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Abstract

An important concern of macroeconomic analysis is how interest rates affect the cash balance demanded at a certain level of nominal income. In fact, the interest-rate-elasticity of the liquidity demand determines the effectiveness of monetary policy, which is useless under *absolute* liquidity preference, i.e. when the money demand is perfectly elastic. An actuarial approach is developed in this paper for dealing with *random* income. Assuming investors face liquidity constraints, a level of surplus exists which maximises expected value. Moreover, the *optimal* liquidity demand is expressed as a *Value at Risk* and the *comonotonic* dependence structure determines the amount of money demanded by the economy. As a consequence, the interest-rate-elasticity depends on the kind of risks and expectations. The more unstable the economy, the greater the interest-rate-elasticity of the money demand. Moreover, part of the adjustment to reestablish the short-run monetary equilibrium may be performed through volatility shocks.

Key words: Money demand; Monetary policy; Economic capital; Distorted risk principle; Value-at-Risk.

IME-Classification: IE10, IE20, IE50.
JEL-Classification: E41, E44, E52, G11.

1 Introduction

The primary role of money is to allow the exchange of goods and services. Since payments and expenditures do not usually occur at the same maturities, economic agents are forced to maintain a stock of cash in order to fulfil assumed liabilities and operational spending. On these grounds, according to the *transactions motive*, the demand for money is regarded as being in proportion to the volume of transactions, which in turn may be considered as proportional to income (see Howells and Bain, 2005). Moreover, when the magnitudes of future cash flows are known with *certainty*, the required surplus can be precisely determined at the starting point. Under conditions of *uncertainty*, capital gains or losses arise and so there is also a *speculative motive* for holding money. Accordingly, investors who are more confident about capital gains prefer to invest more on risk and consequently retain fewer provisions. Conversely, less confident investors demand less risk and so they maintain a higher surplus.

However, capital markets offer a set of alternative securities in which cash balances can be held. Some of these can be regarded as free of default-risk and can thus be considered as *perfect* substitutes to money, with the difference that they also provide a return. Why then should investors keep money at all? Actually, risk-free instruments are equal to money except for availability and offered interest rates correspond to *premiums* for liquidity. In frictional markets, it is not possible to attract or resign all required capital at any given moment and so investors trying to avoid bankruptcy will demand money for *precautionary motives* — to avoid insolvency.

According to the Keynes *liquidity preference* proposition (Keynes, 1935), the money demand is positively affected by the level of income and negatively affected by the return offered by a class of non-risky instruments. The first part of the proposition is a consequence of the transactions motive. To explain the effect of the interest rate, Keynes emphasises the role played by the speculative motive. Thus, decision makers expecting interest rates to rise demand fewer risk-free securities in order to avoid capital losses. On the other hand, when interest rates are expected to fall, more bonds are demanded — in this way, once interest rates collapse and bond prices rise, capital gains can be attained. Therefore, fewer provisions are maintained for higher levels of the interest rate and vice versa.

Still a distinction has to be made between *nominal* and *real* quantities. *Nominal* magnitudes represent flows expressed in monetary units, while *real* quantities are expressed in terms of the goods and services that money can purchase. The level of prices P establishes the connection between real and nominal quantities in such a way that $Y = P \cdot y$, where Y and y denote the nominal and real income respectively. The liquidity demand is then expressed in the following way:

$$L \;=\; P\,y \cdot l(r) \;=\; Y \cdot l(r) \quad with \quad \frac{dl(r)}{dr} < 0$$

The *liquidity preference* function $l(r)$ expresses the ratio between demanded cash balances and nominal income. It is not likely to be constant but it may change slowly over time. The inverse ratio of the liquidity preference function is called *velocity of money*. Let us additionally denote by M the total offer of money, which although mainly controlled by the central bank, can also be altered by private investors. It is traditionally related to a class of *narrow* money, containing currency held by the public, and is denoted by $M1$. Therefore, the short-run monetary equilibrium is given by the *quantity equation*:

$$M \;=\; P\,y \cdot l(r) \;=\; Y \cdot l(r) \tag{1}$$

A change in the nominal quantity of money will require a change in one or more of the variables determining the liquidity demand — i.e. P, y or r — in order to reestablish the monetary equilibrium.

In the quantity equation, liquidity preference plays a role in determining the monetary equilibrium. When prices are regarded as rigid for short-run fluctuations[1] and there is no distinction between real and nominal magnitudes, the whole adjustment is performed in $l(r)$. In addition, if liquidity preference is *absolute*, i.e. if investors are satisfied at a single level of the interest rate, the amount of money might change without a change in either nominal income or interest rates. The amount of money is therefore of no consequence. Under such circumstances, monetary policy is useless for dealing with short-run fluctuations.

Absolute liquidity preference corresponds to the case when the liquidity demand is perfectly elastic with respect to the interest rate. According to Keynes, the degree of elasticity depends on how homogeneous expectations are, where perfect elasticity is obtained when expected and actual values are the same. In this case, money and risk-free securities are perfect substitutes — since no capital gains or losses are expected — and any increase of the amount of money by buying bonds will push their prices up and thus lower the interest rate. Consequently, speculators with firm expectations will absorb the additional cash by selling bonds, a situation that will lower their prices until they reach a level consistent with the original interest rates. Conversely, if the monetary authorities decreased the amount of money by selling bonds, the rate of interest would rise, inducing speculators to absorb all the extra offer of bonds. As a result, attempts by the monetary authorities to change the interest rate would be thwarted by investors holding firm expectations (Friedman, 1970).

The situation is different if prices are flexible and liquidity preference is *non-absolute*. Thus, let us suppose, as Friedman (1970) does, that prices adjust more rapidly than quantities, so rapidly in fact that the price adjustment can be regarded as instantaneous. A monetary expansion then produces a new equilibrium involving a higher price for the same quantity, the higher this response the more inelastic the money demand. In the short-run, production is encouraged until prices are reestablished at their original level. In the long-run, new producers enter the market and existing plants are expanded. Throughout the process, it takes time for output to adjust but no time for prices to do so.

Therefore, the efficacy of monetary policy largely depends on the degree of rigidity of prices and the elasticity of the money demand, as well as on the stability of liquidity preference. There is a consensus among researchers about the existence of a stable long-run relationship, though fluctuations of cash balances in the short-run remain unexplained. Episodes like the *missing money* in the mid-seventies, the great velocity decline in the early eighties, followed by the expansion of narrow money in the mid-eighties, or the *velocity puzzle* of the mid-nineties, still lack a satisfactory explanation (Ball, 2001 and 2002; Carpenter and Lange, 2002; Teles and Zhou, 2005). In accounting for such drawbacks, recent literature has focussed on *uncertainty*, which is supposed to have been incremented after 1980 due to deregulation and financial innovations (Atta-Mensah, 2004; Baum et al., 2005; Carpenter and Lange, 2002; Choi and Oh, 2003; Greiber and Lemke, 2005). Calza and Sousa (2003) have instead centered their attention on idiosyncracy and aggregation.

Deregulation and financial innovation are also given as arguments to support the role of the opportunity cost in accounting for unexplained fluctuations (Ball, 2002; Collins and Edwards, 1994; Duca, 2000; Dreger and Wolters, 2006; Teles and Zhou, 2005). Given that

[1] Keynes claims that this situation applies to conditions of underemployment. At full employment, all adjustment would be in prices.

the distinction between cash and alternative risk-free securities has blurred, the class of money substitutes is extended to consider broader monetary aggregates, such as $M2$ (which includes saving and small-denomination time deposits, as well as retail mutual funds) or $M3$ (which adds mutual funds, repurchase agreements and large-denomination time deposits). According to this view, a stable long-run relationship exits and movements of the interest rate can explain all short-run episodes, as long as the right monetary aggregate is used (Ball, 2002). Teles and Zhou (2005) additionally argue that the term of financial securities determines liquidity and so the set of *Money Zero Maturity* instruments, denoted by MZM, provides an appropriate reference for the opportunity cost.

An extended model is proposed in this paper, according to which liquidity preference is explicitly determined by *uncertainty* and *information.* First the cash demand of a single representative investor is obtained. The classic reference in this respect is Tobin (1958), whose model is presented in *Section 2.* He assumes *perfect market* conditions, meaning that risks are completely characterised by only two parameters, *expected value* and *volatility*, while decision making is represented by a *utility* function. Moreover, lending and borrowing are allowed without restriction. Under such circumstances, a single portfolio — combining risk and equity — maximises expected utility and so liquidity preference is obtained as a consequence of uncertainty and aversion-to-risk. By contrast, as obtained in the extended model, under *imperfect* competition investors face liquidity constraints and consequently, in *Section 3* equity is treated as an additional liability. In addition, the behaviour of investors is determined by the transformation of probabilities according to an *informational* parameter. Then the expected return of the fund is maximised when the mathematical expectation of the residual exposure (a measure of the cost of assuming bankruptcy) plus the opportunity cost of capital, is minimised. In this way, I follow Dhaene et al. (2003), who on these terms develop a mechanism for capital allocation (see also Goovaerts et al., 2005).

In fact, under liquidity constraints, individual liquidity demands are given by the *quantile* functions — i.e. the *Value-at-Risk* — of the random variables describing risks, where the arguments are occupied by the net opportunity returns on capital. Moreover, uncertainty and expectations explicitly affect the amounts of demanded cash balances (see *Equations 2* and *3*). When looking for the aggregated surplus in *Section 4*, capital is supposed to be provided by a central authority or financial intermediaries acting in a competitive market, in such a way that a single interest rate is required for lending (see *Equation 4*). Hence the situation is similar to the case of a centralised conglomerate distributing capital among subsidiaries (Dhaene et al., 2003; Goovaerts et al., 2005) and the opportunity cost of money is related to the average return over a class of money substitutes. Thus monetary aggregates are determinants of liquidity preference in the model. Finally, within a Gaussian setting, the aggregate exposure is also normally distributed and its volatility is equal to the weighted average of individual volatilities (as established in *Equation 5*). Therefore, aggregation also plays a role in the determination and stability of the liquidity demand. The final remarks are given in *Section 5.*

2 Liquidity Preference in Perfect Markets

According to the precautionary motive, to avoid insolvency investors maintain a stock of money that in frictionless markets can be modified at any time by lending and borrowing. Hence managers who maximise value demand no equity, which is actually the proposition established by Modigliani and Miller (1958). However, risk-averse customers are sensible to fluctuations and, as long as the business activities of financial intermediaries — which accordingly are said to be *opaque* — are not observed by outsiders, a pressure is established to be perceived as default-free. Under such circumstances, the value of the guarantee is given by the price of an *option* putted on the aggregate exposure, with exercise price equal to the amount of demanded cash (Merton, 1997).

In the model of Tobin (1958), risk-averse investors show liquidity preference as behaviour towards uncertainty. Decisions are based on estimations of the probability distributions characterising the price movements of assets transacted in *perfect markets*, such that the following conditions are fulfilled. **(PM1)** Only two measures completely describe risks: expected return and *volatility* — expressed as the standard deviation of the series of returns. **(PM2)** Lending and borrowing are allowed at any moment for a common risk-free interest rate — at least to a desired extent. **(PM3)** At any point of time, investors share expectations concerning the future performance of securities and thus portfolios — the *efficient market's hypothesis*.

Let X denote the aggregate percentage — random — gain or loss of a mutual fund and, on account of the transactions motive, let the cash demanded be regarded as proportional to the level of income $\overline{Y}$, such that $L = \overline{Y} \cdot l$. Since no liquidity constraints are imposed, such surplus may be invested in the risk-free security to earn the return r, an instrument that can be sold at any time funds are required — for in perfect markets there is always a buyer who agrees to take any quantity at the market price. Hence the total claim is determined by the random variable $Y = (1 - l) \cdot X + l \cdot r$ and its expected value is equal to:

$$\mu_Y = E[Y] = (1 - l) \cdot \mu_X + l \cdot r$$

Moreover, assuming the risky claim follows a Gaussian distribution with volatility σ_X, the volatility of the portfolio is given by:

$$\sigma_Y = \left(E\left[(Y - \mu_Y)^2\right]\right)^{\frac{1}{2}} = (1 - l) \cdot \sigma_X$$

Solving for l in both equations, a linear relationship is established between the expected return and the volatility of the portfolio:

$$\mu_Y = r + \frac{\mu_X - r}{\sigma_X} \cdot \sigma_Y = r + \Psi_X \cdot \sigma_Y$$

This relationship determines the set of *efficient* portfolios — in the sense that for any combination outside the line, it is always possible to build a new fund providing the same expected return and a lower risk, or the same risk but a higher return. Higher volatility offers investors the chance of large capital gains at the price of equivalent chances of large capital losses, since Gaussian risks are symmetric. On the contrary, portfolios of low volatility insure investors against capital losses but offer little prospect of gains. Therefore, a higher return can be obtained only if more risk is assumed. The term $\Psi_X := \frac{\mu_X - r}{\sigma_X}$ denotes the *Sharpe Ratio*, which gives the rate at which investors agree to assume more risk in order to obtain greater return (Sharpe, 1966).

The way preferences affect portfolio decisions can then be analysed in the plane (σ_Y, μ_Y), where the indifference curves of risk-lovers should present negative slope, as long as such individuals accept a lower expected return if there is a chance to obtain additional gains. By contrast, averse-to-risk investors do not take more risk unless they are compensated by a greater expected return and consequently, their indifference curves have positive slopes. Moreover, *more* is regarded as *better*, so that indifference curves located to the upper left corner of the plane are related to higher utilities. Therefore, for any risk-aversion profile, the optimal combination is determined by the (tangency point of) intersection between the unique indifference curve representing preferences and the line of efficient portfolios.

The efficient portfolio can be explicitly determined by introducing a utility function satisfying the Von Neumann and Morgenstern (1944) axioms. Thus *averse-to-risk* investors are characterised by *concave* utility functions (such that $U''(y) < 0$) while *risk-lovers* by *convex* utility functions (such that $U''(y) > 0$). Moreover, the marginal utility $U'(y)$ is positive over the whole range — *more is better*. In this way, every utility function — and so the risk-profile of the investor — induces a level of utility to each portfolio, and *rational* decision making under uncertainty is determined by the maximisation of the expected utility:

$$E\left[U(Y)\right] = \int U(y)\ dF_Y(y)$$

If Φ denotes the probability distribution function of a *normalised* Gaussian distribution — with mean and volatility equal to zero and one respectively — the slope of the respective indifference curve is given by (Tobin, 1958):

$$\frac{d\mu_Y}{d\sigma_Y} = -\frac{\int \left(\frac{y-\mu_Y}{\sigma_Y}\right)\ U'(y)\ d\Phi\left(\frac{y-\mu_Y}{\sigma_Y}\right)}{\int U'(y)\ d\Phi\left(\frac{y-\mu_Y}{\sigma_Y}\right)}$$

As justified in the previous analysis, the optimal combination is obtained when the slope of the indiference curve is equal to the Sharpe ratio:

$$\frac{d\mu_Y}{d\sigma_Y} = \frac{\mu_X\ -\ r}{\sigma_X}$$

In this way, the demand for money in terms of the interest rate is deduced: $l = l(r)$. Unfortunately, the sign of the dependence is ambiguous. So for quadratic utility functions (as in Tobin, 1958) negative and positive elasticities are respectively obtained for interest rates lower and higher than the squared volatility of the aggregate claim.

3 Rational Demand for Cash under Liquidity Constraints

The model of Tobin is generalised in this section, at the time that its main features are maintained, i.e. *uncertainty* is characterised by probability distributions and *rational* behaviour is determined by maximising the expected value of income. However, liquidity constraints and differing expectations about risks are allowed, which means that markets are regarded as *imperfect* and accordingly described by the following conditions. **(IM1)** Risks are taken from a general class of probability distributions which economic agents distort according to their information and knowledge when taking decisions. **(IM2)** Investors face liquidity constraints at borrowing and lending. **(IM3)** Information is not fairly distributed and managers have to expend effort to correctly asses prices. Consequently, investors keep different expectations about risks.

Let the parameter θ denote the state of information of an investor holding a mutual fund whose percentage return is represented by the random variable X. Because of the precautionary motive, a guarantee L is maintained for a determined period of time in order to avoid bankruptcy. Since under liquidity constraints non-risky assets cannot be converted to money at any moment without incurring in additional costs — cash and risk-free securities are not perfect substitutes — it is not possible to invest the surplus on a current account to obtain the return r. Instead, equity is regarded as an additional liability and — based on the transactions motive, as in *Section 2* — the size of the guarantee is expressed as a proportion of the *level* of income $\overline{Y}$, such that $L = \overline{Y} \cdot l$, where l represents the proportion of income assigned to the non-risky asset. Hence the percentage capital return of the total portfolio can be expressed as $Y = X - l - r \cdot l$ and its expected value is given by:

$$\overline{Y} \cdot \mu_{\theta,Y} = \overline{Y} \cdot E_\theta[Y] = \big(\overline{Y} \cdot \mu_{\theta,X} - L\big) - r \cdot L = \overline{Y} \cdot [\ (\mu_{\theta,X} - l) - r \cdot l\ ]$$

Thus, decisions are affected by the *percentage* return on income:

$$\mu_{\theta,Y} := E_\theta[Y] = (\mu_{\theta,X} - l) - r \cdot l$$

In giving a meaning to the informational parameter θ, let us stress the fact that *expectations* are wanted to be modified — and not *tastes* — and then probability beliefs should be transformed. Moreover, the type can be given a broader meaning. So, while the utility function is intended to account for the *psychological response* of decision makers, the distortion parameter is supposed to be also determined by *information* and *knowledge*, in the sense that — as stated by De Finetti (1975) — *information and knowledge permit a limitation of expectations* and so the perception of uncertainty should depend on both. Consequently, let us define[2]:

$$\mathrm{E}_\theta\,[X] = \int x\ dF_{\theta,X}(x) = \int G_{\theta,X}(x)\ dx := \int G_X(x)^{\frac{1}{\theta}}\ dx$$

The *cumulative* and *decumulative probability distribution* functions have been introduced, $F_{\theta,X}(x) = P_\theta\,[X \leq x] = 1 - P_\theta\,[X > x] = 1 - G_{\theta,X}(x)$. Whenever $\theta > 1$ the expected value of risk is overestimated and underestimated when $\theta < 1$, in this way respectively accounting for the behaviour of *risk-averse* and *risk-lover* investors.

[2]The *proportional hazards distortion* is introduced (Wang, 1995), so called since it is obtained by imposing a safety margin to the *hazard rate* of risk in a multiplicative fashion: $h_{\theta,X}(x) = \frac{1}{\theta} \cdot h_X(x) := \frac{1}{\theta} \cdot \frac{d}{dx}\ ln\ G_X(x)$, with $\theta > 0$. Other distortions can be used instead. In the general case, a *distortion function* is defined — over the unit interval — and an axiomatic description is provided for the distorted price (see Wang et al., 1997 and Wang & Young, 1998). *Averse-to-risk* investors are then characterised by *concave* — while *risk-lovers* by *convex* — transformations. All the analysis that follows is maintained in the same terms under this general setting (see also Mierzejewski, 2006).

Notice, however, that individuals react differently depending on the sign of the capital return. Actually, when a loss is suffered, cash is demanded to avoid default, while in the case a gain is obtained the surplus can be used to pay current liabilities or assigned to new investments. Hence the value of the contingent claim is simultaneously determined by two markets which usually work under different conditions, for the borrowing and lending needs of the economy use to differ. On these grounds, we should more properly write:

$$\mu_{\theta,Y} = E_\theta \left[\, (X-l)_+ \right] - E_\theta \left[\, (X+l)_- \right] - r \cdot l = \Delta(l) - r \cdot l$$

The term $\Delta(l) := E_\theta \left[\, (X-l)_+ \right] - E_\theta \left[\, (X+l)_- \right]$ represents the *economic margin* obtained because of financial intermediation, while $E_\theta \left[\, (X+l)_- \right]$ accounts for the *cost of assuming bankruptcy* — a role that can be adopted by the own investor, an insurance company or a central authority. Rational subjects maximise value and accordingly the rational demand is determined by the first order condition[3]:

$$\frac{\partial}{\partial l} \, E_\theta \left[\, (X-l)_+ \right] - \frac{\partial}{\partial l} \, E_\theta \left[\, (X+l)_- \right] - r = -G_{\theta,X}\left(l^*\right) + F_{\theta,X}\left(-l^*\right) - r = 0$$

Since $F_{\theta,X}\left(-l\right) = P_\theta \left[\, X \leq -l \, \right] = P_\theta \left[\, -X > l \, \right] = G_{\theta,-X}\left(l\right)$, the following is an equivalent characterisation:

$$G_{\theta,-X}\left(l^*\right) - G_{\theta,X}\left(l^*\right) = r$$

The firm attains a maximum value as long as the expected income $\Delta(l) - r \cdot l$ is a non-decreasing and concave function, which can be mathematically assured if $\Delta'(l) > r$ and $\Delta''(l) < 0$ or equivalently, if $G_{\theta,-X}\left(l\right) - G_{\theta,X}\left(l\right) > r$ and $G'_{\theta,-X}\left(l\right) - G'_{\theta,X}\left(l\right) < 0$. The first inequality implies that, for a given amount of equity, the marginal loss due to financial intermediation is greater than its opportunity cost — and accordingly there are incentives to maintain a surplus. The second condition ensures concavity.

The rational money demand is thus determined in such a way the marginal gain minus the marginal loss on capital — i.e. the instantaneous benefit of liquidity — equals the marginal return of the sure investment. Therefore, the optimal capital allocation involves an optimal exchange of a sure return and a flow of probability and it is the mass accumulated in the tails of the distribution what matters. No explicit relationship is obtained for the cash demand, but a numerical procedure can be implemented to find the solution.

Decision makers mainly concerned about the speculative and the precautionary motives respectively focus on the terms $E_\theta\left[(X-l)_+\right]$ and $E_\theta\left[(X+l)_-\right]$. Let us accordingly assume that capital decisions are taken by risk managers who minimise bankruptcy and rely on the *average* value of the insured return. Applying a Taylor series around zero:

$$E_\theta \left[\, (X-l)_+ \right] \;\approx\; E_\theta \left[X_+ \right] + \left(\frac{\partial E_\theta \left[(X-l)_+ \right]}{\partial l} \left(l = 0 \right) \right) \cdot l$$

[3]Applying the Leibnitz rule, such that $\frac{d}{dl} \int_{u(l)}^{v(l)} H(l,x) \; dF_{\theta,X}(x) = \int_{u(l)}^{v(l)} \frac{d}{dl} H(l,x) \; dF_{\theta,X}(x) + H\left(l, v(l)\right) \cdot v'(l) - H\left(l, u(l)\right) \cdot u'(l)$ (see, for example, Churchill, 1958) the relationship is obtained by noticing that: $E_\theta \left[\, (X-l)_+ \right] = \int_l^\infty (x-l) \; dF_{\theta,X}(x)$ and $E_\theta \left[\, (X+l)_- \right] = -\int_{-\infty}^{-l} (x+l) \; dF_{\theta,X}(x)$.

Since the term $\frac{\partial E_\theta\left[(X- l)_+\right]}{\partial l}(l=0)$ represents the marginal reduction in insured capital gains produced when attracting the first unit of equity, it can be regarded as a *premum for solvency*:

$$r_{\theta,X} := -\frac{\partial E_\theta\left[(X-l)_+\right]}{\partial l}(l=0) = G_{\theta,X}(0) = P_\theta\left[X>0\right] \tag{2}$$

Hence the following expression is obtained for the expected percentage income:

$$\mu_{\theta,Y} = E_\theta\left[X_+\right] - E_\theta\left[\ (X+l)_-\right] - (r+r_{\theta,X})\cdot l$$

Under such conditions, *precautionary* investors that maximise value minimise bankruptcy costs. Applying Lagrange optimisation, we obtain that decision makers attracts funds until the marginal return of risk equals the total cost of capital:

$$-\frac{\partial}{\partial l}\ E_\theta\left[\ (X+l)_-\right] - (r+r_{\theta,X}) = G_{\theta,-X}\left(l^*\right) - (r+r_{\theta,X}) = 0$$

Equivalently, it can be said that investors stop demanding money at the level at which the marginal expected gain in solvency equals its opportunity cost. Thus the optimal cash demand is given by:

$$l_{\theta,X}\left(r+r_{\theta,X}\right) \ = \ G^{-1}_{\theta,-X}\left(r+r_{\theta,X}\right) \tag{3}$$

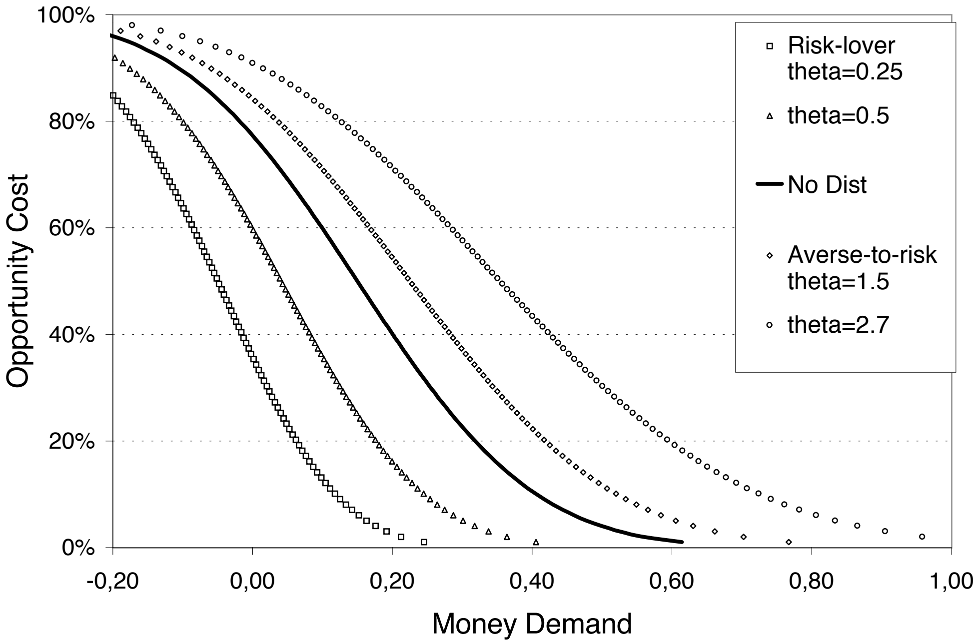
From this expression, the money demand follows a decreasing and — as long as the distribution function describing uncertainty is continuous — continuous path, whatever the kind of risks and distortions (as as can be verified in *Figures 1* and *2*). The minimum and maximum levels of surplus are respectively demanded when $(r+r_{\theta,X}) \geq 1$ and $(r+r_{\theta,X}) \leq 0$.

In *Figure 1*, the optimal demands for cash under Gaussian risks are depicted, distorted according to different informational types. Averse-to-risk investors (characterised by $\theta > 1$) underestimate the cost of money and consequently prefer to keep a higher surplus. Risk-lovers (characterised by $\theta < 1$) behave in the opposite direction. In *Figure 2*, the money demand is shown for non-distorted Gaussian probability distributions with the same mean and different volatilities. Within this class, the exposure to risk is completely determined by volatility and the lower its level the greater the efficiency, for the same expected return is offered by every portfolio. Accordingly, in *Figure 2* the liquidity demand is less and more elastic the lower and the higher the variability of the fund respectively, or equivalently, liquidity preference is less and more sensible to the cost of capital when decision makers respectively face a safer and riskier environment. Moreover, as a consequence of the symmetry of the Gaussian distribution, the demand curves intersect at the point $r + r_{\theta,X} = 0.5$. At this level, there is an equal chance of obtaining a capital gain or loss for every volatility and so the same amount of cash is demanded, equal to the expected value of the fund[4].

In practical applications, intermediaries face operational and administrative costs, at the time that a premium over the risk-free interest rate is asked for lending in secondary markets. Hence, the return $r + r_{\theta,X}$ can be interpreted as a *net* opportunity cost. Though

[4]Actually, if Φ denotes the probability function of a *normalised* Gaussian distribution, the money demand is given by $l_{\theta,X} = \mu + \sigma_{\theta,X}\cdot\Phi^{-1}\left(r+r_{\theta,X}\right)$ (Dhaene et al., 2002). Noting that $\Phi^{-1}(0.5) = 0$ the claim is justified.

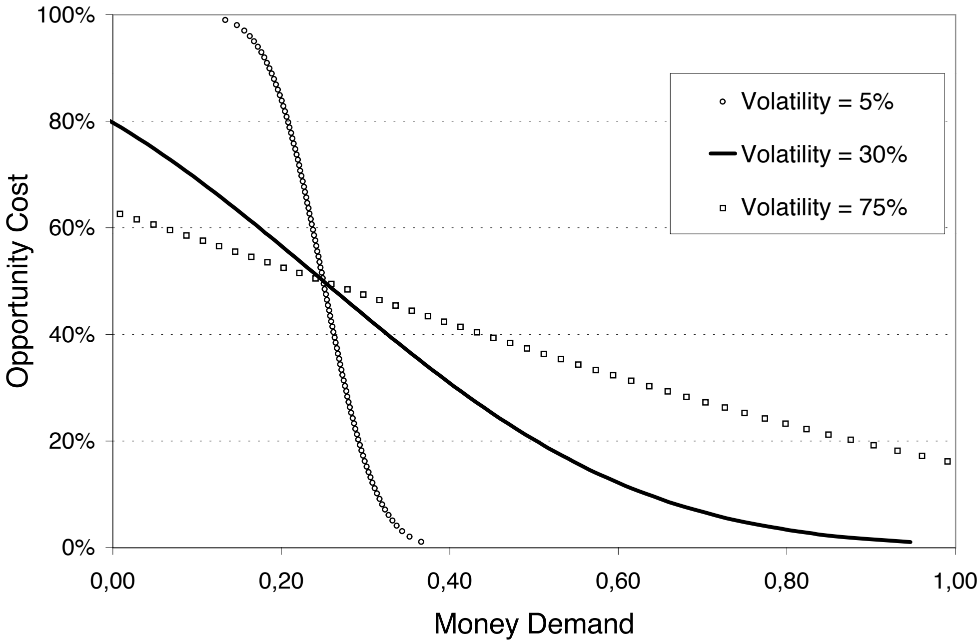
Figure 1: Distorted Liquidity Demand
for Gaussian Risks (mean 15% and vol 20%)



environmental facts, such as the perception of credit quality and gains in efficiency because of improvements on analysis and administration, are expected to evolute on time, we can regard them as softly modified — and not a matter of *speculation*. Also the risk attitude of managers is supposed to remain more or less unchanged — as long as they are emotionally stable. Therefore, the parameter θ is expected to remain stable and consequently, as long as the probability distribution of the random variable X is also stable, the capital decisions of investors should remain more or less the same and the economy as a whole should behave accordingly.

However, if probability distributions evolve on time, so does the premium for solvency $r_{\theta,X}$. Actually, this can be the case after a monetary expansion — which can be performed by the central bank as well as by the entrance of new investors — since part of the extra money is used to buy financial securities and if the increment in demand is high and persistent enough to induce the price to rise *more frequently*, the probability of obtaining capital gains, $P_\theta\left[X > 0\right] = G_{\theta,X}(0) = r_{\theta,X}$, is pushed to increase. In a similar way, a monetary contraction can press the premium for solvency to decrease. This situation might in turn impel decision makers to actualise expectations and so the informational parameter θ might be modified. But this adjustment is supposed to be produced with a certain delay — for a time is required for analysis — while the opportunity cost may be *instantaneously* altered. In other words, prices are supposed to adjust more rapidly than quantities (as in Friedman, 1970). Therefore, changes in the stock of money may induce instability *from within* in secondary markets. Adjustments are performed along a stable money demand relationship, though the process may be reinforced by structural modifications once expectations are actualised.

Figure 2: Non-distorted Liquidity Demand
for Gaussian Risks and Different Volatilities



4 Liquidity Shocks in Capital Markets

In the extended model presented in the previous section, the rational money demand has been determined by the *quantile* (or inverse) function of the probability distribution representing risks, modified according to knowledge and information. A particular feature of the model is that a *solvency premium*, equal to the marginal reduction in capital gains resulted from attracting the first unit of capital, affects the opportunity cost of money. This means that interest rates are corrected into conformity with *expectations* and the *prospects* of risks. However, as long as firms are allowed to attract funds at the same interest rate — independently of the composition of their portfolio and their credit quality — the adjustment is performed in accordance to a *common* set of information θ, representing the *average* knowledge of the market:

$$r^* \ = \ r + r_{\theta,X} \tag{4}$$

Already Keynes (1935), on the grounds of the speculative motive, claimed the cash demand depends on the return expected to prevail over a longer period, regarded as an *anticipated* interest rate. This fact is stressed by monetarists (Friedman, 1970). Though r^* cannot be interpreted as an *anticipated* value in the same terms, it is also affected by *expected* capital gains.

In *Equation 3*, both the probability distribution and the informational parameter alter the money demand by affecting the decumulative probability function and the *corrected* interest rate r^*. Such effects can be regarded as *quantity* and *price* adjustments respectively.

Moreover, since the first adjustment involves a *structural* modification, it is supposed to be performed after the second one, which corresponds to a movement along a stable relationship. In this way, a fundamental assumption that is frequently implicit in macroeconomic analysis, i.e. that prices adapt more rapidly than quantities, is incorporated in the model.

In order to obtain an expression for the cash balance demanded by the whole economy, let us assume that economic agents hold aggregate exposures characterised by the random variables $X_1, \cdots, X_n$. Capital is supplied by a central authority at a single interest rate r^* (or, equivalently, secondary markets are regarded as competitive such that financial intermediaries are *price takers*) relying on the informational parameter θ and the uncertainty introduced by the *market* portfolio X^*. Then the aggregate money demand is given by:

$$l_{\theta,-X^*}(r^*) \;=\; \sum_{i=1}^{n} G^{-1}_{\theta,-X_i}(r^*) \;=\; G^{-1}_{\theta,-X^*}(r^*)$$

The second equality is a mathematical identity as long as the process of capital gains and losses of the market portfolio is described by the *comonotonic sum* $X^* = X_1^c + \cdots + X_n^c$, where $(X_1^c, \ldots, X_n^c)$ represents the *comonotonic random vector* with same marginal distributions as $(X_1, \ldots, X_n)$ and *comonotonicity* characterises an extreme case of dependence, when no benefit can be obtained from diversification[5]. Thus *precautionary* investors rely on the most pessimistic case, when the failure in any single firm spreads all over the market. When differing expectations are allowed among decision makers, the aggregate money demand is given by:

$$l_{\theta_1,\ldots,\theta_n,-X^*}(r^*) \;=\; \sum_{i=1}^{n} G^{-1}_{\theta_i,-X_i}(r^*) \;=\; G^{-1}_{\theta_1,\ldots,\theta_n,-X^*}(r^*)$$

where $G_{\theta_1,\ldots,\,\theta_n,-X^*} = \left(\sum_{i=1}^{n} G^{-1}_{\theta_i,-X_i}\right)^{-1}$ denotes the distribution function of the comonotonic sum when marginal distributions are given by $(G_{\theta_1,-X_1}, \ldots, G_{\theta_n,-X_n})$.

Let M denote the total offer of money in the economy and let us analise how the monetary equilibrium is established in the short-run:

$$M \;=\; \overline{Y} \cdot l_{\theta_1,\ldots,\theta_n,-X^*}(r^*)$$

As long as the expectations of decision makers and the stochastic nature of risks remain unchanged, the adjusted rate of interest r^* or the level of income $\overline{Y}$ are forced to vary for the liquidity demand to fit a given stock of money. Hence, to avoid undesired fluctuations, the central bank can modify the interest rate to adapt the liquidity preferences of investors to a new equilibrium while keeping stable the level of income. However, as explained in *Section 3*, when the processes of assets returns are not stationary, part of the adjustment might be prevented by the premium for solvency. This means that a stronger or a weaker movement of the interest rate may be needed to reestablish the monetary equilibrium.

The mechanism can be regarded as an adjustment to *inflationary* and *deflationary* trends in capital markets. Actually, if part of the cash available after a monetary expansion is invested on assets, the total transactions spending, expressed in nominal units, will increase,

[5] The inverse probability distribution of the comonotonic sum is given by the sum of the inverse marginal distributions (Dhaene et al., 2002).

thus putting a pressure on prices to rise — though not necessarily affecting the *real* side of the economy. As a consequence, the probability of capital gains will increase and so will the premium for solvency, in this way stimulating decision makers to keep fewer provisions. By contrast, after a monetary contraction, security prices might fall as a consequence of the contraction of demand, independently of the *real* prospects of investments.

As times pass by, liquidity preference is affected by the new stochastic regime of the market portfolio — characterised by the random variable X^* — and so the interest-rate-elasticity of the money demand may change. Moreover, individual exposures will be more or less altered depending on the variations in the amounts of money spent on the respective assets. In the medium-term investors actualise their expectations according to the new market conditions. Since different combinations of the informational parameters $\theta_1, \ldots, \theta_n$ may lead to the same cash balance, the model allows multiple equilibria. Therefore, even if the authority succeeded in stabilising the level of income, the new equilibrium may involve a different distribution of resources in the economy.

The dependence of the liquidity demand on the variability of income becomes explicit in a Gaussian setting. Let us assume in the following that individual exposures are distributed as Gaussians with mean zero and volatilities $\sigma_1, \ldots, \sigma_n$, while the contributions of individual exposures to the market portfolio are given by the coefficients $\lambda_1, \ldots, \lambda_n$, with $0 \leq \lambda_i \leq 1 \; \forall i$, such that $\overline{Y}_i = \lambda_i \cdot \overline{Y}$ and $\overline{Y} = \overline{Y}_1 + \cdots + \overline{Y}_n$. Volatilities are expressed as proportions of the levels of income and can be interpreted as the volatilities of different funds as well as the distorted volatilities of a same Gaussian exposure — or some intermediate case. Since the mean returns of funds are equal to zero — more generally, as long as the mean returns of funds are the same — volatilities completely describe risks and efficiency is characterised by low volatility. Under such conditions, the comonotonic sum is also a Gaussian random variable with mean zero and volatility (Dhaene et al., 2002):

$$\sigma^* = \sum_{i=1}^{n} \lambda_i \cdot \sigma_i \tag{5}$$

On these grounds, the weighted average volatility describes the uncertainty of the market portfolio. Consequently, high volatility may be induced by a single group, as a negative externality to more efficient companies and so the possibility of *contagion* naturally arises in the model. In the same way, stability may be inherited by less efficient institutions when low volatility predominates. In this context, *aggregation* is a determinant of the total cash balance demanded by the economy.

Since the *quantile* function of a Gaussian random variable can be express in terms of the standard normal distribution Φ (Dhaene et al., 2002), the short-run monetary equilibrium is described by the following equation:

$$M = \overline{Y} \cdot l_{\sigma^*}(r^*) = \overline{Y} \cdot \sigma^* \, \Phi^{-1}(1 - r^*) \tag{6}$$

Therefore, the monetary equilibrium can be reestablished by modifying the level of nominal income $\overline{Y}$, the market volatility σ^* or the interest rate r^*. As already stated, only r^* is expected to change in the short-run. Monitoring and analysis induce investors to eventually incorporate the new regime of X^* in decision making and possibly modify expectations, both determinants of σ^*. During the process, the demand for goods and services may also be expanded or contracted depending on how spending is affected by the new stock of money. Hence part of the adjustment may be performed on the level of nominal income $\overline{Y}$.

The difference between the classic and the extended model presented in this paper, can be appreciated by comparing *Equations 1* and *6*. Thus, while in *Equation 1* the elasticity of income with respect to the stock of money exclusively depends on the interest rate through the liquidity preference function, in *Equation 6* it is also affected by volatility. In addition, if $\overline{y}$ represents the level of *real* income, the *new* short-run equilibrium can be written in real terms as:

$$M \;=\; P\,\sigma^* \cdot \Phi^{-1}\left(1 - r^*\right) \cdot \overline{y}$$

Therefore, monetary policy cannot be only seen as the determination of the interest rate in order to control the level of prices. To stabilise the product it is also required to control the market volatility and the opportunity cost of money. A proper monetary policy should then consider a combination of P, σ^* and r^* compatible with a given level of income.

Since *Equation 6* is not likely to fit with exactitude to empirical observations, the level of σ^* can be found that preserves the monetary equilibrium for given values of M, $\overline{Y}$ and r^*, which can be regarded as the *induced* volatility. A tentative criterion for monetary policy may then be to determine the level of interest rates ensuring a given inflation and induced market volatility. Additionally, the *non-distorted* volatility σ can be estimated by the *standard deviation* of the random variable X^* representing the capital losses of the market portfolio. A measure of the degree of distortion performed by the market can thus be obtained by comparing the induce with the non-distorted volatility[6].

As stated in *Section 1*, the efficacy of monetary policy depends on the interest-rate-elasticity, as well as on the stability, of the money demand. Instability is disregarded by monetarists on the grounds of the *rational expectations hypothesis*, according to which, rational investors form their expectations by making the most efficient use of all information provided by past history (Modigliani, 1977). Consequently, errors can only occur in short-terms and so the economy is inherently stable. Keynes (1936) assumes, on the opposite, that liquidity preference is affected by the speculative motive and that economic agents, when forming their expectations, are controlled by *animal spirits*. Under such conditions, money markets should be pretty unstable.

As can be seen from *Equation 6*, the *point interest-rate-elasticity* of the money demand is affected by the level of interest rates in the extended model, but not by market volatility:

$$e \;=\; \frac{r^*}{l_{\sigma^*}\left(r^*\right)} \cdot \frac{dl_{\sigma^*}\left(r^*\right)}{dr^*} \;=\; \frac{r^*}{\Phi^{-1}\left(1 - r^*\right)} \cdot \frac{d\Phi^{-1}}{dr^*}\left(1 - r^*\right) \tag{7}$$

However, recall that the *slope* of the money demand depends on volatility (see *Figures 1* and *2*) in such a way that liquidity preference is more *absolute* for less stable economies. Moreover, the opportunity cost of money, represented by the return offered by a given class of money substitutes, is determined by open market operations performed by investors who expend effort in analysis to appropriately estimate the prospects of business opportunities. Thus movements along a stable long-run relationship may induce instability. The *rational*

[6]Specifically, the informational parameter θ^* can be found for which the volatility σ^* is related to the transformed probability distribution of X^*.

expectations hypothesis is maintained in this way, but with different consequences. Additionally, the market volatility may remain unchanged at the time that individual exposures are evolving, since multiple configurations of the parameters $\sigma_1, \ldots, \sigma_n$ are compatible with the same level σ^*. Consequently, stability is also related to the way the allocation of resources is altered by monetary shocks. In the presence of frictions and limitations in the access to information, some companies or industries might be severely affected and in turn the economy as a whole might be affected.

The function $\frac{d\Phi^{-1}}{dr^*}$ attains a maximum when $r^* = 0.5$ and hence so does the magnitude of the interest-rate-elasticity. Consequently, monetary policy is most efficient at this level. As long as the opportunity cost approaches to any of the extreme cases $r^* = 1$ or $r^* = 0$ the liquidity demand becomes more elastic and it turns *absolute* for $r^* \geq 1$ and $r^* = 0$. Recall that capital decisions involve the exchange between a sure return and a confidence probability level. Thus in the first case a capital gain is expected with probability equal or greater than one, i.e. it is produced with certainty. Accordingly, investors not only do not keep provisions for this asset, they actually agree to take new liabilities to provide the guarantee and they are satisfied whatever the required volume of extra funds. By contrast, when $r^* = 0$ capital losses are expected with certainty and so investors agree on any level of surplus in order to insure the exposure to risk.

5 Conclusions

Since no liquidity restrictions are expected in perfect markets, it is always possible to attract or release funds under such conditions and accordingly, rational decision makers demand no cash balances, as long as the value of the portfolio does not depend on the level of surplus (Modigliani and Miller, 1958). In a Gaussian setting, risks are completely characterised by the expected return and volatility, and a line of *efficient* portfolios exists, in the sense that, for any combination of risk and equity outside the class, there is an alternative portfolio ensuring the same return but less risk or the same level of exposure and a higher return. Moreover, risk-aversion determines a set of preferred combinations of risk and equity. Therefore, a single fund maximises the expected utility of investors showing a given degree of risk-aversion. On these grounds, Tobin (1958) regards liquidity preference as behaviour towards risk.

A model is presented in this paper, referred to as the *extended* or the *imperfect competition* model, to characterise the liquidity preference of investors facing liquidity constraints. Under such circumstances, a level of surplus exists that maximises value and the *rational* money demand is determined by the *quantile* function — a measure of the probability accumulated in the *tail* of the distribution function — of the random variable representing the capital losses of the residual exposure (see *Equation 3*). In this way, an equivalence is established between a confidence level and the opportunity cost of capital and the optimal amount of cash is determined by the exchange of a sure return and a flow of probability. An informational parameter, affecting the opportunity cost of money, represents the expectations of decision makers. Averse-to-risk and risk-lover investors respectively under and overestimate the cost of capital and so they respectively demand more and less equity.

The quantile function, better known as *Value-at-Risk* or *VaR* in the Risk Management literature, has become very popular among practitioners and researchers. This is in part a consequence of simplicity, since in a Gaussian setting the VaR is proportional to volatility which in turn can be estimated by the *standard deviation*, a measure of the average variability of a random variable. Alternatively, the VaR answers the question of how much can be lost in the next trading period for a given level of probability — the *confidence* level — and so it can be interpreted as a bound for expected losses. Given that the VaR then provides a clear intuition, risk managers have naturally incorporated it into decision making.

The importance attached to liquidity preference in macroeconomic analysis is a consequence of the fact that it determines the short-run monetary equilibrium of the economy. In the classic approach, which is based on a partial equilibrium of the economy (Lucas, 2000; Teles and Zhou, 2005), the amount of money which is compatible with given levels of nominal income and interest rates can be obtained from *Equation 1.* According to the *extended* model presented in this paper, the aggregate money demand of the economy is given by the sum of the liquidity preferences of investors, mathematically characterised by the *comonotonic sum* of individual exposures. The aggregate money demand is thus expressed as a Value-at-Risk but referred to a *market portfolio* which relies on the most pessimistic case, when no gain can be obtained from diversification. In a Gaussian setting, the comonotonic sum is also a Gaussian variable, whose volatility is equal to the weighted average of individual volatilities (see *Equations 5* and *6*). In this way, the classic model is extended allowing a correction for risk.

Attempts have been made in recent literature to introduce uncertainty in the determination of the money demand, as part of an effort to explain the erratic path followed by demanded cash balances since 1980. In Carpenter and Lange (2002), the money demand is supposed to be affected by volatility in equity markets, a fact that is justified by the role of money as a capital asset. As a matter of fact, growth and innovations in the financial industry have reduced transaction costs and thus increased the substitutability between equities and money. Two indicators of market risk conditions, the volatility in equity prices and the revisions to earning expectations, are then introduced into a linear regression and both are statistically significant over the period starting in the early nineties. Greiber and Lemke (2005) argue that liquidity preference has been stimulated by increased macroeconomic uncertainty and low asset yields. Macroeconomic uncertainty is treated as an unobserved process reflected in several economic indicators, such as financial market data, loss and volatility measures, as well as on business and consumer sentiments. In this way, it is proved that measures of uncertainty help to explain the increase in cash balances in the euro area — specifically in the aggregate $M3$ — over the period 2001 to 2004.

Choi and Oh (2003) derive a money demand function from a general equilibrium framework. In so doing, they suppose that monetary and output variables are determined by log-normal processes, while investors preferences are given by a discounted expected-utility operator. Liquidity preference is then determined by the maximisation of utility — subject to constraints representing budget and market conditions — and in this way, given a level of nominal interest rates, is negatively and positively affected by output and monetary uncertainty respectively. The model allows to satisfactorily explain the cases of missing money and velocity shifts already mentioned in *Section 1.* Other models that consider the role of macroeconomic uncertainty in the determination of the money demand can be found in Atta-Mensah (2004) and Baum et al. (2005).

Within the *imperfect competition* framework, the total stock of money M, the level of income $\overline{Y}$, the interest rate r^* and the market volatility σ^* are all determinants of the short-run monetary equilibrium (see *Equation 6*). Any of the last three variables may change together with the monetary regime. Thus, as long as part of the funds available in the economy are spent on capital assets, an adjustment in the opportunity cost r^* is expected in the short-run — stimulated by the modification of the stochastic nature of capital gains — which is supposed to *instantaneously* affect liquidity preference (see *Equations 4* and *2*). In the medium-term, investors correct their volatility estimations and so part of the adjustment may be performed through volatility shocks. An important feature of the mechanism is that volatility changes, motivated by flows of funds, determine expectations and not the opposite, though liquidity preference might also be affected by a *purely* informational shock — not supported by any change on the probability distributions characterising risks.

Recall that liquidity preference is not affected in the same way by capital gains and losses. Thus, while positive returns affect the opportunity cost of money and so determine a movement along a stable relationship, the precautionary attitude of decision makers depends on negative returns and so does the *shape* of the money demand (see *Equation 3*). The first adjustment is supposed to occur instantaneously, while the second one is performed gradually, for it takes time to investors to internalise new market conditions. In practice, we are induce to believe that both decisions should be related to different markets. Accordingly, the cost of equity r^* is represented by the average return over a class of securities, other than cash, that can be regarded as substitutes to money. On the other hand, the liquidity preference function depends on the series of returns over a set of instruments representative of assumed exposures. Then the variability showed by a representative index of this class determines the *market volatility* σ^*.

The stability of the money demand has been analysed by Teles and Zhou (2005), who notice that though nominal interest rates have decreased considerably in the last quarter century, the monetary aggregate $M1$ has increased very little in the same period (see also Ball, 2001). A comparison between the evolution of the different monetary aggregates $M1$, $M2$, $M3$ and MZM (see *Figure 4* in Teles and Zhou, 2005) shows that $M1$ has actually grown at a lower rate than other monetary aggregates since 1980 and has remained roughly constant after 1994 — $M1$ and MZM have respectively incremented in a five and nine percent since 1980. This fact represents a break in the money demand as estimated by classic methods. Actually, different periods can be estimated using different elasticities (three periods are proposed: 1900-1979, 1980-1994 and 1995-2003). A stochastic general equilibrium model (based on the model of Lucas, 2000) allows Teles and Zhou (2005) establishing a relationship for the money demand depending on the difference between the interest rates offered by a class of nominal bonds and a class of money substitutes. They estimate the model for the measures $M1$ and MZM and find that changing the monetary aggregate measure preserves the long-run relationship of the money demand up to a constant factor.

Ball (2001 and 2002) claims that short-run fluctuations in money holdings are produced by movements in near-money returns, such as saving accounts, money market deposits and money market mutual funds. Such instruments are included in the class of MZM and a *cointegrating* relation is proposed, with the real income and the opportunity cost as explanatory variables and a stochastic and stationary error-term accounting for deviations. Then *non-cointegration* is rejected when sufficient lags of the variables are introduced. Moreover, deviations from a *long-run state* are smaller when the return in near-money is considered instead of $M1$. A similar model is proposed by Dreger and Wolters (2006), but using $M3$ as the measure of the opportunity cost and introducing inflation as an additional explanatory variable.

As a general rule, it can be said that after the innovations and regulatory reforms that has taken place since 1980, components of the *transactions* aggregate $M1$ bear interest, while components of other *savings* aggregates are used for transactions. In this way, most *velocity shocks* occurred after 1980 can be explained by increased volatility in near-money returns and can thus be interpreted as movements along a money demand curve. However, as stated by Teles and Zhou (2005), when changing the monetary aggregate the parameters of the money demand relationship are also modified, in particular the interest-rate-elasticity (they report an estimated elasticity 0.32 for the entire period 1900-2003 and elasticities 0.26, 0.12 and -0.07 for the subperiods 1900-1979, 1980-1994 and 1995-2003 respectively) suggesting that an structural change takes place. Such a reasoning makes sense within the *imperfect competition* framework, for risk explicitly affects liquidity preference.

Calza and Sousa (2003) have pointed out three main factors to explain why the money demand has been more stable in the euro area than in other large economies: specific conditions in the capital markets of different countries, the impact of financial innovation and gains in stability obtained from aggregation. Idyosincratic behaviour — both at the investor and sectorial levels — is accounted by the informational parameter in the *imperfect competition* model. Regarding financial innovation, the degree of concentration and the role of Germany are given by Calza and Sousa as possible explanations of why its influence has been weaker in the euro area. Moreover, the lack of synchronisation of shocks to national money demands guarantee a more stable environment. But most importantly, the fact that Germany has a large weight in the $M3$ aggregate for the euro area and that the money demand has been historically stable in that country contributes to the overall stability of the euro area money demand. In other words, the stability of the German economy is shared by the rest of the economies in the block, as a positive externality.

Such a conclusion is readily obtained from the *extended* model for, as stated in *Equation 5*, market variability is determined by the mean value of individual volatilities. Hence, the market volatility will be mainly determined by a single institution or sector, in the case it contributes more to the aggregate exposure. In particular, stability can be induced in the whole market in this way. Moreover, the system accepts multiple equilibria, since different combinations of individual volatilities may lead to the same market variability. The terms under which market volatility shocks affect individual expectations about risks will be determined by specific conditions, such as aggregation, restrictions in the access to credit, the distribution of information within the market and the skills and knowledge of investors. Thus, changes in the aggregate monetary stock may induce intermediaries to favour bigger or more efficient companies, a situation that may become more difficult the availability of funds and possibly increment more the riskiness of less productive sectors of the economy. In this way, within the *imperfect competition* framework, a broader meaning is given to instability.

Finally, a word can be said about the interest-rate-elasticity of the money demand. As deduced from *Equation 1*, monetary policy is more efficient the more inelastic the liquidity preference function, such that under perfect inelasticity investors are satisfied with the same amount of cash for every level of the interest rate and so all adjustment is expected on the level of income. On the contrary, under *absolute* liquidity preference, i.e. when the money demand is *perfectly* elastic, investors are satisfied at a single level of the interest rate and so all the adjustment is expected from liquidity preference and none on the level of income. In the later case, monetary policy is useless (Friedman, 1970; Modigliani, 1977; Howells and Bain, 2005).

As established in *Equation 7*, in the *extended* model the point interest-rate-elasticity depends on the level of the interest rate when risks are Gaussian. Looking at *Figures 1* and *2*, we notice the money demand follows a linear path across a wide range, but it becomes parallel to the x-axis when approaching to it, meaning that it becomes more inelastic when the opportunity cost approaches zero. In a similar way, the money demand becomes parallel to the axis $x = 1$ when the opportunity cost approaches one. Therefore, since the liquidity demand curve never touches the x-axis — as long as the random variable representing the aggregate exposure is not bounded — there is no maximum level for cash balances, which is in contradiction with common beliefs. This novel result is a consequence of the equivalence established in the *imperfect competition* setting between the opportunity cost and the confidence level (see *Equation 3*). Thus, when the return on money substitutes approaches zero decision makers react as if the probability of obtaining capital gains was close to zero and accordingly, they are forced to insure their residual exposures using only cash balances. On the other hand, when the opportunity cost approaches one, the probability of obtaining capital losses is close to zero and so fewer provisions are demanded.

Therefore, whatever the level of elasticity that predominates over the medium range of the interest rate — i.e. no matter the level of stability of the economy in the medium range — liquidity preference becomes absolute when the opportunity cost approaches zero. Since the return in money substitutes is expected to be low in a depressed economy, lowering the level of interest rates becomes investors more sensible to the cost of capital and eventually stimulates them to demand all available cash balances for precautionary purposes. Under such circumstances, the economy is found in a *liquidity trap* and any attempt of the monetary authority to stimulate it by lowering the level of the interest rate will only do matters worse. On the contrary, when the return on money substitutes is high — a situation that might be stimulated by deregulation and financial innovation — less cash balances are demanded than extrapolated by a linear relationship, a fact that could explain some *missing money* events.

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