On the relationship of persistence and number of breaks in volatility: new evidence for three CEE countries

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Abstract: *In this article, we contribute to the discussion of volatility persistence in the presence of sudden changes. We follow previous research, particularly Wang and Moore (2009), who analysed stock market returns in five Central and Eastern European countries using the Iterated Cumulative Sum of Squares (ICSS) algorithm for detecting multiple breaks and the test (IT) proposed by Inclán and Tiao (1994). We complement this analysis by using the* κ_1 *and* κ_2 *statistic introduced by Sansó et al. (2004), which lead us to the hypothesis that the estimated persistence in volatility depends inversely on the number of breakpoints in volatility. We explored this claim through a simulation study, where by randomizing an increasing number of breakpoints over the sample, we estimated kernel density of the persistence measure. The results confirmed the relationship between persistence and the number of breakpoints. It also showed that the use of break detection algorithms leads to lower persistence estimates, even within the class of models with an equal number of breaks. Therefore, the overall decrease in persistence can be attributed both to the number of breaks and their position, as suggested by the chosen break detection tests*.

Key words: volatility persistence, GARCH model, ICSS procedure, CEE stock markets

JEL classification: G15, C22

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I. Introduction

The persistence of volatility describes the effect of volatility changes on the properties of the following observations in the series. Poterba and Summers (1986) concluded that the persistence of volatility shocks is low; they estimated the half-life to be less than six months. Lamoureux and Lastrapes (1990) analysed GARCH properties and persistence of volatility with regard to sudden changes. Hamilton and Susmel (1994) studied autoregressive conditional heteroskedasticity and changes in regime. They fitted GARCH models with various error distributions (Student, Gaussian and GED) and the Markov switching model of Hamilton (1989), which takes into account sudden changes. They concluded that overall persistence is better explained by classifying the data into different time period regimes. A similar approach was used by Kholodilin and Yao (2006).

Aggarwal *et al.* (1999) analysed US and several Latin American, South American and Asian emerging markets, identifying sudden changes by the IT test. They reported very high volatility persistence using standard GARCH models. However, these results were not confirmed in models adjusted for IT breaks, which also made most of the coefficients insignificant.

The possibility of volatility persistence explained by the presence of long memory in financial time series has been explored by Byers and Peel (2001). Marcelo *et al.* (2008) explored volatility shifts in the Spanish market using IT test. Using weekly data from the Canadian stock exchange, Maliq *et al.* (2005) verified persistence overestimation using IT test and concluded that breaks in volatility may be easy to overlook, a problem that lead to high persistence estimates reported in earlier articles. They also emphasized the advantage of using IT test because the breaks are not exogenously provided by the researcher, which could lead to bias through individual judgement, but are endogenously estimated within the modelling framework.

In a recent study, Wang and Moore (2009) analysed the CEE-3 countries, as well as Slovakia and Slovenia, using similar methodology to preceding authors. The choice of weekly data for CEE-3 countries makes their study particularly similar to ours; however, they only employ the IT test for the detection of breaks.

II. Methodology and data

We used weekly data for the period of 6 April 2005 to 7 April 2010 for stock market indices of three Central and Eastern Europe (CEE-3 henceforth) countries: the PX for the Czech Republic, BUX for Hungary and WIG for Poland. The choice was due to the economic development of these countries. The transitional period of these post-communist countries in early 1990 has been followed by diverse economic events like privatization, depreciations of local currencies, or recently, entry into the European Union and significant financial crisis in Hungary.

The log returns for the indices were calculated on a Wednesday-to-Wednesday basis to exclude calendar artefacts. In cases of missing data, the closing values were imputed from the next day with valid prices from the progression “Tuesday, Thursday, Monday, Friday”. Every week in our sample had a valid price.

We did not follow the standard practice of joint estimation for both mean and variance equations. We considered the basic GARCH model as a restricted version of a general model allowing for breaks. The two models are nested, which allows their comparison. However, this would not be true in the case of simultaneous estimation of mean and variance equations, where residuals for the two models would be different. Therefore, we used standard ARMA modelling to obtain the residuals for further analyses.

The presence of unit roots in all series was tested using Phillips–Perron (PP), ADF-GLS, KPSS and Zivot–Andrews (ZA) tests. With exception of the KPSS tests in some cases, all tests suggested stationarity. These exceptions may be the result of model misspecification by not including possible breaks in level and trend. As a consequence, all mean equations included exogenous variables for breaks indicated by the ZA test[1].

Autocorrelation of the residuals presents a problem for the GARCH estimates and the IT test. In previous studies, the mean equation was modelled as an AR(1) process; however, this was not sufficient in our case. Therefore, we chose the minimal ARMA order that lead to residuals with no autocorrelation for up to 60 lags, as indicated by the Ljung–Box *Q*-statistic.

The volatility shifts were identified using three different techniques. Following Inclán and Tiao (1994), let ε_t be a series of residuals with zero mean and variance σ_t^2, where $t = 1,2,...,T$ and T is the number of observations. Denote the cumulative sum of squares $C_0 = 0$ and $C_k = \sum_{t=1}^{k} \varepsilon_t^2$ for $k = 1,2,...,T$. Then IT test statistics is $|D_k|\sqrt{T/2}$:

$$D_k = \frac{C_k}{C_T} - \frac{k}{T}, \quad k = 1,2,...,T \tag{1}$$

The same algorithm can be used unaltered for κ_1 and κ_2 statistics, as suggested by Sansó *et al.* (2004):

$$\kappa_1 = \sup_k \left| T^{-1/2} B_k \right| \tag{2}$$

where

$$B_k = \frac{C_k - \frac{k}{T} C_T}{\sqrt{\hat{\eta}_4 - \hat{\sigma}^4}} \tag{3}$$

and $\hat{\eta}_4 = T^{-1} \sum_{t=1}^{T} \varepsilon_t^4$, $\hat{\sigma}^2 = T^{-1} C_T$ for $k \in \{1,2,...,T\}$.

The calculation of κ_2 is given by

$$\kappa_2 = \sup_k \left| T^{-1/2} G_k \right| \tag{4}$$

where

$$G_k = \hat{\omega}_4^{-1/2} \left(C_k - \frac{k}{T} C_T \right) \tag{5}$$

[1] A dummy variable for a break in the constant was assigned the value 0 prior to and 1 after the occurrence of a break. An exogenous variable indicating a break in trend was set to zero prior and grew linearly after the break. The detailed results of unit root tests are available upon request

Sansó *et al*. (2004) give two possible ways to estimate $\hat{\omega}_4$ consistently, a nonparametric estimator

$$\hat{\omega}_4 = \frac{1}{T}\sum_{t=1}^{T}\left(\varepsilon_t^{\,2} - \hat{\sigma}^2\right)^2 + \frac{2}{T}\sum_{t=1}^{m} w(l,m)\sum_{t=l+1}^{T}\left(\varepsilon_t^{\,2} - \hat{\sigma}^2\right)\left(\varepsilon_{t-l}^2 - \hat{\sigma}^2\right) \tag{6}$$

and a parametric estimator

$$\tilde{\omega}_4 = (1-\sum_{j=1}^{p}\lambda_j)^{-2}T^{-1}\sum_{t=1}^{T}e_t^2 \tag{7}$$

where λ_j and e_t are from the regression

$$\xi_t = \hat{\delta} + \sum_{j=1}^{p}\hat{\lambda}_j\xi_{t-j} + e_t \tag{8}$$

with $\xi_t = \varepsilon_t^2 - \hat{\sigma}^2$.

In our analysis, both alternatives produced the same results, with the exception of WIG, where only the parametric estimator found any breaks. The $w(l,m)$ in Equation 6 is a Bartlet lag window, given by $w(l,m) = 1 - l/(m+1)$. The lag length was calculated according to Newey and West (1994) as $l = \lfloor 4(T/100)^{1/5} \rfloor$, and p in *(8)* was chosen using AIC information criteria. The critical values for each statistic were obtained from a response-surface provided by Sansó *et al*. (2004) because they performed better in small samples[2].

For each series, we estimated a basic GARCH(1,1) model and models, which take into account the changes in volatility, as shown in Table 1. We denoted the number of breaks found in a particular case N_T. These breaks partitioned our observations into groups corresponding to regimes, during which we considered the variance to be constant. Let $\{t_{(1)}, t_{(2)}, ..., t_{(N_T)}\}$ be the set of indices corresponding to the time at which a break is indicated, where $1 \le t_{(1)} < t_{(2)} < ... < t_{(N_T)} \le T$. We also set $t_{(0)} = 1$ and $t_{(N_T+1)} = T+1$. We defined the indicator function

$$D_i(t) = \begin{cases} 1; & 1 \le i \le N_T \wedge t_{(i)} \le t < t_{(i+1)} \\ 0; & \text{otherwise} \end{cases} \tag{9}$$

Using the indicator function as a dummy variable, we formulated a model with breaks as

$$\sigma_t^{\,2} = \alpha_0 + \sum_{i=1}^{p}\alpha_i\varepsilon_{t-i}^2 + \sum_{i=1}^{q}\beta_i\sigma_{t-i}^2 + \sum_{i=0}^{N_T}\gamma_i D_i(t) \tag{10}$$

The persistence of volatility in a GARCH(1,1) is given by $\alpha_1 + \beta_1$.

III. Empirical results

The identified shifts in volatility are presented in the following figure (for detailed results see the Appendix).

[2] The entire procedure was conducted using R software; source code is available upon request.

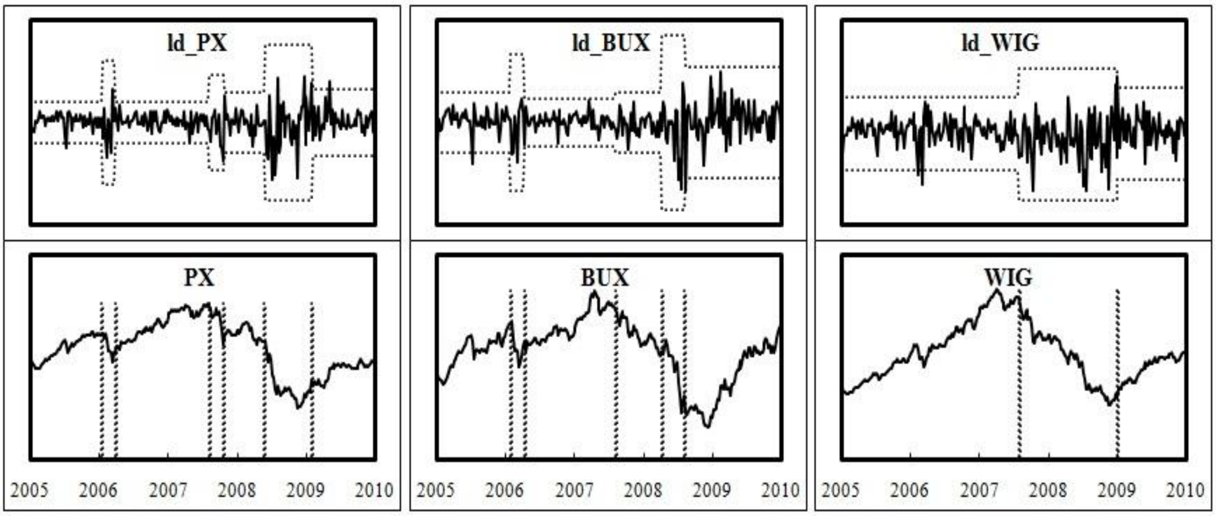

Figure 1. The shifts in volatility for the analysed series

Note: The first row indicates the log returns for the stock indices, with +/- 3 standard deviations calculated for the given regime. The bottom row shows the original values of market indices, with breaks indicated by the IT test.

From Table 1, we can see that GARCH coefficients for all market indices are statistically significant and that the estimated persistence is close to 0.9, indicating lasting effects of sudden changes in variance. When we compare the results with the three models (incorporating these changes by means of dummy variables), we see that the results are clearly different. Similar to Aggarwal *et al.* (1999), some of the ARCH terms are no longer significant. It is also clear that the estimated persistence is lower, particularly with Hungarian BUX.

Table 1. Volatility model estimates

	Single GARCH					*ICSS(IT)*			
	α	***β***	***α+β***			***α***	***β***	***α+β***	***percentile***
px	0.248***	0.704***	0.952		**px**	0.096	0.497***	0.593	0.90%
	(0.080)	(0.074)				(0.066)	(0.140)		[6]
bux	0.208**	0.693***	0.901		**bux**	0.015	0.394**	0.409	0.40%
	(0.083)	(0.110)				(0.051)	(0.190)		[5]
wig	0.141*	0.744***	0.886		**wig**	0.087	0.607***	0.694	4.20%
	(0.073)	(0.128)				(0.066)	(0.161)		[2]
	ICSS(κ_1)					*ICSS(κ_2)*			
	α	***β***	***α+β***	***percentile***		***α***	***β***	***α+β***	***percentile***
px	0.211***	0.544***	0.756	1.40%	**px**	0.231***	0.591***	0.821	8.40%
	(0.075)	(0.111)		[2]		(0.077)	(0.104)		[1]
bux	0.041	0.453**	0.494	1.50%	**bux**	0.131*	0.563***	0.694	0.40%
	(0.063)	(0.192)		[5]		(0.076)	(0.170)		[2]
wig[a]	0.087	0.607***	0.694	4.20%	**wig**	0.108	0.632***	0.740	4.70%
	(0.066)	(0.161)		[2]		(0.072)	(0.155)		[1]

[a] breaks in this case are the same as using IT test; The percentile of the calculated volatility persistence from the corresponding EDF, with the number of breaks in parenthesis.

For all series, the lowest persistence was reported when the IT test was used for identification of breakpoints in variance. As the results suggest, IT is also the test that tends to have the largest number of breaks for every series. There appears to be a link between the number of regimes and the resulting persistence of the series. To test this hypothesis, we conducted a simulation where we randomized an increasing number of breakpoints over our sample, and we used these random breakpoints to calculate the persistence of the series using the methodology described above.

For each series (BUX, PX and WIG) we made 1000 iterations for 6 samples, obtained by randomly generating 1,2,…,6 breaks from a uniform distribution. We only kept cases with the required number of breakpoints in each sample. After calculating the GARCH models adjusted for breaks, we discarded the cases where the estimates resulted in α_1, $\beta_1<0$, $\alpha_1+\beta_1>1$ or where the optimization algorithm failed to converge.

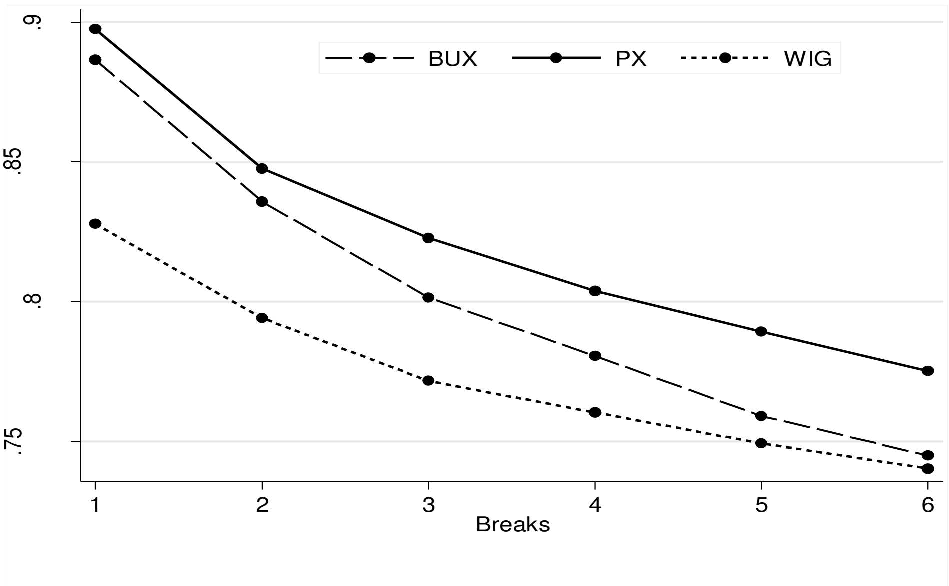

Figure 2. Medians for the persistence measures as a function of the number of breakpoints

Our simulation confirms that by adding breakpoints at random positions in the series, the persistence tends to decrease systematically (see Figure 2). Nevertheless, when the breakpoints were chosen according to IT, κ_1 or κ_2, the estimated persistence was reduced even further. Clearly, the reduction in estimated persistence cannot be solely attributed to the particular breakpoints found by these procedures. To correctly describe the net effect of these procedures, one should compare the persistence to the simulation results obtained for the same number of breaks.

Figure 3 shows the kernel densities of the distribution of $\alpha_1 + \beta_1$. The percentile of the calculated volatility persistence from the corresponding empirical distribution function is shown in Table 1. The volatility persistence estimated by including adjustments for breaks in the variance equation, as indicated by IT, κ_1 or κ_2 is lower not only when compared to the GARCH estimate but also when compared to the persistence distribution from our simulation, as all are below the 10th percentile.

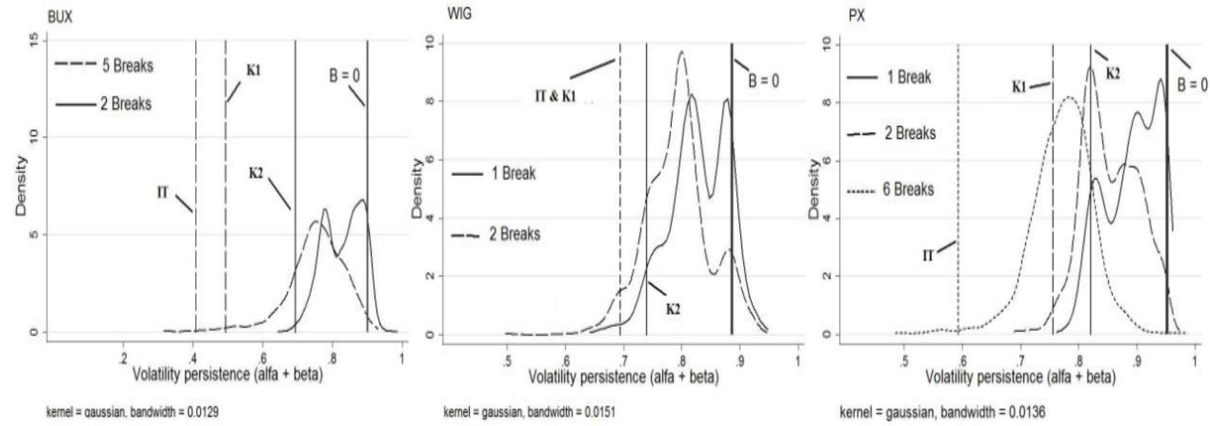

Figure 3. Kernel densities for volatility persistence with breaks at random positions

Note: Vertical bars correspond to the persistence calculated using IT, κ_1 and κ_2. B = 0 indicates no breaks.

IV. Conclusions

This article analyses the volatility persistence in stock index returns of CEE-3 countries. We confirm the findings of previous studies by estimating high persistence in volatility. When adjusting the model by explicit treatment of endogenously identified volatility breaks, the estimated persistence decreases substantially.

We used three different procedures for identifying volatility breaks. This allowed us to demonstrate a connection between the number of breakpoints and the reduction in persistence. We explored this hypothesis by replicating the estimation procedure using a varying number of breakpoints randomly scattered through the sample.

Our contribution, based on the sample of CEE-3 countries, suggests two conclusions. First, we confirm the existence of the inverse relationship between persistence and number of breakpoints. Therefore, the choice of the break identification procedure directly influences the magnitude of the persistence found. Second, even randomly-generated breakpoints lead to lower estimated persistence. Thus, the overall reduction in persistence must be attributed to both the number and the position of the breaks found.

These results are more pronounced when using an algorithm that generates more breakpoints, such as IT, which was used by most earlier studies.

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Appendix

Shifts in volatility, identified by IT, κ_1 and κ_2 tests

PX					
ICSS(IT)		*ICSS(κ_1)*		*ICSS(κ_2)*	
obs.	date	obs.	date	obs.	date
55	26.4.2006	143	2.1.2008	143	2.1.2008
65	5.7.2006	226	5.8.2009		
136	14.11.2007				
147	30.1.2008				
178	3.9.2008				
214	13.5.2009				

BUX					
ICSS(IT)		*ICSS(κ_1)*		*ICSS(κ_2)*	
obs.	date	obs.	date	obs.	date
57	10.5.2006	67	19.7.2006	171	16.7.2008
67	19.7.2006	136	14.11.2007	188	12.11.2008
136	14.11.2007	171	16.7.2008		
171	16.7.2008	182	1.10.2008		
188	12.11.2008	188	12.11.2008		

WIG					
ICSS(IT)		*ICSS(κ_1)*		*ICSS(κ_2)*	
obs.	date	obs.	date	obs.	date
135	7.11.2007	135	7.11.2007	135	7.11.2007
209	8.4.2009	209	8.4.2009		