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Franchise Fee, Tax/Subsidy Policies and Economic Growth

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Abstract

In this paper, we take a new look at the effects of the subsidy policy and the government’s R&D activities in an R&D-based growth model. The government not only subsidizes the R&D cost of the firms but also engages in R&D activities and, in addition, levies a specific tax on the firms producing the final and the intermediate goods, respectively, in order to finance the expenditure. We find that in the economy there exist two balanced equilibrium growth paths. In an economy with a high growth path, the government’s subsidy policy and its R&D activities will crowd out the private R&D activities, and hence the fiscal policies are of no help to the economic growth. In other words, the intermediate goods firms play an important role in driving the economic growth. By contrast, in an economy with a low growth path, the government that directly engages in R&D activities plays an important role in economic growth. The fiscal policies of the government have a positive effect on the economic growth.

Keywords: Government’s R&D activities, Specific tax, Subsidy policy, Endogenous growth, R&D

\textit{JEL classification:} L00; O41; O30

1. Introduction

The role of the government cannot be ignored in the endogenous growth model, and accordingly in the 1990s there was an explosion of research on the growth effects of several government activities. In the R&D-driven endogenous growth models, Romer (1990), Segerstrom et al. (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), and Barro and Sala-i-Martin (2004) all find that R&D subsidies encourage firms to devote more resources to R&D activities and as a result there is an increasing

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rate of economic growth in the long run. Jones and Williams (1998, 2000) point out that the decentralized economy typically under-invests in R&D when compared to what is socially optimal when using data for the US economy. Recently, some of the theoretical literature, such as Davidson and Segerstrom (1998), Segerstrom (2000), and Zeng and Zhang (2007) that discusses the role of government policy in the field of the R&D-based endogenous growth model, has focused on subsidies for R&D because monopoly pricing and knowledge spillovers may result in too little private R&D. Şener (2008) studies the determinants of an optimal R&D policy, under a wide range of empirically relevant calibrations and finds that the subsidy rate turns out to be positive and to fluctuate between 5% and 25%.

This paper focuses on the financial resources that come in the form of subsidies out of government revenue. Most of the papers that consider tax policies in the context of an R&D-based growth model or subsidy policies financed by tax revenue usually introduce ad valorem taxation (i.e., a tax proportional to the firm’s revenue/profit) to research the tax effect, in addition to Futagami and Doi (2004) who investigate a specific tax or unit tax (i.e., a tax proportional to the firm’s output) against the backdrop of economic growth, but they do not discuss the subsidy policy in relation to economic growth. It is well known that, in the R&D-driven endogenous growth model, the monopoly profit serves as an engine of economic growth. In this model, the tax incidence under imperfect competition not only gives rise to a price effect, but also to a profit effect that impacts the economic growth. Hence, the tax incidence and subsidy policy in relation to the R&D activities plays an important role in driving the endogenous growth model.

A large number of studies in the taxation literature, including Suits and Musgrave (1953), Stern (1987), Hamilton (1999), Atkinson and Stiglitz (1987), Delipalla and Keen (1992), Myles (1996), Keen (1998), Anderson et al. (2001a), Schröder (2004), and Pirttilä (2002), investigate the relative efficiency of specific taxes and ad valorem taxes under an imperfectly competitive static analysis, for example, under Cournot competition and Bertrand competition, and their effect on social welfare. They show that there is a welfare dominance of ad valorem taxes over specific taxes under imperfect competition. In particular, ad valorem taxation leads to the lower consumer price of a good even though firms would exit the market in a monopolistic competition case (Schröder, 2004). These studies almost all focus on social welfare, but not on the economic growth. However, an important point of view regarding ad valorem taxation is that there is an increase in firms exiting the market as compared to a specific tax and the lower firm profits that result. In such an instance,

1 Related empirical studies, for example Cohen and Levin (1989), Griliches (1992), and Nadiri (1993), support the conclusion of under-investment in R&D.

2 See, for example, Lin and Russo (1999), Peretto (2007), and Haruyama and Itaya (2006).
an ad valorem tax is no longer better than a specific tax in the case of the R&D-based endogenous growth model since the economic growth is derived from the monopoly profit due to innovation needs. Because the profits induce the intermediate goods firms to engage in R&D activities, the increase in R&D activities will cause the economy to grow.

On the other hand, Besley (1989) indicates that the output per firm is enhanced by an increase in a specific tax on output if the inverse demand function is convex. Anderson et al. (2001b) show that unit taxation can be welfare-superior in the long run when the market is characterized by Bertrand competition with differentiated products. Doi and Futagami (2004) also introduce a monopolistic competition model in which consumers have a variety of preferences and which shows that a specific tax increases social welfare and that the optimal tax rule is contrary to the inverse elasticity rule. Kitahara and Matsumura (2006) investigate how a specific tax and an ad valorem tax affect equilibrium location choice in a model of product differentiation which includes Hotelling and Vickrey-Salop spatial models. They find that the specific tax affects neither of the firms’ equilibrium location, output quantity, or profits. However, the number of firms is a key point in R&D-driven endogenous growth models because the more firms there are in the intermediate goods market, namely, the more variety there is, the more the economy grows. Hence, the analysis of the specific tax which is the government’s revenue in a dynamic framework associated with economic growth differs from an ad valorem tax which is discussed in traditional theory and thus becomes more interesting.

In this paper, we would like to extend Wang et al. (2010) and to take a new look at the effects of tax and subsidy policies and the government’s R&D activities in an R&D-based growth model. Let us consider a government that imposes a specific tax on both final goods and intermediate goods to finance the subsidies and expenditure on R&D activities. Under a successively monopolistic competition model this paper in following Wang et al. (2010) deals with franchise contract bargaining for vertical integration, a subsidy to reduce the R&D cost of the firms and increase the government’s R&D activities due to the too few private R&D activities in a decentralized economy, and a specific tax to correct the market power of the producer.

We present a four-period model. In the first period, the government levies specific taxes on final goods and intermediate goods to finance the government expenditure, to engage in R&D activities and to subsidize the R&D costs of the firms. In the second period, the final goods firms and the intermediate goods firms bargain over the franchise contract including over the franchise fee and the price of the intermediate goods according to Nash efficient bargaining. In other words, the upstream and downstream industries will vertically integrate to eliminate the double
marginalization through the franchise contract. In the third period, the final goods firms determine the prices of the final goods to maximize their profits. In the fourth period, the consumers decide the expenditure plan to maximize their utility. We proceed by solving the model backward.

2. The Model
The model is an extension of the endogenous growth model with the increasing variety model of Grossman and Helpman (1991, chapter 3) and Wang et al. (2010). We consider an imperfectly competitive final goods market and the government not only implements a tax/subsidy policy but also engages in R&D activities. There are four agents in this model, the household, the final goods producers, the intermediate goods firms and the government. In this model, R&D investment creates new types of intermediate goods for final production. The price of intermediate goods is determined by the negotiation between the intermediate goods firms and final goods firms. The government levies a specific tax to finance the subsidy for too little R&D and engages in R&D activities. The household chooses a consumption/investment plan.

2.1 Households
The individuals supply labor service, \( L \), that is supplied inelastically, and consumption loans in competitive labor and imperfectly competitive product markets. The representative household’s preferences are defined over an infinite horizon

\[
V = \int_0^{\infty} e^{-\rho t} U(C(t)) dt
\]

where

\[
U(C) = \ln C
\]

\[
C \equiv m \left( \frac{1}{m} \int_{j=0}^{m} c_j^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1
\]

Eqs. (1) and (2) indicate that utility is a unitary elasticity function and is discounted by a constant pure rate of time preference \( \rho \). \( C \) is a composite consumption good which consists of a bundle of closely-related product varieties according to Eq. (3). This type of monopolistic competition CES functional form follows Dixit and Stiglitz (1977), \( m \) is the number of different varieties, and \( c_j \) is a consumption good of variety \( j \). Commodities supplied by different producers are imperfect substitutes with constant elasticity of substitution \( \sigma \). \( j \in [0, m] \) represents the varieties

\[3\] To simplify our notation, the time arguments will all be dropped.
produced by different downstream firms.

The budget constraint, which describes how the household invests the new assets, is equal to the rate of return \( r \) earned on assets and total labor income plus the profit the household receives from the downstream firms minus total spending on consumption goods. It is therefore given by

\[
\dot{a} = ra + wL + m\Pi - E
\]

where

\[
E = PC = \int_{j=0}^m p_j c_j dj
\]

\( E \) is total spending on consumption goods, and \( P \) is the aggregate consumption price index. \( p_j \) is the after-tax price of consumption good \( j \). \( w \) is the wage rate which is common to all sectors in the economy since labor is assumed to be perfectly mobile. \( \Pi \) is the after-tax profits of the firms of the final goods sector. \( a \) is the household assets which is the value of the stock of the blueprints, \( a = p_A n \) and \( \dot{a} = p_A \dot{n} + n\dot{p}_A \), where \( p_A \) is the after-subsidy cost or price of a new blueprint \( \dot{n} \). Therefore, the budget constraint may be rewritten as

\[
p_A \dot{n} + n\dot{p}_A = rp_A n + wL + m\Pi - PC
\]

First, the representative household chooses the optimal consumption and investment plan to maximize its discounted utility, Eq. (1), subject to the budget constraint, Eq. (6). The current-value Hamiltonian associated with this decision problem is given by

\[
H = \ln C + \lambda \left( \frac{rp_A n + wL + m\Pi - PC - n\dot{p}_A}{p_A} \right)
\]

where \( \lambda \) is the co-state variable for \( n \). The first-order conditions are\(^4\)

\[
\frac{1}{C} = \frac{\lambda}{p_A} P
\]

\[
\dot{\lambda} = \rho \lambda - r \lambda + \frac{\dot{p}}{p} \lambda
\]

By combining these two expressions, we obtain the Keynes-Ramsey rule:

\[
\frac{\dot{C}}{C} = r - \rho \frac{\dot{P}}{P}
\]

Secondly, the household chooses its consumption levels for each available product variety, \( c_j \), in order to maximize the utility of Eq. (2), given the definition of

\(^4\) The transversality condition is \( \lim_{t \to \infty} \lambda t e^{-\rho t} = 0 \) to ensure that neither debts nor assets will be left at the end of the planning horizon.
composite consumption in Eq. (3) and the budget constraint in Eq. (5). The solutions for the consumption of variety \( j \) are obtained:

\[
c_j = m^{-1} \left( \frac{P_j}{P} \right)^{-\sigma} C
\]

where

\[
P = \left( m^{-1} \int_0^m p_j^{1-\sigma} \, dj \right)^{1-\sigma}
\]

Eq. (11) gives the downward-sloping demand curve for goods \( c_j \) which is faced by the final goods firms. Eq. (12) expresses the aggregate consumption price index.

### 2.2 The Final Goods Sector

We consider a production economy with imperfectly competitive product markets. The consumption goods are produced by monopolistically competitive firms. Each consumption good is supposed to be produced by a single firm, that is, \( m \) also represents the number of firms which produce industry \( j \) goods. Therefore, a composite final good \( Y \) which is equal to consumption goods, Eq. (3), can be represented as

\[
Y = m \left( \frac{1}{m} \int_{j=0}^m y_j^{\sigma-1} \, dj \right)^{\sigma-1}
\]

\( Y \) is produced by monopolistically competitive firms. Each firm produces \( y_j \) by using a continuum of intermediate goods \( x_i \). According to Spence (1976) and Dixit and Stiglitz (1977), the production function of firm \( j \) is

\[
y_j \equiv \left( \int_0^n x_i^{\frac{1}{\alpha}} \, di \right)^{\frac{1}{\alpha}}, \quad \alpha > 1
\]

where \( x_i \) represents the amount of intermediate goods \( i \) used by firm \( j \). Each intermediate good \( x_i \) is produced by a single firm, and intermediate goods are not perfect substitutes. \( i \in [0, n(t)] \) is the range of intermediate goods existing at time \( t \). \(-1/(1-\alpha)\) represents the elasticity of substitution between final goods.

The producer \( j \) in the final goods sector chooses a price to maximize its profit

\[
\Pi_j = q_j \left( \int_0^n x_i^{\frac{1}{\alpha}} \, di \right)^{\frac{1}{\alpha}} - \int_0^n \hat{p}_j x_i \, di - n\hat{f}_j
\]

subject to the demand function in Eq. (11) and the clearness condition for the final goods market, \( y_j = c_j \). \( q_j \) is the price of the final goods, \( \hat{p}_j \) is the after-tax price
of the intermediate goods $i$, and $f_j$ represents the franchise fee that the final goods producer $j$ has to pay to the upstream firm in order to obtain the right and know-how to produce the final good by using these intermediated goods.

We assume that the government levies a specific tax on each final good and intermediate good, and that each tax is constant over time for analytical simplicity. The consumption goods price and the intermediate goods price become

$$p_j = q_j + \tau_y$$

(16)

$$\hat{p}_y = p^*_y + \tau^x$$

(17)

where $\tau_y$ represents the specific tax imposed on the final goods and is the same for all $j$. $\tau^x$ represents the specific tax imposed on the intermediate goods $i$ and is the same for all $i$.

From the perspective of symmetry, we have $x_{ij} = x_j$, $p^*_y = p^*_x$, $\forall i$, in equilibrium. The production function in Eq. (14) becomes $y_j = n^a x_j$. Then we substitute $x_{ij} = x_j = n^{-a} y_j$, $y_j = c_j$ into Eq. (15) and subject to Eqs. (16) and (17).

The maximizing profit function of firm $j$ becomes

$$\max_{q_j} \Pi_j = \left[q_j - n^{-a} \left(p^*_y + \tau^y\right)\right] n^{-1} \left(\frac{q_j + \tau^x}{p}\right)^{-\sigma} Y - n f_j$$

(18)

To maximize the profit, the typical final goods firm $j$ will charge a monopolistic markup price to the consumers as follows

$$q_j = \frac{\sigma \left[n^{-a} \left(p^*_y + \tau^y\right)\right] + \tau_y}{\sigma - 1}$$

(19)

The pricing rule depends on the elasticity of substitution ($\sigma$) of the final goods firms, the prices of intermediate goods ($p^*_x$), and the specific taxes ($\tau^x$, $\tau_y$). Then the consumption goods price becomes

$$p_j = \frac{\sigma}{\sigma - 1} \left[n^{-a} \left(p^*_y + \tau^y\right) + \tau_y\right]$$

(20)

Substituting Eq. (19) into Eq. (18) yields the profit function of a typical downstream firm $j$.
\[ \Pi_j = \left[ \frac{1}{\sigma - 1}(n^{-\alpha}(p_j^r + \tau^r) + \tau_y) \right]m^{-1}(\sigma - 1)(n^{-\alpha}(p_j^r + \tau^r) + \tau_y) \frac{\sigma}{p} (\sigma - 1)^{-\alpha}Y - nf_j \] (21)

### 2.3 The Intermediate Goods and the R&D Sectors

The typical intermediate firm produces its differentiated goods with a technology that requires one unit of labor per unit of intermediate goods \((x_i = l_i^x)\). Each intermediate goods firm produces and sells a slightly unique variety of goods \(x_i\) to each final goods firm to maximize its profit since the good is protected by an infinitely-lived patent, taking the actions of all other producers in the intermediate goods sector as given

\[
\pi_i = p_i^r x_i - w l_i^x + mf_j
\] (22)

where \(l_i^x\) is the amount of labor used by firm \(i\), and \(f_i\) is the franchise fee received from the final goods firm. Since a variety \(x_i\) is needed by the final goods firms and the production function of the final goods firms is \((y_j = n^a x_j)\), we have

\[ l_i^x = mn^{-\alpha} y_j \]

to substitute into Eq. (22). In addition, since \(y_j = c_j\), the profit function of intermediate goods in Eq. (22) may be re-presented as

\[
\pi_i = (p_i^r - w)mn^{-\alpha}m^{-1}(\sigma - 1)(n^{-\alpha}(p_j^r + \tau^r) + \tau_y) \frac{\sigma}{p} (\sigma - 1)^{-\alpha}Y + mf_j
\] (23)

R&D technology is such that, to develop a new idea, a researcher needs a quantity of labor to develop ideas. The production function in the R&D sector is given by

\[
\hat{n} = nL_A
\] (24)

where \(L_A\) is the amount of labor hired in the R&D sector which is from the R&D firms \((L_R)\) and the government sector \((L_G)\), \(L_A = L_R + L_G = vL_A + (1-v)L_G\), \(v\) is the proportion of labor employed in the R&D sector between the R&D firms and the government, \(\hat{n}\) is the number of new blueprints created for a given period of time, and \(n\) refers to the positive spillovers in the production of blueprints. The more workers the R&D sector employs or the more varieties of goods the intermediate goods market has, the more new blueprints are produced per unit of time.

The research sector’s after-subsidy profit flow is given by

\[
\pi_A = p_A\hat{n} - v(1-s)wL_A
\] (25)
where $s$ is a fraction of all research expenses paid by the government. Such a subsidy to R&D lowers the private cost. By substituting the production function, Eq. (24), into Eq. (25) and due to the property of perfect competition in the R&D sector ($\pi_A = 0$), the blueprint cost or value is as follows

$$p_A = \frac{v(1-s)w}{n}$$

Eq. (26) indicates that the value of the blueprint is equal to its cost.

Anyone can have free entry into the business of being an inventor as long as the R&D cost secures the net present value of the profit in intermediate goods, that is

$$p_A = \int_0^\infty \pi e^{-(\omega-1)} d\omega$$

(27)

Differentiating the free entry condition in Eq. (27) with respect to time, we obtain

$$r = \frac{\pi}{p_A} + \frac{\dot{p}_A}{p_A}$$

(28)

where $\pi$ is the profit flow given by Eq. (23). Eq. (28) is a non-arbitrage condition which states that the rate of return on bonds, $r$, equals the rate of return to investing in R&D. The R&D rate of return equals the profit rate, $\pi/p_A$, plus the rate of capital gain or loss, $\dot{p}_A/p_A$.

### 2.4 Decentralized Contract Bargaining

In this period, firm $j$ producing final goods and firm $i$ producing intermediate goods bargain over the franchising contract $(p^*, f)$ simultaneously.

The division of the rent between firm $j$ producing final goods and firm $i$ producing intermediate goods, using Eq. (21) and Eq. (23), is obtained by maximizing the following Nash product

$$\max_{p^*, f} N = (\Pi_j - \Pi^0_j)^{\theta} (\pi_i - \pi^0_i)^{1-\theta}$$

s.t.

$$\Pi_j = \left[ \frac{1}{\sigma - 1} \left(n^{1-a}(p^*_i + \tau^*_i) + \tau_j \right) \right] m^{-1} \left( \frac{\sigma - 1}{\sigma - 1} \left(n^{1-a}(p^*_i + \tau^*_i) + \tau_j \right) \right)^{-\sigma} Y - mf_j \quad (21)$$

$$\pi_i = (p^*_i - w)mm^{-a}m^{-1} \left( \frac{\sigma - 1}{\sigma - 1} \left(n^{1-a}(p^*_i + \tau^*_i) + \tau_j \right) \right)^{-\sigma} Y + mf_j \quad (23)$$

$\Pi^0_j$ is the profit of firm $j$ which is constant when the bargaining breaks down, namely, the minimum profit of the final goods firm. $\pi^0_i$ is the profit of firm $i$.

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5 We use Leibniz’s rule for the differentiation of a definite integral.
which is constant when the bargaining breaks down, namely, the minimum profit of the intermediate goods firm. That is to say, if the bargaining breaks down, the downstream and upstream firms will mark up their prices by marginal cost, respectively.\(^6\) \(\theta\) describes the bargaining power of firm \(j\) and lies in the interval \([0, 1]\). With \(\theta = 0\), the model indicates that the intermediate goods firm \(i\) has full bargaining power to decide the intermediate goods price completely. To keep the analysis simple, we assume an identical bargaining power for all final goods firms with decentralized status. The same is true for all of the intermediate goods firms.

The decentralized bargaining means that there is no coordination between different bargaining units. All bargains take place simultaneously and the bargaining partners take all other intermediate goods prices and franchise fees as given.

According to the Nash bargaining solutions that are derived by maximizing Eq. (29), firm \(j\) and firm \(i\) select an optimal franchise fee and intermediate price as follows

\[
p^* = w \\
\pi^* = \frac{\theta \Pi^0}{m} + \frac{(1-\theta)}{n} \times \left[ \frac{1}{\sigma - 1} \left( (w + \tau') + \tau'h^{\alpha - 1} \right) \right]n^{1-\alpha}Y - (1-\theta)\frac{\Pi^0}{n}
\]

Eqs. (30) and (31) describe the optimal bargaining contract in a vertically connected imperfectly competitive market structure. Eq. (30) is the pricing rule for intermediate goods, resulting from competition between the final goods firm and the intermediate goods firm, with both firms simultaneously engaging in optimization. The bargaining contract in our model is unlike the traditional franchise contract, in which the final goods firm does not have any bargaining power to determine the contract’s content. The prices of the intermediate goods are equal to marginal cost which is unrelated to the bargaining power. This is an efficient result. Because the aggregate rent/franchise fee is maximized by setting the prices of the intermediate goods equal to their marginal cost, this result is interpreted as stemming from the negotiations between the intermediate goods firm and the final goods firm or the competition between the upstream and downstream industries. They obtain the maximum aggregate rent at first and then extract the extra rent, respectively, according to their bargaining power through the franchise fee. This result, which characterizes the interaction of firms in this market structure, reflects the economic consequence that double marginalization does not occur. This is a vertical integration outcome through franchise contract bargaining. Unlike traditional models, in this paper the prices of intermediate goods are determined by negotiation, the intermediate goods firms charge a price based on marginal cost and not on markup to the final goods firms and then extract the profit through the franchise fee (Eq. (31)). Since vertical integration takes place, an

\(^6\) The \(\Pi^0\) and \(\pi^0\) are constant here.
inelasticity demand function of intermediate goods appears. Inside the square brackets on the right-hand side of Eq. (31) is the corporate income of firm \( j \) per unit of final good. The optimal franchise fee depends on the bargaining power \( \theta \). Firm \( i \) will extract all the rent if firm \( j \) has no bargaining power (\( \theta = 0 \)). Similarly, the rent will vanish if firm \( i \) has no bargaining power (\( \theta = 1 \)).

Substituting Eq. (30) into the consumption goods price, Eq. (20), is given respectively as

\[
p_j = \frac{\sigma}{\sigma-1} \left[ ((w + \tau') + \tau_j n^{\alpha-1}) n^{1-\alpha} \right]
\]

Then, by substituting the results, Eqs. (30)-(32), into Eqs. (21) and (23), the profits can be written as

\[
\Pi_j = \frac{\theta}{m} \frac{1}{\sigma-1} ( (w + \tau') + \tau_j n^{\alpha-1} ) n^{1-\alpha} Y
\]

\[
\pi_i = \frac{1-\theta}{n} \frac{1}{\sigma-1} ( (w + \tau') + \tau_j n^{\alpha-1} ) n^{1-\alpha} Y
\]

where the subscript 0 denotes the value of the bargaining breakdown. If firm \( j \) is weaker than firm \( i \) in terms of the bargaining power of the franchising contract, the rent will be distributed more to the patent-holder in the intermediate goods market.

Since we would like to analyze the global economy, we assume that the numbers of firms in the intermediate goods and final goods markets are the same for the economics of the bargaining and the breakdown in negotiations, \( m_0 = m \), \( n_0 = n \). Furthermore, the R&D activities take place in the first period of the game structure, namely, the government’s expenditure on R&D activities takes place first. We then assume that \( s_0 = s \), \( \tau_0 = \tau' \), \( \tau_{j0} = \tau_j \). Therefore, Eqs. (33) and (34) are rewritten as

\[
\Pi = \left[ \theta Y - (\theta - \frac{(1-\theta)\sigma}{\sigma-1}) Y_0 \right] \frac{1}{m} \frac{1}{\sigma-1} ( (w + \tau') + \tau_j n^{\alpha-1} ) n^{1-\alpha}
\]

\[
\pi = \left[ (1-\theta)Y + (\theta - \frac{(1-\theta)\sigma}{\sigma-1}) Y_0 \right] \frac{1}{n} \frac{1}{\sigma-1} ( (w + \tau') + \tau_j n^{\alpha-1} ) n^{1-\alpha}
\]

Moreover, by substituting Eq. (32) into the aggregate consumption price index, Eq. (12), is given respectively as

\[
P = \frac{\sigma}{\sigma-1} ( (w + \tau') + \tau_j n^{\alpha-1} ) n^{1-\alpha}
\]
2.5 Government

The government cannot borrow and thus satisfies the budget constraint

\[ m \tau, n^{a-1} nx + \tau' mnx = S + G = vswL_A + (1 - v)wL_A \]  (36)

where \( m \tau, n^{a-1} nx + \tau' mnx \) is total tax revenues, \( S = vswL_A \) is the subsidy to defray the R&D cost of the firms, and \( G = (1 - v)wL_A \) is government expenditure to employ labor in the R&D sector. In considering the decomposition of government expenditures from the upstream and downstream industries, \( \tau' mnx \), and \( m \tau, n^{a-1} nx \), we assume

\[ m \tau, n^{a-1} nx = g(1 - v)(1 - s))wL_A \]  (36a)

\[ \tau' mnx = (1 - g)(1 - v)(1 - s))wL_A \]  (36b)

where the parameter \( 0 < g < 1 \) is the share of government expenditure financed by tax revenues from the final goods market and \( 1 - g \) is the share of the government expenditure financed by tax revenues from the intermediate goods market. We consider that the parameters \((g, s, v)\) are fixed and the vector of tax rates must adjust endogenously. This will allow our results to easily show how the government’s R&D subsidy policy and the government’s R&D activities affect the dynamics of growth.

3. Market Equilibrium with a Fixed Subsidy Rate

To determine the aggregate dynamics of this economy, we impose two conditions: labor market clearing and the final goods market. The labor market equilibrium condition states that total labor demand is equal to total labor supply, i.e., the optimal allocation of the given supply of labor \( L \) to the three sectors, \( L_s + L_G + L_R = L \), and that labor is perfectly mobile across the intermediate goods sector and the blueprint industry. Since the quantity of labor allocated to the intermediate goods sector is \( L_s = mn^s \) and that allocated to the R&D industry is \( L_A = n/n \), the labor market equilibrium condition will be rewritten as

\[ L_s + \frac{n}{n} = L \]  (37)

Next, by combining Eq. (3) with \( c = y \) and \( x = l^x \), and considering the clearing condition for the final goods market in the symmetric equilibrium, we have

\[ C = Y = n^{a-1} L_s \]  (38)
3.1 General Equilibrium Dynamics

The macroeconomic model is represented by

\[
\frac{\dot{C}}{C} = r - \rho - \frac{\dot{p}}{P} \tag{9}
\]

\[
P = \frac{\sigma}{\sigma - 1} \left( (w + \tau^s) + \tau, n^{a-1} \right) n^{1-a} \tag{35}
\]

\[
\Pi = \left[ \theta Y - (\theta - \frac{1-\theta}{\sigma}) Y_0 \right] \frac{1}{m} \left[ \frac{1}{\sigma - 1} \left( (w + \tau^s) + \tau, n^{a-1} \right) n^{1-a} \right] \tag{33a}
\]

\[
\pi = \left[ (1-\theta) Y + (\theta - \frac{1-\theta}{\sigma}) Y_0 \right] \frac{1}{n} \left[ \frac{1}{\sigma - 1} \left( (w + \tau^s) + \tau, n^{a-1} \right) n^{1-a} \right] \tag{34a}
\]

\[
p_A = \frac{v(1-s)w}{n} \tag{26}
\]

\[
r = \frac{\pi}{P_A} + \frac{\dot{p}_A}{P_A} \tag{28}
\]

\[
m \tau, n^{a-1} nx + \tau^s mnx = (1 - v(1-s))wL_R \tag{36}
\]

\[
m \tau, n^{a-1} nx = g(1 - v(1-s))wL_A \tag{36a}
\]

\[
\tau^s mnx = (1 - g)(1 - v(1-s))wL_A \tag{36b}
\]

\[
L_n + \frac{\dot{n}}{n} = L \tag{37}
\]

\[
C = Y = n^{a-1} L_s \tag{38}
\]

Assume that a vector of tax rates \((\tau_y, \tau^s)\) is endogenous. Using Eq. (38) and the labor market equilibrium condition \((L_A = L - L_n)\), Eqs. (36a) and (36b) may be rewritten as

\[
\tau_y = \frac{g(1 - v(1-s))w(L - L_n)}{n^{a-1} L_s} \tag{36c}
\]

\[
\tau^s = \frac{(1 - g)(1 - v(1-s))w(L - L_n)}{L_s} \tag{36d}
\]

Substituting Eqs. (36c) and (36d) into Eq. (35), we obtain

\[
P = \frac{\sigma}{\sigma - 1} \left( L_n + (1 - v(1-s))(L - L_n) \right) n^{1-a} w \tag{39}
\]

By multiplying Eq. (39) by \(v(1-s)/n\), we obtain

\[\text{Eq. (38) is the resource constraint of the economy (see Appendix A).}\]
\[ p_A = \frac{v(1-s)\sigma-1}{n} \frac{L_s n^{\alpha-1}}{L_s + (1 - v(1-s))(L - L_s)} P \]  

(40)

Differentiating Eq. (40) with respect to time

\[ \frac{\dot{p}_A}{p_A} = (\alpha - 2) \frac{\dot{n}}{n} + \frac{\dot{L}_x}{L_x} + \frac{\dot{P}}{P} - \frac{v(1-s)\dot{L}_x}{L_s + (1 - v(1-s))(L - L_s)} \]  

(41)

Substituting Eqs. (35), (36c), (36d) and (38) into Eq. (34) and dividing by Eq. (26), we obtain

\[ \frac{\pi}{p_A} = \frac{1}{v(1-s)\sigma} \left[ (1 - \theta)L_x + (\theta - (1 - \theta)\sigma) L_x^0 \right] \frac{L_s + (1 - v(1-s))(L - L_s)}{L_s} \]  

(42)

where \( L_x^0 = \frac{v(1-s)\sigma-1}{v(1-s)\sigma} \rho \) is the quantity of labor employed in the intermediate goods market in a successively imperfectly competitive economy.

Substituting Eqs. (41), (42a) and (28) into Eq. (9), we obtain

\[ \frac{\dot{C}}{C} = \frac{1 - \theta}{\sigma - 1} L_x + \frac{(1 - \theta)(1 - v(1-s))}{(\sigma - 1)v(1-s)} L_s \frac{1}{\sigma - 1} (\theta - (1 - \theta)\sigma) L_x^0 L \]

\[ + \frac{1 - v(1-s)}{(\sigma - 1)v(1-s)} (\theta - (1 - \theta)\sigma) L_x^0 L \]

\[ + (\alpha - 2) \frac{\dot{n}}{n} + \frac{\dot{L}_x}{L_x} - \frac{v(1-s)\dot{L}_x}{L_s + (1 - v(1-s))(L + v(1-s)L_s)} - \rho \]

(43)

Differentiating Eq. (38) with respect to time

\[ \frac{\dot{C}}{C} = (\alpha - 1) \frac{\dot{n}}{n} + \frac{\dot{L}_x}{L_x} \]  

(44)

**Proposition 1.** There is a necessary and sufficient condition that leads the economy to indeterminacy. A high equilibrium and a low equilibrium in the economy will take place.

From Eqs. (43), (44) and (37), we find the dynamic equation for \( L_x \)

\[ \frac{(\sigma - 1)L_x v(1-s)\dot{L}_x}{(1 - v(1-s))L + v(1-s)L_s} \]

\[ = (\sigma - 1) L_x^2 + \frac{(1 - \theta)(\sigma - \theta)v(1-s)}{v(1-s)} L_s + (\theta - (1 - \theta)\sigma) L_x^0 L \]

\[ + \frac{1 - v(1-s)}{v(1-s)} (\theta - (1 - \theta)\sigma) L_x^0 L \]  

(45)

Assume that \( \Omega = \frac{(1 - \theta)(\sigma - \theta)v(1-s)}{v(1-s)} L_s + (\theta - (1 - \theta)\sigma) L_x^0 L \) , and
\[
\Gamma = \frac{1 - v (1 - s)}{v (1 - s)} (\theta - \frac{(1 - \theta) \sigma}{\sigma - 1}) L_x^0 L.
\]

In the steady state \( \dot{L}_x = 0 \), we obtain
\[
L_x = \frac{-\Omega \pm \sqrt{\Omega^2 - 4(\sigma - \theta)\Gamma}}{2(\sigma - \theta)}
\]

Eq. (46) indicates that the economy exhibits an indeterminate solution if \( \Omega < 0 \), \( \Gamma > 0 \). The conditions are as follows
\[
\left(1 - \theta\right) - \left(\sigma - \theta\right) \frac{v (1 - s)}{v (1 - s)} L + \left(\theta - \frac{(1 - \theta) \sigma}{\sigma - 1}\right) L_x^0 \left(\sigma - 1\right) \rho < 0
\]
\[
\frac{1 - v (1 - s)}{v (1 - s)} \left(\theta - \frac{(1 - \theta) \sigma}{\sigma - 1}\right) L_x^0 L > 0
\]

According to Eq. (45), the first-order condition and second-order condition are as follows
\[
\frac{\partial \dot{L}_x}{\partial L_x} = 2(\sigma - \theta) L_x + \left[\left(1 - \theta\right) - \left(\sigma - \theta\right) \frac{v (1 - s)}{v (1 - s)} L + \left(\theta - \frac{(1 - \theta) \sigma}{\sigma - 1}\right) L_x^0 \left(\sigma - 1\right) \rho\right] > 0
\]
\[
\frac{\partial^2 \dot{L}_x}{\partial L_x^2} = 2(\sigma - \theta) > 0
\]

Eqs. (49) and (50) imply that there are two equilibria for \( L_x \) in the steady state. One is stable, namely, the low equilibrium (\( \bar{L}_x \)), and the other is unstable, namely, the high equilibrium (\( \bar{L}_x \)).

### 3.2 Steady State Analysis

In the generalized case, the economy is under indeterminate equilibrium, that is to say, the conditions, Eqs. (47) and (48), are satisfied. In the steady state \( \dot{L}_x = 0 \), and by totally differentiating Eq. (45), the results of the comparative static state are as follows
\[
\frac{\partial L_x}{\partial \theta} = -\frac{1}{2(\sigma - \theta) L_x + \Omega} \times \left[ L_x + \frac{1 - v (1 - s)}{v (1 - s)} L \right] \left[ L_x - \left(1 + \frac{\sigma}{\sigma - 1}\right) L_x^0\right] > 0
\]
\[
\frac{\partial L_x}{\partial s} = \frac{-1}{2(\sigma - \theta) L_x + \Omega} \times \left(\frac{1 - \theta}{v (1 - s)^2}\right) \times
\]
\[
\left[ L_x - \left(\frac{\theta}{1 - \theta} - \frac{\sigma}{\sigma - 1}\right) L_x^0 - \left(\frac{\theta}{1 - \theta} - \frac{\sigma}{\sigma - 1}\right) \left(1 - \frac{v (1 - s)}{v (1 - s) \sigma} L - L_x^0\right)\right] > 0
\]
\[
\frac{\partial L_s}{\partial \nu} = \frac{1}{2(\sigma - \theta) L_x + \Omega} \times \frac{(1 - \theta)L}{v^2(1 - s)} \times \\
\left[ L_x - \left( \frac{\theta}{1 - \theta} - \frac{\sigma}{\sigma - 1} \right) \frac{1}{\sigma} L_x - \left( \frac{\theta}{1 - \theta} - \frac{\sigma}{\sigma - 1} \right) \left( \frac{1 - v(1 - s)}{v(1 - s)\sigma} L - L_x^0 \right) \right] > 0
\]

(52)

\[
\frac{\partial L_s}{\partial \sigma} = \frac{1}{2(\sigma - \theta) L_x + \Omega} \times \\
\left[ (L - L_x + \rho) L_x - \frac{1 - \theta}{\sigma - 1} (L_x + \frac{1 - v(1 - s)}{v(1 - s)} L)L_x^0 \right] > 0
\]

(53)

\[
\left[ - \frac{1 - \theta}{\sigma - 1} (\frac{\theta}{1 - \theta} - \frac{\sigma}{\sigma - 1}) (\frac{1}{v(1 - s)} L + \rho) (L_x + \frac{1 - v(1 - s)}{v(1 - s)} L) \right] < 0
\]

(54)

From Eq. (37), in the steady state the growth rates of innovation depend on the state of \( L_x \) such that

\[
\check{\gamma}_n = L - \check{L}_x
\]

(55)

otherwise

\[
\hat{\gamma}_n = L - \hat{L}_x
\]

(56)

where \( \gamma_n = \dot{n}/n \). Eq. (55) denotes the low balanced equilibrium growth rate of innovation and Eq. (56) the high balanced equilibrium growth rate of innovation. Hence, the effects of the fiscal and exogenous variables on the balanced equilibrium growth rate depend on Eqs. (51)-(54). In addition, the growth rate depends on the size of the population; this is the well-known, although controversial, scale effect of R&D-based endogenous growth models. \(^8\)

**Proposition 2.** The fiscal policies of the government, direct expenditure on R&D activities and subsidies to defray the R&D costs of firms, have entirely different effects on the economy according to whether there is a high balanced growth path or a low balanced growth path. In addition, the bargaining power between the firms producing intermediate goods and final goods also has reverse effects on the economy.

When the economy is in a low equilibrium where labor is employed by intermediate goods firms, \( \check{L}_x \), the denominator of the first fraction on the right-hand

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\(^8\) Jones (1995).
side of Eqs. (51)-(54), is negative, i.e., \( 2(\sigma - \theta)L_x + \Omega < 0 \). On the other hand, when the economy is in a high equilibrium where labor is employed by intermediate goods firms, \( \bar{L}_x \), the denominator of the first fraction on the right-hand side of Eqs. (51)-(54) is positive, i.e., \( 2(\sigma - \theta)L_x + \Omega > 0 \). Hence, it is well known that the effects of the exogenous parameters are entirely reversed between the high balanced growth path economy and low balanced growth path economy, regardless of whether the values inside the square brackets are positive or negative.

4. Special Case
In order to obtain definite results, in regard to Eqs. (51)-(53), we make some legitimate assumption as follows

\[
L_x < (1 + \frac{\sigma}{\sigma - 1})L_x^0 \tag{57}
\]

\[
\frac{1 - v(1 - s)}{v(1 - s)\sigma} < \frac{L_x^0}{L} \tag{58}
\]

Eq. (57) assumes that after bargaining the quantity of labor employed by intermediate goods firms \( (L_x) \) will not be more than double the quantity of labor employed when bargaining breaks down \( (L_x^0) \).\(^9\) Besides, Eq. (58) assumes that the ratio of labor employed by intermediate goods firms when bargaining breaks down \( (L_x^0/L) \) is more than just some rate.\(^10\)

According to Eqs. (57) and (58), we obtain definite results which are interpreted in the following sections.

4.1 High Balanced Growth Path Economy
When the economy is in a low equilibrium where labor is employed by intermediate goods firms, \( \bar{L}_x \), the denominator of the first fraction on the right-hand side of Eqs. (51)-(53) is negative, i.e., \( 2(\sigma - \theta)L_x + \Omega < 0 \). At this time, the economy has a high balanced growth path \( (\bar{\gamma}_x) \). Therefore, the effects of the parameters on the economic growth are as follows

\[
\frac{\partial \bar{\gamma}_x}{\partial \theta} = -\frac{\partial \bar{L}_x}{\partial \theta} < 0 \tag{59}
\]

\(^9\) If \( \sigma = 12 \) (Wang and Wen, 2008), we obtain \( L_x < 2.1L_x^0 \).

\(^10\) If \( s = 0.15 \) (Sener, 2008), \( v = 0.5 \), and we obtain \( 0.11 < \frac{L_x^0}{L} \) which is feasible for our study.
\[ \frac{\partial \hat{\gamma}_u}{\partial s} = -\frac{\partial \hat{L}_u}{\partial s} < 0 \] (60)

\[ \frac{\partial \hat{\gamma}_u}{\partial v} = -\frac{\partial \hat{L}_u}{\partial v} > 0 \] (61)

Eq. (59) illustrates that increasing the bargaining power of intermediate goods firms will increase the high balanced growth rate of innovation. That is to say, the intermediate goods firm plays an important role in a high balanced growth path economy in boosting the rate of economic growth. In addition, Eq. (60) indicates that the government’s policy of subsidizing the R&D cost of the firms has a negative effect on the high balanced growth rate. In other words, the government raises the rate of the subsidy which will cause the growth rate to slow down. Moreover, if the government directly increases the expenditure on the R&D activities, it will decrease the growth rate, too (Eq. (61)). Therefore, the government’s expenditure on R&D will crowd out the private R&D activities.

4.2 Low Balanced Growth Path Economy

When the economy is in a high equilibrium where labor is employed by intermediate goods firms, \( \hat{L}_s \), the denominator of the first fraction on the right-hand side of Eqs. (51)-(53) is positive, \( 2(\sigma - \theta)\hat{L}_s + \Omega > 0 \). At this time, the economy is on a low balanced growth path (\( \gamma_n \)). Therefore, the effects of the parameters on the economic growth are as follows

\[ \frac{\partial \hat{\gamma}_u}{\partial \theta} = -\frac{\partial \hat{L}_u}{\partial \theta} > 0 \] (62)

\[ \frac{\partial \hat{\gamma}_u}{\partial s} = -\frac{\partial \hat{L}_u}{\partial s} > 0 \] (63)

\[ \frac{\partial \hat{\gamma}_u}{\partial v} = -\frac{\partial \hat{L}_u}{\partial v} < 0 \] (64)

Eq. (62) illustrates that an increase in the bargaining power of final goods firms will increase the low balanced growth rate of innovation. That is to say, the final goods firms play an important role in a low balanced growth path economy to enhance the rate of economic growth. In addition, Eq. (63) indicates that the effect of a government’s subsidy policy on the R&D cost of the firms in a low balanced growth path economy is positive. In other words, a government that raises the ratio of the subsidy to the R&D cost of the firms will cause the growth rate to speed up. Moreover, if the government directly increases its expenditure on R&D activities, it will increase the growth rate, too (Eq. (64)). Hence, the government plays an important role in enhancing the rate of economic growth, and the policies on R&D activities are helpful
5. Conclusion

In this paper, we take a new look at the effects of a government’s subsidy policy and its R&D activities in an R&D-based growth model. The government not only subsidizes the R&D cost of the firms but also engages in R&D activities, and a specific tax is levied on the firms that produce the final goods and intermediate goods to finance the expenditure.

We find that the economy is characterized by two balanced equilibrium growth rates which comprise a high balanced growth equilibrium and a low balanced growth equilibrium. In a high growth rate economy the government’s subsidy policy and the R&D activities will crowd out the private R&D activities, and hence the fiscal policy is of no help to the economic growth. In other words, the intermediate goods firms play an important role in driving the economic growth, and the stronger the bargaining power of the intermediate goods firms is, the more the economy grows. On the contrary, in a low growth path economy the government that directly engages in R&D activities plays an important role in economic growth. The fiscal policies of the government have a positive effect on the economic growth.

This paper finds evidence of entirely different effects on a high growth rate economy and a low growth rate economy. In different economies, the government and the firms that manufacture intermediate goods and final goods play different roles in the process of economic growth.

Appendix A

From household’s budget constraint

\[ p_A \dot{n} + n \dot{p}_A = rp_A n + wL + m\Pi - PC \]  
(A.1)

Substituting the zero profit condition: \( p_A \dot{n} = v(1-s)wL_A \), labor market equilibrium:

\[ L_x + L_A = L \], and non-arbitrage condition: \( r = \frac{\pi}{p_A} + \frac{\dot{p}_A}{p_A} \) into Eq. (A.1)

\[ v(1-s)wL_A + n \dot{p}_A = (\frac{\pi}{p_A} + \frac{\dot{p}_A}{p_A}) p_A n + w(L_x + L_A) + m\Pi - PC \]  
(A.2)

Rewriting Eq. (A.2) to

\[ v(1-s)wL_A = n \pi + w(L_x + L_A) + m\Pi - PC \]  
(A.3)

Substituting \( \pi \) and \( \Pi \), namely, Eqs. (33a) and (34a), into Eq. (A.3), we obtain

\[ v(1-s)wL_A = Y \frac{1}{\sigma - 1} ((w + \tau^*) + \tau, n^{\alpha \tau})^{1-\alpha} + w(L_x + L_A) - PC \]  
(A.4)

Since the taxes are endogenous, substituting the government budget constraint
\[ \tau_y = \frac{g(1-v(1-s))wL_A}{n^{\alpha-1}L_x}, \text{ and } \tau^* = \frac{(1-g)(1-v(1-s))wL_A}{L_x} \] into Eq. (A.4), we obtain
\[ v(1-s)wL_A = Y \frac{1}{\sigma-1} \left( \frac{L_x + (1-v(1-s))L_A}{L_x} \right) n^{1-\alpha} + w(L_x + L_A) - PC \] (A.5)

Owing to \( Y = n^{\alpha-1}L_x \),
\[ v(1-s)wL_A = \frac{1}{\sigma-1} \left( n^{1-\alpha}Y + (1-v(1-s))L_A \right) w + w(n^{1-\alpha}Y + L_A) - PC \] (A.6)

Rewriting to
\[ v(1-s)wL_A = \frac{\sigma}{\sigma-1} n^{1-\alpha}wY + \frac{1}{\sigma-1} (1-v(1-s))wL_A + wL_A - PC \] (A.7)

Substituting \( P = \frac{\sigma}{\sigma-1} \frac{L_x + (1-v(1-s))wL_A}{L_x} n^{1-\alpha}w \) into (A.7), we obtain
\[ 0 = \frac{\sigma}{\sigma-1} n^{1-\alpha}wY - \frac{\sigma}{\sigma-1} n^{1-\alpha}wC \] (A.8)

Hence, we obtain the resource constraint as follows
\[ Y = C \] (A.9)

References
Delipalla, S., and M. Keen (1992) The comparison between ad valorem and specific


