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**RENEGOTIATION AND RELATIVE PERFORMANCE EVALUATION:  
WHY AN INFORMATIVE SIGNAL MAY BE USELESS**

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**KEYWORDS:** Informativeness, Monitoring, Renegotiation, Principal-Agent Model.

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## RENEGOTIATION AND RELATIVE PERFORMANCE EVALUATION:

### WHY AN INFORMATIVE SIGNAL MAY BE USELESS

Holmström's (1979) *informativeness criterion* is a seminal result in agency theory.<sup>1</sup> It tells us that a signal is useful in incentive contracting if and only if it is *informative*, i.e., conveys information not already included in the outcome of the action. This criterion has been applied to understand management control issues including the controllability principle and relative performance evaluation. Field studies, such as those by Merchant (1987) and Maher (1987), however, have not confirmed the consistent application of the informativeness criterion. Merchant found that while the informativeness criterion suggests the controllability principle, i.e., evaluation of a manager's performance should be based only on factors controllable by the manager, the principle is often disregarded in practice. As an application of the principle, interfirm relative performance evaluation (RPE) should be used to filter the common uncertainty faced by firms in the same industry. On the contrary, Maher found that the use of interfirm RPE is rare in practice.<sup>2</sup>

The goal of this study is to reconcile the discrepancy between the theory and practice of interfirm RPE based on a new development of agency theory. Recently, the theory is moving in the direction of understanding the effects of limited commitment by contracting parties.<sup>3</sup> Along these lines, this paper investigates the value of monitoring information in *renegotiable contracts*.<sup>4</sup> Allowing contracts to be renegotiable creates a cost of using monitoring information that was ignored in Holmström's (1979) analysis. When this cost is taken into account, the sufficiency part of the informativeness criterion no longer holds. Instead, whether a signal is useful in incentive contracting will depend on the quality of the signal's information. I show that a signal is useless if the information quality is sufficiently poor, and useful if sufficiently rich. In light of this result, violations of the controllability principle and the rare use of interfirm RPE are rationalizable. They can be understood as resulting from signals with sufficiently poor information despite their informativeness.

This paper synthesizes the models of Holmström (1979) and Fudenberg and Tirole (1990). My model differs from Fudenberg and Tirole's two-action, renegotiable contract model mainly in the placement of the renegotiation stage. They placed it between the agent's choice of action and the outcome. In contrast, renegotiation is allowed in my model only after the outcome is known and before a signal of the action

arrives. Since my focus is on the signal, this seems to be the most tractable way to start.

Additionally, one can argue that in reality opportunities for the contracting parties to renegotiate are limited. A natural point at which renegotiation issues are likely to arise is right after the principal learns some up-to-date information about the agent's performance. For example, a company usually holds only a small number of board-of-directors meetings each year to approve the most important decisions, which might include top managers' compensation packages. A typical moment to discuss this issue (and hence to *renegotiate*) is after the company's annual performance (which corresponds to an *outcome*) has been summarized in the annual report pending for approval. This moment often occurs earlier than knowing a competitor's performance (which corresponds to a *signal*) which can be found in its annual report.

Because of the timing of the renegotiation stage, the principal in my model cannot induce the *high* (i.e., more productive) action with certainty if she uses the signal's information to design a contract. This result is analogous to why any non-randomized action, other than the least productive one, is not implementable in Fudenberg and Tirole's model. If the high action in my model could be induced by a signal-contingent contract, the principal would be able to infer the action accurately in equilibrium. Then his only concern at the renegotiation stage would be to provide better insurance to the agent. As a result, the best replacement contract to be offered by the risk-neutral principal would have to be a full insurance contract. Foreseeing this, the agent would have benefitted from choosing the low action instead, contradicting the supposition that the high action and a signal-contingent contract could constitute an equilibrium.<sup>5</sup>

Since the principal cannot induce the high action with a signal-contingent contract, the only way to utilize the signal's information is to settle for a less productive, randomized action.<sup>6</sup> The implementation of a randomized action is essential because it will endow the agent with private information at the renegotiation stage. Consequently, risk sharing will no longer be the principal's only concern at that point. The information asymmetry created by a randomized action will provide an incentive for the principal to use the signal to induce the agent's disclosure of the private information about the action actually taken.<sup>7</sup>

This incompatibility between the high action and the use of the signal's information creates a cost of using the signal. The cost appears as a reduction in the principal's expected revenue due to inducing a randomized action. When the signal carries poor information, the benefit from using it to better motivate the agent will be small. As a result, the signal's benefit may not justify the cost of using it. Under such

circumstances, the principal would rationally ignore the signal in designing contracts despite its informativeness. This "insufficiency of informativeness" result is in sharp contrast to Holmström's informativeness criterion.

The rest of this paper is organized as follows. The formal structure of the model is introduced in Section 1. Preliminary results are derived in Section 2. The cost and benefit of using a signal's information for contracting are explained in Section 3. Section 4 presents the main results. Section 5 discusses how the results can reconcile the discrepancy between the theory and practice of interfirm RPE. Concluding remarks are given in Section 6, with mathematical proofs contained in the appendix.

## 1. Model Structure

In my model, a principal hires an agent to perform a task, which is accomplished by taking an action  $a$  in the feasible action set  $A \equiv \{L, H\}$ . The agent's choice of action can be governed by a probabilistic decision rule with the probability distributions representing such rules constituting the set of feasible randomized actions. Any randomized action is uniquely characterized by its probability of choosing  $H$ , denoted by  $\alpha$ , with  $\alpha = 1$  and  $\alpha = 0$  representing the choice of non-randomized action  $H$  and  $L$ , respectively.

The agent's action affects the probability distribution of a jointly observed outcome,  $x$ , which is interpreted as the principal's revenue. The set of possible outcome values is  $X \equiv \{x_1, x_2, \dots, x_M\}$  with  $x_1 < x_2 < \dots < x_M$  and  $M \geq 2$ . The parties also jointly observe a signal  $y$  of  $a$ . The set of possible signal values is  $Y \equiv \{B, G\}$ . Let  $\pi(x, y|a)$  denote the joint probability mass function of  $x$  and  $y$  given  $a$ . The marginal probability mass function of  $x$  given  $a$  is  $p(x|a) \equiv \sum_y \pi(x, y|a)$ , and the conditional probability mass function of  $y$  given  $x$  and  $a$  is  $q(y|x, a) \equiv \pi(x, y|a) / \sum_y \pi(x, y|a)$ .

The contracting parties have von-Neumann-Morgenstern preferences. The principal is assumed to be risk-neutral and his utility payoff is the revenue  $x$  less the compensation cost  $s$ . The agent's utility depends both on the compensation received and the action taken. Specifically, the agent is assumed risk-averse and work-averse with additively separable preferences:  $U(s) - D(a)$ , where  $U' > 0$ ,  $U'' < 0$ , and  $D(L) < D(H)$ . Let  $\Phi$  denote the inverse of the agent's utility-for-income function  $U$ .

ASSUMPTION 1: *The domain of the inverse utility-for-income function  $\Phi$  is  $(-\infty, \bar{u})$  with  $\lim_{u \rightarrow -\infty} \Phi'(u) = 0$  and  $\lim_{u \rightarrow \bar{u}} \Phi'(u) = \infty$ .*

Examples of utility functions whose inverses satisfy Assumption 1 are the logarithmic and negative exponential functions. The domain of  $\Phi$  is assumed unbounded from below to ensure that the individual rationality constraints in some programs studied here are always binding. The assumption on the limits guarantees that optimal solutions exist in such programs.

ASSUMPTION 2: (a)  $\sum_{x \leq \tilde{x}} p(x|L) > \sum_{x \leq \tilde{x}} p(x|H) \forall \tilde{x} \in X \setminus \{x_M\}$ ; (b)  $p(x|H) > 0 \forall x \in X$ .

Part (a) of this assumption says that a change from action  $L$  to  $H$  will reduce the chance of observing a smaller outcome value. In other words, the high action will generate a statistically larger outcome value, in the sense of first-order stochastic dominance. Part (b) rules out a shifting-support problem.

ASSUMPTION 3: *The likelihood ratio  $p(x|L)/p(x|H)$  is decreasing in  $x$ .*

This *monotone likelihood ratio condition* ensures that a larger outcome value is stronger evidence for the selection of the high action. It implies Assumption 2(a), but is needed only for the proof of Lemma 2 and Proposition 3.

ASSUMPTION 4: (a)  $q(B|x,L) > 1/2 > q(B|x,H) \forall x \in X$ ; (b)  $q(B|x,H) > 0 \forall x \in X$ .

This is the counterpart of Assumption 2 for the signal  $y$ . Because  $G$  is more likely to be observed with action  $H$  taken, the principal will tend to believe action  $H$  is the agent's choice when  $G$  is observed. Thus,  $G$  is called the *good* signal value; similarly,  $B$  is called the *bad* signal value.<sup>8</sup> Assumption 4 differs from Assumption 2 in that the probabilities  $q(B|x,L)$  and  $q(B|x,H)$  are required to be greater than and less than  $1/2$ , respectively, regardless of the outcome value. That is to say, the bad signal value will have more than a fifty-fifty chance of occurring if action  $L$  is taken, as will the good signal value if action  $H$  is taken. This seems to be a reasonable assumption, whose main function is to simplify the appearance, but not the essence, of the results derived below.

ASSUMPTION 5: *When only the outcome is used for contracting, it is suboptimal to induce the low action,  $L$ , which is nonetheless better than not to contract with the agent at all.*

The first part of this assumption, i.e., the suboptimality of the low action, is needed to derive the sufficiency part of the informativeness criterion. So it is assumed here to allow equal-footing comparisons. The second part of the assumption, i.e., the inferiority of no contracting to the implementation of the low action, is not critical but can simplify the exposition of the analysis.

ASSUMPTION 6: *The risk endurance  $\varphi = \Phi'/\Phi''$  of the agent is a weakly decreasing and concave function.*

This risk aversion measure defined with the inverse utility-for-income function  $\Phi$  is the counterpart of a more common measure, namely the *risk tolerance*:  $\tau = -U'/U''$ . Concavity of  $\varphi$  is equivalent to saying that the risk tolerance of the agent is a concave function.<sup>9</sup> Because  $\Phi'$  is an increasing function,  $\varphi$  being decreasing implies  $\Phi'$  is convex. This last condition, which has been used by Jewitt (1988) to justify the first-order approach, is essential to my analysis. Although  $\varphi' \leq 1$  is enough to guarantee the convexity of  $\Phi'$ , the stronger assumption used here allows some results to be presented in a simpler form.<sup>10</sup>

To conclude this section, let me specify the timing of the events in the model:

1. The principal offers the agent an initial contract  $C^1$ , which consists of possibly multiple compensation schemes from which the agent may pick one after the realization of the outcome.<sup>11</sup>
2. The agent either accepts or rejects  $C^1$ . In case of rejection, the game ends with the principal and the agent getting their reservation payoffs, 0 and  $\bar{V}$ , respectively.
3. If the agent accepts the contract, he will take an action  $a$  guided by the realization of a randomized action  $\alpha$  chosen by him. Subsequently, the outcome is known by both parties.
4. The renegotiation stage begins. The principal leads the take-it-or-leave-it bargaining game at this stage by proposing a replacement contract,  $C^2$ , to substitute for the initial contract. Like  $C^1$ , this proposal may consist of multiple compensation schemes from which the agent may pick one.
5. The agent ends the bargaining game by accepting or rejecting  $C^2$ . If the agent accepts this contract, it becomes the final contract; otherwise, the initial contract is the final contract.
6. The agent chooses a compensation scheme from the final contract. Once the signal value  $y$  is known, the agent will be paid according to the compensation scheme selected. The game then concludes.

The event sequence is summarized with the time line in Figure 1.

**Insert Figure 1 around here.**

In the next section, I will first present a full-commitment benchmark result, namely, the sufficiency of informativeness.<sup>12</sup> Then I will analyze the replacement contract design problem and derive the renegotiation-proofness constraints for later analysis.

## 2. Preliminaries

Because action  $L$  is the least costly action in my model, there is no incentive problem for inducing this action. It is straightforward to confirm that the optimal contract for inducing action  $L$  is a non-contingent contract with compensation  $U_L^* \equiv D(L) + \bar{V}$  in utility, whether full commitment is possible or whether the signal is used for contracting. The associated expected compensation cost equals  $\Phi_L \equiv \Phi(U_L^*)$ .

Let  $U_H^*(X, Y) \equiv (U_H^*(x, y))_{x \in X, y \in Y}$  denote the optimal full-commitment contract for inducing action  $H$  and  $U_H^*(X) \equiv (U_H^*(x))_{x \in X}$  the corresponding contract when only the outcome is used for contracting. The compensation costs associated with these contracts are  $\Phi_H \equiv \sum_x \sum_y \Phi(U_H^*(x, y)) \pi(x, y | H)$  and  $\Phi_H^X \equiv \sum_x \Phi(U_H^*(x)) p(x | H)$ . With full commitment, the only effect of using the signal is greater flexibility in designing contracts. Thus, the expected cost associated with  $U_H^*(X, Y)$  can never exceed that associated with  $U_H^*(X)$ , i.e.,  $\Phi_H \leq \Phi_H^X$ . Together with Assumption 5, this implies  $R_H - \Phi_H \geq R_H - \Phi_H^X > R_L - \Phi_L > 0$ , where  $R_a \equiv \sum_x xp(x | a)$  is the principal's expected revenue conditional on the agent's choice of action  $a$ . Therefore, action  $H$  is optimal in the full-commitment model with both the outcome and signal available for contracting.<sup>13</sup>

Given below is the condition characterizing the unique optimal full-commitment contract:

$$\Phi(U_H^*(x, y)) = \lambda + \mu [1 - \pi(x, y | L) / \pi(x, y | H)] \quad \forall x \forall y,$$

where  $\lambda$  and  $\mu$  are the multipliers for the *individual rationality* and *incentive compatibility* constraints. Because both constraints are binding at the optimum,  $\lambda$  and  $\mu$  are positive. This implies the optimal full-commitment contract will utilize the signal  $y$  if and only if the likelihood ratio  $\pi(x, y | L) / \pi(x, y | H)$  depends on  $y$  for some  $x$ . This condition on the ratio will be satisfied if  $y$  is *informative*, as defined below:

DEFINITION 1:  $y$  is *informative* about  $a$  given  $x$  if  $q(y | x, a)$  depends on  $a$  for some  $x$  and  $y$ .<sup>14</sup>

Since the optimal full-commitment contract is unique, a signal-independent contract cannot be optimal if the signal is informative. In other words, utilizing an informative signal will enhance contracting efficiency. This full-commitment benchmark result reiterates the sufficiency part of Holmström's (1979) informativeness criterion for the present setting with renegotiation considerations. Formally, the result is stated as the following proposition:

PROPOSITION 1: *With full commitment, the signal  $y$  is valuable if it is informative.*

In Section 4, I will present results showing that the *sufficiency of informativeness* established with full commitment does not extend to renegotiable contracts. This is due to the cost of using the signal's information arising from the renegotiation-proofness constraints derived next.

Because contracts are renegotiable, solving the contract design problem requires first solving the replacement contract design problem at the renegotiation stage. At this stage, the agent has private information, i.e., he knows which action he has taken, while the principal has a probabilistic belief about that choice. The action chosen becomes the agent's type, and the focus is now on risk sharing, not moral hazard. In particular, the principal's problem at this stage is to find a menu of replacement contracts that will efficiently share the agent's risk with respect to his compensation from the original contract. So with renegotiable contracts, an adverse selection problem is embedded in the original moral hazard problem.

In my model, renegotiation may occur only after the outcome  $x$  is known and has only one round. Therefore, I can apply the *revelation principle* (Myerson (1979) and Harris and Townsend (1981)) to confine the choice of an outcome-contingent final contract to the set of interim incentive-compatible contracts. A contract in this set is a pair of compensation schemes, one for each type of the agent, that provides no incentive for any type to select the scheme intended for another type. Specifically, a final contract has the following form:  $C(x) \equiv (U_H(x,y))_{y \in Y}, (U_L(x,y))_{y \in Y}$ , where  $U_a(x,y)$  is the compensation in utility received by the agent, when  $y$  is observed after his choice of the compensation scheme intended for type  $a$  and after the observation of  $x$ . By the *renegotiation-proofness principle* (Hart and Tirole (1988) and Fudenberg and Tirole (1990)), the choice of an initial contract can be confined to the set of renegotiation-proof contracts. This means only initial contracts of the form  $C(X) \equiv (C(x))_{x \in X}$  need to be considered. Because the initial contract

is signed before the outcome arrives, the contract cannot be a direct function of the realized  $x$ . Instead, it specifies for every  $x$  a final contract,  $C(x)$ , to offer at the renegotiation stage.

Given an initial contract  $C(X) \equiv ((U_H(x,y))_{y \in Y}, (U_L(x,y))_{y \in Y})_{x \in X}$ , the replacement contract design problem is represented by the following program:

$$\begin{aligned}
(\text{oRS-rp}) \quad & \text{Min}_{u_a(x,y) \forall y \forall a} \alpha(x) \sum_y \Phi(u_H(x,y))q(y|x,H) + (1-\alpha(x)) \sum_y \Phi(u_L(x,y))q(y|x,L) \\
\text{subject to} \quad & \\
\text{IIC}(H): \quad & \sum_y u_H(x,y)q(y|x,H) \geq \sum_y u_L(x,y)q(y|x,H) \\
\text{IIC}(L): \quad & \sum_y u_L(x,y)q(y|x,L) \geq \sum_y u_H(x,y)q(y|x,L) \\
\text{IIR}(H): \quad & \sum_y u_H(x,y)q(y|x,H) \geq \sum_y U_H(x,y)q(y|x,H) \\
\text{IIR}(L): \quad & \sum_y u_L(x,y)q(y|x,L) \geq \sum_y U_L(x,y)q(y|x,L),
\end{aligned}$$

where  $\alpha(x) \equiv \alpha p(x|H) / (\alpha p(x|H) + (1-\alpha)p(x|L))$  is the posterior probability revised from the prior probability  $\alpha$ , based on the  $x$  observed before the renegotiation. The constraints IIC are *interim incentive compatibility (IIC)* constraints which guarantee that no type can benefit from selecting a scheme intended for another type. The constraints IIR are *interim individual rationality (IIR)* constraints which ensure that each type can get at least his interim reservation payoff by accepting the proposed contract. This payoff is determined by what he obtains from the initial contract, i.e.,  $\sum_y U_a(x,y)q(y|x,a)$  for a type- $a$  agent, because the agent can always insist on the initial contract. As the action taken is irreversible, the disutility  $D(a)$  is not a consideration of the agent at the renegotiation stage and hence does not appear in the constraints here.

Recall that  $R_a \equiv \sum_x xp(x|a)$  is the principal's expected revenue, conditional on the agent's choice of action  $a$ . So her expected revenue from inducing randomized action  $\alpha$  is  $R(\alpha) \equiv \alpha R_H + (1-\alpha)R_L$ . Given that  $\alpha$  is induced, the principal's expected profit from proposing an IIC and IIR replacement contract,  $((u_a(x,y))_{y \in Y})_{a \in A})_{x \in X}$ , is  $R(\alpha) - [\alpha(x) \sum_y \Phi(u_H(x,y))q(y|x,H) + (1-\alpha(x)) \sum_y \Phi(u_L(x,y))q(y|x,L)]$ . Because  $R(\alpha)$  does not depend on her choice of a replacement contract, her objective at the renegotiation stage is to minimize the expected compensation cost, i.e.,  $\alpha(x) \sum_y \Phi(u_H(x,y))q(y|x,H) + (1-\alpha(x)) \sum_y \Phi(u_L(x,y))q(y|x,L)$ .

**DEFINITION 2:** An initial contract  $C(X)$  is a *renegotiation-proof* contract for randomized action  $\alpha$  if for every  $x \in X$ , the final contract  $C(x)$  specified in the initial contract is an optimal solution of the replacement

contract design problem given itself.

The following lemma completely characterizes a class of renegotiation-proof contracts. It has been proven that any other renegotiation-proof contracts are weakly dominated by some contracts in this class (see Yim (1995) for details). Thus, the search for an optimal renegotiation-proof contract can be confined to this class without loss of generality.

LEMMA 1: *Suppose  $C(X)$  is an initial contract comprising final contracts satisfying the following conditions: (i) upward-sloping-scheme (US):  $U_H(x,G) \geq U_H(x,B)$ , (ii) flat-scheme (FS):  $U_L(x,G) = U_L(x,B)$ , and (iii) same-expected-reward (SER):  $\sum_y U_H(x,y)q(y|x,L) = \sum_y U_L(x,y)q(y|x,L)$ . Then  $C(X)$  is a renegotiation-proof contract for randomized action  $\alpha$  if and only if the following no-net-gain (NNG) condition is fulfilled:*

$$(1-\alpha)\Phi'(\sum_y U_L(x,y)q(y|x,L))\delta(x)\rho(x) \geq \alpha[\Phi'(U_H(x,G)) - \Phi'(U_H(x,B))] \quad \forall x \in X,$$

where  $\delta(x) = q(B|x,L)/q(B|x,H) - q(G|x,L)/q(G|x,H)$  and  $\rho(x) = p(x|L)/p(x|H)$ .

To understand why the final contracts of an optimal contract should satisfy conditions US, FS, and SER, it is helpful to draw an analogy between the replacement contract design problem at the renegotiation stage and Stiglitz's (1977) insurance model. They are closely related because the objective of the replacement contract design problem is to provide better insurance to the different types of agent. Details of such an analogy can be found in Section 2B of Fudenberg and Tirole (1990).

The intuition behind condition NNG is as follows. If  $U_H(x,G)$  and  $U_H(x,B)$  are too far apart (i.e., the payoffs vary too much with  $y$ ), a high-action agent will be exposed to too much risk due to the signal's randomness. Consequently, there will be room for the principal to extract an insurance premium from the agent by providing him with more insurance. This can be done by reducing the sensitivity of the compensation scheme for a high-action agent to the signal. Specifically, the change involves an  $\varepsilon/q(G|x,H)$  reduction in  $U_H(x,G)$  and an  $\varepsilon/q(B|x,H)$  increment in  $U_H(x,B)$ , where  $\varepsilon > 0$ , such that the expected compensation to a high-action agent is left unchanged. Because  $q(G|x,H) > q(G|x,L)$ , the expected compensation to a low-action agent choosing the contract intended for a high-action agent will then increase by  $\delta(x)\varepsilon$ . To maintain the incentive compatibility of the contract, the expected compensation  $\sum_y U_L(x,y)q(y|x,L)$  to a low-action agent must be raised by the same amount. If the agent is of the high-action

type, the expected reduction in compensation cost from such marginal adjustments of the contract is  $(\Phi'(U_H(x,G)) - \Phi'(U_H(x,B)))\varepsilon$ . If the agent is of the low-action type, the expected increment in compensation cost is  $\Phi'(\sum_y U_L(x,y)q(y|x,L))\delta(x)\varepsilon$ . On outcome realization  $x$ , the probabilities that the agent is of the high-action type and that he is of the low-action type are  $\alpha(x)$  and  $1-\alpha(x)$ , respectively. Since  $\alpha(x) \equiv \alpha p(x|H)/(ap(x|H)+(1-\alpha)p(x|L))$ , the principal's expected net gain from adjusting the contract is  $[\alpha(\Phi'(U_H(x,G))-\Phi'(U_H(x,B))) - (1-\alpha)\Phi'(\sum_y U_L(x,y)q(y|x,L))\delta(x)\rho(x)] \times \varepsilon p(x|H)/(ap(x|H)+(1-\alpha)p(x|L))$ . This must be non-positive in order to ensure the initial contract's renegotiation-proofness. Such a *no-net-gain* requirement is condition NNG, as given in the lemma.

In the next section, I will state the initial contract design problem. It differs from that of a full-commitment model due to incorporation of the renegotiation-proofness constraints, i.e., conditions US, FS, SER, and NNG. I show that a necessary and sufficient condition for the signal to be useful is the suboptimality of the high action. The implications of this result for the cost and benefit of using the signal's information are then discussed.

### 3. Cost and Benefit of Using the Signal's Information

For the moment, suppose the principal wants to induce a genuine randomized action  $\alpha \in (0,1)$ . The optimal renegotiation-proof contract, hereafter the *optimal contract*, can then be identified by solving the following program:

$$\begin{aligned}
\text{(oRs)} \quad & \text{Min}_{U_a(x,y) \forall y \forall x \forall a} \alpha \sum_x \sum_y \Phi(U_H(x,y))\pi(x,y|H) + (1-\alpha) \sum_x \sum_y \Phi(U_L(x,y))\pi(x,y|L) \\
\text{subject to} \quad & \\
\text{SER:} \quad & \sum_y U_L(x,y)q(y|x,L) = \sum_y U_H(x,y)q(y|x,L) \quad \forall x \\
\text{FS:} \quad & U_L(x,G) = U_L(x,B) \quad \forall x \\
\text{US:} \quad & U_H(x,G) \geq U_H(x,B) \quad \forall x \\
\text{NNG:} \quad & (1-\alpha)\Phi'(\sum_y U_L(x,y)q(y|x,L))\delta(x)\rho(x) - \alpha[\Phi'(U_H(x,G))-\Phi'(U_H(x,B))] \geq 0 \quad \forall x \\
\text{AIC:} \quad & \sum_x \sum_y U_H(x,y)\pi(x,y|H) - D(H) = \sum_x \sum_y U_L(x,y)\pi(x,y|L) - D(L) \\
\text{AIR:} \quad & \sum_x \sum_y U_L(x,y)\pi(x,y|L) - D(L) \geq \bar{V},
\end{aligned}$$

where  $\pi(x,y|a) \equiv q(y|x,a)p(x|a)$ ,  $\delta(x) \equiv q(B|x,L)/q(B|x,H) - q(G|x,L)/q(G|x,H)$ , and  $\rho(x) \equiv p(x|L)/p(x|H)$ .

Constraint AIC is the *ex ante incentive compatibility* constraint ensuring that genuine randomization between

$H$  and  $L$  is not against the agent's interest. Note that the contract must give the agent at least his *ex ante* reservation payoff, or he will simply refuse the offer. Therefore, the *ex ante individual rationality* constraint

$$\alpha[\sum_x \sum_y U_H(x,y)\pi(x,y|H) - D(H)] + (1-\alpha)[\sum_x \sum_y U_L(x,y)\pi(x,y|L) - D(L)] \geq \bar{V}$$

should be included. This is equivalent to constraint AIR in the presence of AIC. When all the constraints are satisfied by a contract  $U_A(X,Y)$ , it is optimal for the agent to accept this contract and take the action guided by the randomization  $\alpha$ . Since  $\alpha$  is the probability of taking action  $H$ , the expected compensation cost to be minimized with the choice of an optimal contract is

$$\alpha \sum_x \sum_y \Phi(U_H(x,y))\pi(x,y|H) + (1-\alpha) \sum_x \sum_y \Phi(U_L(x,y))\pi(x,y|L).$$

Although  $\alpha \in (0,1)$  has been supposed, program (oRs) can also be used to identify optimal contracts for inducing degenerate randomized actions  $\alpha = 1$  and  $\alpha = 0$  (see Yim (1995) for details). Let  $\hat{U}_A^\alpha(X,Y) \equiv (((\hat{U}_a^\alpha(x,y))_{y \in Y})_{a \in A})_{x \in X}$  denote an optimal solution of program (oRs) for any given  $\alpha \in [0,1]$ . The minimized expected compensation cost, denoted by  $\hat{\Phi}(\alpha)$ , is the objective function value of the program evaluated at  $\hat{U}_A^\alpha(X,Y)$ . Because  $R(\alpha) \equiv \alpha R_H + (1-\alpha)R_L$  is the expected revenue from inducing randomized action  $\alpha$ , maximizing the principal's expected profit  $R(\alpha) - \hat{\Phi}(\alpha)$  yields the optimal randomized action.

Constraint NNG implies when  $\alpha = 1$ ,  $U_H(x,G) = U_H(x,B)$  for all  $x$ , i.e.,  $y$  is not used for contracting. Thus, if the signal is useful,  $\alpha = 1$  must be suboptimal. On the other hand, if  $\alpha = 1$  is suboptimal, some interior  $\alpha$  must be optimal, and the optimal contract must utilize the signal's information. Otherwise, such a signal-independent contract and the interior  $\alpha$  would be optimal in a full-commitment model in which only the outcome is available for contracting. This contradicts the fact that any interior  $\alpha$  is suboptimal in such a model, and therefore the suboptimality of the high action is a necessary and sufficient condition for the signal to be useful.

PROPOSITION 2: *The signal  $y$  is useful if and only if the degenerate randomized action  $\alpha = 1$ , i.e., action  $H$ , is suboptimal.*

This result implies that the net benefit of using the signal's information is tied to the net gain from inducing a genuine randomized action, as opposed to the non-randomized high action. With full commitment,  $\alpha = 1$  is optimal. Although this remains implementable with renegotiable contracts, the renegotiation-

proofness constraints rule out using the signal's information to induce  $\alpha = 1$ . To utilize the information, the principal must settle for some less productive, randomized action  $\alpha < 1$ . This will reduce her expected revenue from  $R(1)$  to some lower level  $R(\alpha) \equiv \alpha R_H + (1 - \alpha)R_L$ . In return, the benefit of using the signal is a lower expected compensation cost  $\hat{\Phi}(\alpha)$  as compared to  $\hat{\Phi}(1) \equiv \Phi_H^X \equiv \sum_x \Phi(U_H^*(x))p(x|H)$ , where  $U_H^*(X)$  is the optimal full-commitment contract for inducing action  $H$  with only the outcome available for contracting. The usefulness of the signal  $y$  is determined by the value of the expected net gain from inducing  $\alpha < 1$ , i.e.,  $(\Phi_H^X - \hat{\Phi}(\alpha)) - (R_H - R(\alpha))$ . If this is positive for some  $\alpha$ , then  $\alpha = 1$  is suboptimal and the signal is useful; otherwise, the signal is useless for contracting.<sup>15</sup>

Figure 2 illustrates the cost and benefit comparison that determines the signal's usefulness. The line  $\alpha\Phi_H^X + (1 - \alpha)\Phi_L$  represents the expected compensation cost resulting from the principal's randomization over inducing action  $H$  with probability  $\alpha$  and action  $L$  with probability  $1 - \alpha$ , given that full commitment is possible and both the outcome and the signal are used for contracting. When only the outcome is used for contracting under full commitment, the expected compensation cost is represented by the line  $\alpha\Phi_H^X + (1 - \alpha)\Phi_L$ . This line is above  $\alpha\Phi_H + (1 - \alpha)\Phi_L$  for all  $\alpha > 0$  because the signal's informativeness implies  $\Phi_H^X > \Phi_H$ .

Since  $\alpha\Phi_H + (1 - \alpha)\Phi_L$  is attained with unrestricted use of the signal's information, it provides a lower bound to the minimum expected compensation cost achievable with a renegotiation-proof contract, i.e.,  $\hat{\Phi}(\alpha)$ . In general, it is not clear which of  $\hat{\Phi}(\alpha)$  and  $\alpha\Phi_H^X + (1 - \alpha)\Phi_L$  is lower.  $\hat{\Phi}(\alpha)$  utilizes the signal's information, but  $\alpha\Phi_H^X + (1 - \alpha)\Phi_L$  has the benefit of having the risk-neutral principal perform the randomization. Nevertheless, it is certain that when  $\alpha = 0$  or  $1$ , neither has any advantage over the other, and hence they are equal at these end points.

**Insert Figure 2 around here.**

In the figure, there is a dashed line passing through  $\hat{\Phi}(1) \equiv \Phi_H^X$  and parallel to the expected revenue line  $R(\alpha) \equiv \alpha R_H + (1 - \alpha)R_L$ . The height of this dashed line represents the value  $\Phi_H^X - (R_H - R(\alpha))$ . Therefore, the vertical difference between the dashed line and the line representing  $\hat{\Phi}(\alpha)$  is the expected net gain  $(\Phi_H^X - \hat{\Phi}(\alpha)) - (R_H - R(\alpha))$  resulting from inducing  $\alpha < 1$  instead of  $\alpha = 1$ . The figure illustrates the situation in

which some positive expected net gains exist. These gains are represented by the shaded area.<sup>16</sup> When such an area exists,  $\alpha = 1$  is suboptimal and the signal is useful.

Because  $\hat{\Phi}(\alpha)$  is bounded from below by  $\alpha\Phi_H + (1-\alpha)\Phi_L$ , the saving in expected compensation cost from inducing some  $\alpha < 1$ , i.e.,  $\Phi_H^X - \hat{\Phi}(\alpha)$ , can never exceed  $\alpha(\Phi_H^X - \Phi_H) + (1-\alpha)(\Phi_H^X - \Phi_L)$ . This means the expected net gain satisfies:

$$\begin{aligned} (\Phi_H^X - \hat{\Phi}(\alpha)) - (R_H - R(\alpha)) &\leq \alpha(\Phi_H^X - \Phi_H) + (1-\alpha)(\Phi_H^X - \Phi_L) - (R_H - R(\alpha)) \\ &= \alpha(\Phi_H^X - \Phi_H) - (1-\alpha)[(R_H - R_L) - (\Phi_H^X - \Phi_L)]. \end{aligned}$$

Note that this upper bound on the expected net gain is negative for all  $\alpha$ 's lower than

$$\alpha_0 \equiv [(R_H - R_L) - (\Phi_H^X - \Phi_L)] / [(R_H - R_L) - (\Phi_H - \Phi_L)].$$

Therefore, any gainful randomized action, if it exists, has to be greater than this cutoff point. In the figure, this is the intersection of the dashed line and the line representing  $\alpha\Phi_H + (1-\alpha)\Phi_L$ .

When the signal carries very poor information about the action,  $q(y|x,a)$  will be nearly independent of  $a$ . As a result,  $U_H^*(X,Y)$  will be almost identical to  $U_H^*(X)$ , and  $\Phi_H$  will move towards  $\Phi_H^X$ . Because the value of  $\Phi_H^X$  and the slope of the dashed line are unaffected by this change, it will result in an  $\alpha_0$  very close to  $\alpha = 1$ . The shaded area that represents positive expected net gains might then disappear. This suggests a signal with very poor information about the action would be useless. Results in the next section confirm this conjecture.

#### 4. Main Results

Sufficient conditions for the signal to be useless and useful are given in this section. One nice feature of these conditions is that the effects of the signal's information are captured in an informativeness measure called the *degree of informativeness vector*. Using the Euclidean norm to summarize the information quality, I find that the signal is useless when it carries sufficiently poor information, i.e., when the norm is sufficiently small. An analogous result for usefulness also holds.<sup>17</sup>

To obtain a sufficient condition for the signal to be useless, I look for a lower bound of  $\hat{\Phi}(\alpha)$ . If this lower bound is everywhere weakly higher than the dashed line  $\Phi_H^X - (R_H - R(\alpha))$ , the shaded area will not exist. That means  $\alpha = 1$  is optimal and the signal is useless.

A component of this lower bound is the minimized value  $\Phi^X(\alpha)$  of the following program:

$$\begin{aligned}
& \text{(oRs-L)} && \text{Min } \sum_x \Phi(\bar{U}_L(x))[\alpha p(x|H) + (1-\alpha)p(x|L)] \\
& \text{subject to} && \bar{U}_L(x) \forall x \\
& \text{NE-L:} && \sum_x \bar{U}_L(x)p(x|H) - D(H) = \sum_x \bar{U}_L(x)p(x|L) - D(L) \\
& \text{AIR-L:} && \sum_x \bar{U}_L(x)p(x|L) - D(L) \geq \bar{V}.
\end{aligned}$$

This is the program for finding an optimal full-commitment contract for inducing  $\alpha \in (0,1]$  using the outcome alone. Because every feasible contract in this program corresponds to a feasible contract in program (oRs), and because for such contracts the objective values of the two programs are the same, it follows that program (oRs-L)'s minimized value must be weakly higher than program (oRs)'s, i.e.,  $\Phi^X(\alpha) \geq \hat{\Phi}(\alpha)$ . When  $\alpha = 1$ , the two programs are identical and their minimized values are the same, i.e.,  $\Phi^X(1) = \hat{\Phi}(1)$ .

Let  $U_H^\alpha(X) \equiv (U_H^\alpha(x))_{x \in X}$  denote the optimal solution of program (oRs-L) for  $\alpha \in (0,1]$ , and  $\theta^\alpha$  and  $\kappa^\alpha$  the associated multipliers for constraints AIR-L and NE-L, respectively. Define

$$\rho^\alpha(x) \equiv p(x|L)/[\alpha p(x|H) + (1-\alpha)p(x|L)],$$

which is a generalization of the likelihood ratio  $\rho(x) \equiv p(x|L)/p(x|H)$ . Moreover, define

$$\Delta(x) \equiv [q(B|x,L)/q(B|x,H) - 1][1 - q(G|x,L)/q(G|x,H)] \quad \forall x$$

and

$$\Delta(X) \equiv (\Delta(x))_{x \in X}.$$

These are called the *degree of informativeness* of  $y$  on outcome value  $x$  and the *degree of informativeness vector*. Finally, define

$$\bar{x} \equiv \arg \max \{ \Delta(x) \mid x \in X \}.$$

This is the outcome value associated with the maximum component of the degree of informativeness vector.

The following result provides a sufficient condition for uselessness.

LEMMA 2: *The signal  $y$  is useless if*

$$(R_H - \Phi_H^X) - (R(\alpha) - \Phi^X(\alpha)) \geq 2(1-\alpha)(\kappa^\alpha/\alpha - \theta^\alpha)\rho^\alpha(x_1)\Delta(\bar{x})\varphi(D(L)+\bar{V}) \quad \forall \alpha \in (\alpha_0, 1).$$

Clearly, the signal's informativeness affects the condition only through  $\Delta(x)$ . Since

$$\Delta(x) \equiv [q(B|x,L)/q(B|x,H) - 1][1 - q(G|x,L)/q(G|x,H)] \quad \forall x,$$

its value is smaller if  $q(B|x,L)/q(B|x,H)$  is smaller while  $q(G|x,L)/q(G|x,H)$  is larger. This happens when the

signal  $y$  is quasi-garbled with a stochastic matrix.<sup>18</sup> Intuitively, this means that if the signal's information gets worse, the condition in the lemma will become less stringent. When the signal carries sufficiently poor information, as represented by a small enough Euclidean norm  $\|\Delta(X)\|$  of its degree of informativeness vector, the condition will be satisfied and the signal is useless.

PROPOSITION 3: *For some  $\varepsilon > 0$ , the condition in Lemma 2 will be satisfied, provided  $\|\Delta(X)\| < \varepsilon$ . Thus, the signal  $y$  is useless if it carries sufficiently poor information about the action.*

This proposition shows that the condition in Lemma 2 is not vacuous and thus establishes the *insufficiency of informativeness* result. Analogous results for usefulness also hold. To show that  $\alpha = 1$  is suboptimal and hence the signal is useful requires the identification of one  $\alpha < 1$  at which the expected net gain  $(\Phi_H^X - \hat{\Phi}(\alpha)) - (R_H - R(\alpha))$  from inducing this randomized action is positive. This will happen if the minimized compensation cost  $\hat{\Phi}(\alpha)$  has an upper bound, say  $\bar{\Phi}(\alpha)$ , satisfying the following conditions: (i)  $\bar{\Phi}(1) = \Phi_H^X$ ; (ii)  $\bar{\Phi}'(1^-) > R'(1^-)$ .

Because  $\bar{\Phi}(\alpha) \geq \hat{\Phi}(\alpha)$ ,  $(\Phi_H^X - \bar{\Phi}(\alpha)) - (R_H - R(\alpha))$  is a lower bound of the expected net gain  $(\Phi_H^X - \hat{\Phi}(\alpha)) - (R_H - R(\alpha))$ . Condition (i) ensures that this lower bound equals zero at  $\alpha = 1$ . Then condition (ii) implies its value will increase as  $\alpha$  decreases from 1. Together, these conditions guarantee that the lower bound has a positive value for some  $\alpha$  very close to 1. Consequently, the expected net gain from inducing this  $\alpha$  must also be positive. That means,  $\alpha = 1$  is suboptimal and the signal is useful.

In the appendix, I show that such a  $\bar{\Phi}(\alpha)$  satisfying conditions (i) and (ii) can be found if the condition in the following lemma holds. Consequently, it is a sufficient condition for the signal to be useful.

LEMMA 3: *The signal  $y$  is useful if  $(R_H - \Phi_H^X) - (R_L - \Phi_{H|L}^X) < (\kappa^1 - \theta^1) \sum_x \rho(x) \Delta(x) \varphi(U_H^*(x)) p(x|L)$ , where  $\Phi_{H|L}^X \equiv \sum_x \Phi(U_H^*(x)) p(x|L)$ .*

If  $q(G|x,H)$  or  $q(B|x,L)$  gets larger for some  $x$  with their values being constant for all other  $x$ , the signal will become more informative and consequently the degree of informativeness vector,  $\Delta(X)$ , will become larger. This will result in a larger value on the right hand side of the condition in Lemma 3. The condition thus becomes less stringent. When  $q(G|x,H)$  gets close to 1 for all  $x$  with  $q(B|x,L)$  being constant

for all  $x$ ,  $q(B|x,H) = 1 - q(G|x,H)$  will approach zero, resulting in an arbitrarily large degree of informativeness vector. Such a vector will have an arbitrarily large Euclidean norm,  $\|\Delta(X)\|$ , and the condition in Lemma 3 will surely be met. Intuitively, this means that when the signal carries sufficiently rich information, as represented by a large enough  $\|\Delta(X)\|$ , the signal will be useful for incentive contracting despite the renegotiation consideration.

PROPOSITION 4: *For some  $K > 0$ , the condition in Lemma 3 will be satisfied, provided  $\|\Delta(X)\| > K$ . Thus, the signal  $y$  is useful if it carries sufficiently rich information about the action.*

In the next section, I will discuss how my results bear on some management control practices which appear to be at odds with conventional agency theory.

### **5. Implication for Relative Performance Evaluation**

In the managerial accounting literature, Holmström's informativeness criterion has been applied to understand the *controllability principle* in responsibility accounting. Antle and Demski (1988) showed that the principle can be modified to mesh with the informativeness criterion. That is to say, a manager should be *held responsible* for a variable if and only if it is *controllable* by him in the sense that the variable is informative about the manager's effort. This appealing interpretation of the controllability principle, however, does not seem to be confirmed by field studies, such as Merchant (1987). The companies in his study persistently held their managers accountable for a number of uncontrollable factors including economic and competitive conditions. This is puzzling because according to the theory, the companies should be able to benefit from adjusting for these uncontrollables in evaluating their managers' performance.

In a different field study, Maher (1987) collected similar evidence on violations of the controllability principle. As an application of the principle, interfirm *relative performance evaluation* (RPE) should be used to filter the common uncertainty faced by firms in the same industry. Because such industry-specific uncertainty is rather common, the use of interfirm RPE should be very noticeable. However, Maher (1987) pointed out that evidence about the use of interfirm RPE to evaluate division managers is sparse. These observations by Merchant and Maher suggest the use of the controllability principle and the practice of interfirm RPE are inconsistent with agency theory.

Such discrepancies between the theory and practice can be explained with the "insufficiency of informativeness" result derived in this paper. When contracts are renegotiable, the usefulness of a signal will depend on its degree of informativeness. I show that it can be optimal to ignore an informative signal when it carries very poor information. So the rare use of interfirm RPE, as observed in practice, will arise if data on the industry peers' performance are not sufficiently informative. Under such circumstances, it is better to ignore the data because its use will tighten the renegotiation-proofness constraints. The implicit cost arising from this will supersede the benefit of using the data for contracting.

## **6. Concluding Remarks**

Does allowing for renegotiation make a difference in our understanding of the use of monitoring information in incentive contracts? I have shown that when contract renegotiation is possible, the usefulness of a signal will depend on the degree of its informativeness. An informative signal can be useless if the benefit of using the information cannot outweigh the cost of using it. This cost exists because the possibility of renegotiation imposes additional constraints on the way a contract can be designed. These constraints will be tightened when the signal's information is utilized in the contract.

Three decades of development of agency theory proves that it is a useful framework for understanding a variety of contracting issues. Nonetheless, there are observed practices that do not fit well with the conventional theory. They include violations of the controllability principle and the rare practice of interfirm relative performance evaluation. This study shows that these practices can also be understood with the agency-theoretic framework, provided the conventional full commitment assumption is replaced with the assumption of renegotiable contracts.

From an empirical standpoint, it remains to be verified that the inability to fully commit indeed plays a prominent role in the issues unexplained by the conventional theory but explained by the results here. For instance, it seems perfectly possible that the rare practice of interfirm RPE is merely a result of bounded rationality or contract writing costs. Unfortunately, we have few empirical studies investigating these factors, and the theoretical research in these areas is even less developed than that concerning commitment. While there might be questions about the importance of the commitment issue, an analytical exercise like this study is still useful for expanding the predictions consistent with agency theory.

## Appendix

Proofs of the preliminary results in Sections 2 and 3 can be found in Yim (1995). Part I of this appendix contains proofs of the main results in Section 4. These proofs are based on properties of an optimal renegotiation-proof contract characterized in Part II of the appendix. The properties are derived from two auxiliary optimization programs specified in Part II and are expressed in terms of notations introduced there. Therefore, understanding the structures of these programs and the meanings of the notations in Part II is a prerequisite for understanding the proofs in Part I.

### I. Proofs of Results in Section 4

**PROOF OF LEMMA 2:** As discussed in Section 3, the expected net gain from inducing some  $\alpha < 1$ , i.e.,  $(\Phi_H^X - \hat{\Phi}(\alpha)) - (R_H - R(\alpha))$ , is non-positive for any  $\alpha$  weakly below  $\alpha_0$ . Therefore, a sufficient condition for the signal to be useless is that some lower bound of the expected net gain is non-positive for every  $\alpha \in (\alpha_0, 1)$ , or equivalently, some lower bound of  $\hat{\Phi}(\alpha)$  is weakly larger than  $\Phi_H^X - (R_H - R(\alpha))$  for every  $\alpha \in (\alpha_0, 1)$ . In this proof, I will show that  $\Phi^X(\alpha) - 2(1-\alpha)(\kappa^\alpha/\alpha - \theta^\alpha)\rho^\alpha(x_1)\Delta(\bar{x})\varphi(D(L)+\bar{V})$  is a lower bound of  $\hat{\Phi}(\alpha)$ . Consequently, the condition

$$\Phi^X(\alpha) - 2(1-\alpha)(\kappa^\alpha/\alpha - \theta^\alpha)\rho^\alpha(x_1)\Delta(\bar{x})\varphi(D(L)+\bar{V}) \geq \Phi_H^X - (R_H - R(\alpha)) \quad \text{for every } \alpha \in (\alpha_0, 1),$$

which is simply the condition in Lemma 2, is a sufficient condition for the signal's uselessness.

It takes four main steps to show that  $\Phi^X(\alpha) - 2(1-\alpha)(\kappa^\alpha/\alpha - \theta^\alpha)\rho^\alpha(x_1)\Delta(\bar{x})\varphi(D(L)+\bar{V})$  is a lower bound of  $\hat{\Phi}(\alpha)$ . In the first step, a lower bound of  $\hat{\Phi}(\alpha)$  is derived based on (i) properties of the optimal solution of program (oRs-L) specified in Section 4, (ii) properties of the renegotiation-proofness constraints discussed in Section 2, and (iii) the convexity of  $\Phi$ . Then in each of the remaining steps, a new lower bound with a simpler structure is derived from the lower bound of the previous step, using properties of programs (oRs-1) and (oRs-2) specified in Part II of the appendix.

**Step 1:**  $\Phi^X(\alpha) + [\sum_x (\kappa^\alpha - (\kappa^\alpha - \alpha\theta^\alpha)\rho^\alpha(x))(\hat{U}_H(x, \neg x, G; \alpha) - \hat{U}_H(x, \neg x, B; \alpha)) \times (q(G|x, H) - q(G|x, L))p(x|H)] +$

$\sum_x [\theta^\alpha p(x|L) + \kappa^\alpha (p(x|H) - p(x|L))] (\hat{U}_L(x; \alpha) - U_H^\alpha(x))$  is a lower bound of  $\hat{\Phi}(\alpha)$ , where

$((\hat{U}_H(x, \neg x, y; \alpha))_{y \in Y})_{x \in X}$ ,  $((\hat{U}_L(x; \alpha))_{y \in Y})_{x \in X}$  is the optimal renegotiation-proof contract characterized in Part II of the appendix.

Consider first the optimal solution  $U_H^\alpha(x)$  of program (oRs-L) for  $\alpha \in (0, 1]$ . The condition characterizing  $U_H^\alpha(x)$  is

$$\Phi'(U_H^\alpha(x)) = \kappa^\alpha/\alpha - (\kappa^\alpha/\alpha - \theta^\alpha)\rho^\alpha(x).$$

Because constraint AIR-L of the program is binding at the optimum, the exact values of  $\theta^\alpha$  and  $\kappa^\alpha$  are determined by constraint NE-L and the binding constraint AIR-L of the program, which are equivalent to the equation system below:

$$\sum_x \Phi'^{-1}(\kappa^\alpha/\alpha - (\kappa^\alpha/\alpha - \theta^\alpha)\rho^\alpha(x))p(x|\alpha) - D(\alpha) = \bar{V} \quad \forall \alpha,$$

where  $\Phi'^{-1}(\bullet)$  is the inverse of  $\Phi'(\bullet)$ . Once  $\theta^\alpha$  and  $\kappa^\alpha$  are determined, so are  $U_H^\alpha(x)$  and  $\Phi^X(\alpha) \equiv \sum_x \Phi(U_H^\alpha(x))[ap(x|H) + (1-\alpha)p(x|L)]$ .

The way I define the Lagrangian guarantees that  $\theta^\alpha$ , i.e., the multiplier for constraint AIR-L, is non-negative. Since  $\rho^\alpha(x)$  is decreasing in  $x$ ,  $\kappa^\alpha/\alpha - \theta^\alpha$  must be positive; otherwise,  $U_H^\alpha(x)$  would be non-increasing in  $x$ , violating constraint NE-L. As  $\kappa^\alpha/\alpha - \theta^\alpha > 0$ ,  $\kappa^\alpha$  must be positive. In short,  $\theta^\alpha \geq 0$ ,  $\kappa^\alpha/\alpha - \theta^\alpha > 0$ , and  $\kappa^\alpha > 0$ .

The convexity of  $\Phi$  implies

$$\Phi(\hat{U}_H(x, \neg x, y; \alpha)) \geq \Phi(U_H^\alpha(x)) + \Phi'(U_H^\alpha(x))(\hat{U}_H(x, \neg x, y; \alpha) - U_H^\alpha(x))$$

and

$$\Phi(\hat{U}_L(x; \alpha)) \geq \Phi(U_H^\alpha(x)) + \Phi'(U_H^\alpha(x))(\hat{U}_L(x; \alpha) - U_H^\alpha(x)).$$

Thus, a lower bound of  $\hat{\Phi}(\alpha)$  can be obtained as follows:

$$\begin{aligned} \hat{\Phi}(\alpha) &\equiv \alpha \sum_x \sum_y \Phi(\hat{U}_H(x, \neg x, y; \alpha)) q(y|x, H) p(x|H) + (1-\alpha) \sum_x \Phi(\hat{U}_L(x; \alpha)) p(x|L) \\ &\geq \alpha \sum_x \sum_y \Phi(U_H^\alpha(x)) q(y|x, H) p(x|H) \\ &\quad + \alpha \sum_x \sum_y \Phi'(U_H^\alpha(x)) (\hat{U}_H(x, \neg x, y; \alpha) - U_H^\alpha(x)) q(y|x, H) p(x|H) \\ &\quad + (1-\alpha) \sum_x \Phi(U_H^\alpha(x)) p(x|L) \\ &\quad + (1-\alpha) \sum_x \Phi'(U_H^\alpha(x)) (\hat{U}_L(x; \alpha) - U_H^\alpha(x)) p(x|L) \\ &= \sum_x \Phi(U_H^\alpha(x)) [\alpha p(x|H) + (1-\alpha) p(x|L)] \\ &\quad + \alpha \sum_x \Phi'(U_H^\alpha(x)) (\sum_y \hat{U}_H(x, \neg x, y; \alpha) q(y|x, H) - \hat{U}_L(x; \alpha)) p(x|H) \\ &\quad + \sum_x \Phi'(U_H^\alpha(x)) [\alpha p(x|H) + (1-\alpha) p(x|L)] (\hat{U}_L(x; \alpha) - U_H^\alpha(x)). \end{aligned}$$

By definition,  $\sum_x \Phi(U_H^\alpha(x)) [\alpha p(x|H) + (1-\alpha) p(x|L)]$  is  $\Phi^X(\alpha)$ . Moreover, by the *flat-scheme* and *same-expected-reward* renegotiation-proofness constraints,  $\hat{U}_L(x; \alpha) = \sum_y \hat{U}_H(x, \neg x, y; \alpha) q(y|x, L)$ . Therefore, using the characterizing condition  $\Phi'(U_H^\alpha(x)) = \kappa^\alpha/\alpha - (\kappa^\alpha/\alpha - \theta^\alpha) \rho^\alpha(x)$ , I can express the lower bound of  $\hat{\Phi}(\alpha)$  as

$$\begin{aligned} \Phi^X(\alpha) &+ [\sum_x (\kappa^\alpha - (\kappa^\alpha - \alpha \theta^\alpha) \rho^\alpha(x)) (\hat{U}_H(x, \neg x, G; \alpha) - \hat{U}_H(x, \neg x, B; \alpha)) \times (q(G|x, H) - q(G|x, L)) p(x|H)] \\ &+ \sum_x [\theta^\alpha p(x|L) + \kappa^\alpha (p(x|H) - p(x|L))] (\hat{U}_L(x; \alpha) - U_H^\alpha(x)). \end{aligned}$$

Step 2:  $\Phi^X(\alpha) - (\kappa^\alpha - \alpha \theta^\alpha) [\sum_x \rho^\alpha(x) (\hat{U}_H(x, \neg x, G; \alpha) - \hat{U}_H(x, \neg x, B; \alpha)) (q(G|x, H) - q(G|x, L)) p(x|H)]$  is a lower bound of  $\hat{\Phi}(\alpha)$ .

Simplifying the lower bound derived in Step 1 with constraint NE-L and the binding constraint AIR-L of program (oRs-L) in Section 4 yields

$$\begin{aligned} \Phi^X(\alpha) &+ [\sum_x (\kappa^\alpha - (\kappa^\alpha - \alpha \theta^\alpha) \rho^\alpha(x)) (\hat{U}_H(x, \neg x, G; \alpha) - \hat{U}_H(x, \neg x, B; \alpha)) \times (q(G|x, H) - q(G|x, L)) p(x|H)] \\ &+ \theta^\alpha [\sum_x \hat{U}_L(x; \alpha) p(x|L) - D(L) - \bar{V}] - \kappa^\alpha [D(H) - D(L) - \sum_x \hat{U}_L(x; \alpha) (p(x|H) - p(x|L))]. \end{aligned}$$

By constraint AIR-2 of program (oRs-2) in Part II of the appendix, the term  $\theta^\alpha [\sum_x \hat{U}_L(x; \alpha) p(x|L) - D(L) - \bar{V}]$  of the expression above is non-negative. Thus, dropping this term yields the new lower bound of  $\hat{\Phi}(\alpha)$  below:

$$\begin{aligned} \Phi^X(\alpha) &+ [\sum_x (\kappa^\alpha - (\kappa^\alpha - \alpha \theta^\alpha) \rho^\alpha(x)) (\hat{U}_H(x, \neg x, G; \alpha) - \hat{U}_H(x, \neg x, B; \alpha)) \times (q(G|x, H) - q(G|x, L)) p(x|H)] \\ &- \kappa^\alpha [D(H) - D(L) - \sum_x \hat{U}_L(x; \alpha) (p(x|H) - p(x|L))]. \end{aligned}$$

Additionally, constraint AIC-1 of program (oRs-1) in Part II of the appendix and constraint NEL of program (oRs-2) there imply

$$\begin{aligned} &\sum_x (\hat{U}_H(x, \neg x, G; \alpha) - \hat{U}_H(x, \neg x, B; \alpha)) \times (q(G|x, H) - q(G|x, L)) p(x|H) \\ &= \sum_x (\sum_y \hat{U}_H(x, \neg x, y; \alpha) q(y|x, H) - \hat{U}_L(x; \alpha)) p(x|H) \\ &= \sum_x \sum_y \hat{U}_H(x, \neg x, y; \alpha) q(y|x, H) p(x|H) - \sum_x \hat{U}_L(x; \alpha) p(x|H) \end{aligned}$$

$$\begin{aligned}
&= \sum_x \hat{U}_L(x;\alpha)p(x|L) - D(L) + D(H) - \sum_x \hat{U}_L(x;\alpha)p(x|H) \\
&= D(H) - D(L) - \sum_x \hat{U}_L(x;\alpha)(p(x|H)-p(x|L)).
\end{aligned}$$

Therefore, the term  $\kappa^\alpha[D(H) - D(L) - \sum_x \hat{U}_L(x;\alpha)(p(x|H)-p(x|L))]$  in the new lower bound derived above may be substituted by  $\sum_x \kappa^\alpha(\hat{U}_H(x,\neg x,G;\alpha) - \hat{U}_H(x,\neg x,B;\alpha)) \times (q(G|x,H) - q(G|x,L))p(x|H)$ . With this substitution, the new lower bound can be expressed as

$$\Phi^X(\alpha) - (\kappa^\alpha - \alpha\theta^\alpha)[\sum_x \rho^\alpha(x)(\hat{U}_H(x,\neg x,G;\alpha) - \hat{U}_H(x,\neg x,B;\alpha))(q(G|x,H) - q(G|x,L))p(x|H)].$$

**Step 3:**  $\Phi^X(\alpha) - 2(1-\alpha)(\kappa^\alpha/\alpha - \theta^\alpha)[\sum_x \rho^\alpha(x)\Delta(x)\varphi(\hat{U}_L(x;\alpha))p(x|L)]$  is a lower bound of  $\hat{\Phi}(\alpha)$ .

Consider below the component  $\hat{U}_H(x,\neg x,G;\alpha) - \hat{U}_H(x,\neg x,B;\alpha)$  of the lower bound of  $\hat{\Phi}(\alpha)$  derived in Step 2. By constraint SER-1 of program (oRs-1) in Part II of the appendix,

$$\hat{U}_H(x,\neg x,G;\alpha) - \hat{U}_H(x,\neg x,B;\alpha) = (\hat{U}_H(x,\neg x,G;\alpha) - \hat{U}_L(x;\alpha))/q(B|x,L).$$

Moreover, the convexity of  $\Phi'$  ensured by Assumption 6 and constraint NNG-1 of program (oRs-1) imply

$$\begin{aligned}
&\Phi''(\hat{U}_L(x;\alpha))(\hat{U}_H(x,\neg x,G;\alpha) - \hat{U}_L(x;\alpha)) \\
&\leq \Phi'(\hat{U}_H(x,\neg x,G;\alpha)) - \Phi'(\hat{U}_L(x;\alpha)) \\
&\leq (\alpha^{-1} - 1)\Phi'(\hat{U}_L(x;\alpha))\delta(x)\rho(x).
\end{aligned}$$

Thus,

$$\begin{aligned}
&\hat{U}_H(x,\neg x,G;\alpha) - \hat{U}_H(x,\neg x,B;\alpha) \\
&\leq q(B|x,L)^{-1}(\alpha^{-1} - 1)(\Phi'(\hat{U}_L(x;\alpha))/\Phi''(\hat{U}_L(x;\alpha)))\delta(x)\rho(x) \\
&\leq 2(\alpha^{-1} - 1)\varphi(\hat{U}_L(x;\alpha))\delta(x)\rho(x).
\end{aligned}$$

This means that the lower bound of  $\hat{\Phi}(\alpha)$  derived in Step 2 is itself bounded below by

$$\Phi^X(\alpha) - (\kappa^\alpha - \alpha\theta^\alpha)\{\sum_x \rho^\alpha(x)[2(\alpha^{-1} - 1)\varphi(\hat{U}_L(x;\alpha))\delta(x)\rho(x)](q(G|x,H) - q(G|x,L))p(x|H)\}.$$

Consider below the component  $\delta(x)\rho(x)(q(G|x,H) - q(G|x,L))p(x|H)$  of this new lower bound of  $\hat{\Phi}(\alpha)$ .

Because  $\rho(x) \equiv p(x|L)/p(x|H)$  and

$$\begin{aligned}
&\delta(x)[q(G|x,H) - q(G|x,L)] \\
&\equiv [q(B|x,L)/q(B|x,H) - q(G|x,L)/q(G|x,H)][q(G|x,H) - q(G|x,L)] \\
&= [(q(B|x,L) - q(B|x,H))/(q(B|x,H)q(G|x,H))][q(G|x,H) - q(G|x,L)] \\
&= [q(B|x,L)/q(B|x,H) - 1][1 - q(G|x,L)/q(G|x,H)] \\
&\equiv \Delta(x),
\end{aligned}$$

the new lower bound can be expressed as

$$\Phi^X(\alpha) - 2(1-\alpha)(\kappa^\alpha/\alpha - \theta^\alpha)[\sum_x \rho^\alpha(x)\Delta(x)\varphi(\hat{U}_L(x;\alpha))p(x|L)].$$

**Step 4:**  $\Phi^X(\alpha) - 2(1-\alpha)(\kappa^\alpha/\alpha - \theta^\alpha)\rho^\alpha(x_1)\Delta(\bar{x})\varphi(D(L)+\bar{V})$  is a lower bound of  $\hat{\Phi}(\alpha)$ .

Consider below the component  $\sum_x \rho^\alpha(x)\Delta(x)\varphi(\hat{U}_L(x;\alpha))p(x|L)$  of the lower bound of  $\hat{\Phi}(\alpha)$  derived in Step 3. Let  $\bar{x} \equiv \arg \max \{ \Delta(x) \mid x \in X \}$ . As  $\rho^\alpha(x)$  is decreasing in  $x$  and  $\varphi$  is positive, non-increasing, and concave,

$$\begin{aligned}
&\sum_x \rho^\alpha(x)\Delta(x)\varphi(\hat{U}_L(x;\alpha))p(x|L) \\
&\leq \rho^\alpha(x_1)\Delta(\bar{x})\sum_x \varphi(\hat{U}_L(x;\alpha))p(x|L)
\end{aligned}$$

$$\begin{aligned} &\leq \rho^\alpha(x_1)\Delta(\bar{x})\varphi(\sum_x \hat{U}_L(x;\alpha)p(x|L)) \\ &\leq \rho^\alpha(x_1)\Delta(\bar{x})\varphi(D(L)+\bar{V}). \end{aligned}$$

Thus,  $\Phi^X(\alpha) - 2(1-\alpha)(\kappa^\alpha/\alpha - \theta^\alpha)\rho^\alpha(x_1)\Delta(\bar{x})\varphi(D(L)+\bar{V})$  is also a lower bound of  $\hat{\Phi}(\alpha)$ .  $\blacksquare$

PROOF OF PROPOSITION 3: Note that the multipliers  $\kappa^\alpha$  and  $\theta^\alpha$  are finite.<sup>19</sup> Thus, some bound  $b < \infty$  will exist such that  $\kappa^\alpha/\alpha - \theta^\alpha \leq b \quad \forall \alpha \in (\alpha_0, 1)$ . Since  $\rho^\alpha(x_1) \equiv p(x_1|L)/[ap(x_1|H)+(1-\alpha)p(x_1|L)] \leq p(x_1|L)/p(x_1|H) \equiv \rho(x_1)$ ,

$$2(1-\alpha)(\kappa^\alpha/\alpha - \theta^\alpha)\rho^\alpha(x_1)\Delta(\bar{x})\varphi(D(L)+\bar{V}) \leq 2(1-\alpha)b\rho(x_1)\Delta(\bar{x})\varphi(D(L)+\bar{V}) \quad \forall \alpha \in (\alpha_0, 1).$$

So it suffices to prove that for some  $\varepsilon > 0$ ,  $2(1-\alpha)b\rho(x_1)\Delta(\bar{x})\varphi(D(L)+\bar{V}) \leq (R_H - \Phi_H^X) - (R(\alpha) - \Phi^X(\alpha)) \quad \forall \alpha \in (\alpha_0, 1)$  if  $\|\Delta(X)\| < \varepsilon$ . To prove this, consider the program below, which is the program for finding the optimal full-commitment contract  $U_H^*(X)$  for inducing action  $H$  using the outcome alone:

$$\begin{aligned} \text{(FC}^X) \quad & \text{Min}_{U_H(x) \forall x} \sum_x \Phi(U_H(x))p(x|H) \\ \text{subject to} \quad & \\ \text{IC}^X: \quad & \sum_x U_H(x)p(x|H) - D(H) \geq \sum_x U_H(x)p(x|L) - D(L) \\ \text{IR}^X: \quad & \sum_x U_H(x)p(x|H) - D(H) \geq \bar{V}. \end{aligned}$$

Because the optimal solution  $U_H^\alpha(X)$  of program (oRs-L) in Section 4 is a feasible choice of the program above,  $\sum_x \Phi(U_H^\alpha(x))p(x|H)$  must be weakly greater than program (FC<sup>X</sup>)'s minimized value,  $\Phi_H^X$ . Moreover, note that  $\sum_x \Phi(U_H^\alpha(x))p(x|L) \geq \Phi(\sum_x U_H^\alpha(x)p(x|L)) = \Phi(D(L)+\bar{V}) \equiv \Phi_L$ . Therefore,

$$\begin{aligned} \Phi^X(\alpha) &\equiv \sum_x \Phi(U_H^\alpha(x))[ap(x|H)+(1-\alpha)p(x|L)] \\ &\geq \alpha\Phi_H^X + (1-\alpha)\Phi_L. \end{aligned}$$

This implies

$$\begin{aligned} (R_H - \Phi_H^X) - (R(\alpha) - \Phi^X(\alpha)) &\geq (R_H - \Phi_H^X) - [(\alpha R_H + (1-\alpha)R_L) - (\alpha\Phi_H^X + (1-\alpha)\Phi_L)] \\ &\geq (1-\alpha)[(R_H - \Phi_H^X) - (R_L - \Phi_L)]. \end{aligned}$$

Let  $\varepsilon \equiv [(R_H - \Phi_H^X) - (R_L - \Phi_L)]/[2b\rho(x_1)\Delta(\bar{x})\varphi(D(L)+\bar{V})]$ . When  $\|\Delta(X)\| < \varepsilon$ ,  $\Delta(\bar{x})$  must also be less than  $\varepsilon$ . Hence,  $\forall \alpha \in (\alpha_0, 1)$ ,

$$\begin{aligned} 2(1-\alpha)b\rho(x_1)\Delta(\bar{x})\varphi(D(L)+\bar{V}) &= (1-\alpha)\Delta(\bar{x})[(R_H - \Phi_H^X) - (R_L - \Phi_L)]/\varepsilon \\ &< (1-\alpha)[(R_H - \Phi_H^X) - (R_L - \Phi_L)] \\ &\leq (R_H - \Phi_H^X) - (R(\alpha) - \Phi^X(\alpha)). \end{aligned}$$

Consequently, for such an  $\varepsilon$ , the condition in Lemma 2 is satisfied when  $\|\Delta(X)\| < \varepsilon$ . That means, the signal  $y$  is useless if it carries sufficiently poor information about the action.  $\blacksquare$

PROOF OF LEMMA 3: As explained in the text, to show that  $\alpha = 1$  is suboptimal and hence the signal is useful, it suffices to find some upper bound  $\bar{\Phi}(\alpha)$  of the minimized compensation cost  $\hat{\Phi}(\alpha)$  such that (i)  $\bar{\Phi}(1) = \Phi_H^X$ , (ii)  $\bar{\Phi}'(1^-) > R'(1^-)$ . To derive such a  $\bar{\Phi}(\alpha)$ , I use program (oRs-U) stated below. It is modified from program (oRs-2) in Part II of the appendix. Specifically, program (oRs-U) is obtained by strengthening the inequality constraints NEU and AIR-2 of program (oRs-2) to equality constraints. As a result, constraint NEL of program (oRs-2) is

fulfilled automatically, and program (oRs-2)'s objective function reduces to the one given below:

$$\begin{aligned} \text{(oRs-U)} \quad & \text{Min}_{\bar{U}_L(x) \forall x} \alpha [\sum_x \sum_y \Phi(U_N(x,y; \bar{U}_L(x), \alpha)) q(y|x, H) p(x|H)] + (1-\alpha) [\sum_x \Phi(\bar{U}_L(x)) p(x|L)] \\ \text{subject to} \quad & \end{aligned}$$

$$\text{NE-U:} \quad \sum_x \sum_y U_N(x,y; \bar{U}_L(x), \alpha) q(y|x, H) p(x|H) - D(H) = \sum_x \bar{U}_L(x) p(x|L) - D(L)$$

$$\text{AIR-U:} \quad \sum_x \bar{U}_L(x) p(x|L) - D(L) = \bar{V}.$$

Clearly, this is a problem of classical optimization. Suppose  $U_L^*(X) \equiv (U_L^*(x))_{x \in X}$  is an optimal solution of this program. Let  $\bar{\theta}^\alpha$  and  $\bar{\kappa}^\alpha$  be the associated multipliers for constraints NE-U and AIR-U. By the implicit function theorem,  $U_L^*(X)$ ,  $\bar{\theta}^\alpha$ ,  $\bar{\kappa}^\alpha$ , and the minimized value of the program, i.e.,

$$\bar{\Phi}(\alpha) \equiv \alpha \sum_x \sum_y \Phi(U_N(x,y; U_L^*(x), \alpha)) q(y|x, H) p(x|H) + (1-\alpha) \sum_x \Phi(U_L^*(x)) p(x|L),$$

are all differentiable in  $\alpha$ .

Because the constraint set of program (oRs-U) is more stringent than that of program (oRs-2),  $\bar{\Phi}(\alpha)$  is an upper bound of  $\hat{\Phi}(\alpha)$ . When  $\alpha = 1$ ,  $U_N(x,y; \bar{U}_L(x), \alpha)$  will be identical to  $\bar{U}_L(x)$  and program (oRs-U) will be the same as program (oRs-L) in Section 4. Thus,  $U_L^*(X) \equiv U_H^*(X) \equiv U_H^*(X)$  and  $\bar{\Phi}(1) \equiv \Phi^X(1) \equiv \Phi_H^X$ . So condition (i) is satisfied.

In the following, I will show that condition (ii), i.e.,  $\bar{\Phi}'(1^-) > R'(1^-)$ , will be satisfied if the condition given in this lemma is met. Therefore, this condition is a sufficient condition for  $\alpha = 1$  to be suboptimal and the signal to be useful.

Consider first the derivative of  $\bar{\Phi}(\alpha)$ . By the envelope theorem, it is equal to

$$\begin{aligned} \bar{\Phi}'(\alpha) \equiv & \sum_x \sum_y \Phi(U_N(x,y; U_L^*(x), \alpha)) q(y|x, H) p(x|H) - \sum_x \Phi(U_L^*(x)) p(x|L) \\ & + \sum_x \sum_y (\bar{\kappa}^\alpha - \alpha \Phi'(U_N(x,y; U_L^*(x), \alpha))) \times (-\partial U_N(x,y; U_L^*(x), \alpha) / \partial \alpha) q(y|x, H) p(x|H). \end{aligned}$$

Note that  $\sum_y (\partial U_N(x,y; \bar{U}_L(x), \alpha) / \partial \alpha) q(y|x, H) = -\alpha^{-2} \rho(x) \Delta(x) \Phi'(\bar{U}_L(x)) / [\sum_y \Phi''(U_N(x,y; \bar{U}_L(x), \alpha)) q(y|x, H)]$ . When  $\alpha = 1$ ,  $U_N(x,y; \bar{U}_L(x), \alpha)$  is identical to  $\bar{U}_L(x)$ , and program (oRs-U) will be the same as program (oRs-L). Thus,  $U_L^*(X) \equiv U_H^*(X) \equiv U_H^*(X)$ ,  $\bar{\Phi}(1) \equiv \Phi^X(1) \equiv \Phi_H^X$ ,  $\bar{\kappa}^1 \equiv \kappa^1$ , and  $\bar{\theta}^1 \equiv \theta^1$ . Because  $\Phi'(U_H^*(x)) \equiv \kappa^1 - (\kappa^1 - \theta^1) \rho(x)$ , the left-derivative of  $\bar{\Phi}(\alpha)$  at  $\alpha = 1$  should be equal to

$$\bar{\Phi}'(1^-) \equiv \sum_x \Phi(U_H^*(x)) (p(x|H) - p(x|L)) + (\kappa^1 - \theta^1) \sum_x \rho(x) \Delta(x) \varphi(U_H^*(x)) p(x|L).$$

On the other hand,  $R'(\alpha) \equiv R_H - R_L$ . Therefore, a sufficient condition for  $\alpha = 1$  to be suboptimal and the signal to be useful is  $\bar{\Phi}'(1^-) > R_H - R_L$ . Let  $\Phi_{H|L}^X \equiv \sum_x \Phi(U_H^*(x)) p(x|L)$ . The condition can then be expressed as  $(R_H - \Phi_H^X) - (R_L - \Phi_{H|L}^X) < (\kappa^1 - \theta^1) \sum_x \rho(x) \Delta(x) \varphi(U_H^*(x)) p(x|L)$ . ■

PROOF OF PROPOSITION 4: Let  $K = M^{1/2} [(R_H - \Phi_H^X) - (R_L - \Phi_{H|L}^X)] / [(\kappa^1 - \theta^1) \rho(\bar{x}) \varphi(U_H^*(\bar{x})) p(\bar{x}|L)]$ , where  $M$  is the number of  $x$ 's in  $X$ . When  $\|\Delta(X)\| > K$ ,  $\Delta(\bar{x}) \geq (\sum_x \Delta(x)^2 / M)^{1/2} = \|\Delta(X)\| / M^{1/2} > K / M^{1/2} = [(R_H - \Phi_H^X) - (R_L - \Phi_{H|L}^X)] / [(\kappa^1 - \theta^1) \rho(\bar{x}) \varphi(U_H^*(\bar{x})) p(\bar{x}|L)]$ . Thus,

$$\begin{aligned} & (\kappa^1 - \theta^1) \sum_x \rho(x) \Delta(x) \varphi(U_H^*(x)) p(x|L) \\ & \geq (\kappa^1 - \theta^1) \rho(\bar{x}) \Delta(\bar{x}) \varphi(U_H^*(\bar{x})) p(\bar{x}|L) \\ & > (R_H - \Phi_H^X) - (R_L - \Phi_{H|L}^X). \end{aligned} \quad \blacksquare$$

## II. Optimal Renegotiation-Proof Contract: A Partial Characterization

In general, program (oRs) is not a concave program. This can be seen by substituting  $\sum_y U_H(x,y)q(y|x,L)$  for  $U_L(x,y)$ 's using constraints SER and FS. The result is a program with choice variables  $U_H(x,y)$ 's only. Since the size of  $\delta(x)$  varies, it is impossible to guarantee the left hand side of constraint NNG is a concave function. Moreover, it is not clear whether any of the normality conditions for non-linear programming is satisfied by the program.<sup>20</sup> Therefore, I cannot rely on the usual Kuhn-Tucker conditions to characterize an optimal contract.

To deal with this difficulty, I decompose the contract design problem into two stages. First, the principal looks for the best compensation plan  $(U_H(x,G), U_H(x,B))_{x \in X}$  for a type- $H$  agent under the *same-expected-reward, upward-sloping-scheme, no-net-gain*, and AIC constraints, given a compensation plan  $(\bar{U}_L(x), \bar{U}_L(x))_{x \in X}$  for a type- $L$  agent that already takes into consideration the *flat-scheme* constraint. In the second stage, the principal adjusts  $(\bar{U}_L(x), \bar{U}_L(x))_{x \in X}$  optimally under the AIR constraint to finish the optimization process. The details of the optimization problems at these two stages are discussed in the following subsections.

### First-Stage Optimization

Stated below is the program representing the first-stage optimization:

$$\text{(oRs-1) } \quad \text{Min}_{U_H(x,y) \forall y \forall x} \quad \alpha \sum_x \sum_y \Phi(U_H(x,y))q(y|x,H)p(x|H) + (1-\alpha) \sum_x \Phi(\bar{U}_L(x))p(x|L)$$

subject to

$$\text{SER-1:} \quad \sum_y U_H(x,y)q(y|x,L) = \bar{U}_L(x) \quad \forall x$$

$$\text{US-1:} \quad U_H(x,G) \geq U_H(x,B) \quad \forall x$$

$$\text{NNG-1:} \quad (1-\alpha)\Phi'(\bar{U}_L(x))\delta(x)\rho(x) - \alpha[\Phi'(U_H(x,G)) - \Phi'(U_H(x,B))] \geq 0 \quad \forall x$$

$$\text{AIC-1:} \quad \sum_x \sum_y U_H(x,y)q(y|x,H)p(x|H) - D(H) = \sum_x \bar{U}_L(x)p(x|L) - D(L).$$

In finding an optimal compensation scheme  $(U_H(x,G), U_H(x,B))$  for a type- $H$  agent on outcome value  $x$ , the principal should recognize that the weighted average  $\sum_y U_H(x,y)q(y|x,L)$  of the compensation must be restricted to  $\bar{U}_L(x)$  by constraint SER-1. Therefore, the principal can only choose the spread,  $U_H(x,G) - U_H(x,B)$ .

There are two constraints on this spread. The first limits it to be non-negative, but not too big. This constraint is derived by combining constraints US-1 and NNG-1 into the compound inequality

$$0 \leq \Phi'(U_H(x,G)) - \Phi'(U_H(x,B)) \leq (\alpha^{-1} - 1)\Phi'(\bar{U}_L(x))\delta(x)\rho(x).$$

Secondly, coordination in the choice of the spreads for different  $x$ 's is required by constraint AIC-1. Using SER-1, one can rewrite AIC-1 as

$$\sum_y (U_H(x,G) - U_H(x,B))(q(G|x,H) - q(G|x,L))p(x|H) = [\sum_x \bar{U}_L(x)p(x|L) - D(L)] - [\sum_x \bar{U}_L(x)p(x|H) - D(H)].$$

This means the sum of the spreads with weights  $(q(G|x,H) - q(G|x,L))p(x|H)$ 's must be equal to the reduction in the expected utility of the agent selecting scheme  $(\bar{U}_L(x), \bar{U}_L(x))$  after taking action  $H$  instead of action  $L$ .

If constraint AIC-1 in program (oRs-1) were lifted, the principal would for each  $x$  adjust the spread to minimize the component  $\sum_y \Phi(U_H(x,y))q(y|x,H)$  of the expected compensation cost. Since  $\sum_y U_H(x,y)q(y|x,L)$  always equals  $\bar{U}_L(x)$ , a scheme  $(U_H(x,G), U_H(x,B))$  with a larger spread is second-order stochastically dominated by one with

a smaller spread. Therefore, without AIC-1, the principal would eliminate any spread.

When AIC-1 is present, there is a tradeoff among the marginal gains from reducing the spreads. The optimal choice of a spread can end up in one of the following three possibilities:

(i) *Constraint US-1 is binding*: As a result, the spread is zero. This together with constraint SER-1 implies  $U_H(x, G) = U_H(x, B) = \bar{U}_L(x)$ ;

(ii) *Constraint NNG-1 is binding*: In this case, the spread is set to the maximum allowed by the constraint. Therefore,  $U_H(x, G)$  and  $U_H(x, B)$  are determined by this binding constraint and constraint SER-1;

(iii) *Neither constraint US-1 nor constraint NNG-1 is binding*: When this happens, the choice of the spread is an "interior solution." Consequently,  $U_H(x, G)$  and  $U_H(x, B)$  are determined by constraint SER-1 and the first-order condition that characterizes the "interior solution."

A formal expression summarizing the optimal  $U_H(x, y)$ 's under these three cases can be obtained with the Kuhn-Tucker conditions. Figure 3 illustrates why these conditions are a valid characterization of the optimal solution of program (oRs-1). Depicted in the figure is the set of  $(U_H(x, G), U_H(x, B))$ 's fulfilling constraints SER-1, US-1, and NNG-1 of program (oRs-1) for the specific  $x$ . This set is represented by the line segment RP. Clearly, it is a convex set. Because the set of  $(U_H(x, G), U_H(x, B))_{x \in X}$ 's fulfilling constraint AIC-1 is convex, and convexity is preserved under set intersection, program (oRs-1)'s constraint set is also convex. Additionally, the program has a strictly convex objective function. Therefore, the optimal solution is uniquely characterized by the Kuhn-Tucker conditions.<sup>21</sup>

**Insert Figure 3 around here.**

If constraint NNG-1 is binding, the optimal  $U_H(x, y)$ 's, denoted by  $(U_N(x, y; \bar{U}_L(x), \alpha))_{y \in Y}$ , will be determined by constraint SER-1 and the binding *no-net-gain* constraint, i.e.,

$$(1 - \alpha)\Phi'(\bar{U}_L(x))\delta(x)\rho(x) = \alpha[\Phi'(U_H(x, G)) - \Phi'(U_H(x, B))],$$

as discussed in case (ii) above. Denote the multiplier for constraint AIC-1 by  $\eta$  and define  $\psi \equiv \eta/\alpha$ . When neither constraint US-1 and NNG-1 is binding, which is case (iii) discussed above, the optimal  $U_H(x, y)$ 's, denoted by  $(U_I(x, y; \psi, \bar{U}_L(x)))_{y \in Y}$ , will be determined by constraint SER-1 and the following *interior solution* equation originating from the Kuhn-Tucker conditions:  $\Phi'(U_H(x, G))q(B|x, L)/q(B|x, H) - \Phi'(U_H(x, B))q(G|x, L)/q(G|x, H) = \psi\delta(x)$ .

Given below is the optimal  $U_H(x, y)$ 's of program (oRs-1), expressed in terms of  $\psi$ :

$$\bar{U}_H(x,y;\psi,\bar{U}_L(x),\alpha) \equiv \begin{cases} \bar{U}_L(x) & \text{if } \psi \leq \Phi'(\bar{U}_L(x)) \\ U_H(x,y;\psi,\bar{U}_L(x)) & \text{if } \Phi'(\bar{U}_L(x)) \leq \psi \\ & \leq \Phi'(U_N(x,G;\bar{U}_L(x),\alpha)) \\ & + (\alpha^{-1}-1)\Phi'(\bar{U}_L(x))q(G|x,L)/q(G|x,H) \\ U_N(x,y;\bar{U}_L(x),\alpha) & \text{if } \psi \geq \Phi'(U_N(x,G;\bar{U}_L(x),\alpha)) \\ & + (\alpha^{-1}-1)\Phi'(\bar{U}_L(x))q(G|x,L)/q(G|x,H). \end{cases}$$

The derivation of this result can be found in Yim (1995). There I also show that some  $\psi(\bar{U}_L(X),\alpha)$  exists such that constraint AIC-1 is fulfilled by  $((\bar{U}_H(x,y;\psi(\bar{U}_L(X),\alpha),\bar{U}_L(x),\alpha))_{y \in Y})_{x \in X})$ , i.e.,

$$\sum_x \sum_y \bar{U}_H(x,y;\psi(\bar{U}_L(X),\alpha),\bar{U}_L(x),\alpha)q(y|x,H)p(x|H) - D(H) = \sum_x \bar{U}_L(x)p(x|H) - D(L).$$

Consequently,  $((\bar{U}_H(x,y;\psi(\bar{U}_L(X),\alpha),\bar{U}_L(x),\alpha))_{y \in Y})_{x \in X})$  is the optimal solution of program (oRs-1).

Note that for any  $(U_H(x,G),U_H(x,B))$  satisfying the renegotiation-proofness constraints, the value of the left hand side of AIC-1 is bounded between  $\sum_x \bar{U}_L(x)p(x|H) - D(H)$  and  $\sum_x \sum_y U_N(x,y;\bar{U}_L(x),\alpha)q(y|x,H)p(x|H) - D(H)$ . These are, respectively, the value of  $\sum_x \sum_y U_H(x,y)q(G|x,H)p(x|H) - D(H)$  when US-1 is binding at the optimum for all  $x$ 's, and its value when NNG-1 is binding at the optimum for all  $x$ 's. The fulfillment of constraint AIC-1 requires the value of its right hand side, i.e.,  $\sum_x \bar{U}_L(x)p(x|L) - D(L)$ , be inside this range. So the choice of  $\bar{U}_L(X) \equiv (\bar{U}_L(x))_{x \in X}$  must be confined to the following set to ensure program (oRs-1) has a non-empty constraint set:

$$\mathcal{U}_L(\alpha) \equiv \{ (\bar{u}_L(x))_{x \in X} \mid \sum_x \bar{u}_L(x)p(x|H) - D(H) \leq \sum_x \bar{u}_L(x)p(x|L) - D(L) \\ \leq \sum_x \sum_y U_N(x,y;\bar{u}_L(x),\alpha)q(y|x,H)p(x|H) - D(H) \}.$$

This requirement gives rise to two *non-emptiness* constraints in the second-stage optimization, which is discussed in the next subsection.

### Second-Stage Optimization

In the second-stage optimization, the principal chooses a compensation plan for a type- $L$  agent to minimize the minimized value of program (oRs-1), subject to the constraints that  $\mathcal{U}_L(\alpha)$  is non-empty and the expected utility of a type- $L$  agent is no less than his *ex ante* reservation payoff:

$$(oRs-2) \quad \text{Min}_{\bar{U}_L(x) \forall x} \alpha \sum_x \sum_y \Phi(\bar{U}_H(x,y;\psi(\bar{U}_L(X),\alpha),\bar{U}_L(x),\alpha))q(y|x,H)p(x|H) + (1-\alpha) \sum_x \Phi(\bar{U}_L(x))p(x|L)$$

subject to

$$\text{NEL:} \quad \sum_x \bar{U}_L(x)p(x|H) - D(H) \leq \sum_x \bar{U}_L(x)p(x|L) - D(L)$$

$$\text{NEU:} \quad \sum_x \sum_y U_N(x,y;\bar{U}_L(x),\alpha)q(y|x,H)p(x|H) - D(H) \geq \sum_x \bar{U}_L(x)p(x|L) - D(L)$$

$$\text{AIR-2:} \quad \sum_x \bar{U}_L(x)p(x|L) - D(L) \geq \bar{V},$$

where NEL and NEU are called the *non-emptiness (lower bound)* and *non-emptiness (upper bound)* constraints.

Let  $\hat{U}_L(X;\alpha) \equiv (\hat{U}_L(x;\alpha))_{x \in X}$  denote an optimal solution of this program. Define

$$\hat{U}_H(x, \neg x, y; \alpha) \equiv \bar{U}_H(x, y; \psi(\hat{U}_L(X; \alpha), \hat{U}_L(x; \alpha), \alpha))$$

with  $\neg x$  denoting the vector of the elements in  $X \setminus \{x\}$ . This is the optimal solution of programs (oRs-1), evaluated at the optimally selected compensation scheme  $\hat{U}_L(X; \alpha)$  for a type- $L$  agent. Since programs (oRs-1) and (oRs-2) together are equivalent to program (oRs),  $((\hat{U}_H(x, \neg x, y; \alpha))_{y \in Y})_{x \in X}, ((\hat{U}_L(x; \alpha))_{y \in Y})_{x \in X}$  is an optimal solution of this program. Consequently, the minimized expected compensation cost  $\hat{\Phi}(\alpha)$  for  $\alpha \in (0, 1)$  should be equal to

$$\alpha \sum_x \sum_y \Phi(\hat{U}_H(x, \neg x, y; \alpha)) q(y|x, H) p(x|H) + (1-\alpha) \sum_x \Phi(\hat{U}_L(x; \alpha)) p(x|L).$$

Note that  $\hat{\Phi}(1) = \Phi_H^X$  and  $\hat{\Phi}(0) = \Phi_L$ . So the expected compensation cost function  $\hat{\Phi}$  is completely determined once program (oRs-2) is solved. Subsequently, the principal can identify an optimal randomized action  $\hat{\alpha}$  by maximizing the expected profit  $R(\alpha) - \hat{\Phi}(\alpha)$ .

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## Notes

1. The influence of Holmström's work ranges from analytical accounting research such as Baiman and Demski (1980) and Wolfson (1985) to empirical and experimental studies such as Bushman, Indjejikian, and Smith (1996) and Frederickson (1990).

2. Having reviewed the related empirical research (Coughlin and Schmidt (1985), Murphy (1985), Antle and Smith (1986), and Gibbons and Murphy (1990)), Rosen (1992) also concluded that there is only weak evidence on the use of interfirm RPE. A more recent study by Janakiraman, Lambert, and Larcker (1992) has the same conclusion.

3. See, for example, Hart and Tirole (1988), Dewatripont (1989), Laffont and Tirole (1990), and Ma (1991;1994). Applications in accounting include Arya, Glover, and Sivaramakrishnan (1997), Indjejikian and Nanda (1999), and Demski and Frimor (1999).

4. More specifically, my analysis follows the *complete-contracting* approach used by studies such as Hart and Tirole (1988) and Fudenberg and Tirole (1990). Under this approach, renegotiation may be embedded in a contract that “renegotiates for the contracting parties implicitly.” Such a contract eliminates any gain from renegotiation and is therefore renegotiation-proof. It is now clear that the analysis of complete-contracting renegotiation models can be limited to renegotiation-proof contracts without loss of generality; this is the so-called *renegotiation-proofness principle*. As renegotiation need not arise in equilibrium, the *process* of renegotiation is inessential. Instead, the crucial factor is the *possibility* of renegotiation that hinders the power of incentive contracts and hence worsens contracting efficiency. Because it is the possibility, not the process, of renegotiation that matters under the complete-contracting approach adopted here, I use the term “renegotiable contracts” to emphasize that my results are driven by *renegotiable contracts*, not *contract renegotiation*.

5. The high action, however, is implementable in my model if the signal is not used for contracting. In that case, the outcome can still be used to motivate the agent. By contrast, the high action in Fudenberg and Tirole's model is not implementable at all because the outcome there, which corresponds to the signal here, is the only variable available for contracting.

6. A randomized action, or mixed strategy, is a probabilistic decision rule guiding the agent's choice of an action actually taken. Some researchers have regarded it as an artificial theoretical construct lacking practical content, for conscious randomization is rare in real-life decision making. This, however, is only one interpretation among other more reasonable alternatives. Fudenberg and Tirole (1990) have explained how a randomized action

can be understood as a shorthand for modeling an agent with private information about his preference based on Harsanyi's (1973) purification argument. Crawford (1990) has also provided another interpretation based on his definition of "equilibrium in beliefs," a generalization to Nash equilibrium for games with player preferences possibly violating the von-Neumann-Morgenstern independence axiom. With his interpretation, mixed strategy is a probability distribution representing a player's belief about the other's strategy; it need not involve conscious randomization at all.

7. Given the importance of a randomized action in this model, one might be curious about whether *ex ante* randomization by the principal also matters. As the fundamental reason for *ex ante* randomization, namely the non-concavity of the principal and the agent's utility frontier, may also exist here, the optimality of *ex ante* randomization is not ruled out. However, for a better focus, this side-issue is not considered in the paper either.

8. One might be confused by the feature of the assumption that the distribution of  $y$  depends on  $a$  given  $x$ . This seems to suggest  $y$  is just another measure of the agent's performance, not a measure of his competitor's performance. Consequently, it seems inconsistent with the interpretation that  $y$  can be a competitor's annual performance and hence may be used for interfirm RPE. However, in many situations of interest, the action of a company's manager can affect its competitor's performance (e.g., the effort put into an ad campaign increases one's market share and at the same time reduces that of its competitor). So  $y$  can be a measure of the company's and its competitor's performance. Even without such externalities, a competitor's performance  $y$  may still convey information about the agent's action  $a$  conditional on  $x$ . This does not necessarily imply that the competitor's performance is actually influenced by the agent's action.

9. The class of utility functions fulfilling this requirement includes the linear risk tolerance (LRT) family, also known as the hyperbolic absolute risk aversion (HARA) family. In decision theory, the absolute risk aversion coefficient  $-U''/U'$  is a popular measure of risk aversion. Recent research, however, suggests risk tolerance might be the more relevant measure in agency-theoretic analyses; see, for example, Jewitt (1988).

10. Although I have discussed the requirements on  $\varphi$  in terms of its first derivative, its differentiability is not needed to derive any results here. Only the first- and second-order differentiability of  $U$  and hence  $\Phi$  are required. Moreover, Assumption 6 is used to prove Lemma 2 and Proposition 3 only. The proposition, which is the key result here, can be established without Assumption 6 using Berge's (1963) maximum theorem if a convex  $\Phi'$  is assumed directly; see Yim (1995) for a proof.

11. The analysis here uses the complete-contracting approach that does not exclude any possible way of writing an enforceable contract. In particular, the principal may offer a contract contingent on the agent's report on the selected action. Alternatively, the principal can achieve the same effect by offering a menu of compensation schemes composed of all the possibilities of the report-contingent contract and let the agent pick a compensation scheme from the menu. This alternative setup is analytically more convenient and thus used here.

12. A technical assumption referred to by Amershi, Banker, and Datar (1990) as the *all-a-or-no-a* condition is needed to prove the sufficiency of informativeness in Holmström's (1979) model. He discussed this in footnote 21 of his paper. Amershi, Banker, and Datar showed that the sufficiency part of the informativeness criterion does not hold if the technical assumption is violated. This complication does not exist in the two-action, binary-signal setting considered here. The insufficiency of informativeness, however, will still arise because of economic forces related to renegotiation.

13. With full commitment, randomized actions are suboptimal because there is always room to upset a randomized-action prospective equilibrium if it is better to induce action  $H$  than action  $L$ . Similar room might not exist in a renegotiable contract model since inducing the agent to take a more productive action will tighten some renegotiation-proofness constraints.

14. Although this definition of informativeness focuses on a binary action  $a$  with  $a = H$  being the action of interest, it is equivalent to the original definition used by Holmström (1979) based on sufficient statistics (see Yim (1995)).

15. One might wonder whether the principal could use the signal for contracting and at the same time induce an  $\alpha$  arbitrarily close to 1. If this is possible, the loss due to inducing an  $\alpha < 1$  could be made arbitrarily small. It would then be dominated by the gain from utilizing the signal even when it carries little information. However, the renegotiation-proofness constraints US and NNG in program (oRs) prevent this from happening. To use  $y$  for contracting, we must have  $\Phi'(U_H(x,G)) - \Phi'(U_H(x,B)) > 0$  for some  $x$ . Therefore,  $\alpha \leq \Phi'(\sum_y U_L(x,y)q(y|x,L))\delta(x)\rho(x) / [\Phi'(\sum_y U_L(x,y)q(y|x,L))\delta(x)\rho(x) + \Phi'(U_H(x,G)) - \Phi'(U_H(x,B))]$ , which sets an upper bound precluding  $\alpha$  from getting arbitrarily close to 1. So the real issue is whether there is a way to utilize  $y$  such that the gain from it can dominate the loss from inducing the highest  $\alpha$  allowed under the constraints of program (oRs). The main result of this paper shows that some informative signals cannot generate sufficient net gains to justify their use in contracting.

16. In general, the line representing  $\hat{\Phi}(\alpha)$  might have a very different shape, and there might exist many such shaded areas or none at all.

17. Necessary and sufficient conditions for the signal to be useful are not derived because of some technical difficulty arising from the *no-net-gain* constraint, NNG. This constraint upsets the concavity property of the contract design problem, leading to an invalid characterization of the optimal solution by the Kuhn-Tucker conditions. The sufficient conditions provided here are derived based on a partial characterization of the optimal contract. Details of the characterization are discussed in the appendix.

18. I follow the terminology used by Marschak (1972) to define "quasi-garbled" here.

19. See Takayama (1985), 40-44 and 70-72, for this fundamental result of non-linear programming.

20. The conditions stated in the Arrow-Hurwicz-Uzawa theorem (Takayama (1985), p. 92-98) are examples of such normality conditions.

21. The convexity of the constraint set and the objective function implies that any  $(U_H(x, G), U_H(x, B))_{x \in X}$  fulfilling the Kuhn-Tucker conditions is an optimal solution. The strict convexity of the objective function then ensures that the optimal solution is unique.

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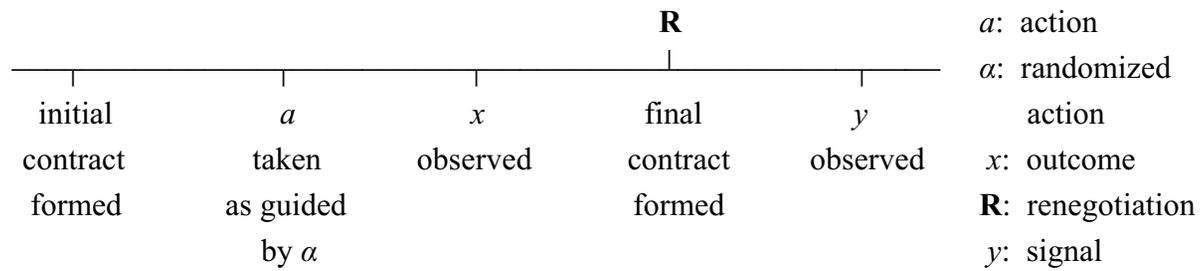
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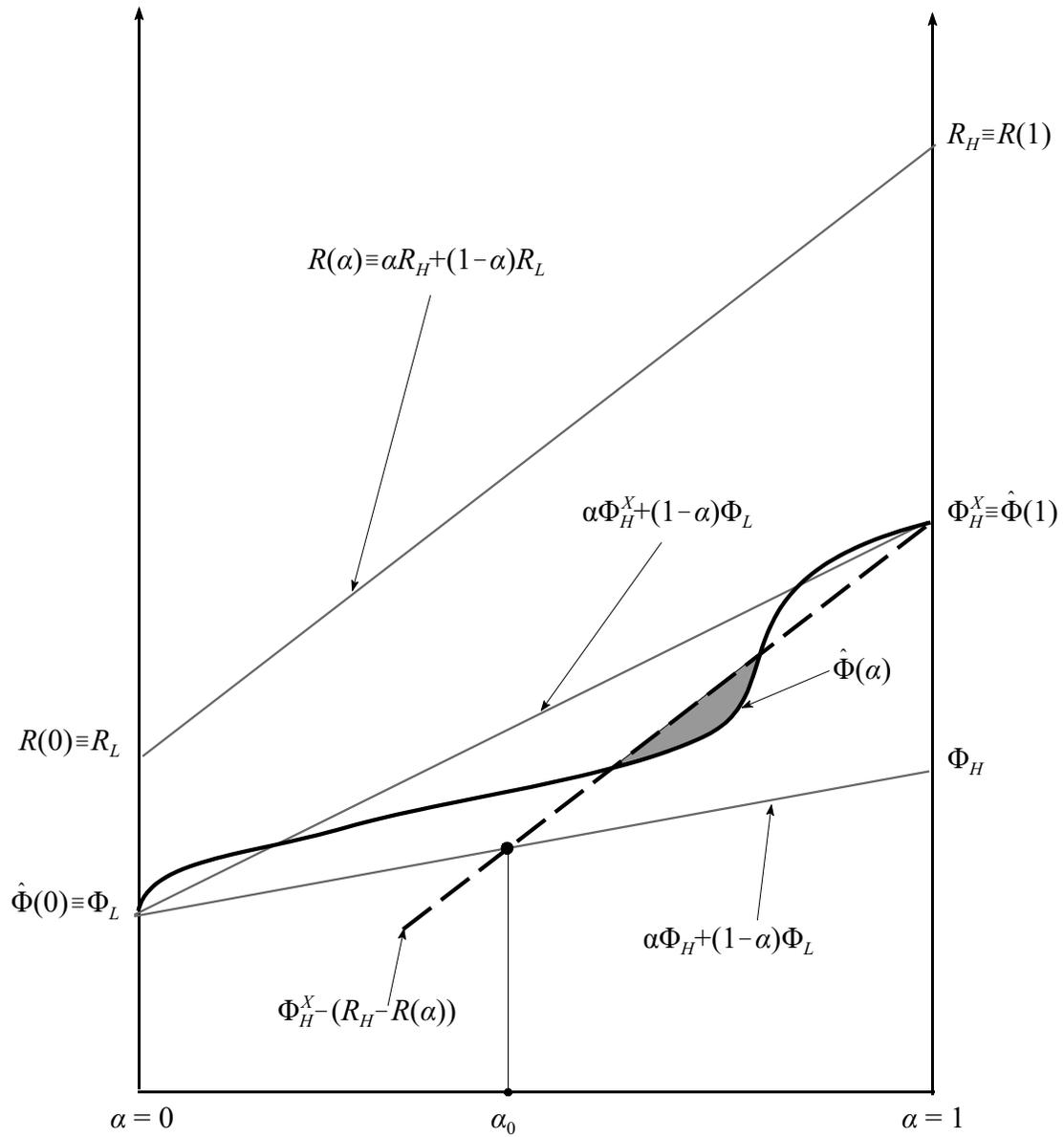
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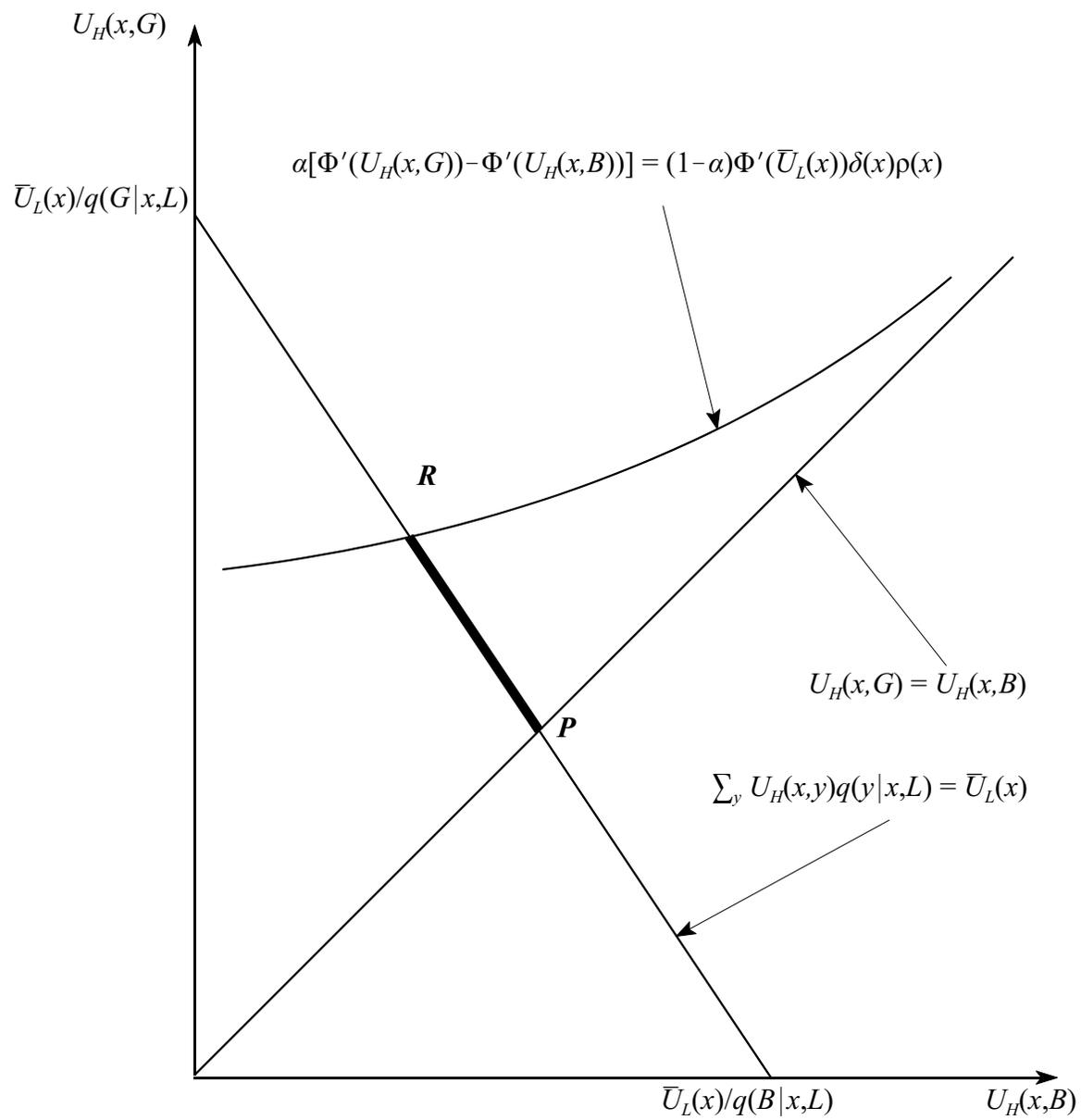
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**Figure 1. Time Line Summarizing the Sequence of the Events in the Model**



**Figure 2. Illustration of the Cost and Benefit Comparison Determining the Usefulness of the Signal**



**Figure 3. Compensation Schemes for a Type-*H* Agent on Outcome Value *x* Fulfilling Constraints SER-1, US-1, and NNG-1 of Program (oRs-1)**