Optimal taxation and monitoring in an economy with matching frictions and underground activities

Lisi, Gaetano

University of Cassino

27 November 2010

Optimal taxation and monitoring in an economy with matching frictions and underground activities

GAETANO LISI *

ABSTRACT This short paper shows the interdependence of taxation and monitoring policy in a search and matching model of equilibrium unemployment with an underground sector. More precisely, from a social welfare standpoint, two options are available to the policy maker: s/he may either substitute a tighter monitoring with a higher penalty or enforce both a higher taxation and an increased monitoring.

JEL CLASSIFICATION: E26, H21, H26, J64
KEYWORDS optimal taxation, tax evasion, underground economy, job search theory

The interdependence of taxation and monitoring in a matching framework

The study of the effects of economic policies such as taxation, monitoring and punishment on the size of the underground economy and the level of involuntary unemployment is the focus of matching-type models with an underground sector.[1] However, a great many of these models do not discuss how the policy parameters can be optimally set by a policy maker who wants to maximize social output (shadow employment produces positive added value so some tolerance should be expected from an efficiency standpoint). As an exception, Boeri and Garibaldi (2002) show that the optimal taxation problem does not affect the policy-maker's choice with respect to the optimal monitoring of underground activities, whereas optimal taxation is affected by the optimal choice of monitoring (the 'separability result'). Instead, this paper shows that the optimal monitoring problem is also affected by the

---

* *Correspondence Address*: Gaetano Lisi, Department of Economic Sciences, University of Cassino, via S. Angelo, I-03043 Cassino (FR), Italy. E-mail: gaetano.lisi@unicas.it. Comments are welcome.

[1] See for example Fugazza and Jacques (2004), Bouev (2002, 2005), Boeri and Garibaldi (2002, 2006), Kolm and Larsen (2003, 2008), Albrecht et al. (2009).

optimal choice of taxation. Precisely, there are two options available to the policy maker: s/he may either substitute a tighter monitoring with a higher penalty or enforce both a higher taxation and an increased monitoring.

To make our point as simply as possible, we consider a basic matching framework *à la* Pissarides (2000) with a continuum of homogeneous workers of measure one. The creation of employment occurs in a labour market with matching frictions. As usual (see Pissarides, 2000; Petrongolo and Pissarides, 2001), an aggregate matching function is used to summarize these frictions. Precisely, the number of job matches formed per unit of time is $m = m(u,v)$, where u is the number of unemployed workers and v is the number of vacancies. The matching function is strictly increasing but concave in both arguments and displays constant returns to scale. It follows that the labour market tightness is given by $\theta_i = v_i / u$, where the subscript $i \in \{f,s\}$, with f = *formal* and s = *shadow*, denotes the type of firm (see below). Hence, $q(\theta_i) \equiv m\{v_i,u\}/v_i = m\{1,\theta_i^{-1}\}$ and $g(\theta_i) \equiv m\{v_i,u\}/u = m\{\theta_i,1\}$, with $i \in \{f,s\}$, are the probability of filling a vacancy and of finding a job, respectively.[2]

We consider two types of firms, thus forming two sectors. Formal firms (with $i = f$) have to pay taxes τ, whereas shadow firms (with $i = s$) enjoy tax evasion.

To ensure that unemployment exists in steady state, it is assumed that job destruction occurs at the exogenous rate δ. Furthermore, since the underground activities are detected and repressed by the government at the exogenous rate ρ, the overall job destruction rate in the shadow sector is $(\delta + \rho)$. Therefore, in steady state these matching and job destruction rates allow us to describe labour market flows:

$$\dot{n}_f = g(\theta_f)\cdot(1 - n_f - n_s) - \delta \cdot n_f \tag{1}$$

$$\dot{n}_s = g(\theta_s)\cdot(1 - n_f - n_s) - (\delta + \rho)\cdot n_s \tag{2}$$

where n_f and n_s are the steady state employment rates, and $1 - n_f - n_s = u$ is the unemployment identity.

The Bellman equations specified to find infinite horizon steady-state solutions are: [3]

Value of ...	*Underground sector*	*Regular sector*
a vacancy	$rV_s = -c_s + q(\theta_s)\cdot(J_s - V_s)$	$rV_f = -c_f + q(\theta_f)\cdot(J_f - V_f)$
a filled job	$rJ_s = y_s - w_s + (\delta + \rho)\cdot(V_s - J_s)$	$rJ_f = y_f - w_f - \tau + \delta\cdot(V_f - J_f)$

[2] Standard technical assumptions are assumed, i.e. $\lim_{\theta \to 0} q(\theta) = \lim_{\theta \to \infty} g(\theta) = \infty$, and $\lim_{\theta \to 0} g(\theta) = \lim_{\theta \to \infty} q(\theta) = 0$.

[3] The unemployed cannot search for jobs in both sectors at the same time (i.e. there is a directed search). However, irrespective of the sector, if an unemployed person fails to find a job, s/he falls back into the same unemployment pool.

searching for a job	$rU_s = b + g(\theta_s)\cdot(W_s - U_s)$	$rU_f = b + g(\theta_f)\cdot(W_f - U_f)$
being employed	$rW_s = w_s + (\delta + \rho)\cdot(U_s - W_s)$	$rW_f = w_f + \delta\cdot(U_f - W_f)$

where r is the exogenous discounted rate; c_i is the vacant job cost; y_i is the match productivity, w_i is the wage rate and b is the benefit of being unemployed.

As usual (see Pissarides, 2000), the equilibrium value of labour market tightness in both sectors is given by the *free-entry condition* or *zero profit condition* (i.e. $V_i = 0$) for firms. An explicit form of tightness can be obtained by using the Cobb-Douglas specification of matching function, i.e. $m_i = v_i^{1-a}\cdot u^a$ (with $0 < a < 1$), which is usual in the literature (Petrongolo and Pissarides 2001): [4]

$$\theta_f = \left(\frac{y_f - w_f - \tau}{c_f\cdot(r+\delta)}\right)^{\frac{1}{a}} \tag{3}$$

$$\theta_s = \left(\frac{y_s - w_s}{c_s\cdot(r+\delta+\rho)}\right)^{\frac{1}{a}} \tag{4}$$

with $\theta_i \in (0,\infty)\ \forall i$, because of the positive and finite parameters, and $\partial\theta_f / \partial\tau < 0$, $\partial\theta_s / \partial\rho < 0$, $\partial^2\theta_f / \partial\tau^2 > 0$, $\partial^2\theta_s / \partial\rho^2 > 0$ (note that $y_f - w_f - \tau > 0$ and $y_s - w_s > 0$ are necessary conditions to ensure that regular and shadow production takes place).

As usual, wages are the outcome of a bilateral matching problem described by the *Nash bargaining solution*,

$$w_i = arg\,max\left\{(W_i - U_i)^{\beta_i}\cdot(J_i - V_i)^{1-\beta_i}\right\} \Rightarrow (W_i - U_i) = \frac{\beta_i}{(1-\beta_i)}\cdot(J_i - V_i),\ \text{ with } i \in \{f, s\} \tag{5}$$

where $\beta_i \in (0,1)$ is the bargaining power of workers.

Therefore, for given policy parameters τ and ρ, equations (1) – (5) together with the unemployment identity define a steady state equilibrium (with $\dot{n}_f = \dot{n}_s = 0$).

Now we derive the optimal level of taxation τ and monitoring ρ in a context in which a policy-maker maximizes the social output, where the latter also includes the output and the vacancy cost in the shadow sector. Furthermore, we consider the case in which taxation is used by the benevolent social planner to finance the benefit of being unemployed, i.e. $b\cdot u = \tau$. Following the textbook of Pissarides (2000), the social welfare function for an

[4] The matching function (i.e. the matching elasticity) of the two sectors may differ, but evidence is lacking in this regard.

infinitely-lived economy is equal to the net output per job, minus vacancies costs, plus the benefit of being unemployed. The policy-maker thus maximizes the following programme: [5]

$$\Omega = \int_0^\infty e^{-rt}\left[\left(y_f - \tau\right)\cdot n_f + y_s \cdot n_s - c_f \cdot \theta_f \cdot u - c_s \cdot \theta_s \cdot u + \tau\right]dt$$

subject to:

$$\dot{n}_f = g\left(\theta_f\right)\cdot u - \delta \cdot n_f$$

$$\dot{n}_s = g\left(\theta_s\right)\cdot u - (\delta + \rho)\cdot n_s$$

$$u = 1 - n_f - n_s$$

Let λ and μ be *co-state variables* (i.e. the *shadow values*) at the time t, so that the *Hamiltonian* (*H*) is:

$$H = \Big\{ \; e^{-rt}\cdot\left[\left(y_f - \tau\right)\cdot n_f + y_s \cdot n_s - c_f \cdot \theta_f \cdot \left(1 - n_f - n_s\right) - c_s \cdot \theta_s \cdot \left(1 - n_f - n_s\right) + \tau\right] + \ldots$$

$$\ldots + \lambda \cdot \left[g\left(\theta_f\right)\cdot\left(1 - n_f - n_s\right) - \delta \cdot n_f\right] + \mu \cdot \left[g\left(\theta_s\right)\cdot\left(1 - n_f - n_s\right) - (\delta + \rho)\cdot n_s\right] \Big\}$$

The solution to this dynamic maximization problem requires that: [6]

$$\frac{\partial H}{\partial \tau} \Rightarrow -n_f - c_f \cdot \frac{\partial \theta_f}{\partial \tau}\cdot u + 1 + \lambda \cdot \frac{\partial g\left(\theta_f\right)}{\partial \tau}\cdot u = 0 \qquad \text{(I)}$$

$$\frac{\partial H}{\partial \rho} \Rightarrow -c_s \cdot \frac{\partial \theta_s}{\partial \rho}\cdot u + \mu \cdot \left[\frac{\partial g\left(\theta_s\right)}{\partial \rho}\cdot u - n_s\right] = 0 \qquad \text{(II)}$$

$$\frac{\partial H}{\partial n_f} \Rightarrow \left(y_f - \tau\right) + c_f \cdot \theta_f + c_s \cdot \theta_s - \lambda \cdot \left[g\left(\theta_f\right) + \delta\right] - \mu \cdot g\left(\theta_s\right) = -\left(\dot{\lambda} - r \cdot \lambda\right) \qquad \text{(III)}$$

$$\frac{\partial H}{\partial n_s} \Rightarrow y_s + c_f \cdot \theta_f + c_s \cdot \theta_s - \lambda \cdot g\left(\theta_f\right) - \mu \cdot \left[g\left(\theta_s\right) + (\delta + \rho)\right] = -(\dot{\mu} - r \cdot \mu) \qquad \text{(IV)}$$

From the previous conditions, the result of interdependence between the optimal choice of taxation and the optimal choice of monitoring is straightforward, since conditions (I) – (IV) set up a set of four equations of four unknowns (τ, ρ, λ, μ). Furthermore, the sign of the relationship between taxation and monitoring crucially depends on λ and μ.

In steady state (with $\dot{\lambda} = \dot{\mu} = 0$), by using (III) and (IV) algebraic manipulations give:

$$\lambda = \lambda(\tau, \rho) = \frac{g\left(\theta_s\right)\left(y_f - \tau - y_s\right) + (r + \delta + \rho)\cdot\left(y_f - \tau + \theta_s \cdot c_s + \theta_f \cdot c_f\right)}{(r + \delta)\cdot\left(\rho + g\left(\theta_s\right) + r + \delta\right) + g\left(\theta_f\right)\cdot(r + \delta + \rho)} \qquad \text{(III b)}$$

---

[5] The social planner is not interested in wages, since wages only determine the output's distribution and distributional considerations are excluded from the social welfare function. Furthermore, the social planner is subject to the same matching constraints as firms and workers. Hence, the evolution of employment constrains social choices as well as private ones.

[6] Besides $\lim_{t \to \infty} \lambda \cdot e^{-rt} \cdot n_f = 0$ and $\lim_{t \to \infty} \mu \cdot e^{-rt} \cdot n_s = 0$.

$$\mu=\mu(\tau,\rho)=\frac{g(\theta_f)(\tau-y_f+y_s)+(r+\delta)\cdot(y_s+\theta_s\cdot c_s+\theta_f\cdot c_f)}{(r+\delta)\cdot(\rho+g(\theta_s)+r+\delta)+g(\theta_f)\cdot(r+\delta+\rho)} \qquad \text{(IV b)}$$

with $\lambda>0$, $\mu>0$, and,

$$\frac{\partial\mu}{\partial\tau}=\frac{\left(g(\theta_f)+(r+\delta)\cdot c_f\cdot\frac{\partial\theta_f}{\partial\tau}+\frac{\partial g(\theta_f)}{\partial\tau}\cdot(\tau-y_f+y_s)\right)}{g(\theta_f)\cdot(r+\delta+\rho)+(r+\delta)\cdot(r+\delta+\rho+g(\theta_s))}$$

$$-\frac{(r+\delta+\rho)\cdot\frac{\partial g(\theta_f)}{\partial\tau}\cdot((r+\delta)\cdot(y_s+c_s\cdot\theta_s+c_f\cdot\theta_f)-g(\theta_f)(y_f-\tau-y_s))}{(g(\theta_f)\cdot(r+\delta+\rho)+(r+\delta)\cdot(r+\delta+\rho+g(\theta_s)))^2}>0$$

$$\frac{\partial\mu}{\partial\rho}=\frac{(r+\delta)\cdot c_s\cdot\frac{\partial\theta_s}{\partial\rho}}{g(\theta_f)\cdot(r+\delta+\rho)+(r+\delta)\cdot(r+\delta+\rho+g(\theta_s))}$$

$$-\frac{\left(g(\theta_f)+(r+\delta)\cdot\left(\frac{\partial g(\theta_f)}{\partial\rho}+1\right)\right)\cdot((r+\delta)\cdot(y_s+c_s\cdot\theta_s+c_f\cdot\theta_f)-g(\theta_f)\cdot(y_f-\tau-y_s))}{(g(\theta_f)\cdot(r+\delta+\rho)+(r+\delta)\cdot(r+\delta+\rho+g(\theta_s)))^2}<0$$

under the following two conditions: (i) $y_f-\tau-y_s=0$, which is no better than an equilibrium condition, namely, in equilibrium an entrepreneur operates indifferently in one of the two sectors if $y_f-\tau=y_s$; [7] (ii) $(r+\delta)$ is sufficiently small, which is a realistic condition since small calibration values are usual in the literature. More precisely, $(r+\delta)$ ranges between 0.112 (Shimer, 2005) and 0.18 (Boeri and Garibaldi, 2006).

Therefore, total differentiation of condition (II) gives: [8]

$$\frac{d}{d\tau}\left(-c_s\cdot\frac{\partial\theta_s}{\partial\rho}\cdot u+\mu(\tau,\rho)\cdot\left[\frac{\partial g(\theta_s)}{\partial\rho}\cdot u-n_s\right]\right)=\frac{\partial\mu(\tau,\rho)}{\partial\tau}\cdot\left[\frac{\partial g(\theta_s)}{\partial\rho}\cdot u-n_s\right]<0$$

and,

$$\frac{d}{d\rho}\left(-c_s\cdot\frac{\partial\theta_s}{\partial\rho}\cdot u+\mu(\tau,\rho)\cdot\left[\frac{\partial g(\theta_s)}{\partial\rho}\cdot u-n_s\right]\right)=$$

$$-c_s\cdot\frac{\partial^2\theta_s}{\partial\rho\,\partial\rho}\cdot u+\frac{\partial\mu(\tau,\rho)}{\partial\rho}\cdot\left[\frac{\partial g(\theta_s)}{\partial\rho}\cdot u-n_s\right]+\mu(\tau,\rho)\cdot\left[\frac{\partial^2 g(\theta_s)}{\partial\rho\,\partial\rho}\cdot u-\underbrace{\frac{\partial n_s}{\partial\rho}}_{<0}\right]$$

which is $\begin{cases}>0 & \textit{if } c_S \textit{ is sufficiently small}\\ <0 & \textit{if } c_S \textit{ is sufficiently large}\end{cases}$

[7] Wages depend on the bargaining power of workers and labour market tightness. Therefore, it is not necessarily true that $w_f > w_s$.
[8] The sign of $\partial\lambda/\partial\tau$ and $\partial\lambda/\partial\rho$ remains indeterminate.

Eventually, we obtain a negative relationship between taxation and monitoring policy if c_s is sufficiently small (very realistic); whereas, the relationship could be positive if c_s were sufficiently large (not very realistic).[9] As a result, from the social welfare standpoint, two options are available to the policy maker: s/he may either substitute a tighter monitoring with a higher fine (the penalty is usually assessed on the level of taxes) or enforce both a higher taxation and an increased monitoring (this could be recommended in countries like Italy where public spending is very inelastic, public debt is very large and tax morale is very low).

References

[1] Albrecht, James, Lucas Navarro, and Susan Vroman (2009), "The Effects of Labour Market Policies in an Economy with an Informal Sector," *Economic Journal*, 119(539), July, 1105-1129.

[2] Boeri, Tito, and Pietro Garibaldi (2002), "Shadow Activity and Unemployment in a Depressed Labour Market," *CEPR Discussion Papers*, 3433, June.

[3] Boeri, Tito, and Pietro Garibaldi (2006), "Shadow Sorting ", *Fondazione Collegio Carlo Alberto Working Paper Series*, 10, May.

[4] Bouev, Maxim (2002), "Official Regulations and the Shadow Economy: A Labour Market Approach", *William Davidson Institute Working Papers Series*, 524, December.

[5] Bouev, Maxim (2005), "State Regulations, Job Search and Wage Bargaining: A Study in the Economics of the Informal Sector", *William Davidson Institute Working Papers Series*, 764, April.

[6] Fugazza, Marco, and Jean-Francois Jacques (2004), "Labour market institutions, taxation and the underground economy," *Journal of Public Economics*, 88(1-2), January, 395-418.

[7] Gërxhani, Klarita (2004), "The Informal Sector in Developed and Less Developed Countries: A Literature Survey", *Public Choice*, Springer, 120(3-4), 09, 267-300.

[8] Kolm, Ann-Sofie, and Birthe Larsen (2003), "Wages, Unemployment, and the Underground Economy", *CESifo Working Paper Series*, 1086, November.

[9] Kolm, Ann-Sofie, and Søren Bo Nielsen (2008), "Under-reporting of Income and Labor Market Performance", *Journal of Public Economic Theory*, 10(2), 04, 195-217.

[10] Petrongolo, Barbara, and Christopher A. Pissarides (2001), "Looking into the Black Box: A Survey of the Matching Function," *Journal of Economic Literature*, 39(2), June, 390-431.

[11] Pissarides, Christopher A. (2000), *Equilibrium Unemployment Theory* (2nd Edition), The MIT Press.

[12] Shimer, Robert (2005), "The Cyclical Behavior of Equilibrium Unemployment and Vacancies", *American Economic Review*, 95(1), 25-49.

[9] The vacancy job cost in the underground sector should be very low, since ease of entry is often one of the criteria used to define the informal sector (Gërxhani, 2004).