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Fraud Detection and Financial Reporting and Audit Delay

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Andrew Yim[†]

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Abstract

I formulate a model to emphasize the fraud detection role of auditors in the financial market, providing a theoretical framework to examine the likelihood of and market reaction to a financial reporting and audit delay. The model has an auditor considering whether to perform extended audit procedures after observing a red flag generated from regular audit procedures. An audit delay is represented by the event of extending audit procedures and manifested as a financial reporting delay observed by the market. I find that the equilibrium likelihood of a delay decreases when the reliability of regular and extended audit procedures improves and/or when the ex ante probability of fraud reduces. My result on the market reaction to a delay suggests that while a negative average reaction is intuitive and has been documented, the reaction can be positive for an individual firm. I derive a closed-form condition indicating when a positive reaction is possible. Specifically, a delay can be good news to the market when the ex ante probability of fraud, the imprecision of a red flag, and the effectiveness of extended audit procedures for detecting fraud are all high. The result is new in the literature. I also discuss the model's empirical implications with suggestions for regression equation specifications. (*JEL* M42/G32/K42)

Keywords: *Audit delay, financial reporting lag, extended audit procedures, red flag, fraud detection, SAS 82, SAS 99, business ethics.*

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Abstract

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1 Introduction

On June 4, 2009 came the news of the alleged fraud in Countrywide Financial, the largest mortgage loan provider in the US before the credit crunch hit (Morgenson, 2009). The allegation to Countrywide reminds people of a long expected but unfulfilled role of auditors in the financial market: fraud detection. In this paper, I formulate a model to emphasize this role and relate it to financial reporting and audit delays that have long been studied in the literature.¹

Consistent with Statement on Auditing Standards No. 99 (SAS 99), the model has an auditor considering whether to perform extended audit procedures, depending on whether a red flag is observed after regular audit procedures. Plenty of examples suggest that financial reporting delays are often the consequence of extending audit procedures to investigate accounting irregularities.² A financial reporting delay thus suggests to outside investors that a red flag was observed by the auditor and has triggered extended audit procedures.³

The difference between regular and extended audit procedures can be related to the context of the Bagnoli, Kross, and Watts (2002) study. They observe that firms voluntarily disclose in advance the expected earnings announcement dates, and the market reacts adversely to each day of delay. Presumably the extent of regular audit procedures is ascertained during the audit planning phase and should be communicated to a client at the beginning of the whole process. Hence, the extent of regular audit procedures is likely to be a factor affecting the client firm's expectation on when it can make the earnings an-

¹Prior models of fraud detection such as Matsumura and Tucker (1992) have not related the fraud detection role of auditors to financial reporting and audit delays.

²For instance, "Sensormatic Electronics Corp.'s accounting firm has expanded the scope of its annual audit, forcing the company to delay its annual earnings report and raising the possibility of downward revisions to past earnings" (Wall Street Journal, 1995). Another example is Royal Ahold, a Dutch retail group with subsidiaries such as Stop & Shop in the US and Albert Heijn in the Netherlands. "The audit of Ahold's consolidated 2002 has been extended because of a number of new internal investigations at US Foodservice (the unit at the heart of the financial scandal) Deloitte & Touche, which is carrying out the audit, said that, while important progress had been made in these investigations, various delays in their completion had placed the resumption of important parts of the total audit some four to six weeks behind schedule. ... Based on the information received to date, intentional accounting irregularities involving earnings management and misapplications of generally accepted accounting principles were found ..." (Decision News Media SAS, 2003).

³In a conversation with a recently retired Big4 audit firm partner, he confirmed that red flags observed during audits can trigger additional audit work and result in financial reporting delays of clients. He explained that clients constantly give pressure to urge completing the audits as soon as possible but additional audit work, if deemed necessary, is important to auditors' risk management.

nouncement. A delay occurs when the auditor extends audit procedures in response to some red flag observed after performing regular audit procedures. Since the delay is triggered by some red flag, it changes the market's expectation about the existence of fraud in the firm. Consequently, the market might react adversely to the delay. The analysis of this paper formalizes these intuitions. It also yields a new result that is not easy to discover without rigorously analyzing a formal model. Specifically, I show that a *positive* reaction to a delay is possible when it is perceived by market investors as good news under certain circumstances.⁴

The model of this paper has an auditor with a client firm managed by an entrepreneur referred to as an "insider." The insider needs capital to invest in a project. He is interested in raising external capital. Outside investors and the auditor are aware of the possibility that the insider can secretly divert some resources from the firm for his private benefit. The outside investors and auditor therefore behave strategically to guard against the potential (but not immediately noticeable) misappropriation, say, by accepting only properly priced equity offering terms and by extending audit procedures adequately.⁵

In the model, an audit delay is represented by the event of extending audit procedures. Such a delay might not be immediately observable to outside investors in reality. I therefore assume an audit delay is manifested as a financial reporting delay, although the latter is not explicitly modeled here. The first main result of the paper concerns the equilibrium likelihood of a financial reporting and audit delay. I find that the delay likelihood decreases when the reliability of regular and extended audit procedures improves and/or when the prior probability of having a dishonest insider reduces. Intuitively speaking, the more reliable audit procedures are, the lower the chance of observing a false-positive red flag and the higher the chance of catching a dishonest insider. He is thus less aggressive in choosing the extent of diversion. Consequently, the auditor has less incentive to perform extended

⁴In Table 5 of Bagnoli, Kross, and Watts (2002), they report that the mean and median cumulative market-adjusted returns (CARs) of the entire group of late-announcer firms are negative. However, they also document that the third quartile of CARs is positive and growing as the delay lengthens, suggesting that a delay indeed may be perceived by the market as good news.

⁵SAS 99 defines fraud as an intentional act resulting in a material misstatement in financial statements and distinguishes between two types of fraud: (i) those arising from fraudulent financial reporting (e.g., falsification of accounting records); (ii) those from misappropriation of assets (e.g., fraudulent expenditures, or theft of assets). The model of this paper discusses diversion of a firm's resources, which is closer to the latter.

audit procedures frequently. As a result, a delay occurs less often. If the prior probability of having a dishonest insider is lower, it triggers a red flag less often. A delay is thus less often seen.

In this model, outside investors worry about an insider secretly diverting firm resources for his private benefit. The diverted resources will become irrecoverable if the fraud is not discovered timely by the auditor. When outside investors see a financial reporting delay, which in this model is due to an audit delay, they know two things have happened. First, the auditor must have observed some *red flag* of potential fraud; otherwise, extended audit procedures would not have been triggered. So outside investors should revise the expected firm value *downward*. However, an audit delay also means the auditor is doing *extra work*, increasing the chance of discovering a fraud and recovering any diverted resources. For this reason, outside investors should revise the expected firm value *upward*. Whether the overall market reaction is negative or positive depends on the relative magnitudes of these opposing effects.

The key result of the paper is a closed-form condition indicating when a positive market reaction to a delay is possible. If the red flag generated from regular audit procedures is not so informative, a delay may simply be a false alarm, and there is little to worry about by outside investors. When the prior probability of having a dishonest insider is high, a delay resulting from a red flag observed by the auditor can be good. The delay is particularly good when extended audit procedures are very effective in detecting fraud, leaving little chance for a fraud to sneak through. In short, a delay can be good news to outside investors when the ex ante probability of fraud, the imprecision of a red flag, and the effectiveness of detecting fraud are all high.

Besides the main results above, the paper also provides a discussion about the equilibrium relation between outside ownership and audit fee. In this model, the strategic interaction between the auditor and her client leads to an equilibrium audit fee independent of the outside ownership. When the outside ownership is lower, the interest of the insider is better aligned with that of the firm. One might think that the fraud risk should be lower and hence the auditor would not extend audit procedures so often. Extra costs that constitute the audit fee would be incurred less frequently, presumably resulting in a lower audit fee.

This however is not the complete story. When the auditor worries less because of lower outside ownership, she has a weaker incentive to extend audit procedures. Anticipating a lower probability of extended audit, a dishonest insider worries less about being caught and has a stronger “induced” incentive to divert resources. In the end, the two effects balance out and have no net impact on the equilibrium audit fee in this model.

Despite the lack of a causality running from outside ownership to audit fee, they are positively associated with each other. The association is due to both a direct and an indirect effect. The direct effect is that the payout of a higher audit fee results in a lower year-end firm value. Hence, outside investors demand more favorable equity offering terms up front, in terms of higher outside ownership for any given level of capital raised. In addition, exogenous factors like the reliability of regular and extended audit procedures affect both the audit fee and the diversion rate chosen by a dishonest insider. Any changes in such factors that lead to a higher diversion rate also raise the audit fee. Anticipating a higher diversion rate, outside investors again demand more favorable equity offering terms up front. This second effect is an indirect way for outside ownership to be associated with the audit fee positively.

This paper makes three contributions. First, I formulate a model to provide a theoretical framework to examine the likelihood of and market reaction to a financial reporting and audit delay. Empirical studies have examined such issues extensively.⁶ However, rarely have the studies developed hypotheses for testing using any formal theoretical model. Prior analytical models on financial reporting timeliness follow the voluntary disclosure literature, assuming a financial reporting delay is a voluntary, strategic decision (e.g., Gennotte and Trueman 1996). These models are completely different from mine that emphasizes the fraud detection role of auditors in the financial market and the involuntariness of a financial reporting delay arising from an audit delay.

Second, my result on the market reaction to a delay suggests that while a negative average

⁶Research in the audit delay literature includes Lambert, Brazel, and Jones (2008), Ettredge, Li, and Sun (2006), Johnson, Davies, and Freeman (2002), Knechel and Payne (2001), McLelland and Giroux (2000), Bamber, Bamber, and Schoderbek (1993), Dwyer and Wilson (1989), and Ashton, Willingham, and Elliott (1987). Studies investigating the determinants and effects of financial reporting timeliness include Owusu-Ansah and Leventis (2006), Sengupta (2004), Begley and Fischer (1998), Alford, Jones, and Zmijewski (1994), Chambers and Penman (1984), and Givoly and Palmon (1982). Krishnan and Yang (2009) study audit report lags as well as earnings announcement lags together.

reaction is intuitive and has been documented by empirical studies, the reaction does not have to be negative for an individual firm. I derive a closed-form condition indicating when a positive market reaction to a delay is possible. This is a new result not previously discussed in the literature.

Besides relating the fraud detection role of auditors to financial reporting and audit delays, another main objective of the paper is to address the need of empirical researchers. This is done by deriving theoretical results in closed form that give clearer guidance on empirical research designs. To serve this purpose, more specific functional forms and exogenous parameters are used in the model to capture the fundamental first-order effects of relevant economic forces. I provide a detailed discussion of the empirical implications of the model, including suggestions for regression equation specifications. With this, I open a dialog with empirical researchers and stimulate closer interaction between empirical and analytical researchers. This is the third contribution of the paper.

The rest of the paper is organized as follows. The next section briefly reviews the background of SAS 99 that has inspired the model of this paper. The model setup is described in section 3, with different parties' equilibrium decisions analyzed in section 4. Empirical-oriented readers can skip section 4 and jump directly to section 5, where I discuss the model's empirical implications. Such readers can go back to the preceding section for a glance of the analysis when doing so is necessary to understand the empirical implications. Section 6 provides a discussion relating this paper to other studies. The paper concludes with remarks in section 7. An illustrative example of an empirical test design is given in appendix A. Technical proofs and derivations are relegated to appendix B.

2 Background

The debate on the fraud detection role of auditors in the financial market has a long history. It was initially framed by the accounting profession as an expectation gap issue. Advocates argued that the assumed integrity of the management is a necessary starting point for an audit engagement. This debate cooled down slightly in the 90's. It has gained attention again following the collapse of Arthur Andersen, one of the biggest five accounting firms in

the world. Its collapse was closely tied to the accounting scandals of a number of companies, including the telecommunication giant WorldCom and especially the energy giant Enron.

The American Institute of Certified Public Accountants (AICPA) first officially emphasized the fraud detection role of auditors in the release of SAS 82 in 1997. This statement superseded SAS 53 *The Auditor's Responsibility to Detect and Report Errors and Irregularities* released in 1988. Entitled *Consideration of Fraud in a Financial Statement Audit*, SAS 82 is the first SAS mentioning fraud in the title. The then Securities and Exchange Commission (SEC) Chief Accountant Michael H. Sutton praised SAS 82 for “referring to fraud by name, for the first time, instead of by the more euphemistic term ‘irregularities’ and for describing over 40 fraud risk factors that auditors must consider in planning and performing an audit.” (*Journal of Accountancy*, Feb. 1997)

In October 2002, partly in response to the accounting scandals and the Sarbanes-Oxley Act (SOX) that followed, the AICPA released SAS 99. This statement superseded SAS 82 but with the same title. The updated standard provides more guidelines on how auditors should structure an audit plan to better serve the fraud detection function. According to the standard, auditors are required to “consider *whether [misstatements identified by audit test results] may be indicative of fraud*. ... If the auditor believes that the misstatement is or may be the results of fraud, and either has determined that the effect could be material to the financial statements or has been unable to evaluate whether the effect is material, the auditor *should ... [a]ttempt to obtain additional audit evidence* to determine whether material fraud has occurred or is likely to have occurred, and, if so, its effect on the financial statements and the auditor’s report thereon.” (AU Section 316, paragraphs 75 and 77; emphasis added.)

In short, this standard requires an auditor to consider whether misstatements identified by audit test results may be indicative of fraud and if so, the auditor should attempt to obtain additional audit evidence to clarify the situation. Considering red flags from audit test results and obtaining additional audit evidence are decisions auditors should think about carefully in assessing fraud risks. They are also two important elements of the model formulated in the next section to understand the fraud detection role of auditors in the financial market.

3 A Model of Extended Audit for Fraud Detection

The model formulated here has a setup similar to the Newman, Patterson, and Smith (2005) (hereafter NPS) modification of the Shleifer and Wolfenzon (2002) model. I depart from the NPS model mainly in the strategic consideration of extending audit procedures contingent on the red flag observed, if any.⁷ In the following, I first outline the basic setup, followed by a description of the sequence of events in the model.

The model has an auditor with a client firm managed by an entrepreneur, or “insider” in NPS’s terminology. The insider needs capital to invest in a project with a known rate of return g on any invested amount. He is interested in raising an endogenously determined amount of external capital, denoted by K , to add to the exogenously endowed wealth he has, denoted by W , to invest in the project. Outside investors and the auditor are aware of the possibility that he can secretly divert some resources from the firm for his private benefit. As a result, the firm value suggested by the firm’s financial statements actually overstates the underlying true value. The outside investors and auditor therefore behave strategically to guard against the potential (but not immediately noticeable) misappropriation, say, by accepting only properly priced equity offering terms and by extending audit procedures adequately. All parties in the model are risk neutral.

Below is the sequence of events in the model:

1. The nature determines the type of the insider: honest or dishonest. The prior probability of having a dishonest insider is θ , where $0 < \theta < 1$. An honest insider never diverts the firm’s resources, whereas a dishonest one will make the diversion decision strategically.⁸

⁷Patterson and Smith (2007) have analyzed a model with a “two-round” audit similar to the one examined here. They study the effects of SOX on an auditor’s choice of effort in performing control tests (the “first-round” audit) before doing substantive tests (the “second-round” audit). The auditor in their model always performs both rounds of audit. Therefore, their setup is more like a refined modeling of the regular audit procedures of my model, which are performed unconditionally. By contrast, the auditor in my model may or may not perform extended audit procedures, depending on the outcome of regular audit procedures.

Patterson and Smith’s model, however, is more general in the sense that they allow audit choices in both rounds, which lets them address the issues in their paper. Smith, Tiras, and Vichitlekarn (2000) analyze the interaction between internal control assessments and substantive testing using a similar model also with audit choices in both rounds. In contrast, I simplify the “two-round” audit setup by allowing an audit choice only in the second round. This facilitates deriving closed-form results that are useful for guiding empirical hypotheses for testing.

⁸It is not new in the economics literature to make the parsimonious assumption of two exogenous types of

2. The insider simultaneously chooses a non-contingent audit fee, F , the amount of external capital to raise, K , and the fraction of ownership to sell, λ . For simplicity, only pure strategies in choosing F , K , and λ that constitute the pooling equilibrium is considered here.⁹ The choice of F , K , and λ is constrained by competitive audit and financial market conditions. A competitive audit market requires that the audit fee F leaves an auditor zero expected profit. Moreover, a competitive financial market requires that outside investors' total cost of investment is equal to the value of the investment.¹⁰ Further details of this condition will be provided shortly.
3. The insider offers the audit fee F , payable after the audit, to an auditor to secure her services for a year-end audit.¹¹ The insider also offers the equity offering terms K and λ to outside investors. Given competitive audit and financial market conditions and anticipating the insider's equilibrium choices, the auditor accepts the audit engagement at the fee offered, and outside investors accept the equity offering terms.
4. The insider invests $K + W$ in the project, earns the return, and chooses the proportion δ of the year-end (after-audit-fee) firm value $\Pi = (1 + g)(K + W) - F$ to divert.
5. At the end of the year, the auditor follows standard practice to perform regular audit procedures. For simplicity, the cost of these procedures is normalized to zero. The

players with one being "irrational." An example is the widely cited paper by Kreps, Milgrom, Roberts, and Wilson (1982). Another example is Erard and Feinstein (1994), of which the authors assume that there is an exogenous fraction of inherently honest taxpayers. This assumption is made to cope with some perplexing empirical facts often described as the "Why people pay tax" puzzle in the tax compliance literature.

Accounting studies have also recognized that some people do behave honestly when they should behave strategically given the monetary incentives. A widely cited example is Evans, Hannan, Krishnan, and Moser (2001). Mounting evidence of similar "irrational" behaviors have been documented in the experimental economics literature, with the ultimatum game being an often cited example (see Camerer and Thaler 1995).

⁹A separating equilibrium cannot exist. If an honest insider had a way (e.g., by choosing a high reputation auditor) to separate himself from a dishonest insider, the auditor and outside investors would not require "premiums" in the audit fee and equity offering terms to price-protect themselves, and the auditor would never extend audit procedures. Given competitive financial and audit markets, these terms obtained by a honest insider must be more favorable than what can be obtained by a dishonest insider in such a hypothetical separating "equilibrium." Consequently, a dishonest insider is always better off by mimicking whatever a honest insider does to pool with him. This invalidates the supposition that a separating equilibrium exists.

¹⁰Here the "competitive financial market" condition means outside investors are under perfect competition to provide capital to the firm. It does not mean outside investors themselves face a capital supply curve that is perfectly elastic.

¹¹That the audit fee is payable after the audit is an inessential difference from NPS's setup. This change, however, is more consistent with the incentive of the insider to grow the "pie" (firm value) as large as possible, regardless of his incentive to divert resources from the firm.

procedures may result in a red flag for potential fraud. The chance of a *false positive* (i.e., red flag on despite no fraud) is $p = \Pr\{\text{red flag} \mid \text{no fraud}\} > 0$.¹² For simplicity, the chance of a *false negative* (i.e., red flag off despite fraud) is assumed to be zero, i.e., $n = \Pr\{\text{no red flag} \mid \text{fraud}\} = 0$. I require that the red flag is an informative signal of potential fraud so that false positives are less likely than true positives, i.e., $p < 1$.¹³

6. Conditional on whether a red flag is observed, the auditor makes the decision x on extending audit procedures ($x = 1$) or not ($x = 0$). She incurs an extra cost C in extending audit procedures.
7. The auditor cannot give a qualified opinion unless audit evidence supports the opinion. More definite, though not completely conclusive, evidence is available only when extended audit procedures are performed. So the auditor can only issue an unqualified opinion if extended audit procedures have not been performed.
8. With probability $q > 0$, the evidence obtained from extended audit procedures is a true positive (i.e., proving the existence of fraud). While with probability $1 - q$ the evidence can also be a false negative (i.e., suggesting no fraud despite fraud), it can never be a false positive (i.e., showing fraud despite no fraud).¹⁴
9. The audit opinion is issued based on whether evidence of fraud has been obtained.¹⁵

¹²The assumption of a non-zero probability of a false positive is consistent with the observation that “while symptoms of fraud (“red flags”) are observed frequently, the presence of such issues is not necessarily indicative of fraud” (see Hogan, Rezaee, Riley, and Velury, 2008).

¹³The results of the paper are essentially unchanged under the more general assumption of $n \geq 0$ with the *false-to-true-positive likelihood* ratio $\frac{p}{1-n} < 1$. This likelihood ratio requirement is equivalent to the monotone likelihood ratio property (MLRP) often assumed in moral-hazard agency models. To see this, first note that the relation between the existence of fraud and the observation of a red flag is analogous to that between a binary action and a binary outcome in a moral-hazard setting. Thus, the following is equivalent to the MLRP:

$$\frac{\Pr\{\text{red flag} \mid \text{fraud}\}}{\Pr\{\text{red flag} \mid \text{no fraud}\}} > \frac{\Pr\{\text{no red flag} \mid \text{fraud}\}}{\Pr\{\text{no red flag} \mid \text{no fraud}\}},$$

which can be written as $\frac{p}{1-n} < 1$.

¹⁴NPS assume the auditor does only a one-round audit with the fraud detection probability q depending on the amount of resources invested in the audit. In contrast, the auditor here always performs regular audit procedures, with the possibility of also performing extended audit procedures. To focus on this unique aspect of the model, I simplify other aspects with assumptions like an exogenous q . Closed-form results can also be obtained without affecting the fundamental structure of the equilibrium if alternatively q is proportional to the diversion rate δ .

¹⁵This assumes the auditor follows the GAAS to document her work properly and finds it prohibitively costly to commit a criminal offense by ignoring evidence of fraud.

10. If fraud is discovered, the insider must return the diverted resources and also bear a penalty equivalent to a monetary fine proportional to the amount of resources diverted, i.e., $b\delta\Pi$, where $b > 0$. The parameter b indicates the severity of the penalty to an insider committing fraud, as in NPS's model. If fraud exists but is not discovered by the auditor, it will nonetheless have a non-zero chance to be discovered in the future, resulting in some liability cost to the auditor. The present value of the expected liability cost is assumed to be proportional to the diversion rate, i.e., $a\delta$, where $a > 0$. The parameter a is referred to as the penalty multiplier for the auditor, as in NPS's model.¹⁶

The model structure and parameters are common knowledge to the insider, outside investors, and auditor. The timeline in Figure 1 summarizes the sequence of events in the model.

When the auditor decides whether to take the offer F for the audit engagement and subsequently determines whether to extend audit procedures ($x = 1$) or not ($x = 0$), she knows the insider's choice of K and λ , which are set to ensure outside investors are not worse off by investing in the firm. The auditor does not observe the privately chosen diversion rate δ but will make decisions based on an equilibrium conjecture about it. The insider chooses δ after securing the auditor but before she comes in to do the year-end audit. When he makes the diversion decision, he knows everything except for whether the auditor will observe a red flag and her decision on x that follows. Such information about the red flag and extended audit are not known to outside investors either when making the investment decision in response to the insider's equity offering terms. Nor do they observe F and δ .¹⁷ They nonetheless make the decision based on equilibrium conjectures about choices not known to them at that time. Anticipating equilibrium behaviors of other parties and himself, the insider chooses F , K , and λ at the very beginning.

To detail the competitive financial market condition, let $R(K)$ denote outside investors' *total* cost of arranging the capital to invest in the firm. For example, if $R(K)$ is the total cost

¹⁶Like NPS, I make the simplifying assumption that the insider's and auditor's expected penalties are deadweight losses unrelated to outside investors' payoff.

¹⁷The assumption that outside investors cannot observe the audit fee *at the moment of* making the investment decision is consistent with the US regulatory requirement of disclosing the audit fee *after* the year end. Whether the audit fee is observable actually is inessential here because in equilibrium all players can correctly anticipate the audit fee as though it was observable.

(i.e., principal plus interest) of getting K dollars from banks, $\frac{R(K)}{K} - 1$ would be the borrowing interest rate. It is thus natural to assume that $R(K) \geq K$ and $R(0) = 0$. Although strictly speaking the “cost of capital” for outside investors to arrange K dollars should be $\frac{R(K)}{K} - 1$, I will simply call R the cost of capital function.¹⁸ A competitive financial market requires that the equity offering terms K and λ leave outside investors indifferent between investing or not investing in the firm. That is to say, the total cost of investment, $R(K)$, equals the value of the investment, i.e., the fraction λ of the expected year-end (after-audit-fee) firm value.

To ensure that the insider’s equilibrium choice of equity offering terms exists and is an interior solution characterized by the first-order condition of maximization, I assume that $R' > \frac{R}{K} > 1$ for all $K > 0$, with $R'(0) < (1+g)(1-\theta)$ and $\lim_{K \uparrow \infty} \frac{R(K)}{K} > 1+g$.¹⁹ In words, these require, firstly, the marginal cost of capital R' to be greater than the average cost of capital $\frac{R}{K}$ (say, because it is increasingly more costly to borrow more money). Secondly, $\frac{R}{K} > 1$ means it is not costless to borrow money. The assumption $R'(0) < (1+g)(1-\theta)$ is satisfied if it costs nearly nothing to borrow a tiny amount of money. Finally, $\lim_{K \uparrow \infty} \frac{R(K)}{K} > 1+g$ is met if the cost of capital becomes prohibitively high when the amount borrowed is enormous.

For the model to be interesting, the audit procedures have to be *sufficiently reliable*. Specifically, this means $q > \frac{1}{1+b}$, which requires that the probability q of detecting a fraud that exists is sufficiently high. The probability has to be high enough such that given the severity of the penalty to a fraudster, represented by b , the insider would rather not divert

¹⁸ Assumptions on $R(K)$ to be stated shortly capture the fact that no individual nor the whole economy can provide an unlimited amount of capital at a constant average cost (e.g., at a constant borrowing interest rate charged by banks), even though everyone in the model is assumed to be risk neutral. NPS simply assume $R(K) = K$. I do not make this simplifying assumption because it can lead to non-existence of equilibrium in my model.

¹⁹ These assumptions are met if the cost of capital function is as follows:

$$R(K) = \frac{e^{\rho(1+r)K} - 1}{\rho}$$

with $\rho > 0$, $r > 0$, and $1 - \theta > \frac{1+r}{1+g}$. This functional form subsumes, as a limiting special case, the usual specification of $(1+r)K$, which is assumed in the NPS model.

It is common in economic analysis to make certain technical assumptions to ensure the existence of an interior solution. The reason is that many analytical techniques are based on the first-order condition, which does not hold if an optimization problem only has corner solutions. This important issue sometimes is not given sufficient attention in accounting literature, and the required technical assumptions are not made explicit. This paper, however, makes explicit such assumptions.

any resources if anticipating $x = 1$ with certainty. Otherwise, it would be in the auditor's interest to always extend audit procedures without conditioning on the observation of a red flag. The model would become indistinguishable from the setting already studied by Patterson and Smith (2007).

It will be clear shortly that $[1 + p(\frac{1}{\theta} - 1)]^{-1}$ is the posterior probability of the existence of a fraud after observing a red flag. Given that a fraud exists, the probability of detecting it is q . Note that a is essentially the auditor's expected liability cost of not detecting a fraud with a diversion rate of $\delta = 1$. Therefore, the condition

$$C < \frac{aq}{1 + p(\frac{1}{\theta} - 1)}, \quad (1)$$

referred to as *affordable audit cost*, ensures that after observing a red flag, extending audit procedures reduces the expected liability cost related to a "full-diversion" fraud by an amount that exceeds the extra cost of audit. This condition is sufficient to deter "full diversion," preventing the uninteresting corner case of $\delta = 1$ to constitute an equilibrium. Throughout my analysis, I assume the condition holds.

To simplify the analysis, I also assume that the insider has *sufficient wealth* to invest in the project so that at the year end he has enough firm resources to pay for the audit fee, even without raising any external capital in the very beginning. Specifically, this means

$$(1 + g)W > a\varphi, \quad (2)$$

where

$$\varphi = a^{-1} \left(\frac{C}{q} \right) [p + (1 - p)\theta]. \quad (3)$$

It will be clear shortly that the $a\varphi$ on the right hand side of (2) is equal to the equilibrium audit fee determined in the model. The constant φ summarizes the effects of several parameters of the model and is larger when (i) the penalty multiplier a that constitutes the auditor's expected liability cost is smaller; (ii) the "unit" audit cost C/q of the extended audit procedures is larger; (iii) the false-positive probability p that captures the imprecision of

the red flag as an early warning is higher; (iv) the prior probability, θ , of having a dishonest insider is higher. I will come back to the intuitive interpretation of the constant φ later.

In the next section, I derive the perfect Nash equilibrium of the model using backward induction. I start with the analysis of the auditor's extended audit decision and the insider's diversion decision. The analysis of the insider's choice of the audit fee, the amount of capital to raise, and the fraction of ownership to sell then follows. Because separating equilibrium strategies do not exist (see footnote 9), an honest insider will simply select the audit fee and equity offering terms that would be chosen by a dishonest insider. Therefore, I only need to analyze the decisions from the angle of a dishonest insider.

4 Equilibrium of the Model

4.1 Diversion and Extended Audit Decisions

After observing a red flag, the auditor weighs the incremental benefit against the incremental cost to decide whether it is gainful to extend audit procedures. Given any conjecture δ on a dishonest insider's diversion decision, the incremental benefit of extending audit procedures is a reduction in expected liability cost equal to $a\delta q\Pr\{\text{fraud}|\text{red flag}\}$. In equilibrium, this benefit must be equal to the incremental cost C .

The equilibrium diversion rate chosen by a dishonest insider is given by the condition below:

$$\delta^* = \frac{C}{aq\Pr\{\text{fraud}|\text{red flag}\}}. \quad (4)$$

Since an honest insider never diverts any resources, the prior probability of fraud is simply the prior probability of having a dishonest insider. By the Bayes' rule,

$$\Pr\{\text{fraud}|\text{red flag}\} = \frac{1}{1 + p(\frac{1}{\theta} - 1)}. \quad (5)$$

Hence,

$$\delta^* = \frac{C/q}{a [1 + p(\frac{1}{\theta} - 1)]^{-1}}. \quad (6)$$

This expression of the equilibrium diversion rate shows clearly its determinants. They are (i) the “unit” audit cost C/q of the extended audit procedures; (ii) the penalty multiplier a for the auditor when a fraud was not detected; (iii) the reliability of the red flag as an indicator of fraud, captured by the posterior probability of having a dishonest insider, i.e., $[1 + p(\frac{1}{\theta} - 1)]^{-1}$.

Given the equilibrium conjecture δ^* , the auditor is indifferent between choosing $x = 1$ or $x = 0$. Let ω denote her randomized strategy of extending audit procedures after observing a red flag, i.e., $\omega = \Pr\{x = 1|\text{red flag}\}$.²⁰ The auditor will not extend audit procedures unless a red flag is observed. Thus, the only way to detect a fraud that exists is to follow through the sequence of observing a red flag, extending audit procedures, and finding evidence of fraud. In other words, a dishonest insider knows that the chance of being caught is $q\omega$.

If a dishonest insider successfully diverts a fraction of the firm’s resources without being detected, his payoff is the sum of (i) diverted resources that he can enjoy personally, $\delta\Pi$, and (ii) his share of the firm value after the diversion, $(1 - \lambda)(1 - \delta)\Pi$. However, if the diversion is detected by the auditor, he must return the diverted resources and bear the penalty. His payoff becomes $(1 - \lambda)\Pi - b\delta\Pi$. Given any conjecture ω , the expected payoff of a dishonest insider selecting a diversion rate δ is as follows:²¹

$$[\delta\Pi + (1 - \lambda)(1 - \delta)\Pi](1 - q\omega) + [(1 - \lambda)\Pi - b\delta\Pi]q\omega. \quad (7)$$

In order for δ^* given by (6) to constitute an equilibrium, the diversion rate must satisfy the first-order condition below:

$$\lambda(1 - q\omega) = bq\omega. \quad (8)$$

²⁰Modeling this decision of the auditor as a randomized strategy is consistent with the highly stochastic nature of individual-level economic choices now widely recognized in the literatures of psychology, decision theory, experimental economics, and accounting (see e.g. Rieskamp 2008, Wilcox 2009, Loomes 2005, and Fischbacher and Stefani 2007).

²¹Like NPS, I assume that if the fraud is not detected by the auditor, the diverted resources will not be recoverable, say, because the dishonest insider will flee from the country.

This condition uniquely determines the auditor's equilibrium randomized strategy of extending audit procedures, characterized by

$$\omega^* = \frac{\lambda}{(\lambda + b)q}. \quad (9)$$

In the following subsections, I continue the analysis by determining the equilibrium audit fee, capital to raise, and fraction of ownership to sell.

4.2 Audit Fee

The auditor's expected profit prior to performing regular audit procedures is

$$F - \{C\omega[\theta + (1 - \theta)p] + a\delta\theta(1 - q\omega)\}. \quad (10)$$

The first term inside the curly brackets is the expected cost of extending audit procedures. The cost is incurred only when a red flag is observed, which happens with probability $\theta + (1 - \theta)p$, followed by the outcome $x = 1$, which occurs with probability ω . The second term inside the curly brackets is the expected liability cost. The auditor will bear the liability cost $a\delta$ if a fraud exists but is undetected during the audit. This has a chance of $\theta(1 - q\omega)$ to occur.

Given a competitive audit market and anticipating the insider's and auditor's equilibrium choices, the equilibrium audit fee F^* will be set such that the auditor in equilibrium earns zero expected profit. In other words, the fee is determined by substituting the equilibrium ω^* and δ^* into (10) and then setting it to zero to solve for the equilibrium F^* . This is the audit fee offered by the insider and accepted by the auditor in equilibrium:

$$F^* = a\delta^*\theta, \quad (11)$$

where δ^* is given by (6).

The equilibrium audit fee does not depend on ω^* because in equilibrium δ^* is chosen to make the auditor indifferent between extending audit procedures or not. Since the cost of performing regular audit procedures has been normalized to zero, the audit fee only needs

to be high enough to cover the auditor’s expected liability cost, as though she had never considered extending the audit procedures.

Substitute δ^* into (11), the equilibrium audit fee becomes

$$F^* = \left(\frac{C}{q}\right) [p + (1 - p)\theta]. \quad (12)$$

The false-positive probability p measures the imprecision of the red flag as an early warning. Ideally one would like regular audit procedures to have a low p so that false positives occur rarely. In other words, the red flag is more informative when p is smaller. Consequently, the decision on extending audit procedures can be made more efficiently. The equilibrium audit fee can thus be lower. Similarly, F^* is lower when the extended audit procedures are more effective in detecting fraud, i.e., q is higher. In a society with a higher moral standard, people are more likely to be honest, i.e., θ is lower, which results in a lower equilibrium audit fee.

It is useful to recognize that $F^* = a\varphi$ and $\varphi = \delta^*\theta$, where the constant φ is defined in (3). Note that $\varphi = F^*/a$ is between 0 and 1. This ratio may be interpreted as a *fraud risk premium* for getting the auditor engaged in providing the “assurance” services. To see why, consider the limiting case where both θ and δ^* are nearly 1. The auditor in equilibrium is indifferent between performing and not performing extended audit procedures. She anticipates that her expected payoff would be equivalent to that without extending the audit procedures. Since θ and δ^* are nearly 1, it is almost certain that she would need to bear the “full diversion” liability cost, i.e., a . Therefore, she would not accept the audit engagement unless the audit fee F^* is as much as a . In general, the audit fee is set according to the anticipated fraction of the “full diversion” liability cost to be paid by the auditor. This fraction is expected to be lower if the fraud risk facing the auditor, captured by θ and δ^* , are lower.

A priori, one might expect that through $\omega^* = \frac{\lambda}{(\lambda+b)q}$, the audit fee F^* should be affected by the ownership sold to outside investors, λ . This endogenous variable may be seen as a measure of the misalignment of interest between the insider and other shareholders. If $\lambda = 0$, the insider is the only shareholder of the firm and has no incentive to divert resources.

The diversion incentive is strongest when $\lambda = 1$. In this model, the strategic interaction between the auditor and her client can result in an equilibrium audit fee independent of λ . When λ is lower, one might think that the fraud risk should be lower and hence the auditor would not extend audit procedures so often. Extra costs would be incurred less frequently, resulting in a lower audit fee. This however is not the complete story. When the auditor worries less because of a lower λ , she has a weaker incentive to extend audit procedures. Therefore, a dishonest insider worries less about being caught and has a stronger “induced” incentive to divert resources. In the end, the two effects balance out and have no net impact on the equilibrium audit fee.²²

I summarize the observations about the equilibrium audit fee as the following proposition.

Proposition 1 (Effects on equilibrium audit fee). *The equilibrium audit fee F^* decreases when (i) the reliability of regular and/or extended audit procedures improves, i.e., p is lower and/or q is higher, or (ii) the chance of having a dishonest insider reduces, i.e., θ is lower. Formally,*

$$\frac{\partial F^*}{\partial p} > 0, \quad \frac{\partial F^*}{\partial q} < 0, \quad \text{and} \quad \frac{\partial F^*}{\partial \theta} > 0.$$

4.3 Capital Raised and Ownership Sold

Given any conjectures about ω , δ , and F , outside investors’ expected benefit from accepting the insider’s equity offering terms is

$$\lambda(1 - \delta)\Pi\theta(1 - q\omega) + \lambda\Pi[1 - \theta(1 - q\omega)], \quad (13)$$

²²The discussion here cautions against regression analysis that uses an endogenous variable like λ to explain another endogenous variable like F . Ideally, equations specified for regression analysis should have only exogenous variables as explanatory variables and endogenous variables as dependent variables.

If the model here is used as a framework to interpret empirical findings, any statistical relationship discovered between outsider ownership and audit fee suggests one of the following: (i) the model is inadequate; (ii) the relationship is spurious; (iii) the causality runs from F to λ . I will in section 5.1 show that the last possibility is consistent with the model.

where $\Pi = (1+g)(K+W) - F$ is the year-end (after-audit-fee) firm value.²³ The probability $\theta(1-q\omega)$ in the first term above is the chance of both having a fraud (i.e., dishonest insider) and not having it detected during the audit. If this event occurs, the value of the shares owned by outside investors is only $\lambda(1-\delta)\Pi$. If the insider is honest, or if the fraud of a dishonest insider is detected during the audit, outside investors have a claim on the undiverted firm, which is the $\lambda\Pi$ in the second term above.

A competitive financial market implies that in equilibrium K and λ will be chosen to equate the expected benefit in (13) with outsider investors' total cost of arranging the capital, i.e. $R(K)$. This condition is expressed as the equation below:

$$R(K) = \mu[(1+g)(K+W) - F], \quad (14)$$

where

$$\mu = \lambda(1 - \Delta) \quad (15)$$

with $\Delta = \delta\theta(1-q\omega)$ referred to as the *effective* diversion rate. This rate Δ differs from the diversion rate δ because there is a chance that the insider is honest. Even if he is dishonest, diversion may be detected during the audit and therefore unsuccessful. Given the effective diversion rate Δ , it is intuitive to refer to μ as the *effective* ownership sold to outside investors (or simply effective outside ownership).

By choosing the effective outside ownership, μ , a dishonest insider indirectly chooses the fraction of ownership sold, λ , and the amount of capital that can be raised, K . To see more clearly how these variables are related to each other, consider the solutions $K(\mu)$ and $\lambda(\mu)$ of the equations below, which are (14) and (15) evaluated at the equilibrium F^* , δ^* , and ω^* :

$$R(K) = \mu\Pi(K), \quad (16)$$

$$\mu = \lambda [1 - \varphi(1 - q\omega^*)], \quad (17)$$

²³As in the NPS model, I assume that unless a fraud is discovered during the year-end audit, the diverted resources are unrecoverable even though the fraud may be eventually discovered some time in the future. Any future attempt to recover the diverted resources, including filing a lawsuit against the auditor, will end up in a negligible net gain to outside investors. It is thus omitted here.

where $\Pi(K) = (1 + g)(K + W) - a\varphi$, $\varphi = a^{-1} \left(\frac{C}{q} \right) [p + (1 - p)\theta]$, and $\omega^* = \lambda/[(\lambda + b)q]$.²⁴ The second equation above can also be expressed as

$$\lambda^2 + b(1 - \varphi)\lambda = \mu(\lambda + b). \quad (18)$$

Recall that given the equilibrium probability ω^* of extending audit procedures, a dishonest insider is indifferent among alternative choices of the diversion rate. Therefore, in deciding the fraction of ownership λ to sell, his concern is simply to maximize the value of his retained ownership,

$$(1 - \lambda)\Pi(K), \quad (19)$$

as though he had no plan to divert resources. Through (16) and (18), choosing an optimal effective outside ownership, μ^* , also determines the optimal fraction of ownership to sell, λ^* , and the optimal amount of capital to raise, K^* . A complete characterization of the optimal μ^* , λ^* , and K^* is given in Lemma 3 stated and proved in the appendix.

In the next section, I examine some implications of the model, namely how the reliability of regular and extended audit procedures and the ex ante probability of fraud may affect the likelihood of and market reaction to a financial reporting and audit delay.

5 Delay Likelihood and Market Reaction

5.1 Effects on Delay Likelihood

The empirical literature on financial reporting and audit delays has a long history (see footnote 6). However, rarely have such studies developed empirical tests using a formal theoretical model. In the following I explore some implications of my model, with the intention to develop empirically testable hypotheses not yet investigated in the literature.

I focus on two interesting aspects that have been repeatedly examined in the literature,

²⁴ $K(\mu)$ exists because of the following reasons: First of all, the assumption of sufficient wealth, i.e., $(1 + g)W > a\varphi$, implies $R(K)/\Pi(K) > 0$. Note that $R(K)/\Pi(K) = 0$ for $K = 0$. Moreover, $R(K)/\Pi(K) > 1$ for sufficiently large K . Since $R(K)/\Pi(K)$ is continuous in K , there exists a bounded $K(\mu)$ solving $R(K)/\Pi(K) = \mu$ for any given μ between 0 and 1.

To see the existence of $\lambda(\mu)$, simply note that given the assumptions I make, the quadratic equation (18) has only one root between 0 and 1 for any given μ between 0 and 1.

namely the determinants and the effects of financial reporting and audit delays.

Consider first the likelihood of an audit delay, manifested as a financial reporting delay. The delay occurs if and only if the event $\{x = 1\}$ happens, i.e., the auditor has chosen to extend audit procedures. Conditional on a red flag, the event happens with probability ω^* . From outside investors' perspective, a red flag will be observed if (i) the insider is dishonest, or (ii) he is honest but the red flag is a false positive. Case (i) has a chance of θ to occur, and case (ii) a chance of $p(1 - \theta)$. Thus the equilibrium likelihood of a delay is

$$\Pr\{x = 1\} = \omega^*[\theta + p(1 - \theta)] \quad (20)$$

$$= \frac{\lambda^* [p + (1 - p)\theta]}{(\lambda^* + b)q}. \quad (21)$$

To see clearly what determine the likelihood, it is convenient to look at the log transformation of the relation above:

$$\log\Pr\{x = 1\} = \log\left(\frac{\lambda^*}{\lambda^* + b}\right) + \log[p + (1 - p)\theta] - \log q. \quad (22)$$

If the equilibrium outside ownership λ^* could be held constant,²⁵ then the delay likelihood is higher when the ex ante probability of fraud is higher (i.e., θ is higher), the imprecision of a red flag is higher (i.e., p is higher), and the effectiveness of extended audit procedures for fraud detection is lower (i.e., q is lower). A higher ex ante probability of fraud triggers a red flag more often. It is thus more likely to see a delay. A red flag appears more often if the red flag is less precise, i.e., false alarms occur more often. A delay thus appears more often. If the effectiveness of extended audit procedures is lower, the chance of seeing a delay is higher because extended audit procedures must be performed more often to provide the same equilibrium level of deterrence to a dishonest insider.

The log-transformed relation can be used as a basis for specifying a regression equation to empirically test the effects of the above-mentioned factors on the delay likelihood. There are two caveats. First, finding adequate proxies for the informativeness of regular audit

²⁵The λ^* can be held constant if the change in θ , p , or q discussed here is balanced out by a covariation in C to fix the value of φ that enters the equation system (16) and (18). All these factors affect λ^* only through φ .

procedures and effectiveness of extended audit procedures might be difficult. If auditors in a competitive audit market are believed to have similar audit technologies, the $-\log q$ may be treated as the intercept to be estimated. Similarly, with a proxy for θ included in the regression equation, the estimated coefficient may be interpreted as reflecting the level of p .²⁶

Finding a firm-level proxy for θ is also challenging. Suppose that the chance of having a dishonest insider is related to the insider's moral standard. Arguably this may be traced back to his family and education background. Suppose one believes that whether the insider grew up in primary and high schools with strong religious heritages might be relevant. Then observable characteristics like these can be considered proxies for θ . Alternatively, one can argue that θ is closely related to the inherent risk of a firm. Then variables like industry, capitalization, and beta can also be used as proxies for the ex ante probability of fraud.²⁷

The second caveat of specifying a regression equation based on (22) is the endogeneity of the outside ownership λ^* . Statistically holding it constant might be possible but it correlates with other factors in the relation through φ that enters the equation system (16) and (18). For example, the estimated coefficient of θ can be biased if λ^* is included directly as an explanatory variable. To mitigate the bias, a 2SLS regression procedure may be used to first regress λ^* on θ and other control variables. Then the predicted value of λ^* can be included as an explanatory variable in the regression equation specified based on (22).

Alternatively, one can consider a reduced-form regression equation based on (22) that includes the exogenous determinants of the outside ownership but not λ^* itself. Because some of these determinants are already in (22), the predicted signs of such factors need to be determined more carefully than by merely assuming that λ^* is being held constant. The next proposition provides the result to sign the effects of those factors. The proposition follows directly from Lemma 4 stated and proved in the appendix. The lemma requires the assumptions stated below to show the uniqueness of the optimal solution μ^* , λ^* , and K^* and thereby unambiguously sign the relation between the equilibrium outside ownership λ^*

²⁶The signs of this coefficient and the intercept can be determined by linearizing the term $\log [p + (1 - p)\theta]$ using Taylor's expansion. If one believes a Big4 audit firm means q is higher and p is lower, a Big4 indicator variable can be included to interact with a proxy for θ and also as a standalone control to test the effects of p and q , whose data usually are difficult to obtain.

²⁷I thank an anonymous referee for suggesting this view and the proxies.

and the fraud risk premium φ .²⁸

Assumption 1 (Regularity condition).

$$(1+g)W > a\varphi \left(1 + \frac{2 \left[\left(1 + \frac{1}{b}\right)^2 - \varphi \right]}{(1+b)(1-\varphi)} \right), \quad (23)$$

where $\varphi = a^{-1} \left(\frac{C}{q} \right) [p + (1-p)\theta]$.

Assumption 1 is a regularity condition on the magnitudes of the parameters in the model. It ensures the satisfaction of the second-order condition of maximization by every solution of the first-order condition and thereby establishes the uniqueness of a solution. Note that the expression in the brackets on the right of (23) is decreasing in b and approaches 1 for a very large b . Because $(1+g)W > a\varphi$, the assumption will hold if b is sufficiently large.

The second assumption concerns requirements on the curvature of the cost of capital function.

Assumption 2 (Curvature of the cost of capital function). *The cost of capital function satisfies the following conditions:*

$$(i) \quad \frac{R'''}{R''} \geq \frac{R''}{R'} \geq \frac{(1+g)}{a(1-\varphi)}; \quad (24)$$

(ii)

$$R' - \frac{R'''}{R''}R > \mu_1(1+g) \quad \text{for all } K \text{ with } R(K) \leq \mu_1\Pi(K), \quad (25)$$

where $\mu_1 = \frac{1+b(1-\varphi)}{1+b}$, $\Pi(K) = (1+g)(K+W) - a\varphi$, and $\varphi = a^{-1} \left(\frac{C}{q} \right) [p + (1-p)\theta]$.

Proposition 2 (Association between outside ownership and audit fee). *If Assumptions 1 and 2 hold, $d\lambda^*/d\varphi > 0$. Hence, for variations in φ driven by variations in θ , p , and/or q ,*

²⁸These assumptions are met by the cost of capital function given in footnote 19, provided the parameter r of the function satisfies the condition below:

$$\frac{1+r}{1+g} > \max \left\{ \frac{1+b(1-\varphi)}{1+b}, \frac{1}{a(1-\varphi)} \right\},$$

where $\varphi = a^{-1} \left(\frac{C}{q} \right) [p + (1-p)\theta]$.

the equilibrium outside ownership λ^* is positively associated with the equilibrium audit fee F^* , i.e.,

$$\left(\frac{d\lambda^*}{d\varphi}\right) \left(\frac{\partial F^*}{\partial \varphi}\right) > 0.$$

Recall that $F^* = a\varphi$. Therefore, by Proposition 1, $\partial\varphi/\partial\theta > 0$, $\partial\varphi/\partial p > 0$, and $\partial\varphi/\partial q < 0$. Moreover, $\frac{\lambda^*}{\lambda^*+b}$ is increasing in λ^* . With $d\lambda^*/d\varphi > 0$, the signs of the exogenous factors' effects on the delay likelihood remain the same, regardless of holding λ^* constant or not. This result is formally stated as the proposition below.

Proposition 3 (Effects on delay likelihood). *Suppose Assumptions 1 and 2 hold. The equilibrium delay likelihood decreases when (i) the reliability of the regular and/or extended audit procedures improves, i.e., p is lower and/or q is higher, or (ii) the ex ante probability of fraud reduces, i.e., θ is lower. Holding constant these factors and the penalty multiplier b for a dishonest insider, the equilibrium delay likelihood is positively associated with the equilibrium outside ownership λ^* .²⁹ Formally,*

$$\begin{aligned} \frac{dPr\{x = 1\}}{dp} &> 0, & \frac{dPr\{x = 1\}}{dq} &< 0, \\ \frac{dPr\{x = 1\}}{d\theta} &> 0, & \text{and } \frac{\partial Pr\{x = 1\}}{\partial \lambda^*} &> 0. \end{aligned}$$

5.2 Market Reaction to Delay

To see how the financial market should react to a delay, I look at the difference in the expected values of the firm right before and right after a delay. Recall that if the insider is dishonest and the fraud is not detected during the year-end audit, the diverted resources will be irrecoverable. Consequently, outside investors' ownership of the firm will be worth $\lambda^*(1 - \delta^*)\Pi(K^*)$ only. However, if the insider is honest, or if he is dishonest but the fraud is detected during the audit, the shares owned by outside investors will be worth $\lambda^*\Pi(K^*)$. Thus, all that matters to outside investors is the posterior probability $\Pr\{\text{undetected, fraud} | x = 1\}$ of having undetected fraud conditional on a delay, relative to the unconditional prior

²⁹When θ , p , q , and b are held constant, the variations in λ^* are due to changes in C and/or a .

probability, i.e.,

$$\theta(1 - q\omega^*) = \frac{b\theta}{\lambda^* + b}. \quad (26)$$

Note that a delay could not have occurred unless a red flag was observed. Therefore,³⁰

$$\Pr\{\text{undetected, fraud}|x = 1\} = \frac{1 - q}{1 + p(\frac{1}{\theta} - 1)}. \quad (27)$$

Intuitively, this means conditional on a delay, the posterior probability of having undetected fraud is lower when the extended audit procedures are more effective in detecting fraud, i.e., q is higher. Note that $[1 + p(\frac{1}{\theta} - 1)]^{-1}$ is the posterior probability of having a dishonest insider after observing a red flag, i.e., $\Pr\{\text{fraud}|\text{red flag}\}$. Suppose the regular audit procedures that generate a red flag are more informative, i.e., p is lower, or the ex ante probability of fraud is higher, i.e., θ is higher. Then $\Pr\{\text{fraud}|\text{red flag}\}$ is higher and consequently the posterior probability of having undetected fraud after observing a delay is also higher.

Let V_b and V_a denote respectively the expected market values of the shares owned by outside investors before and after observing a delay. The market reaction to a delay can be represented by the percentage deviation of V_a from V_b , denoted by v :

$$v \equiv \frac{V_a - V_b}{V_b} = \frac{\lambda^*\Pi(K^*)[1 - \delta^*\Pr\{\text{undetected, fraud}|x = 1\}]}{\lambda^*\Pi(K^*)[1 - \delta^*\Pr\{\text{undetected, fraud}\}]} - 1 \quad (28)$$

$$= \frac{\delta^* \left[\frac{b\theta}{\lambda^* + b} - \frac{1 - q}{1 + p(\frac{1}{\theta} - 1)} \right]}{1 - \frac{b\varphi}{\lambda^* + b}} \quad (29)$$

$$= \frac{\varphi \left[1 - (1 - \theta)(1 - p) - (1 - q) \left(1 + \frac{\lambda^*}{b} \right) \right]}{\left(1 + \frac{\lambda^*}{b} - \varphi \right) [p + (1 - p)\theta]}. \quad (30)$$

Note that by (18), $1 + \frac{\lambda^*}{b} - \varphi = \frac{\mu^*(\lambda^* + b)}{\lambda^*b} > 0$. Hence, the denominator of the expression for v above is positive. Below is the last result of the paper.

Proposition 4 (Market reaction to delay). *The market reaction to a delay, measured by the percentage deviation of the post-delay market value of the outside ownership from the*

³⁰The derivation of the posterior probability is provided in the appendix.

pre-delay market value, is equal to

$$v = \frac{\varphi [1 - (1 - \theta)(1 - p) - (1 - q)(1 + \frac{\lambda^*}{b})]}{(1 + \frac{\lambda^*}{b} - \varphi) [p + (1 - p)\theta]}, \quad (31)$$

which is positive if and only if

$$(1 - \theta)(1 - p) + (1 - q) \left(1 + \frac{\lambda^*}{b}\right) < 1. \quad (32)$$

The inequality above provides a necessary and sufficient condition for observing a *positive* market reaction to a delay. If such a market reaction is observed, a delay is perceived by outside investors as good news. When will this happen?

If the red flag generated from regular audit procedures is imprecise, the delay so triggered may well be just a false alarm to outside investors. When the ex ante probability of fraud is high, observing a red flag by the auditor can actually be good to outside investors because the red flag may trigger an extended audit. This is especially good when the extended audit is very effective in detecting fraud, leaving little chance for a fraud to sneak through. Thus a delay can be good news to outside investors when the ex ante probability of fraud, the imprecision of a red flag, and the effectiveness of extended audit procedures for detecting fraud are all high.³¹

The condition in the proposition above can serve as a basis for specifying a logistic regression equation. The dependent variable of the regression is binary with the value equal to 1 for a positive market reaction observed following a delay. The condition suggests that if the equilibrium outside ownership λ^* could be held constant, the odd of seeing a positive market reaction increases with θ , p , and q . On the other hand, holding constant θ , p , q , and b , an increase in the equilibrium outside ownship λ^* (e.g., driven by variations in C , a , g , and/or W) makes the positive reaction condition harder to be met. If taking into account

³¹It might appear that if a financial reporting delay can result in a positive market response, then the firm would always delay financial reporting even when there is no audit delay, seemingly invalidating the result that a financial reporting delay can be good news to market investors. This argument, however, assumes the firm has no cost in adopting such a strategy, which is not true. After the annual report is released, the date of the audit report would suggest whether the firm has deliberately delaying financial reporting. Anticipating that the initial positive market response to a financial report delay would reverse, the firm has little incentive to mimic a delay.

the effects of θ and p on the equilibrium outside ownership, only q has an unambiguously positive effect on the odd of seeing a positive market reaction to a delay.

Even for q , its effect on the magnitude of the market reaction, $|v|$, cannot be unambiguously determined. This ambiguity is due to the appearance of λ^* both in the top and bottom components of v . However, if b is very high in a country, given that λ^* cannot exceed 1, one would expect the changes in $\frac{\lambda^*}{b}$ as a result of the changes in other factors could be negligible. In such circumstances, a higher q would result in a lower φ and accordingly could lead to a smaller top component and a larger bottom component of the expression of $|v|$, assuming that v is negative. That is to say, when the market reaction to a delay is negative, outside investors perceive a delay as less of bad news if extended audit procedures are more effective in detecting fraud, giving outside investors stronger protection.

6 Discussion

The two-round audit process in the model is probably the simplest way to capture the considerations prescribed in SAS 99 (see the discussion in section 2). The formulation provides a parsimonious structure to relate the fraud detection role of auditors to financial reporting and audit delays, serving the purpose of obtaining closed-form results to guide empirical research.

Compared to NPS's continuous audit effort formulation, the binary audit choice model here has some dissimilar results. For example, in this model a greater incentive to extend audit procedures owing to a higher penalty multiplier a is neutralized by a new equilibrium diversion rate that continues to make the auditor indifferent between extending or not extending the audit. In the end, the equilibrium audit fee does not depend on a , unlike the case in the NPS model. For a similar reason, outside ownership λ affects the audit fee in their model but not here.

To clearly differentiate my contributions from NPS's, I have chosen not to include any analysis about the effects of the penalty multipliers, a and b , on the equilibrium of my model. While mine is similar to NPS's model, the strategic consideration of extending audit procedures is unique. Moreover, I have made additional assumptions, about a non-linear

(total) cost of capital function, that allow proving the uniqueness of the equilibrium and unambiguously signing some relations in the comparative static analysis. Recognizing these differences between the two models, I believe it should be interesting to examine in future research the effects of the penalty multipliers on the equilibrium.

The Knechel and Payne (2001) study reports that “audit report lag is decreased by the potential synergistic relationship between MAS and audit services.” The model of this paper provides a theoretical foundation to interpreting the empirical finding in this way. Specifically, I have shown that $d\Pr\{x = 1\}/dp > 0$ and $d\Pr\{x = 1\}/dq < 0$. If indeed a synergistic relationship between MAS and audit services results in a less imprecise red flag generated by regular audit procedures and more effective extended audit procedures for detecting fraud, it will reduce the likelihood of a delay. Intuitively speaking, this is consistent with a reduction in the audit report lag.³² What my model has not incorporated is the potential judgmental bias that might result from a closer business relationship with an audit client purchasing also MAS. Extending the model in this direction provides another avenue for future research.

Ettredge, Li, and Sun (2006), p. 5, argue that “[a]uditors ... need to extend their scope of work and perform additional substantive tests to compensate for the control weakness. ... The extended audit effort due to control weakness should lead to longer audit delay.” However, I believe that such a delay needs not be significant if the auditors have prepared enough manpower to do the additional tests *concurrently*. Presumably, the control weaknesses are identified mainly in the planning phase. It can be well ahead of the peak period around the fiscal year end when the auditors are completely tied up and unable to easily find more new hires to provide the extra help. Such a control weakness driven delay might not be as unanticipated as a delay due to contingent extended audit procedures discussed in this paper.³³

³²In its present form, the model cannot directly talk about audit report lag because the delay is modeled as a 0-1 event with a fixed delay duration. A modification to it using some hazard-rate function modeling technique would better fit the model to the empirical literature on audit report lag.

³³For control weaknesses discovered during the audit after the planning phase, they are equivalent to the red flag discussed in this paper and can lead to a delay not fully anticipated.

7 Concluding Remarks

This paper contributes to the literature of financial reporting and audit delays by providing a theoretical model to examine the likelihood of and market reaction to a delay. I have chosen to make the least departure from the NPS model, which is useful in understanding closely related issues like the role of auditing in investor protection. The key departure from their model is a two-round audit process. This modeling strategy lets a reader understand what this main departure can do. Specifically, the departure allows (i) formalizing intuitions and deriving predictions consistent with what are known in the literature (e.g., the positive association between audit fee and outside ownership) and (ii) on top of this, adding an interesting new insight that is hard to discover without the help of such a model (i.e., the possibility of a positive market reaction to a delay).

I attempt to reach out to an empirical-oriented audience by discussing the model's empirical implications in detail, including specifics about how the model can provide suggestions for regression equation specifications. I wish that by taking a step towards this direction, empirical and analytical researchers can work closer together to advance knowledge of the field. If empirical researchers find it easier to formulate a testable hypothesis with this paper than without it, I consider this attempt a success.

I have discussed in length the empirical implications of the model. Any empirical test based on this model is a joint test of the model predictions and assumptions. Keeping in mind this caveat can avoid mistaking spurious findings as evidence for supporting the model. For example, the model assumes fraud is the main issue behind a delay. In reality, there are other reasons leading to delays. In constructing the sample for an empirical test, it is important to take into consideration such reasons. I elaborate on this point in appendix A where an illustrative example of an empirical test design is discussed.

To facilitate deriving closed-form results useful for guiding empirical hypothesis testing, simplifying assumptions have been made to keep the model tractable. For example, unlike prior models with a “two-round” audit, I do not allow the auditor to make continuous audit choices. Allowing continuous choices potentially could make the model useful for addressing a number of additional questions: e.g., When is it more efficient for the auditor to focus

on the first-round audit and when is it more efficient to focus on the second? How does the auditor respond to an increase in the strictness of the legal environment when there are two rounds of audit instead of one? Are the effort levels in the two rounds substitutes or complements? While these questions are interesting, they are not the focus of this paper and are left for future research.

I have also assumed that honesty is exogenously determined in the model. Recent development in economic theory has begun analyzing endogenous honesty behavior using “lying cost” models (e.g., Kartik 2008). Advocates argue that business ethics education in MBA programs should be strengthened to reduce unethical behaviors of future business leaders. Suppose business ethics education is effective. The model here then suggests if such education reduces θ (i.e., the prior probability of having a dishonest insider) from 5% to 4% (a one-percent difference), it will save society in audit cost by a percentage approximately equal to $(1 - p) / [1 - 0.95(1 - p)]$.³⁴ Investigation along this line provides an avenue for quantitative business ethics research.

Suppose a government has one extra dollar to be used on reducing fraud. Should it be spent on reducing the imprecision of a red flag, i.e., p , or on enhancing the effectiveness of extended audit procedures for detecting fraud, i.e., q ? Or would it be even better to spend the extra dollar on decreasing the chance of having a dishonest insider, i.e., θ , say, by providing better business ethics education that works? These are interesting questions unanswered here. However, the model of this paper can provide a vehicle for future research to examine these and some other interesting questions.

³⁴Formally,

$$\left. \frac{\partial \log F^*}{\partial \theta} \right|_{\theta=0.05} = \frac{1 - p}{1 - 0.95(1 - p)}.$$

References

- ALFORD, A. W., J. J. JONES, AND M. E. ZMIJEWSKI (1994): "Extensions and violations of the statutory SEC form 10-K filing requirements," *Journal of Accounting and Economics*, 17(1-2), 229–254.
- ASHTON, R. H., J. J. WILLINGHAM, AND R. K. ELLIOTT (1987): "An Empirical Analysis of Audit Delay," *Journal of Accounting Research*, 25(2), 275–292.
- BAGNOLI, M., W. KROSS, AND S. G. WATTS (2002): "The Information in Management's Expected Earnings Report Date: A Day Late, a Penny Short," *Journal of Accounting Research*, 40(5), 1275–1296.
- BAMBER, E. M., L. S. BAMBER, AND M. P. SCHODERBEK (1993): "Audit structure and other determinants of audit report lag: An empirical analysis," *Auditing: A Journal of Practice & Theory*, 12(1), 1–23.
- BEGLEY, J., AND P. E. FISCHER (1998): "Is there Information in an Earnings Announcement Delay?," *Review of Accounting Studies*, 3(4), 347–363.
- CAMERER, C., AND R. H. THALER (1995): "Anomalies: Ultimatums, Dictators and Manners," *The Journal of Economic Perspectives*, 9(2), 209–219.
- CHAMBERS, A. E., AND S. H. PENMAN (1984): "Timeliness of Reporting and the Stock Price Reaction to Earnings Announcements," *Journal of Accounting Research*, 22(1), 21–47.
- DECISION NEWS MEDIA SAS (2003): "Ahold delays audit results as fraud deepens," May 26, <http://www.foodanddrinkeurope.com/content/view/print/81368>.
- DWYER, P. D., AND E. R. WILSON (1989): "An empirical investigation of factors affecting the timeliness of reporting by municipalities," *Journal of Accounting and Public Policy*, 8(1), 29–55.
- ERARD, B., AND J. S. FEINSTEIN (1994): "Honesty and Evasion in the Tax Compliance Game," *The RAND Journal of Economics*, 25(1), 1–19.
- ETTREDGE, M. L., C. LI, AND L. SUN (2006): "The impact of SOX Section 404 internal control quality assessment on audit delay in the SOX Era," *Auditing: A Journal of Practice and Theory*, 25(November), 1–23.
- EVANS, J. H., R. L. HANNAN, R. KRISHNAN, AND D. V. MOSER (2001): "Honesty in Managerial Reporting," *The Accounting Review*, 76(4), 537–559.
- FISCHBACHER, U., AND U. STEFANI (2007): "Strategic Errors and Audit Quality: An Experimental Investigation," *The Accounting Review*, 82(3), 679.
- GENNOTTE, G., AND B. TRUEMAN (1996): "The Strategic Timing of Corporate Disclosures," *The Review of Financial Studies*, 9(2), 665–690.
- GIVOLY, D., AND D. PALMON (1982): "Timeliness of Annual Earnings Announcements: Some Empirical Evidence," *The Accounting Review*, 57(3), 486–508.

- HOGAN, C. E., Z. REZAEI, J. RILEY, AND U. K. VELURY (2008): "Financial Statement Fraud: Insights from the Academic Literature," *Auditing: A Journal of Practice & Theory*, 27(2), 231–252.
- JOHNSON, L. E., S. P. DAVIES, AND R. J. FREEMAN (2002): "The effect of seasonal variations in auditor workload on local government audit fees and audit delay," *Journal of Accounting and Public Policy*, 21(4-5), 395–422.
- KARTIK, N. (2008): "Strategic Communication with Lying Costs," *Review of Economic Studies*, forthcoming, December 12, 2008.
- KNECHEL, W. R., AND J. L. PAYNE (2001): "Additional Evidence on Audit Report Lag," *Auditing: A Journal of Practice & Theory*, 20(1), 137–146.
- KREPS, D. M., P. MILGROM, J. ROBERTS, AND R. WILSON (1982): "Rational cooperation in the finitely repeated prisoners' dilemma," *Journal of Economic Theory*, 27(2), 245–252.
- KRISHNAN, J., AND J. S. YANG (2009): "Recent Trends in Audit Report and Earnings Announcement Lags," *SSRN eLibrary*.
- LAMBERT, T. A., J. F. BRAZEL, AND K. L. JONES (2008): "Unintended Consequences of Accelerated Filings: Do Changes in Audit Delay Lead to Changes in Earnings Quality?," *SSRN eLibrary*, November.
- LOOMES, G. (2005): "Modelling the Stochastic Component of Behaviour in Experiments: Some Issues for the Interpretation of Data," *Experimental Economics*, 8(4), 301.
- MATSUMURA, E. M., AND R. R. TUCKER (1992): "Fraud Detection: A Theoretical Foundation," *The Accounting Review*, 67(4), 753–782.
- MCLELLAND, A. J., AND G. GIROUX (2000): "An empirical analysis of auditor report timing by large municipalities," *Journal of Accounting and Public Policy*, 19(3), 263–281.
- MORGENSON, G. (2009): "S.E.C. Accuses Countrywide's Ex-Chief of Fraud," *The New York Times*.
- NEWMAN, D. P., E. R. PATTERSON, AND J. R. SMITH (2005): "The Role of Auditing in Investor Protection," *Accounting Review*, 80, 289–313.
- OWUSU-ANSAH, S., AND S. LEVENTIS (2006): "Timeliness of corporate annual financial reporting in Greece," *European Accounting Review*, 15(2), 273–287.
- PATTERSON, E. R., AND J. R. SMITH (2007): "The Effects of Sarbanes-Oxley on Auditing and Internal Control Strength," *The Accounting Review*, 82(2), 427–455.
- RIESKAMP, J. (2008): "The probabilistic nature of preferential choice," *Journal of Experimental Psychology: Learning, Memory, and Cognition*, pp. 1446–1465.
- SENGUPTA, P. (2004): "Disclosure timing: Determinants of quarterly earnings release dates," *Journal of Accounting and Public Policy*, 23(6), 457–482.
- SHLEIFER, A., AND D. WOLFENZON (2002): "Investor protection and equity markets," *Journal of Financial Economics*, 16, 3–28.

- SMITH, J. R., S. L. TIRAS, AND S. S. VICHITLEKARN (2000): "The interaction between internal control assessment and substantive testing in audits for fraud," *Contemporary Accounting Research*, 17(2), 327.
- WALL STREET JOURNAL (1995): "Sensormatic's Stock Skids 17% on News of Expanded Audit," September 1.
- WILCOX, N. T. (2009): "'Stochastically more risk averse:' A contextual theory of stochastic discrete choice under risk," *Journal of Econometrics*, forthcoming.

Appendix

A Illustrative Example of an Empirical Test Design

The purpose of this appendix is to demonstrate how a theoretical result of the paper can provide an easier starting point for formulating an empirical test design. It is not intended to be a detailed cookbook for one to follow step by step to conduct an empirical study.

In the main text, I have taken the strategy to explore what a key departure from the NPS model can do. Therefore the endogenous choice of the external capital K and outside ownership λ in their model is maintained. However, the *positive-reaction* condition (32) that predicts whether there will be a positive market reaction to a delay is unaffected if K and λ are predetermined. Below I consider this simplified version of the model. The sample constructed accordingly should be larger because most firms do not have equity offering in most years.

The positive-reaction condition predicts that with everything else being equal, if a positive reaction to a delay occurs to a firm with an ex ante probability of fraud θ , it should also occur to firms with higher θ 's. To construct a sample for testing this hypothesis, it is important to exclude observations with known reasons for delays unrelated to fraud concerns, e.g., due to mergers and acquisitions. Holding constant b , i.e., the penalty multiplier for a fraudster, should not be a problem if the sample observations are under a single jurisdiction. This is true for many countries but not necessarily for the US because the states can differ in their choices of b . Because the outside ownership λ is observable, an attempt to hold it constant can be made by ranking firms by decile so that the test can be done based on observations in each decile.

Holding constant the imprecision p and the effectiveness q of the two-round audit process is a challenge because they are not directly observable. Suppose both p and q is highly correlated with a single observable quality of auditors. Then ranking auditors by decile based on the quality and testing the hypothesis using observations in each decile can provide a way to reduce the variations in the unobservable p and q . Besides the Big4 versus non-Big4 classification, the accuracy rate of an auditor's going-concern opinions can be another

candidate to proxy for the observable quality.

Finding proxies for θ is less difficult. One can twist the interpretation of the model by arguing that insiders are always dishonest. However, whether they have the opportunity to divert firm resources for private benefit depends crucially on the corporate governance and internal control environments. With this alternative interpretation, θ can be proxied by a variety of corporate governance variables like CEO-chairman duality, the number of independent directors, and the size of the board. It can also be proxied by data on internal control weaknesses.

With a proxy for θ , one can test a weak version of the hypothesis by arguing as follows: A positive reaction to a delay might be rare. However, even when the usual negative reaction to a delay is observed, the response should be weaker if the news is less bad. The positive-reaction condition provides a way to classify good and bad news. If θ is high, the news of a delay is not as bad as when θ is low because the market is “psychologically prepared” and not shocked by the delay. Based on this reasoning, one can partition the sample by the median value of a proxy for θ . A weaker negative market reaction is thus expected for the above-median subsample.

B Proofs and Derivations

The proposition below qualitatively summarizes some results in Lemma 1 concerning how the capital raised, K , and ownership sold, λ , are related to the effective outside ownership, μ . The lemma is stated and proved in this appendix after a brief discussion of the implication of the proposition given below.

Proposition 5 (Relations among capital raised, ownership sold, and effective outside ownership). *The amount of capital raised K and fraction of ownership sold λ , as functions of the effective outside ownership μ , have the following properties:*

(i) *The amount of capital raised is strictly increasing in the effective outside ownership, i.e., $K' > 0$;*

(ii) *The fraction of ownership sold is strictly increasing and concave in the effective outside ownership, i.e., $\lambda' > 0$ and $\lambda'' < 0$.*

The monotonicity of K and λ as functions of μ imply that the amount of capital raised is positively related to the fraction of ownership sold. The relation highlights the tension between growing the “pie” bigger and retaining a larger slice of the “pie.” Given the fraud risk to outside investors, the insider is unable to raise more capital to grow the “pie” bigger (a higher year-end firm value) unless he accepts a smaller slice of the “pie” (a lower retained ownership $1 - \lambda$, or equivalently, a higher effective outside ownership μ).

Lemma 1. *The amount of capital raised K and fraction of ownership sold λ , as functions of the effective outside ownership μ , have the following properties:*

(i) *The amount of capital raised is strictly increasing in the effective outside ownership. Specifically,*

$$K' = \frac{\Pi(K)}{R' - \mu(1 + g)} > 0 \quad (33)$$

with

$$K'(0) = \frac{\Pi(0)}{R'(0)}, \quad (34)$$

where $\Pi(0) = (1 + g)W - a\varphi$, and

$$\frac{K''}{K'} = \frac{2(1 + g) - R''K'}{R' - \mu(1 + g)}; \quad (35)$$

(ii) *The fraction of ownership sold is strictly increasing and concave in the effective outside ownership. Specifically,*

$$\lambda' = \frac{(\lambda + b)^2}{(\lambda + b)^2 - b^2\varphi} > 0 \quad (36)$$

with

$$\lambda'(0) = \frac{1}{1 - \varphi} \quad (37)$$

and

$$\frac{\lambda''}{\lambda'} = -\frac{2b^2\varphi(\lambda + b)}{[(\lambda + b)^2 - b^2\varphi]^2} < 0; \quad (38)$$

(iii) *The shape of the fraction of ownership sold, as a function of the effective outside ownership, has the following “bounds”:*

$$\frac{\lambda''}{\lambda'} \geq -\frac{2\varphi}{b(1-\varphi)^2} \quad (39)$$

and

$$\frac{\lambda'}{1-\lambda} \geq \frac{(1+b)^2}{(1+b)^2 - b^2\varphi}. \quad (40)$$

Proof. (**Lemma 1**) Differentiating both sides of (16) with respect to μ yields

$$R'K' = \Pi(K) + \mu(1+g)K'. \quad (41)$$

The first derivative of $K(\mu)$ thus equals

$$K' = \frac{\Pi(K)}{R' - \mu(1+g)}, \quad (42)$$

By assumption, $\Pi(0) = (1+g)W - a\varphi > 0$ and $R' > R/K$. Given a competitive financial market, $R = \mu\Pi$ and $R' > \mu\Pi/K = \mu[(1+g)K + \Pi(0)]/K > \mu(1+g)$. Hence, the first derivative K' is strictly positive for all $K > 0$. Recall that $R(0) = 0$. When $\lambda = 0$, both μ and K must also be zero. Hence

$$K'(0) = \frac{\Pi(0)}{R'(0)}. \quad (43)$$

To derive K''/K' , I differentiate both sides of (41) with respect to μ once more to get

$$R''(K')^2 + R'K'' = 2(1+g)K' + \mu(1+g)K''. \quad (44)$$

Rearranging the terms gives

$$\frac{K''}{K'} = \frac{2(1+g) - R''K'}{R' - \mu(1+g)}. \quad (45)$$

Now differentiating both sides of (18) with respect to μ , I get

$$2\lambda\lambda' + b(1-\varphi)\lambda' = \lambda + b + \mu\lambda'. \quad (46)$$

Rearranging the terms gives the first derivative of $\lambda(\mu)$ below:

$$\lambda' = \frac{\lambda + b}{2\lambda - \mu + b(1 - \varphi)}. \quad (47)$$

Recall that by definition $\mu = \lambda [1 - \varphi(1 - q\omega^*)]$, which can be expressed as

$$\mu = \lambda \left[1 - \frac{b\varphi}{(\lambda + b)} \right]. \quad (48)$$

Since $\lambda - \mu = b\varphi\lambda/(\lambda + b)$, the first derivative of $\lambda(\mu)$ can be written as

$$\lambda' = \frac{(\lambda + b)^2}{(\lambda + b)^2 - b^2\varphi} > 0. \quad (49)$$

When $\lambda = 0$, $\lambda'(0) = 1/(1 - \varphi)$. To derive λ''/λ' , I differentiate both sides of (46) with respect to μ once more to get

$$2(\lambda')^2 + 2\lambda\lambda'' + b(1 - \varphi)\lambda'' = 2\lambda' + \mu\lambda''. \quad (50)$$

Rearranging the terms gives

$$\frac{\lambda''}{\lambda'} = \frac{2(1 - \lambda')}{2\lambda - \mu + b(1 - \varphi)}. \quad (51)$$

Note that

$$1 - \lambda' = \frac{\lambda - \mu - b\varphi}{2\lambda - \mu + b(1 - \varphi)} \quad (52)$$

and $\lambda - \mu = b\varphi\lambda/(\lambda + b)$. Therefore,

$$\frac{\lambda''}{\lambda'} = - \frac{2b^2\varphi}{(\lambda + b)[2\lambda - \mu + b(1 - \varphi)]^2} \quad (53)$$

$$= - \frac{2b^2\varphi(\lambda + b)}{[(\lambda + b)^2 - b^2\varphi]^2}. \quad (54)$$

To complete the proof of this lemma, I derive below the lower bounds of λ''/λ' and

$\lambda'/(1 - \lambda)$. Consider first the following function:

$$h(\lambda) = \frac{(\lambda + b)}{[(\lambda + b)^2 - b^2\varphi]^2}. \quad (55)$$

Since $\lambda''/\lambda' = -2b^2\varphi h(\lambda(\mu)) < 0$ and $\lambda' > 0$, the ratio λ''/λ' reaches its lowest value when $h(\lambda)$ reaches its highest value. Differentiating h once gives

$$h' = -\frac{3(\lambda + b)^2 + b^2\varphi}{[(\lambda + b)^2 - b^2\varphi]^3} < 0. \quad (56)$$

This means h reaches its maximum at $\lambda = 0$, or equivalently at $\mu = 0$. Hence, the following is a lower bound of λ''/λ' :

$$\left. \frac{\lambda''}{\lambda'} \right|_{\mu=0} = -\frac{2\varphi}{b(1 - \varphi)^2}. \quad (57)$$

Note that when $\lambda = 1$, equation (18) implies μ equals

$$\mu_1 = \frac{1 + b(1 - \varphi)}{1 + b}. \quad (58)$$

Since $\lambda'' < 0$, λ' reaches its lowest value at $\mu = \mu_1$. Thus,

$$\frac{\lambda'}{1 - \lambda} \geq \lambda' \geq \lambda'(\mu_1) = \frac{(1 + b)^2}{(1 + b)^2 - b^2\varphi}. \quad (59)$$

□

Lemma 2. *Suppose Assumptions 1 and 2 hold. Then the following conditions also hold for any μ , $\lambda(\mu)$, and $K(\mu)$ satisfying the FOC:*

$$\frac{R''}{R' - \mu(1 + g)} > \frac{(1 + g)b}{a[\lambda + b(1 - \varphi)]} \quad (60)$$

and

$$\frac{R''K'}{R' - \mu(1 + g)} + \frac{\lambda''}{\lambda'} > 0. \quad (61)$$

Proof. (**Lemma 2**) First, recall that $K' > 0$. So the inverse function of K , denoted by $\mu(K)$, exists and has $\mu' = 1/K'$. Next, consider the function below:

$$J(K) = \frac{R''}{R' - \mu(K)(1+g)} \quad (62)$$

with

$$J' = \frac{R'''[R' - \mu(K)(1+g)] - R''[R'' - \mu'(1+g)]}{[R' - \mu(K)(1+g)]^2}. \quad (63)$$

The first derivative is positive if and only if the following inequality holds:

$$[R'''R' - (R'')^2] + (1+g)\left[\frac{R''}{K'} - \mu(K)R'''\right] > 0. \quad (64)$$

The first term above is non-negative under Assumption 2. To show that $J(K)$ reaches its minimum at $K = 0$, it suffices to prove that $R'' > \mu(K)K'R'''$.

Given the competitive financial market condition, $R(K) = \mu\Pi(K)$, and that $K' = \Pi(K)/[R' - \mu(1+g)]$,

$$\mu(K)K' = R(K)/[R' - \mu(1+g)]. \quad (65)$$

Thus, proving $R'' > \mu(K)K'R'''$ is equivalent to proving

$$R' - \frac{R''R}{R''} > \mu(K)(1+g). \quad (66)$$

This condition is met under Assumption 2 because $\mu(K)$ must not exceed μ_1 , the maximum possible value of μ (which is constrained by the corresponding value of λ that is at most 1).

In conclusion, $J(K)$ is increasing in K and reaches its minimum at $K = 0$,

Since $R(0) = 0$ and $\Pi(0) > 0$, $\mu(K) = R(0)/\Pi(0) = 0$. Consequently,

$$\frac{R''}{R' - \mu(K)(1+g)} \geq \frac{R''}{R'} \geq \frac{(1+g)}{a(1-\varphi)} \geq \frac{(1+g)b}{a[\lambda + b(1-\varphi)]}, \quad (67)$$

where the second inequality is due to Assumption 2.

According to the FOC,

$$K' = \left(\frac{\lambda'}{1-\lambda}\right) \frac{\Pi(K)}{1+g}. \quad (68)$$

Hence,

$$\frac{R''K'}{R' - \mu(K)(1+g)} \geq \left[\frac{(1+g)}{a(1-\varphi)} \right] \left(\frac{\lambda'}{1-\lambda} \right) \frac{\Pi(K)}{1+g} \quad (69)$$

$$\geq \left[\frac{(1+g)W - a\varphi}{a(1-\varphi)} \right] \left(\frac{\lambda'}{1-\lambda} \right) \quad (70)$$

$$\geq \left[\frac{(1+g)W - a\varphi}{a(1-\varphi)} \right] \left(\frac{(1+b)^2}{(1+b)^2 - b^2\varphi} \right). \quad (71)$$

The second inequality above uses the fact that $\Pi(K)$ is smallest at $K = 0$. The last inequality uses the result of Lemma 1.

Note that the condition in Assumption 1 is equivalent to $\frac{(1+g)W - a\varphi}{a} > \frac{2\varphi}{(1-\varphi)} \left[\frac{(1+b)^2 - b^2\varphi}{b^2(1+b)} \right]$.

Thus, by Lemma 1,

$$\left[\frac{(1+g)W - a\varphi}{a(1-\varphi)} \right] \left(\frac{(1+b)^2}{(1+b)^2 - b^2\varphi} \right) \quad (72)$$

$$> \frac{2\varphi}{(1-\varphi)^2} \left[\frac{(1+b)^2 - b^2\varphi}{b^2(1+b)} \right] \left(\frac{(1+b)^2}{(1+b)^2 - b^2\varphi} \right) \quad (73)$$

$$= \frac{2\varphi(1+b)}{b^2(1-\varphi)^2} \quad (74)$$

$$> \frac{2b^2\varphi(\lambda+b)}{[(\lambda+b)^2 - b^2\varphi]^2} \quad (75)$$

$$= \left(\frac{-\lambda''}{\lambda'} \right). \quad (76)$$

In conclusion, for any μ , $\lambda(\mu)$, and $K(\mu)$ satisfying the FOC,

$$\frac{R''K'}{R' - \mu(1+g)} + \frac{\lambda''}{\lambda'} > 0. \quad (77)$$

□

Lemma 3 (Optimal choice of equity offering terms). *The optimal amount of capital to raise K^* , fraction of ownership to sell λ^* , and effective outside ownership μ^* exist and satisfy the conditions of the following equation system:*

$$R(K) = \mu[(1+g)(K+W) - a\varphi], \quad (78)$$

$$\mu = \lambda \left[1 - \frac{b\varphi}{\lambda + b} \right], \quad (79)$$

and

$$\frac{(1 - \lambda)(1 + g)}{R'(K) - \mu(1 + g)} = \frac{(\lambda + b)^2}{(\lambda + b)^2 - b^2\varphi}, \quad (80)$$

where $\varphi = a^{-1} \left(\frac{C}{q} \right) [p + (1 - p)\theta]$. Additionally,

$$0 < \mu^* < \mu_1 \equiv \frac{1 + b(1 - \varphi)}{1 + b} \quad \text{and} \quad \mu^* < \lambda^* < 1. \quad (81)$$

The optimal K^* , λ^* , and μ^* are unique if any solution of the equation system satisfies also the condition below:

$$\frac{R''K'}{R' - \mu(1 + g)} + \frac{\lambda''}{\lambda'} > 0, \quad (82)$$

which is met under Assumptions 1 and 2.

The first condition of the equation system above is the competitive financial market condition discussed earlier. It specifies the pricing of the equity offering, linking the amount of capital raised K to the effective outside ownership μ . The second condition describes how the effective outside ownership μ is determined by the ownership sold λ and the effective diversion rate $\frac{b\varphi}{\lambda + b}$.

The last condition of the equation system is a re-expression of the first-order condition of maximization that characterizes the optimal effective outside ownership. In its original form, the condition describes how the marginal benefit and marginal cost of increasing the effective outside ownership balance out with each other at optimum. On the one hand, a marginal increase in μ raises the external capital by an amount of K' . Each dollar of such an increase is multiplied by $1 + g$ through the investment project. Only $1 - \lambda$ of the extra $(1 + g)K'$ belongs to the insider given his retained ownership. The diversion decision does not show up in this calculation because given the equilibrium extended audit decision, the insider is indifferent among alternative choices of the diversion rate and acts as though he had no plan to divert resources. The insider's marginal cost of increasing μ is the reduction in his claim on the year-end firm value $\Pi(K)$ by an amount of λ' . What the first-order

condition says is that in equilibrium the marginal benefit equals the marginal cost:

$$(1 - \lambda)(1 + g)K' = \lambda'\Pi(K) \quad (83)$$

By Lemma 1, $K'/\Pi(K) = 1/[R' - \mu(1 + g)]$ and $\lambda' = (\lambda^* + b)^2/[(\lambda^* + b)^2 - b^2\varphi]$. The first-order condition can thus be expressed as the third condition of the equation system in Lemma 3.

Proof. (Lemma 3) Given the one to one mappings from μ to λ and K , it is convenient to determine the optimal pair of (λ, K) by maximizing a dishonest insider's expected payoff over the domain of μ . To do so, I substitute $\lambda(\mu)$ and $K(\mu)$ into (19) to obtain the following objective function that incorporates the restrictions of (16) and (18):

$$\pi = (1 - \lambda(\mu))\Pi(K(\mu)). \quad (84)$$

The definition of μ , namely (17), implies that $0 < \mu < \lambda$ for all $\lambda > 0$ and $\mu = 0$ for $\lambda = 0$. When $\lambda = 1$, equation (18) implies μ equals

$$\mu_1 = \frac{1 + b(1 - \varphi)}{1 + b}. \quad (85)$$

So the optimal μ , denoted by μ^* , is determined by maximizing π over the domain $[0, \mu_1]$.

Since $\lim_{K \uparrow \infty} R(K)/K > 1 + g$, a sufficiently large K violates the competitive financial market condition $R(K) = \mu\Pi(K)$. Thus, π is bounded. This together with the continuity of π ensures an optimal μ^* exists. Below I show that μ^* cannot be at the corners of the domain $[0, \mu_1]$. Therefore, it must be a solution of the following first-order condition (FOC):

$$\pi' = (1 - \lambda(\mu))(1 + g)K' - \lambda'\Pi(K(\mu)) = 0. \quad (86)$$

The corresponding optimal λ^* and K^* can be obtained by solving equations (16) and (18) given the $\mu = \mu^*$.

Note that $\mu = 0$ implies $\lambda = 0$ and $K = 0$. Hence,

$$\pi'(0) = (1 + g)K'(0) - \lambda'(0)\Pi(0) \quad (87)$$

$$= \left[\frac{(1 + g)}{R'(0)} - \frac{1}{(1 - \varphi)} \right] \Pi(0), \quad (88)$$

where $\Pi(0) = (1 + g)W - a\varphi$. The second equality above uses the fact that $K'(0) = \Pi(0)/R'(0)$ and $\lambda'(0) = 1/(1 - \varphi)$, according to Lemma 1. By assumption, $(1 + g)(1 - \theta) > R'(0)$. Thus, $\pi'(0) > 0$.³⁵ In other words, some sufficiently small μ is strictly better than the corner at zero. When $\mu = \mu_1$, $\lambda = 1$, which implies $\pi'(\mu_1) = -\lambda'(\mu_1)\Pi(K(\mu_1)) < 0$. So the other corner is not optimal either.

Because neither of the corners of the domain $[0, \mu_1]$ is optimal, the optimal μ^* must be an interior solution characterized by the first-order condition. Note that π' is continuous over the domain $[0, \mu_1]$. As $\pi'(0) > 0$ and $\pi'(\mu_1) < 0$, π' must cut across the zero line at some point from above. Only such a solution of the FOC satisfies the second-order condition of a maximum and is an optimal μ^* .

Next I show that if every μ that solves the FOC also satisfies (82) in the proposition, it must then satisfy $\pi'' < 0$ as well. In other words, π' always cuts across the zero line from above. This necessarily means there is only one μ^* that solves $\pi' = 0$. Hence this is the unique optimal choice that maximizes the objective function π . To prove this, consider the second derivative of π :

$$\pi'' = -2(1 + g)\lambda'K' + (1 - \lambda)(1 + g)K'' - \lambda''\Pi(K). \quad (89)$$

For any μ that solves the FOC, i.e. $(1 - \lambda)(1 + g)K' = \lambda'\Pi(K)$, the second derivative of π at that μ is given by

$$\pi'' = -\lambda' \left[2(1 + g)K' - \left(\frac{K''}{K'} - \frac{\lambda''}{\lambda'} \right) \Pi(K) \right]. \quad (90)$$

Since $K' = \Pi(K)/[R' - \mu(1 + g)]$ and $K''/K' = 2(1 + g) - R''K'/[R' - \mu(1 + g)]$, the second

³⁵It suffices to merely assume $(1 + g)(1 - \varphi) > R'(0)$. The stronger assumption replaces the more complicated expression of φ by a single parameter θ .

derivative becomes

$$\pi'' = -\lambda'\Pi(K) \left[\frac{2(1+g)}{R' - \mu(1+g)} - \frac{K''}{K'} + \frac{\lambda''}{\lambda'} \right] \quad (91)$$

$$= -\lambda'\Pi(K) \left[\frac{R''K'}{R' - \mu(1+g)} + \frac{\lambda''}{\lambda'} \right], \quad (92)$$

which is strictly negative under Assumptions 1 and 2.

To complete the proof, I only need to rewrite the first-order condition in the form as provided in the lemma. Substituting $K' = \Pi(K)/[R' - \mu(1+g)]$ into the condition gives

$$\lambda' = \frac{(1-\lambda)(1+g)}{R' - \mu(1+g)}. \quad (93)$$

Using a result of Lemma 1 to substitute for λ' on the left hand side, I obtain

$$\frac{(1-\lambda)(1+g)}{R' - \mu(1+g)} = \frac{(\lambda+b)^2}{(\lambda+b)^2 - b^2\varphi}. \quad (94)$$

Recall that by definition $\mu = \lambda[1 - \varphi(1 - q\omega^*)]$, which can be expressed as follows:

$$\mu = \lambda \left[1 - \frac{b\varphi}{(\lambda+b)} \right]. \quad (95)$$

On top of these two conditions, add the one below that is due to a competitive financial market:

$$R(K) = \mu\Pi(K), \quad (96)$$

where $\Pi(K) = (1+g)(K+W) - a\varphi$. Altogether they form a system of three equations with three unknowns that determines the optimal μ^* , λ^* , and K^* . \square

Lemma 4. *The equilibrium outside ownership λ^* and effective outside ownership μ^* are related to the fraud risk premium φ as follows:*

$$\left(\frac{d\lambda^*}{d\varphi} \right) = \left[\left(\frac{d\mu^*}{d\varphi} \right) + \frac{b\lambda^*}{(\lambda^*+b)} \right] \left[1 - \frac{b^2\varphi}{(\lambda^*+b)^2} \right]^{-1}, \quad (97)$$

$$\left(\frac{d\mu^*}{d\varphi}\right) = \frac{\frac{\lambda^*[\lambda^*+b(1-\varphi)]}{(\lambda^*+b)} \left\{ \frac{aR''}{R' - \mu^*(1+g)} - \frac{(1+g)b}{\lambda^*+b(1-\varphi)} \right\}}{R''K' + \left(\frac{\lambda''}{\lambda'}\right) [R' - \mu^*(1+g)]}, \quad (98)$$

where $\varphi = a^{-1} \left(\frac{C}{q}\right) [p + (1-p)\theta]$. Suppose Assumptions 1 and 2 also hold. Then $d\mu^*/d\varphi \geq 0$ and $d\lambda^*/d\varphi > 0$.

Proof. (**Lemma 4**) Substituting the optimal K^* , λ^* , and μ^* into (78) and (79), I obtain the following identities:

$$R(K^*) = \mu^*[(1+g)(K^* + W) - a\varphi]; \quad (99)$$

$$\mu^* = \lambda^* \left[1 - \frac{b\varphi}{(\lambda^* + b)} \right]. \quad (100)$$

Totally differentiating both sides of the identities with respect to φ yields the following:

$$\left(\frac{dK^*}{d\varphi}\right) R' = \left(\frac{d\mu^*}{d\varphi}\right) \Pi(K^*) + \mu^* \left[\left(\frac{dK^*}{d\varphi}\right) (1+g) - a \right]; \quad (101)$$

$$\left(\frac{d\mu^*}{d\varphi}\right) = \frac{d\lambda^*}{d\varphi} \left[1 - \frac{b\varphi}{(\lambda^* + b)} \right] - b\lambda^* \left[\frac{1}{(\lambda^* + b)} - \frac{\left(\frac{d\lambda^*}{d\varphi}\right) \varphi}{(\lambda^* + b)^2} \right]. \quad (102)$$

Rearrange the terms and recognize that $K'(\mu^*) = \Pi(K^*)/[R'(K^*) - \mu^*(1+g)]$. The identities become

$$\left(\frac{dK^*}{d\varphi}\right) = \left(\frac{d\mu^*}{d\varphi}\right) K' - \frac{\mu^* a}{R' - \mu^*(1+g)}; \quad (103)$$

$$\left(\frac{d\lambda^*}{d\varphi}\right) \left[1 - \frac{b^2\varphi}{(\lambda^* + b)^2} \right] = \left(\frac{d\mu^*}{d\varphi}\right) + \frac{b\lambda^*}{(\lambda^* + b)}. \quad (104)$$

To obtain the third identity for deriving the total derivatives, I substitute $K' = \Pi(K)/[R' - \mu(1+g)]$ into the FOC (86) evaluated at the optimal values to get

$$(1 - \lambda^*)(1+g) = \lambda'(\mu^*)[R'(K^*) - \mu^*(1+g)]. \quad (105)$$

Totally differentiating both sides of this with respect to φ gives the following:

$$-\left(\frac{d\lambda^*}{d\varphi}\right)(1+g) = \left(\frac{d\mu^*}{d\varphi}\right)[R' - \mu^*(1+g)]\lambda'' + \left(\frac{dK^*}{d\varphi}\right)R''\lambda' - \left(\frac{d\mu^*}{d\varphi}\right)(1+g)\lambda'. \quad (106)$$

Use (103) to substitute for $dK^*/d\varphi$ and rearrange the terms. The identity becomes

$$\left(\frac{d\lambda^*}{d\varphi}\right)\frac{1+g}{\lambda'} = -\left(\frac{d\mu^*}{d\varphi}\right)\left[R''K' + \left(\frac{\lambda''}{\lambda'}\right)[R' - \mu^*(1+g)] - (1+g)\right] + \frac{\mu^*aR''}{R' - \mu^*(1+g)}. \quad (107)$$

Multiplying both sides of (104) by $(1+g)/\lambda'$ and then substituting (107) into it, I obtain

$$\begin{aligned} & \left(\frac{d\mu^*}{d\varphi}\right)\left\{\left(\frac{1+g}{\lambda'}\right) + \left[R''K' + \left(\frac{\lambda''}{\lambda'}\right)[R' - \mu^*(1+g)] - (1+g)\right]\left[1 - \frac{b^2\varphi}{(\lambda^* + b)^2}\right]\right\} \\ & = \frac{\mu^*aR''}{R' - \mu^*(1+g)}\left[1 - \frac{b^2\varphi}{(\lambda^* + b)^2}\right] - \frac{b\lambda^*}{(\lambda^* + b)}\left(\frac{1+g}{\lambda'}\right). \end{aligned} \quad (108)$$

Recall that by definition $\mu = \lambda[1 - \varphi(1 - q\omega^*)]$, which is equivalent to

$$\mu = \lambda\left[1 - \frac{b\varphi}{(\lambda + b)}\right]. \quad (109)$$

Moreover, by Lemma 1,

$$\lambda' = \frac{(\lambda + b)^2}{(\lambda + b)^2 - b^2\varphi}. \quad (110)$$

Thus, the left hand side of (108) can be written as

$$\left(\frac{d\mu^*}{d\varphi}\right)\left[\frac{(\lambda^* + b)^2 - b^2\varphi}{(\lambda^* + b)^2}\right]\left\{R''K' + \left(\frac{\lambda''}{\lambda'}\right)[R' - \mu^*(1+g)]\right\},$$

and the right hand side can be written as

$$\begin{aligned}
& \lambda^* \left\{ \frac{aR''}{R' - \mu^*(1+g)} \left[1 - \frac{b\varphi}{(\lambda^* + b)} \right] \left[1 - \frac{b^2\varphi}{(\lambda^* + b)^2} \right] - \frac{(1+g)b[(\lambda^* + b)^2 - b^2\varphi]}{(\lambda^* + b)^3} \right\} \\
= & \lambda^* \left[\frac{[\lambda^* + b(1-\varphi)][(\lambda^* + b)^2 - b^2\varphi]}{(\lambda^* + b)^3} \right] \left\{ \frac{aR''}{R' - \mu^*(1+g)} - \frac{(1+g)b}{\lambda^* + b(1-\varphi)} \right\}.
\end{aligned}$$

Eliminating a common term that is strictly positive, the identity becomes

$$\left(\frac{d\mu^*}{d\varphi} \right) = \frac{\frac{\lambda^*[\lambda^* + b(1-\varphi)]}{(\lambda^* + b)} \left\{ \frac{aR''}{R' - \mu^*(1+g)} - \frac{(1+g)b}{\lambda^* + b(1-\varphi)} \right\}}{R''K' + \left(\frac{\lambda''}{\lambda'}\right) [R' - \mu^*(1+g)]}. \quad (111)$$

By Assumption 2, the bottom expression on the right hand side is strictly positive. This means $d\mu^*/d\varphi \geq 0$ if and only if

$$\frac{aR''}{R' - \mu^*(1+g)} \geq \frac{(1+g)b}{\lambda^* + b(1-\varphi)}. \quad (112)$$

According to the proof of Lemma 2 provided in the appendix, the inequality is satisfied under Assumption 2. This also implies $d\lambda^*/d\varphi > 0$, given (104). \square

Derivation. (Posterior probability of undetected fraud conditional on a delay)

Because a delay necessarily means a red flag has been observed,

$$\Pr\{\text{undetected, fraud}|x = 1\} \quad (113)$$

$$= \Pr\{\text{undetected}|\text{fraud}, x = 1\}\Pr\{\text{fraud}|x = 1\} \quad (114)$$

$$= (1 - q)\Pr\{\text{fraud}|x = 1\} \quad (115)$$

$$= (1 - q) \left(\frac{\Pr\{x = 1|\text{fraud}\}\Pr\{\text{fraud}\}}{\Pr\{x = 1\}} \right) \quad (116)$$

$$= (1 - q) \left(\frac{\Pr\{x = 1|\text{fraud}\}\theta}{\omega^*[\theta + p(1 - \theta)]} \right) \quad (117)$$

$$= (1 - q) \left(\frac{\Pr\{x = 1, \text{red flag}|\text{fraud}\}\theta}{\omega^*[\theta + p(1 - \theta)]} \right) \quad (118)$$

$$= (1 - q) \left(\frac{\omega^*\theta}{\omega^*[\theta + p(1 - \theta)]} \right) \quad (119)$$

$$= \frac{1 - q}{1 + p\left(\frac{1}{\theta} - 1\right)}. \quad (120)$$

Table 1: List of parameters and variables

θ	=	prior probability of having a dishonest insider
p	=	chance of observing a false-positive red flag (i.e., red flag on despite no fraud)
n	=	chance of observing a false-negative red flag (i.e., red flag off despite fraud)
q	=	probability of detecting fraud that exists
C	=	extra cost incurred as a result of extending audit procedures
x	=	decision on extending audit procedures ($x = 1$) or not ($x = 0$)
ω	=	probability of the auditor's randomized choice of x , conditional on observing a red flag
δ	=	diversion rate, i.e., the proportion of the firm's resources to divert
Δ	=	$\delta\theta(1 - q\omega)$, referred to as <i>effective diversion rate</i>
λ	=	fraction of ownership to sell (also referred to as <i>outside ownership</i>)
μ	=	$\lambda(1 - \Delta)$, referred to as <i>effective outside ownership</i>
K	=	amount of external capital to raise
$R(K)$	=	outside investors' <i>total</i> cost of arranging K dollars of capital to invest in the firm; $\frac{R(K)}{K} - 1 =$ "cost of capital" for raising K dollars
R	=	(total) cost of capital function
g	=	constant rate of return on any amount of capital invested in the firm's project
W	=	endowed wealth of the insider
F	=	non-contingent audit fee, payable after the audit
Π	=	$(1 + g)(K + W) - F$, i.e., year-end (after-audit-fee) firm value
b	=	penalty multiplier for an insider committing fraud
a	=	penalty multiplier for an auditor failing to discover fraud during an audit
φ	=	$a^{-1} \left(\frac{C}{q} \right) [p + (1 - p)\theta]$, referred to as <i>fraud risk premium</i> .

Figure 1: Timeline of events in the model

