetime welfare of an individual living in country i born after period t_0.

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