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Abstract

Political economy aspects make progressive income taxation and taxation of capital income imperative in practise. International tax competition and profit shifting, in turn, put pressure on corporate and capital taxes. Hence, the scope for a politically feasible change-over to a status of improved taxation is little. We provide an extended dynamic general equilibrium model and analyze politically feasible recent reform proposals referring neutrality. We then propose an alternative tax reform that, in contrast to these proposals, guarantees even growth neutrality, without necessarily jeopardizing political feasibility.

Keywords: Dynamic general equilibrium models; taxation; tax reform; decision neutrality; ACE; dual income tax

JEL-code: H21, H24, H25

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1 Introduction

Economists have been advocating tax systems, that assure decision neutrality, for quite a long time. Well-known proposals are the "ACE/EXPEP"[1] system (Wenger, 1983; Boadway and Bruce, 1984; Institute for Fiscal Studies, 1991), the "Flat Tax" (Hall and Rabushka, 1995), and the "Simple Tax" (Rose, 2002, 2005). Given the assumptions of the famous Production Efficiency Theorem (Diamond and Mirrlees, 1971), decision neutrality is an important building block of welfare optima. Though these assumptions do not hold in reality, Devereux and Sørensen (2005, p. 26) conclude that, due to political economy-reasons and a lack of required information at the policy-makers side, "it seems preferable to stick to the traditional goal of tax neutrality." Sørensen (2007) discusses eight alternative proposals and concludes that the fundamental decision to make is whether the "normal" return of capital should be taxed. However, representing rather comprehensive and fundamental reforms, these proposals ignore the political economy aspects of tax reform. Politicians often do not believe in winning elections with these proposals on their agenda – and thus these proposals fail to gain political support.

Especially a progressive income tax schedule with maximum rates well above the corporate tax rate seems to be "not negotiable", because progressive income taxes provide a broadly accepted instrument of redistribution. Varian (1980) already showed that in a world with uncertainty the optimal tax may involve a progressive tariff,[2] and in any case

[1]This is a combination of an *Allowance for Corporate Equity (ACE)* and *Extended Personal Equity Plans (EXPEP)*. The latter exempt the return from savings made out of taxed income from the personal income tax base. The concept of notional interest deduction is based on the theoretical work of Wenger (1983), Boadway and Bruce (1984), and Bond and Devereux (1995). As far as the profit tax base is concerned, the term *Allowance for Corporate Equity (ACE)* is often used, as for the first time in Institute for Fiscal Studies (1991). From 1994 to 2000, Croatia ran a purely consumption-based tax system with an ACE profit tax (cf. Rose and Wiswesser, 1998; Keen and King, 2003). It has also been tested in Brazil (Klemm, 2007). Recently, Belgium introduced a notional interest deduction at the corporate level (Gerard, 2006). An overview of (partial) ACE systems in practice is given by Klemm (2007).

[2]Sinn (1990) discusses the problems of such an insurance argument in an Tiebout-like tax competition. The powerful theorem of Atkinson and Stiglitz (1976) teaches us that a government should not levy capital taxes in order to achieve equity but rather should make use of progressive labor income taxes.

inequality can be harmful for growth (Alesina and Rodrik, 1994; Persson and Tabellini, 1994). Moreover, if multinationals can shift profits across borders via transfer pricing to avoid taxes,[3] capital taxes can be second-best (Haufler and Schjelderup, 2000). It follows that tax reform efforts have to accept particular constraints in practise.

In a globalized world, tax reforms have to address the problems of international tax competition (e.g. McLure, 1986; McGuire, 1986; Wildasin, 1986; Haufler and Schjelderup, 2000; Sørensen, 2007). A single country cannot afford rising the corporate tax rate too far above international average.[4] So the scope in which the change-over from status quo to an improved tax system has to move in is small. In many countries, government's response has been decreasing corporate tax rates while broadening tax base ("Tax-Rate-Cut-cum-Base-Broadening").[5] This may guarantee tax revenue in tax competition by small rates, and provide a second-best level of public good supply (e.g. Haufler, 2001; Becker and Fuest, 2009). The system of personal income taxation is left unchanged in large part.

In Switzerland and Germany interesting compromise proposals have been developed that promise a higher level of decision neutrality without being obviously politically infeasible: the Swiss ACE Dual Income Tax (SADIT)[6] (Keuschnigg, 2004; Keuschnigg and Dietz, 2007) and the German Dual Income Tax (GDIT)[7] (Sinn, 2003; Spengel and Wiegard, 2004; German Council of Economic Experts, 2006). We analyze the two proposals with respect to decision neutrality and growth in a dynamic general equilibrium model. We

[3]See Hines and Rice (1994) for empirical evidence on transfer pricing activities of American multinationals.

[4]Devereux et al. (2002) find that on average corporate tax rates have fallen across EU and G7 countries since the early 1980s. They argue that a fall in the cost of income shifting across countries and international competition for profitable investment projects are two possible explanations. Similarly, Weichenrieder (2005) provides evidence of on average declining corporate tax rates in 29 OECD and 45 non-OECD countries from 1985 to 2004.

[5]Sinn (1989) shows how a tax-rate-cut-cum-base-broadening policy can increase capital net outflows when portfolio investment is the dominant element of international capital movements, using the example of the U.S. tax reform in 1986.

[6]The authors label the proposal "Growth-Oriented Dual Income Tax".

[7]The Dual Income Tax originates in the Nordic Dual Income Tax (Sørensen, 1994; Nielsen and Sørensen, 1997; Cnossen, 2000).

then consider a third alternative of reforming taxation, that guarantees an even higher degree of decision neutrality, without necessarily jeopardizing political feasibility.

We provide an extended dynamic general equilibrium model and formulate a neoclassical growth model with taxation in the tradition of Sinn (1987). Notwithstanding Sinn's approach we do not take Fisher's separation theorem (Fisher, 1930) as *a priori* given; hence, there is no well defined market value of a firm to maximize. Similar to Stiglitz (1973) we thus directly maximize a representative household's utility function subject to the accumulation process.[8] Moreover, we consider real as well as financial capital. The reason for this approach is that Sinn's approach applies net interest as discount rate, which implies a tax system characterized by debt financing not to be inferior to any other source of financing a firm's investment. Our approach serves to question formally whether this condition is satisfied. Furthermore, our approach enables us to distinguish between two levels of personal capital income taxation: taxation at the time of realization and at the time of consumptive usage. This is especially important when analyzing consumption-based tax reform proposals, as the alternative reform we propose in this paper.

2 Basic Model

We consider an economy with competitive markets and identical infinitely living individuals. The basic structure of the model is illustrated in figure 1.

Households For the sake of simplicity, the number of individuals at date $t \geq 0$ is constant over time, and the labor supply is fully inelastic and at each date given by L. There is a representative household that earns labor income wL, where w denotes the wage rate. The household owns the representative firm of the economy. Per capita consumption is a flow c of units of a single good. The representative household evaluates the flow of consumption it can enjoy after a certain point in time t in terms of the utility

[8] Cf. also the models by Judd (1985) and Chamley (1986). However, we do not have the restriction of a finite planning horizon, as in Stiglitz (1973).

function

$$\mathcal{U}(t) = \int_t^{\infty} e^{-\rho(v-t)}\; U[c(v)]\; \mathrm{d}v \tag{1}$$

$U(c)$ is a twice differentiable, strictly concave and monotonically increasing function indicating instantaneous felicity. Parameter $\rho > 0$ represents a subjective rate of discount.

The household's stream of gross savings, labeled b^Q, flows to an account at a representative financial intermediary, also owned by the household. The household decides whether to invest part of the resources held at the account into the firm via injections of new equity, labeled b^K, or to leave resources as a bank deposit. The stock of savings deposit, held at the financial intermediary, is labeled Q. Financial assets bear interest at rate r, i.e. the stock Q yields rQ. However, the household can also decide to withdraw resources from the account for consumption purposes. The corresponding flow variable is denoted by a^Q.

Firms The firm's production of the single consumption good is assumed to depend on the level of real capital K and labor L, according to a neoclassically well-behaved production function $f(K, L)$. We assume the Inada conditions to hold.[9] Economic depreciation of real capital is a fixed fraction $\delta > 0$ of the stock K. The firm takes out loans of size d from the financial intermediary, where d measures loans in terms of net values (new loans minus repayments). For simplicity and in order to exclude tax relief for consumptive usage, we assume that loans are solely used for investments. The outstanding stock of debt we denote by D. As a third way of finance the firm can retain profits, denoted by p. The amount of dividends distributed from the firm (in terms of real assets) to the bank account is given as the residual a^K:[10]

$$a^K := f(K, L) - \delta K - wL - rD - p \tag{2}$$

We assume that financial intermediation is costless and that the interest rate on loans equals the rate on deposits. Hence, the financial intermediary does not make any profits.

[9] That is, $\lim_{K\to\infty} f_K(K, L) = \lim_{L\to\infty} f_L(K, L) = 0$ and $\lim_{K\to 0} f_K(K, L) = \lim_{L\to 0} f_L(K, L) = \infty$.

[10] As usual we exclude debt financed dividends, because these are prohibited in most countries and would cause excessive debt in the long run. We also abstain from new equity financed dividends, as distributions beyond the amount of retained profits are usually treated tax-free.

Consequently, consumption is financed by labor income and by withdrawals from the account, a^Q.

Before we continue some remarks on why we interpose a financial intermediary via an account are apposite at this point: As consumption-based tax systems may distinguish between distributed profits that are reinvested and those that are consumed, it is required to interpose the bank account level, Q, because by doing so, we are able to distinguish three levels of taxation: company taxation, capital income taxation at the time of realization, and taxation at the time of consumption.

These preliminaries settled, the representative household maximizes (1) subject to three equations of motion

$$\begin{aligned} \dot{K} &= d + b^K + p, & (3) \\ \dot{Q} &= rQ + b^Q - a^Q - b^K + a^K, & (4) \\ \dot{D} &= d, & (5) \end{aligned}$$

to the budgetary constraint of the household

$$wL + a^Q = c + b^Q \qquad (6)$$

and to the non-negativity conditions $d, p, a^K, a^Q, b^K, b^Q \geq 0$. Equation (3) tells us that net investments of the firm (into real capital K) can be financed either by debt d, by inflows of new equity b^K, or by retaining profits p. Following Sinn (1987) we assume that economic depreciation δK is always financed via retained profits, that is, p represents a net measure of investment: gross investment minus δK. Equation (4), in turn, tells us that the stock of financial assets rises by accumulating interest earnings, by inflows of new savings, b^Q, or by distributed profits and withdrawn equity, a^K, and is reduced by withdrawn deposits used for consumption, a^Q, or for investments in the firm's equity, b^K. The budget constraint (6) tells us that labor income wL and withdrawn deposits a^Q can be used either for consumption c or for new deposits b^Q. Net savings are given by $b^Q - a^Q$. In order to be able to analyze taxation of consumption at the household level we consider b^Q and a^Q separately, instead of using net savings. Finally, due to $a^K \geq 0$, we obtain via

(2) the financial constraint that a firm cannot retain more profit for investment than it generates: $0 \leq p \leq f(K,L) - \delta K - wL - rD$.

2.1 Laissez-faire Optimum

The laissez-faire optimum is determined by utility maximization of the representative household. After substituting for a^K according to (2) and for c according to (6), the resulting current-value Hamiltonian to maximize is given by:

$$\begin{aligned} \mathcal{H} = U(wL + a^Q - b^Q) + \mu^K \left(d + b^K + p\right) + \mu^D d \\ +\mu^Q \left[rQ + b^Q - a^Q - b^K + f(K,L) - \delta K - wL - rD - p\right] \\ +\beta^K b^K + \gamma p + \alpha^K \left[f(K,L) - \delta K - rD - wL - p\right] + \sigma d + \alpha^Q a^Q + \beta^Q b^Q \end{aligned} \tag{7}$$

To allow for boundary solutions involving the financial variables b^K, p, d or a^K being equal to zero, we have to consider the more general first order Kuhn-Tucker conditions $x \cdot \frac{\partial \mathcal{H}}{\partial x_1} = 0$, referring these control variables' Kuhn-Tucker multipliers $x = \in \{\alpha^K, \beta^K, \gamma, \sigma\}$ of the non-negativity constraints:

$$\alpha^K a^K = \beta^K b^K = \gamma p = \sigma d = 0. \tag{8}$$

Using the Kuhn-Tucker multipliers we are able to apply the usual first order conditions (FOC) $\frac{\partial \mathcal{H}}{\partial x} = 0$ for all control variables $x \in \{a^Q, b^Q, b^K, p, d\}$, and $\frac{\partial \mathcal{H}}{\partial X} = -\dot{\mu}^X + \rho \mu^X$ for state variables $X \in \{K, Q, D\}$. Hence we obtain:[11]

$$a^Q \;:\; U'(c) = \mu^Q - \alpha^Q \tag{9}$$

$$b^Q \;:\; U'(c) = \mu^Q + \beta^Q \tag{10}$$

$$b^K \;:\; \mu^K = \mu^Q - \beta^K \tag{11}$$

$$p \;:\; \mu^K = \mu^Q - \gamma + \alpha^K \tag{12}$$

$$d \;:\; \mu^K = -\mu^D - \sigma \tag{13}$$

$$K \;:\; -\dot{\mu}^K + \rho \mu^K = (\mu^Q + \alpha^K)(f_K(K,L) - \delta) \tag{14}$$

$$Q \;:\; -\dot{\mu}^Q + \rho \mu^Q = \mu^Q r \tag{15}$$

[11] Note that because of using debt as negative stock the first order condition is $-\frac{\partial H}{\partial D} = -\dot{\mu}^D + \rho \mu^D$. Moreover, if p equals the complete profits, this equivalent to $a^K = 0$.

$$D \;\; : \;\; -\dot{\mu}^D + \rho\mu^D = -(\mu^Q + \alpha^K)r \tag{16}$$

The transversality conditions[12], in turn, require

$$\lim_{t\to\infty} \mu^Q(t)e^{-\rho t} = 0 \tag{17}$$

From (9) and (10) it follows, that the shadow price of the bank account, μ^Q, is equal to marginal utility $U'(c)$ in optimum, and that the shadow prices α^Q and β^Q must be equal to zero in optimum.[13] The other conditions inform us about optimal financing of investment (conditions (11)-(13)) and optimal size of investment (conditions (14)-(16)).

2.2 Financial and Investment Neutrality

Static decision neutrality comprises, most of all, financial and investment neutrality. Dynamic decision neutrality, in turn, comprises intertemporal or growth neutrality and will be investigated in Section 5.

Financial Neutrality In the case of positive net investments, $\dot{K} > 0$, the household has to decide how to finance the firm's investment optimally, given the three instruments at hand: b^k, p, d. Irrespective of the way of financing, the marginal utility of investing is given by shadow price μ^K. Combining (11) and (12) it holds $\alpha^K = \gamma - \beta^K$ in optimum. We obtain:

Lemma 1

If $\alpha^K = \gamma - \beta^K$ is an optimum condition, $\alpha^K = 0$ holds.

Proof: Given $\beta^K \geq 0$, $\alpha^K > 0$ is only possible if $\gamma > 0$, which corresponds with $p = 0$. This, however, is inconsistent with $\alpha^K > 0$.

□

[12] See (103) in the Appendix.

[13] Analyzing the last of our tax system alternatives, these Kuhn-Tucker multipliers will become crucial. In all other reform proposals, however, it will always be the case that α^Q and β^Q must be equal to zero.

We conclude that constraint $p \leq f(\cdot) - \delta K - rD - wL$ is never binding for the firm: $\alpha^K = 0$.[14] Conditions (11) to (13) state that the respective marginal utility loss of financing of each instrument must be equal to μ^K in an interior solution where all three options are used. In this case, we obtain $\beta^K = \gamma = \sigma = 0$. As long as the marginal disutility of debt, μ^D, is lower than that of the two alternatives, μ^Q, despite optimal investment is financed completely via debt (which corresponds with $-\mu^D = \mu^K$) it is optimal to choose $b^K = p = 0$, so that β^K, $\sigma > 0$. Therefore, if a financing option involves marginal cost higher than marginal benefit μ^K, this option is not used in optimum and the corresponding respective shadow price – β^K, γ, or σ – is strictly positive.

To conclude, for the household to be indifferent between the possible ways of financing investment, it is necessary that $\beta^K = \gamma = \sigma = \alpha^K = 0$ is in line with the FOC, so that a solution $b^K > 0$, $p > 0$ and $d > 0$ exists. Given indifference, this solution cannot be unique, because any feasible combination of b^k, p and d is optimal, including possible (weak) corner solutions. This is illustrated by Figure 2 and explained in further detail in Appendix B.

We thus obtain an analogon to the well-known neutrality result derived by Modigliani and Miller (1958):

Proposition 1

In the laissez-faire optimum, i. e. without taxes, the household is indifferent referring the way of financing investment, that is, $\beta^K = \gamma = \sigma = \alpha^K = 0$ *is in line with the FOC.*

Proof: See appendix.

Our model implies a world without taxation, which is characterized by marginal cost of financing being the same for all of the three instruments or, equivalently, by the indifference of the household referring the way of financing investment. This situation without

[14] Marginal disutility of issuing new equity (b^K) is loosing one unit at the bank account (μ^Q). Retaining profit involves the same marginal cost, because losing one unit dividend reduces the balance by one unit, too. Therefore, if the firm wants to retain more profit than earned, marginal investment can equivalently be financed via issuing new equity, so that the marginal disutility of not being able to retain more profits is zero: $\alpha^K = 0$.

taxes serves as reference scenario.[15] A tax system is then said to be neutral with respect to the financial decision if the household is indifferent with respect to the combination of the three ways of finance. We obtain:

Corollary 1

The criterion of financial neutrality *is met if and only if for all t a solution exists with*

$$\beta^K = \gamma = \sigma = \alpha^K = 0.$$

Investment Neutrality The optimal size of investment is determined by conditions (14)-(16). Because of $\mu^K = \mu^Q = \mu^D$ these conditions express the well-known result that, in the optimum, the marginal return on investment equals the cost of capital, which is equal to the market rate of interest:

$$f_K(K, L) - \delta = r \tag{18}$$

From the household's perspective, (18) states the no arbitrage condition that the yield of the marginal investment into the firm's stock of real capital equals the yield of lending to the financial intermediary, r. Tax systems are neutral referring the investment decision if this condition holds, which leads to:

Corollary 2

The criterion of investment neutrality *is met if and only if equation* (18) *holds for all t.*

From a normative perspective, neutrality can be considered as a necessary precondition of production efficiency if the strong assumptions of the production efficiency theorem hold (Diamond and Mirrlees, 1971).[16] Although these do not hold in practice, Devereux

[15] Already Modigliani and Miller (1958, p. 291-293) outlined that in real world – for instance, because of manager preferences – the firm is not necessarily indifferent between the ways of financing even in a world without taxation. Similarly our results basically only state that the way of financing is irrelevant for the optimal investment decision. As in the literature on optimal taxation, we abstract from such complications.

[16] To see this interpret different ways of financing a firm's investment as different inputs into the production function. As the theorem states, even in a second best scenario no discriminating taxes should be levied on different input factors in order to prevent a distortion in the production process. Such

and Sørensen (2005, p. 26) conclude that, due to political economy reasons and a lack of required information at the policy-makers' side, "it seems preferable to stick to the traditional goal of tax neutrality."

3 Basic Model with Taxation

We will now incorporate the major taxes on all different levels: the consumption level, the level of bank deposit and the firm level. As we are interested in the economic efficiency of different tax systems, we have to rule out income effects. Therefore, we have to close the model and include the use of the tax revenue. As usual (for instance, Haufler and Schjelderup, 2000), we assume that the tax revenue is rebated as lump-sum transfer to the representative household and that the household does not anticipate that a higher tax payment leads to higher transfers. Thus, from the household's perspective, the lump-sum transfers do not depend on any variable he has to determine, directly or indirectly. We abstain from including the corresponding notation, for simplicity.

3.1 Taxation at the Personal Level

There are two possible levels of personal capital income taxation in the model. Capital income taxation at the time of realization comprises a tax on interest earnings at rate τ^r and one on dividend payments at rate τ^a. Throughout the paper $\theta^i := (1 - \tau^i)$ is used as tax factor, which corresponds to the tax rate τ^i, $i = \{r, a, c, w\}$. The insertion of new equity, b^K, may be deductible from the tax base of the personal capital income tax. The general model meets this possibility by factor $\tilde{\theta^a}$, which takes the value θ^a in the case of deductibility, and one otherwise.

A tax on consumption, as the second possible level of personal capital income taxation, is levied on withdrawals from the bank account for the purpose of consumption at rate

distortions may also occur in the case of a tax system lacking investment neutrality, as different industry sectors may face unequal capital costs. Likewise, this can be interpreted as imposing discriminating taxes on sector specific inputs.

τ^c.[17] Labor income, in turn, is taxed at rate τ^w, which represents the marginal income tax rate. The corresponding constraints (4) and (6) thus change to

$$\begin{aligned} \dot{Q} &= \theta^r rQ + b^Q - a^Q - \tilde{\theta^a} b^K + \theta^a a^K && (19) \\ \theta^w wL + \theta^c a^Q &= c + b^Q && (20) \end{aligned}$$

3.2 Company Taxation

First of all a tax base has to be defined. To be able to model a broad set of company tax regimes, two further parameters, α_1 and α_2, are introduced. The former is well-known from Sinn (1987) and stands for accelerated tax depreciation, while the latter allows for a full or partial deduction of equity costs. The general tax base reads as follows:

$$B := f(K, L) - wL - rD - \delta K - \alpha_1 \dot{K} - \alpha_2 r \left[(1 - \alpha_1)K - D\right] \qquad (21)$$

All company tax regimes considered in this paper allow for a deduction of labor costs as well as interest payments to the creditor. Depending on the details of the corresponding tax law, the tax deductible write-off may lead to a higher present value than the true economic depreciation of the capital stock, as given by δ. This difference is captured by α_1. Thus tax depreciation is modeled by continuous economic depreciation, δ, plus an immediate write-off, α_1. If the tax system allows for imputed costs of corporate equity, the tax base is reduced by $\alpha_2 r$ per unit of equity $[(1 - \alpha_1)K - D]$.

Let the profit tax rate be denoted by τ^G. Note that our basic model is affected by company taxation only in eq. (2), which now changes to:

$$\begin{aligned} a^K &= f(K, L) - \delta K - wL - rD - p - \tau^G B \\ &= \theta^G \left[f(K, L) - \delta K - wL - rD\right] - p \qquad (22) \\ &\quad + \tau^G \left\{\alpha_1 \dot{K} + \alpha_2 r[(1 - \alpha_1)K - D]\right\} \end{aligned}$$

[17] Observe that this is not about a consumption tax in terms of a cash-flow tax, which would imply the household's stream of gross savings, b^Q, to be allowed against (personal capital income) tax.

As (3) stays unchanged, (22) can be expressed as:

$$a^K = \theta^G \left[f(K,L) - \delta K - wL - rD\right] - p \tag{23}$$
$$+\tau^G \left\{\alpha_1(b^K + d + p) + \alpha_2 r[(1-\alpha_1)K - D]\right\}$$

3.3 Optimization in the Presence of Taxation

With taxation (2), (4) and (6) have to be replaced by (19), (20) and (23), respectively. The extended Hamiltonian is given by:

$$\begin{aligned}
\mathcal{H} = U(\theta^w wL + \theta^c a^Q - b^Q) \\
+\mu^K \left(d + b^K + p\right) \\
+\mu^Q \Big\{\tilde{\theta}^a b^K + \theta^a \left\{\theta^G[f(K,L) - \delta K - wL - rD] - p \right. \\
\left. + \tau^G \left[\alpha_1(b^K + d + p) + \alpha_2 r[(1-\alpha_1)K - D]\right]\right\} \Big\} \\
+\mu^D d + \alpha^Q a^Q + \beta^Q b^Q + \beta^K b^K + \gamma p + \sigma d \\
+\alpha^K \Big\{\theta^G[f(K,L) - \delta K - wL - rD] - p \\
+\tau^G \left\{\alpha_1(b^K + d + p) + \alpha_2 r[(1-\alpha_1)K - D]\right\} \Big\}
\end{aligned} \tag{24}$$

As first-order conditions we obtain:[18]

$$a^Q \quad : \quad \theta^c U'(c) = \mu^Q - \alpha^Q \tag{25}$$
$$b^Q \quad : \quad U'(c) = \mu^Q + \beta^Q \tag{26}$$
$$b^K \quad : \quad \mu^K = \tilde{\theta}^a \mu^Q - \beta^K - \tau^G \alpha_1(\theta^a \mu^Q + \alpha^K) \tag{27}$$
$$p \quad : \quad \mu^K = \theta^a \mu^Q - \gamma + \alpha^K - \tau^G \alpha_1(\theta^a \mu^Q + \alpha^K) \tag{28}$$
$$d \quad : \quad \mu^K = -\mu^D - \sigma - \tau^G \alpha_1(\theta^a \mu^Q + \alpha^K) \tag{29}$$
$$K \quad : \quad -\dot{\mu}^K + \rho\mu^K = (\theta^a \mu^Q + \alpha^K)[\theta^G(f_K(K,L) - \delta) + \tau^G \alpha_2 r(1-\alpha_1)] \tag{30}$$
$$Q \quad : \quad -\dot{\mu}^Q + \rho\mu^Q = \mu^Q \theta^r r \tag{31}$$
$$D \quad : \quad -\dot{\mu}^D + \rho\mu^D = -(\theta^a \mu^Q + \alpha^K)[\theta^G + \tau^G \alpha_2] r \tag{32}$$

The laissez-faire Kuhn-Tucker conditions given in (8) remain valid. These formulas serve as tools for the following analysis of different tax reform proposals. They are not in-

[18]The transversality condition (17) must hold in an optimum as well.

terpreted here in this general form. Nevertheless one observation should be mentioned, namely that the deviation of tax depreciation from economic depreciation affects all three financial instruments of the firm (b^K, p and d) in exactly the same way. This can be seen from the last summand in (27), (28) and (29), respectively. That is, if a tax system lacks financial neutrality, this is not due to its particular rules of tax write-off. In fact, such an a-neutrality is due to an inappropriate choice of tax *rates*.

3.4 Some Methodological Remarks

In our approach we do not assume Fisher's Separation Theorem (Fisher, 1930) to hold *a priori*. This theorem states that utility maximization implies the maximization of the representative firm's market value. Thereby the market value is determined by the present value of all future cash flows net of tax between the firm and its shareholders. To calculate the present value, the interest rate net of tax is employed as discount rate.

The economic reasoning underlying this approach is that the market value quotes how much capital would have to be alternatively deposited today in a bank account, yielding interest income, in order to generate the same future net cash flows. However, this premises that the alleged alternative investment in financial assets actually is a good alternative, assuming rational behavior of the investor. Suppose, for example, the corporate tax rate falls short of the personal tax rate levied on interest income. To keep things simple there be no further taxes on dividends and capital gains. In such a scenario, it would be rational to invest as much real capital until the difference between its marginal product and the pre-tax interest rate exactly compensates the tax wedge. But if we allow for granting loans at the company level, the marginal gross product of an investment into the firm will never fall short of the pre-tax interest rate. In such a case the investor is capable of a better alternative than a bank deposit on his private account. Thus, in such a case it can hardly be justified to employ the interest rate net of *personal* tax as the right discount rate to determine the market value (Siemers and Zöller, 2006).

In our general model we exclude granting loans at the company level, as we require $d \geq 0$. In equilibrium, it must hold that $Q = D$. Dropping restriction $d \geq 0$ would result in the firm to become a net supplier of debt capital. In market equilibrium, this would imply the household to borrow from the firm. In the simple example from above this would imply that the tax system allows for tax deduction of interest payed on private credits at a higher rate than the firm has to tax its yields. This implies arbitrage for every positive interest rate and no finite equilibrium $D = Q$ will exist. In such a case, $d \geq 0$ generates a "synthetic" equilibrium $D = Q = 0$ if we assume $D = 0$ as initial value. We do not try to interpret this case, which would degenerate our model in large parts. We rather use the corresponding Kuhn-Tucker multiplier σ to question whether a capital market with a well defined interest rate r exists.

Summing up, if $\sigma = 0$ is in line with the first-order conditions, the non-negativity constraint $d \geq 0$ is not binding. In this case a capital market exists when we close the model. To deposit capital in a bank account yielding net interest is a possible alternative to an investment into the firm, restricting the net return of the latter not to fall below the net interest rate. Thus net interest serves as discount rate to determine the firm's market value. On the other hand, $\sigma > 0$ indicates that there exists no capital market in the closed model. In such a case we are not able to define a market value of the firm.

Our approach allows for a third level of taxation, namely to tax withdrawals from bank accounts which are used for consumption. It enables us to analyze the financial decision of a firm in a tax system applying this level of taxation. Furthermore, if it turns out that $\sigma = 0$ holds in such a tax system, we can find a well defined discount rate and, thus, the market value of the firm. Therefore, by applying the utility approach instead of the market value approach we verify the applicability of Fisher's Separation Theorem in our extended setting.

4 Analysis of Different Tax Systems

Since capital is more mobile than the input factor labor, taxation of capital involves stronger allocation distortions and higher revenue losses than wage taxes. Hence, several reform proposals of the last few years combine a moderate level of capital income and corporate taxes, with maintaining the existing (highly) progressive income tax on wages (e.g. Haufler, 2001). In this section, we analyze three proposals of that kind.

4.1 The Swiss ACE Dual Income Tax

The Swiss ACE Dual Income Tax (SADIT) has been proposed by Christian Keuschnigg in 2004.[19] The profit tax allows for a notional interest on a company's equity. The notional interest rate matches the "normal return" in the amount of the market interest rate r. In contrast to a pure consumption-based tax system, the Allowance for Corporate Equity (ACE) is combined with a personal tax on capital income, which comprises returns on interest, dividend payments and capital gains. Interest yields are taxed at the constant rate τ^r, while dividends and capital gains are taxed at a *dynamic* rate τ^a, in order to compensate for the tax free accumulation of normal returns in the firm. The latter increases in time t, beginning at the time of share purchase t_1:

$$\tau^a \equiv 1 - e^{-\tau^r r(t-t_1)} \tag{33}$$

The tax base is the dividend payment or the realization receipts, respectively. Observe that, in the latter case, purchase costs are not deductible. Assuming efficient capital markets, Auerbach and Bradford (2004) show this tax formula to be equivalent to a cash-flow tax at rate

$$\tau^a = 1 - e^{-\tau^r r(t-\bar{t})} \tag{34}$$

with $\bar{t}$ representing a particular calendar date, which is not associated with the date of purchase. It rather applies to all dividends and realization receipts, independent of the underlying share's purchase date. As is inherent to a cash-flow tax, the share purchase at date t_1 provides a deduction at rate $\tau^a = 1 - e^{-\tau^r r t_1}$. The same is true for the injection of new equity into the firm. Without affecting the functioning of the dynamic tax rate, namely the compensation for the tax free accumulation of normal returns in the firm, the starting point $\bar{t}$ can be set to zero.

[19]Cf. Keuschnigg (2004) and Keuschnigg and Dietz (2007).

In our model, we use the second variant of the dynamic tax rate, that is, the cash-flow version. As this implies a deduction of new equity, b^K, from the capital income tax base at the time of insertion, we have to set $\tilde{\theta^a} = \theta^a$. We account for the ACE system on the company level by setting $\alpha_2 = 1$. The conclusions derived from the first-order conditions in the general model reduce to:

$$a^Q \ : \ U'(c) = \mu^Q - \alpha^Q \tag{35}$$

$$b^Q \ : \ U'(c) = \mu^Q + \beta^Q \tag{36}$$

$$b^K \ : \ \mu^K = \theta^a \mu^Q - \beta^K - \tau^G \alpha_1 (\theta^a \mu^Q + \alpha^K) \tag{37}$$

$$p \ : \ \mu^K = \theta^a \mu^Q - \gamma + \alpha^K - \tau^G \alpha_1 (\theta^a \mu^Q + \alpha^K) \tag{38}$$

$$d \ : \ \mu^K = -\mu^D - \sigma - \tau^G \alpha_1 (\theta^a \mu^Q + \alpha^K) \tag{39}$$

$$K \ : \ -\dot{\mu}^K + \rho \mu^K = (\theta^a \mu^Q + \alpha^K)[\theta^G (f_K(K, L) - \delta) + \tau^G r (1 - \alpha_1)] \tag{40}$$

$$Q \ : \ -\dot{\mu}^Q + \rho \mu^Q = \mu^Q \theta^r r \tag{41}$$

$$D \ : \ -\dot{\mu}^D + \rho \mu^D = -(\theta^a \mu^Q + \alpha^K) r \tag{42}$$

We find:

Proposition 2

The SADIT establishes financial neutrality.

Proof: See appendix.

To get the economic intuition of the result we pick up the interpretation of shadow prices. A once and for all reduction of one unit of outstanding debt reduces interest payments from that time. *Ceteris paribus*, this leads to a permanent increase in dividend payments, which are given as a residual according to (23), in the amount of r. In contrast to the tax free world, only the amount net of dividend tax reaches the bank account, i.e. $\theta^a r$. Remember that θ^a decreases monotonically as time elapses, starting at one in $t = 0$ and converging to zero as time goes to infinity. A once and for all increase in Q, on the other hand, leads to a permanent increase in interest yields, which are taxed at the time they accrue at rate τ^r. As the interest tax rate is constant over time, the dividend tax rate falls short of it in a first period of time and exceeds it afterwards. For given before-tax cash flows this implies a higher net income during this first period in the case of dividends.

This tax induced surplus augments the bank account and yields interest income. By construction the increase in the dividend tax rate exactly offsets this advantage. Starting at $t = 0$, which implies a dividend tax rate of zero, the shadow prices under consideration, $-\mu^D$ and μ^Q, are equal. At every later point in time the dividend tax rate drives a wedge between them. This is due to the fact, that the reduction in the stock of debt is assumed to be exogenous, to be in line with the interpretation of shadow prices, rather than preceded by a tax deductible insertion of equity. It thus bears the very tax burden accrued from $t = 0$ until that time as retained profits. But observe that this is only a level effect and constitutes no incentives for further retaining profits or substituting equity for debt.

To see this algebraically take a look at the present value of the tax induced difference in the net income at time t. As discount rate we use the interest rate net of tax, which is justified as $\gamma = 0$ is in line with the first-order conditions. The present value at time t reads as:

$$\int_t^\infty r(\theta^r - \theta^a)e^{-\theta^r r(v-t)}dv \tag{43}$$

Inserting the formula of the dynamic dividend tax rate according to (34) we receive:

$$\int_t^\infty r(\theta^r - e^{-\tau^r rv})e^{-\theta^r r(v-t)}dv = \int_t^\infty \theta^r re^{-\theta^r r(v-t)}dv - e^{-\tau^r t}\int_t^\infty re^{-r(v-t)}dv = \tau^a \tag{44}$$

As claimed above the present value of the tax induced difference in interest and dividend income, given equal before-tax cash flows, is determined by the dividend tax rate at the time of the respective exogenous once and for all changes in debt and bank deposits. As a special case the tax rate and thus the difference in net income vanishes if $t = 0$.

Next we control for investment neutrality and ask how much is optimally invested, given the gross interest rate r. We obtain:

Proposition 3

The SADIT establishes investment neutrality.

Proof: See appendix.

That is, the optimal investment of the firm is the same as without taxation, so that the tax system does not distort the investment decision of the firm. As this is true for any α_1, this result does not rely on any specific tax write-off. To put it differently, the depreciation rules of the tax code do not affect the pre-tax return of the marginal investment and thus the optimally chosen capital stock, given r. This is a well known result of a pure ACE tax system and, as has just been shown, stays true for the combined tax system of the SADIT. To sum up, the SADIT neither distorts the financial nor the investment decision of the firm. Whether the intertemporal allocation is affected, we will investigate in Section 5.

4.2 The German Dual Income Tax

In 2006, the German Council of Economic Experts proposed a Dual Income Tax for Germany (German Council of Economic Experts, 2006) that was similar to the Norwegian Dual Income Tax Reform in 2002.[20] The German Dual Income Tax (GDIT) formally treats corporations and non-incorporated firms differently.

4.2.1 GDIT and Corporations

The council proposes to tax all kinds of capital income – for instance, dividends, capital gains and interest yields – at a rate which significantly falls short of the maximum rate levied on labor income. The main difference to the current German tax system is an indexing of an asset's purchase price or the amount inserted as new equity into a firm at the personal income tax level. The indexing rate equals the market rate of interest net of tax. Thus dividends and capital gains are effectively not taxed until this imputation capacity is exhausted.

We again confine to insertions of new equity and receipts of dividends. Assume an insertion at time t_1 which gives rise to a dividend payment at t_2 that is high enough to exhaust the imputation induced by the insertion. An insertion of b^K in t_1 leads to an amount of $[\tau^a b^K e^{\theta^r r(t_2-t_1)}]$, which can be offset against the dividend tax in t_2. If we use net interest[21] as discount rate, an assumption to be justified later on, the present value

[20] Cf. Sørensen (2005).

[21] For the sake of simplicity we hold r constant. Nothing essential hinges on this assumption.

of this imputation at any point in time t is given by

$$\tau^a b^K e^{\theta^r r(t_2-t_1)} e^{-\theta^r r(t_2-t)} = \tau^a b^K e^{\theta^r r(t-t_1)} \tag{45}$$

As (45) reveals, the present value of an insertion's imputation capacity is independent of the particular date it is offset against dividend tax. Apart from its reference date t, it solely depends on the time of insertion. Thus the present value of an imputation capacity induced by an insertion b^K and valued at the time of insertion is just $\tau^a b^K$, irrespective of the time it is actually offset.[22] It should be clear that in present value terms it would be equivalent to induce a cash flow tax comprising an asset's purchase price and insertions on the one hand as well as realization receipts and dividends on the other hand, rather than to index an asset's purchase price and insertions of new equity until they can be offset against realization receipts or dividends.[23] We employ this equivalence by setting $\tilde{\theta^a} = \theta^a$.

Furthermore the GDIT for corporations can be represented by setting the following tax parameters. The interest tax rate equals the dividend tax rate and also meets the corporate tax rate, $\tau^r = \tau^a = \tau^G$. In order to see how essential this equality is, we start our analysis with a separate parameter for each tax and equalize them after the optimization. As has just been argued, the future imputation induced by an insertion b^K has a present value of $\tau^a b^K$ at the time of insertion ($\tilde{\theta^a} = \theta^a$). On the company level there is no allowance for corporate equity, thus $\alpha_2 = 0$.

[22]The assumption that the imputation capacity is exhausted by one dividend payment at one point in time (t_2), is not restrictive, as we could split the insertion and assign each fraction to an adequate dividend payment at different points in time.

[23]The equivalence is completely in line with the findings of Sørensen (2005), who analyzes the *Shareholder Income Tax* as the respective part of the Norwegian Dual Income Tax.

The first-order conditions of the general model now yield:

$$a^Q \; : \; U'(c) = \mu^Q - \alpha^Q \tag{46}$$

$$b^Q \; : \; U'(c) = \mu^Q + \beta^Q \tag{47}$$

$$b^K \; : \; \mu^K = \theta^a \mu^Q - \beta^K - \tau^G \alpha_1 (\theta^a \mu^Q + \alpha^K) \tag{48}$$

$$p \; : \; \mu^K = \theta^a \mu^Q - \gamma + \alpha^K - \tau^G \alpha_1 (\theta^a \mu^Q + \alpha^K) \tag{49}$$

$$d \; : \; \mu^K = -\mu^D - \sigma - \tau^G \alpha_1 (\theta^a \mu^Q + \alpha^K) \tag{50}$$

$$K \; : \; -\dot{\mu}^K + \rho \mu^K = (\theta^a \mu^Q + \alpha^K) \left\{ \theta^G \left[f_K(K, L) - \delta \right] \right\} \tag{51}$$

$$Q \; : \; -\dot{\mu}^Q + \rho \mu^Q = \mu^Q \theta^r r \tag{52}$$

$$D \; : \; -\dot{\mu}^D + \rho \mu^D = -(\theta^a \mu^Q + \alpha^K) \theta^G r \tag{53}$$

We obtain:

Proposition 4

The GDIT establishes financial neutrality for corporations.

Proof: See appendix.

Analyzing the investment decision of the firm, given r, we obtain:

Proposition 5

The GDIT does not establish investment neutrality for corporations, unless $\alpha_1 = 0$.

Proof: See appendix.

That is, only in the special case of true economic depreciation ($\alpha_1 = 0$) investment neutrality prevails, which is the well-known Johansson-Samuelson Theorem (Samuelson, 1964; Johansson, 1969).

4.2.2 GDIT and Non-Corporate Firms

As already mentioned the GDIT taxes non-corporate firms quite differently. But, as will be shown, the two ways yield equivalent economic results: τ^r stays unchanged, there is no tax on the withdrawals from the firm, $\tau^a_{non-corporate} = 0$, and the firm's return is taxed at the progressive personal tax rate τ^p, i. e. $\tau^G_{non-corporate} = \tau^p$. The tax rates are chosen such that $\theta^p = \theta^G \theta^a$ holds at the top level of the progressive tariff. At the firm level there is a partial allowance for equity capital. As in an ACE system, the stock of equity net of tax write-offs is the relevant quantity. In contrast to a pure ACE the tax rate levied on the imputed interest on equity at the firm level, however, is not zero, but matches the tax rate on interest income τ^r. We translate this partial allowance into the model by setting

$$\alpha_2 = (\tau^G - \tau^r)/\tau^G \tag{54}$$

Observe that, as the tax base is reduced by $\alpha_2 r$ per unit of tax-written down equity capital, this choice of α_2 implies a tax levied on a normal return r given by

$$\tau^G(1-\alpha_2)r = \tau^r r, \tag{55}$$

which justifies the choice of α_2. We obtain:

Proposition 6

The GDIT establishes financial neutrality for non-corporated firms.

Proof: See appendix.

To analyze the investment decision of the firm we apply (30) using the respective parameters for the non-corporate case of the GDIT.

Proposition 7

The GDIT does not establish investment neutrality for non-corporate firms, unless $\alpha_1 = 0$.

Proof: See appendix.

Summing up, though the GDIT treats corporations and non-corporate firms quite differently in a formal sense, the economic impact of the two variants is equivalent as far as the financial decision and the marginal investment is concerned. If we assume τ^p to be at the top level of the progressive tariff and remember that τ^a and $\tau^G_{corporate}$ are chosen such that $\theta^p = \theta^G_{corporate}\theta^a$ holds, even super-normal profits are taxed equally, measured in present values.

4.3 An Alternative ACE Proposal

We now consider an alternative that consists of an ACE at the company level combined with a modified capital income tax at the personal level, which leaves capital income untaxed until it is withdrawn for the purpose of consumption.[24] While ACE components are well known, the implementation of a usage-dependent capital income taxation requires some explanation.

Our basic model in particular fits for this third proposal (cf. Figure 1): A crucial element of the proposal is the introduction of so called “Qualified Bank Accounts” (QBA).[25] A QBA comprises a savings account as well as a custody account as sub-accounts. For the sake of simplicity assume that every financial transaction made by an individual, in particular the insertion of new equity into the firm, is processed using such a QBA. Furthermore every share be hold within the custody sub-account, which ensures dividends to flow into the QBA when distributed. In principle there are four possible inflows to the QBK: new savings, dividends distributed by the firm, capital gains from share transfers and interest on the current stock of savings. None of these inflows is subject to capital income taxation.

New savings are not tax deductible, which would be the case in an S-base cash-flow tax. Possible outflows are insertions of new equity into the firm and withdrawals for consumption. Only the latter is subject to capital income taxation. Thus there is an asymmetry between taxable withdrawals and non-deductible new savings.

As the QBA implies zero taxes levied on interest income accumulated within the account and dividends payed into the account, the corresponding tax parameters in the model vanish, that is, $\tau^r = \tau^a = 0$. Withdrawals from the QBA are taxed at $\tau^c > 0$. On the

[24] See Heidelberger Steuerkreis and RWI Essen (2006) for the basics of the proposal. Siemers and Zöller (2006) further describe the proposal and analyze its neutrality properties in a two-period setting.

[25] In the remainder of the paper we refer to the Heidelberg-RWI proposal as ACE/QBA system. The basic concept of the ACE/QBA system were developed by Manfred Rose, Marco Scholz and Daniel Zöller at the University of Heidelberg. Since 2006 the RWI joins this team. The tax law provisions necessary to introduce the ACE/QBA system in Germany have been developed, too.

company level the ACE implies $\alpha_2 = 1$. The conclusions derived from the first-order conditions in the general model with taxation now reduce to:

$$a^Q \quad : \quad \theta^c U'(c) = \mu^Q - \alpha^Q \tag{56}$$

$$b^Q \quad : \quad U'(c) = \mu^Q + \beta^Q \tag{57}$$

$$b^K \quad : \quad \mu^K = \mu^Q - \beta^K - \tau^G \alpha_1(\mu^Q + \alpha^K) \tag{58}$$

$$p \quad : \quad \mu^K = \mu^Q - \gamma + \alpha^K - \tau^G \alpha_1(\mu^Q + \alpha^K) \tag{59}$$

$$d \quad : \quad \mu^K = -\mu^D - \sigma - \tau^G \alpha_1(\mu^Q + \alpha^K) \tag{60}$$

$$K \quad : \quad -\dot{\mu}^K + \rho\mu^K = (\mu^Q + \alpha^K)[\theta^G(f_K(K,L) - \delta) + \tau^G r(1-\alpha_1)] \tag{61}$$

$$Q \quad : \quad -\dot{\mu}^Q + \rho\mu^Q = \mu^Q r \tag{62}$$

$$D \quad : \quad -\dot{\mu}^D + \rho\mu^D = -(\mu^Q + \alpha^K) r \tag{63}$$

We obtain:

Proposition 8

The ACE/QBA system establishes financial and investment neutrality.

Proof: See appendix.

Its again noteworthy that investment neutrality holds in this system independent of the depreciation parameter α_1. Overall, combining an ACE at the company level with a QBA preserves the well-known intra-temporal neutrality results of a pure consumption-based tax system. It neither distorts the investment and financial decision of a firm, nor does the investment decision depend on the tangible depreciation rules given by tax law.

As far as static efficiency is concerned, the ACE/QBA system does not qualitatively differ from the SADIT. However, it is attained in quite different ways. The SADIT basically adheres to taxing capital income, in particular interest yields, when it accrues. It counterbalances the relative advantage of an ACE tax base, namely to accumulate normal returns tax free on the company level, by adequately increasing the tax rate levied on dividends and capital gains. The ACE/QBA system, in contrast, countervails the relative disadvantage of taxing interest yields, compared to an ACE tax base on the company level. Thus a Qualified Bank Account is primarily employed in order to leave interest yields tax exempt as long as they are accumulated, just as normal returns on assets at the company level are treated given an ACE tax base. Moreover, QBAs also prevent lock-in effects, which would arise if the reallocation of capital from one firm to another would inevitably be charged with a tax. In contrast, under GDIT and SADIT the reallocation of capital triggers capital income taxation on the amount not yet subject to personal income taxation. The SADIT prevents lock-in effects by resetting the dynamic tax rate to zero ($\tau^a = 0$). In the case of the GDIT the base for indexing the purchase costs of the new shares is increased by the taxed amount.

However, the decisive advantage of the alternative is located at its dynamic characteristics, which brings us to the issue of intertemporal or growth neutrality, that is, the effects on the capital accumulation process and thus on economic growth. For this issue it will turn out that the first two FOC of the new ACE/QBA proposal are decisive. From (56) and (57) we conclude that the tax on withdrawals from the bank account, τ^c, potentially drives a wedge between the shadow price of bank deposits, μ^Q, and the marginal utility of consumption, $U'(c)$, depending on the value of β^Q. We obtain:

$$\alpha^Q + \beta^Q = \tau^c U'(c) \tag{64}$$

There are three possible scenarios that fulfill this condition: (i) $\alpha^Q > 0$ and $\beta^Q = 0$; (ii) α^Q, $\beta^Q > 0$; and (iii) $\alpha^Q = 0$ and $\beta^Q > 0$. Given the slack conditions $\alpha^Q a^Q = 0$ and $\beta^Q b^Q = 0$, this corresponds with (i) savings, but no withdrawals from the QBA: $a^Q = 0$ and $b^Q \geq 0$; (ii) no savings and no withdrawals: $a^Q = b^Q = 0$; and (iii) no savings, but withdrawals from the QBA: $a^Q \geq 0$, but $b^Q = 0$. Hence, in case (i) there is no wedge between μ^Q and $U'(c)$, because $\beta^Q = 0$. Note that at every point in time there are either no insertions or no withdrawals from the bank account. However, it might be in line with utility maximization that there are neither insertions nor withdrawals at the same time (especially case (ii)).

It follows that three distinct phases are possible: one of insertion of new savings (phase I: $b^Q \geq 0$, $\beta^Q = 0$), one of internal growth by retaining savings (phase II: $b^Q = a^Q = 0$, β^Q, $\alpha^Q \geq 0$) and one of withdrawing (phase III: $a^Q \geq 0$, $\alpha^Q = 0$). How these possible phases interact and in which way they affect the path of capital accumulation will be analyzed in the next section.

5 Taxation and Economic Growth

In this section we address the question whether the introduced tax systems affect economic growth. For this purpose we have to endogenize the factor prize paths, that is, the interest rate r and the wage rate w. In each period equilibrium requires that demand and supply is balanced such that the factor markets clear. As we assumed inelastic labor supply given by L and competitive factor markets the wage rate is determined by the marginal product of labor: $w(t) = f_L(K(t), L)$. As financial neutrality holds for all of the described tax systems, we can assume without loss of generality that the firm's real capital is completely debt financed. In a closed economy, debt then has to equal savings. Taken together equilibrium condition $Q = D = K$ always holds. Capital demand is in all three tax systems implicitly determined by:

$$r(t) = \frac{f_K(K(t), L) - \delta}{P_K} \tag{65}$$

where variable P_K represents a distortion parameter: $P_K = 1$ holds in the case of investment neutrality (SADIT and the ACE/QBA system) and $P_K = 1 - \alpha_1 \tau^r$ in the case of the GDIT. The supply side is driven by (25), (26) and (31). With $dU'(c)/dt = U''(c)\dot{c}$ we obtain after a few steps

$$\frac{U''(c)}{U'(c)}\dot{c} = \frac{\dot{\mu}^Q + \dot{\beta}^Q}{\mu^Q + \beta^Q} \tag{66}$$

and

$$\frac{\dot{\mu}^Q}{\mu^Q} = \rho - \theta^r r \tag{67}$$

Inserting (65) into (67) yields

$$\frac{\dot{\mu}^Q}{\mu^Q} = \rho - \theta^r \frac{f_K(K(t), L) - \delta}{P_K} \tag{68}$$

As tax revenue is rebated to the household as lump-sum transfer, the whole production net of economic depreciation, $f(K, L) - \delta K$, is either consumed or invested. Thus we obtain a further equation of motion

$$\dot{K} = f(K, L) - \delta K - c \tag{69}$$

which holds in an intertemporal market equilibrium.

5.1 Growth Effects of the SADIT and the GDIT

As has been shown above, $\beta^Q = 0$ holds for all t both within the SADIT and the GDIT. Thus, by equating (66) and (68) we obtain:

$$\frac{U''(c)}{U'(c)}\dot{c} = \rho - \frac{\theta^r}{P_K}[f_K(K(t), L) - \delta]. \tag{70}$$

The dynamic system described by (69) and (70) determines paths $c(t)$ and $K(t)$, which are well-defined and unique for given initial values $c(0)$ and $K(0)$ (Hadley and Kemp, 1973, p. 370-371). For a given initial value $K(0)$ there is only one path, the "stable path", that converges to steady state (Sinn, 1987, p. 245). The stable path is the only path that is compatible with the idea of market equilibrium (Sinn, 1987, p. 377ff).[26]

We restrict the analysis to the properties of the steady state, where $\dot{c} = \dot{K} = 0$ must hold. From (70) we then obtain:

$$\frac{\theta^r}{P_K}[f_K(K(t), L) - \delta] = \rho \tag{71}$$

Observe that, as $\frac{\theta^r}{P_K}[f_K(K(t), L) - \delta] = \theta^r r$, in steady state $\dot{\mu}^Q = 0$ holds as well. This ensures that the transversality condition, given by (17), is satisfied. The steady state is affected by taxation only through $\frac{\theta^r}{P_K}$.

Without taxation, $\frac{\theta^r}{P_K} = 1$. In the case of the SADIT, investment neutrality implies $P_K = 1$ and $\tau^r > 0$. Therefore, $\frac{\theta^r}{P_K} < 1$ and $f_K(K(t), L) - \delta$ has to be greater than in the absence of taxation. This implies a lower steady state level of capital. The steady state level of the capital stock implied by the GDIT depends on the value of α_1, i. e. the rules of tax write-off. Only in the extreme case of immediate write-off, $\alpha_1 = 1$, we have $\frac{\theta^r}{P_K} = \frac{\theta^r}{\theta^r} = 1$, which would imply the same steady state capital stock as with no taxation. In Germany, as in other countries, there is $\alpha_1 < 1$ and the capital stock is lower than that without taxation. Hence, the SADIT as well as the GDIT distort the steady state allocation of capital. We obtain:

Proposition 9

The SADIT as well as the GDIT do not establish growth neutrality.

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[26] Given $K(0)$ and the path of $r(t)$ the maximizing household, who regards $r(t)$ as exogenously given, chooses the initial value of the control variable $c(0)$ such that $(K(0), c(0))$ actually lies on the stable path.

5.2 Growth Effects of the ACE/QBA Proposal

As we have seen in Section 4.3, we can distinguish three different possible phases. We will concentrate on the one that is relevant in the neighborhood of the steady state. As in a steady state $\dot{K} = 0$ holds by definition, no further capital is accumulated. Consequently all kinds of capital income are paid to the QBA, be it in the form of distributed profits or interest income, and no capital is retained or inserted into the firm ($p = b^K = 0$). With $Q = K$ we immediately conclude that $\dot{Q} = \dot{K} = 0$ holds in the steady state as well. No cash flow from the QBA to the firm on the one hand and a constant QBA stock on the other, implies positive withdrawals for consumptive usage in steady state, $a^Q > 0$, and thus $\alpha^Q = 0$. Now imagine we are below the steady state capital stock. The closer we are to the steady state, the smaller is $\dot{K}$. If we are sufficiently close, we can be sure that there is more capital income than will be reinvested. Thus, in a neighborhood of the steady state the same arguments as in the steady state imply $\alpha^Q = 0$ or equivalently $\beta^Q = \tau^c U'(c)$. Hence, (66) reduces to

$$\frac{U''(c)}{U'(c)}\dot{c} = \frac{\dot{\mu}^Q}{\mu^Q} \tag{72}$$

which can be equated to (68). With $\theta^r = 1$ we conclude that

$$\frac{U''(c)}{U'(c)}\dot{c} = \rho - (f_K(K,L) - \delta) \tag{73}$$

holds in the neighborhood of the steady state.[27] As (73) is equivalent to the the case without taxation, the growth path in the neighborhood of the steady state is not altered by the tax system. We obtain that $r(t) = \rho$ must hold in steady state. Thus,

$$f_K(K,L) - \delta = \rho, \tag{74}$$

which matches the steady state condition in the case without taxation. This neutrality result implies by no means, that the whole path of capital accumulation is undistorted by the tax system. To investigate the whole path of capital accumulation, we take a further look at the three distinct phases labeled phase I, II and III (see Section 4.3).

[27]The underlying argument bears analogy to the "new view" of dividend taxation (Sinn, 1991), though not related to a single firm but to the aggregate economy: if enough capital is accumulated through retained capital income, the tax on withdrawals for consumptive usage are sunk cost.

Proposition 10

The optimum consumption path $c(t)$ is continuous. This also holds at the transition from one phase to another.

Proof: See appendix.

We now turn to the final phase of an optimal path of $c(t)$ and $K(t)$. We obtain:

Proposition 11

Only phase III is capable of converging to an optimal steady state.

Proof: See appendix.

It should be mentioned that every path of capital accumulation, that leads to an infinite capital stock, cannot be optimal either. As this implies $\lim_{t\to\infty} r(t) = 0$, $\mu^Q(t) > 0$ grows at rate ρ in the long run. This, again, contradicts the transversality condition (17). Furthermore, we derive the following result:

Proposition 12

Phase I cannot be followed immediately by phase III. Phase III cannot be followed immediately by phase I.

Proof: See appendix.

To sum up, phase I can only be succeeded by phase II, and phase III can only be preceded by phase II. In general, we cannot exclude that phase I and II respectively phase II and III alternate. In any case the final phase of a convergent path $(c(t), K(t))$ has to be of type III, that is, this phase is characterized by withdrawing resources from the QBA for consumption.

A sufficient condition for the existence of at most one phase of each type is given by the following proposition. Here η denotes the elasticity of marginal utility.[28]

Proposition 13

If

$$\frac{f_K(K,L) - \delta - \rho}{f(K,L) - \delta K - \theta^w f_L(K,L)\, L} > \eta\, \frac{f_{LK}(K,L)}{f_L(K,L)} \tag{75}$$

[28] The elasticity is defined as $\eta := -c\,U''(c)/U'(c)$.

holds at every point $c(t) = \theta^w w(t) L$ along the optimum consumption path, neither phase II and III nor phase I and II alternate.

Proof: See appendix.

As Proposition 11 states, paths $c(t)$ and $K(t)$, that satisfy the first order conditions and converge to a steady state, end in a final phase of type III. At least at the junction of a possible phase II and this final phase III, equation (75) must hold.

In the case of a linear-homogeneous Cobb-Douglas production function with a and b representing the output elasticity of capital and labor, respectively, eq. (75) reduces to:

$$f_K(K,L) - \delta - \rho > \eta\, a\,[(1+\tau^w \frac{b}{a}) f_K(K,L) - \delta] \tag{76}$$

If $\eta\, a(1+\tau^w \frac{b}{a}) \geq 1$, which necessarily holds for $\eta\, a \geq 1$, eq. (76) does not hold for any K below to the steady state level.[29] In such a case, no phase II exists, and consequently no phase I does. For any given initial capital stock, the economy starts (and ends) in a unique phase III.

If $\eta\, a(1+\tau^w \frac{b}{a}) < 1$, implying $\eta\, a < 1$, (76) is equivalent to

$$f_K(K,L) > \frac{\delta(1-\eta\, a) + \rho}{1 - \eta\, a\,(1+\tau^w \frac{b}{a})} > \rho \tag{77}$$

As f satisfies the lower Inada-condition, $f_K(0,L) = \infty$, a positive capital stock $\hat{K}$ exists for which equation (76) holds. Furthermore, assuming η to be constant, (76) holds a fortiori for all $K < \hat{K}$ as $f_K(K,L) > f_K(\hat{K},L)$. Consequently, phase II may exist and if this is the case, equation (76) holds for all K left to the junction of phase II and phase III. Thus phase II neither alternates with phase I nor III.

To sum up, if we assume a linear homogeneous Cobb-Douglas production function and a constant elasticity of marginal utility, exactly one phase III and at most one phase I and one phase II exist.

A numerical example of the convergent path for such a case is given by Figure 3. The thick line depicts the process of consumption and capital accumulation. Parameters satisfy the

[29] A capital stock below the steady state level assures $f_K - \delta > 0$. Starting to the right of the steady state necessarily leads to positive withdrawals $a^Q > 0$, thus the economy is in phase III anyway.

inequality in (76). Furthermore, the initial value of K, which is set to unity, is sufficiently small to start in a phase of insertions (phase I). For the sake of comparison, the thin dashed line depicts the respective path under the SADIT, starting at the same initial capital stock. Here, the tax rate on interest income equals the tax rate levied on withdrawals from the QBA in the ACE/QBA system, $\tau^r_{SADIT} = \tau^a_{ACEQBA}$. Assuming true economic depreciation, $\alpha_1 = 0$, and a capital income tax rate of equal magnitude, the dashed line also depicts the convergent path under the GDIT.[30]

6 Conclusion

We introduce an extended dynamic general equilibrium model for investigating decision neutrality of tax systems. In doing so, we apply an alternative way of checking for financial neutrality via shadow prices in a Kuhn-Tucker framework and introduce a stylized financial intermediary. In our framework, non-negativity constraints play also a crucial role in analyzing the growth characteristics of different tax systems.

We apply the model and analyze recent tax reform proposals – the "Swiss ACE Dual Income Tax" (SADIT) and the "German Dual Income Tax" (GDIT) – that accept several exogenous restrictions, in order to be politically feasible, and, at the same time, to take account of international tax competition. We then consider a third alternative – proposed by Heidelberger Steuerkreis and RWI Essen –, that is based on an ACE at the company level combined with so called "Qualified Bank Accounts" (QBA) at the private level.

The developed extended dynamic general equilibrium model is in particular required for investigating the ACE/QBA system, but also generates interesting insights into the functioning of other proposals. In all three tax systems financial neutrality prevails. The SADIT and the ACE/QBA system also establish investment neutrality. Moreover, in contrast to the SADIT and GDIT, the ACE/QBA system establishes additionally intertemporal, respectively, growth neutrality, and thus ensures at least an undistorted level

[30] Accelerated tax depreciation, $\alpha_1 > 0$, leads to a convergent path which lies right of, and $\alpha_1 < 0$ to one which lies left of the dashed line. This is in line with the findings of Sinn (1987).

of the capital stock in and around the steady state. Therefore, in contrast to the other considered proposals, the alternative ACE/QBA system allows for the socially optimal growth rate in the long run.

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Appendix

A Proofs

Proof of Proposition 1: Insert $\beta^K = \gamma = \sigma = 0$ into the FOC. Combining (11) and (12) it follows that $\alpha^K = 0$. Because of (13) we have $\mu^K = \mu^Q = -\mu^D$. As this must be true for all t, we also have $\dot{\mu}^Q = -\dot{\mu}^D$. Insertion of $\dot{\mu}^Q = -\dot{\mu}^D$ into (16) together with $\alpha^K = 0$ shows that (16) is equivalent to (15). Given that the other FOC are fulfilled as well, we found a solution with $\beta^K = \gamma = \sigma = \alpha^K = 0$.

□

Proof of Proposition 2: (35) and (36) guarantee $\alpha^Q = \beta^Q = 0$. Due to Lemma 1, combining (37) and (38) produces $\alpha^K = 0$. It immediately follows that $\beta^K = \gamma$: retaining profits and inserting new equity are not discriminated against each other. Assuming $\gamma = \sigma = 0$, the combination of (38) and (39) generates $\theta^a \mu^Q = -\mu^D$. Differentiation with respect to time t yields

$$-\dot{\mu}^D = \theta^a \dot{\mu}^Q + \dot{\theta}^a \mu^Q \tag{78}$$

Differentiating the dynamic tax factor θ^a according to (34), and substituting for $\dot{\mu}^Q$ and $-\dot{\mu}^D$ according to (41) and (42), respectively, we then obtain

$$\theta^a \mu^Q (r - \rho) = \theta^a \mu^Q (r - \rho), \tag{79}$$

which bears no contradiction. Therefore, $\beta^K = \gamma = \sigma = 0$ is in line with all first-order conditions, and the SADIT establishes financial neutrality.

□

Proof of Proposition 3: Applying $\alpha^K = \beta^K = 0$, we obtain from (37):

$$\mu^K = \theta^a \mu^Q (1 - \tau^G \alpha_1). \tag{80}$$

Differentiation and substituting for $\dot{\mu}^Q/\mu^Q$ according to (41) yields:

$$\dot{\mu}^K/\mu^K = \dot{\mu}^Q/\mu^Q - \tau^r r = \rho - r \tag{81}$$

Rearranging (40) and substituting for μ^Q according to (80) we arrive at:

$$\rho - \dot{\mu}^K/\mu^K = (1-\tau^G\alpha_1)^{-1}[\theta^G(f_K(K,L)-\delta) + \tau^G r(1-\alpha_1)] \tag{82}$$

Using (81) we finally obtain

$$(1-\tau^G\alpha_1)r = \theta^G(f_K(K,L)-\delta) + \tau^G r(1-\alpha_1), \tag{83}$$

which is equivalent to $f_K(K,L) - \delta = r$.

□

Proof of Proposition 4: Analogously to the SADIT, we find $\alpha^Q = \beta^Q = 0$ from (46) and (47), as well as $\alpha^K = 0$ and $\gamma = \beta^K$ from (48) and (49). Testing for $\gamma = \beta^K = \sigma = 0$ yields $\theta^a\mu^Q = -\mu^D$, and differentiation with respect to time $\theta^a\dot{\mu}^Q = -\dot{\mu}^D$. Substituting for $\dot{\mu}^D$ and μ^D in (53) reveals that this is in line with (52) only if $\theta^r = \theta^G$, which holds according to the tax rates proposed by the council. Thus financial neutrality holds.[31]

□

Proof of Proposition 5: Applying $\alpha^K = \beta^K = 0$ to (48) we arrive at:

$$\mu^K = \theta^a\mu^Q(1-\tau^G\alpha_1) \tag{84}$$

Differentiation with respect to t and substituting for $\dot{\mu}^Q/\mu^Q$ according to (52) yields

$$\dot{\mu}^K/\mu^K = \dot{\mu}^Q/\mu^Q r = \rho - \theta^r r \tag{85}$$

Then substituting for $\theta^a\mu^Q$ according to (84) in (51) we obtain:

$$\dot{\mu}^K/\mu^K = \rho - (1-\tau^G\alpha_1)^{-1}\theta^G(f_K(K,L)-\delta) \tag{86}$$

[31] Interestingly, this would also be true for any other dividend tax rate $\tau^a \neq \tau^r$, thus it is not necessary to set $\tau^a = \tau^r$. However, the neutrality result requires $\theta^r r$ to be the rate of indexing the purchase price. That $\theta^r r$ is the right discount rate is justified, as $\sigma = 0$ is in line with the first-order conditions.

Equating (85) and (86) with using $\tau^r = \tau^G$, we obtain:

$$f_K(K, L) - \delta = (1 - \tau^G \alpha_1) r. \tag{87}$$

As (87) shows, the capital cost depend on the regime of tax write-off. Only in the special case of true economic depreciation ($\alpha_1 = 0$) investment neutrality prevails.

□

Proof of Proposition 6: Again $\alpha^K = 0$ holds (Lemma 1). We thus have $\gamma = \beta^K$. Testing for $\gamma = \beta^K = \sigma = 0$ and applying $\tau^a = 0$ then yields $\mu^Q = -\mu^D$ and consequently $\dot{\mu}^Q = -\dot{\mu}^D$. Applying $\theta^a = 1$ in combining (32) and (54) we obtain:

$$-\dot{\mu}^D + \rho \mu^D = -\mu^Q [\theta^G + \tau^G \alpha_2] r = -\mu^Q \theta^r r, \tag{88}$$

which is the relevant equation of motion for μ^D in optimum. As $\mu^Q = -\mu^D$, this is in line with (52), which holds both for the corporate and the non-corporate firm under the GDIT. Thus financial neutrality also holds for non-corporate firms.

□

Proof of Proposition 7: As equation of motion for μ^K in optimum we obtain:

$$\begin{aligned} -\dot{\mu}^K + \rho \mu^K &= \mu^Q [\theta^G (f_K(K, L) - \delta) + \tau^G \alpha_2 r (1 - \alpha_1)] \\ &= \mu^Q [\theta^G (f_K(K, L) - \delta) + (\tau^G - \tau^r) r (1 - \alpha_1)] \end{aligned} \tag{89}$$

As in the case of a corporate firm, but with $\theta^a = 1$, we can derive

$$\mu^K = (1 - \tau^G \alpha_1) \mu^Q \tag{90}$$

¿From (89) and (90) we conclude

$$\dot{\mu}^K / \mu^K = \rho - (1 - \tau^G \alpha_1)^{-1} [\theta^G (f_K(K, L) - \delta) + (\tau^G - \tau^r) r (1 - \alpha_1)] \tag{91}$$

Applying (85), which also holds in the non-corporate case, yields:

$$\begin{aligned} \theta^r r (1 - \tau^G \alpha_1) &= \theta^G (f_K(K, L) - \delta) + (\theta^r - \theta^G) r (1 - \alpha_1) \\ \Leftrightarrow \quad & f_K(K, L) - \delta = (1 - \tau^r \alpha_1) r \end{aligned} \tag{92}$$

Since $\tau^G_{corporate} = \tau^r$, this is exactly the same result as in the corporate case.

□

Proof of Proposition 8: As the QBA ensures that no cash-flow is subject to personal capital income taxation until it is withdrawn for consumption, no tax factor enters eq. (58) to (60). Since these equations do not differ from their counterparts in the tax free world, we obtain $\alpha_K = 0$ and $\beta^K = \gamma = \sigma = 0$ is in line with the first-order conditions. Thus the financial decision of the firm is not altered by the tax system.

Furthermore, the shadow prices of bank deposits and the reduction of debt are equal: $\mu^Q = -\mu^D$. They differ from the shadow price of real capital only by the constant factor, for instance, $\mu^K = \mu^Q(1-\alpha_1\tau^G))$. Hence, they change at the same rate: $\dot{\mu}^K/\mu^K = \dot{\mu}^Q/\mu^Q$. Using (61) and (62), we thus obtain $f_K(K, L) - \delta = r$, which guarantees investment neutrality.

□

Proof of Proposition 10: Observe first that within phase I and III, the continuity directly follows from the continuity of the costate variable μ^Q, equation (56) and equation (57), respectively, combined with $\alpha^Q = 0$ and $\beta^Q = 0$, respectively. During phase II consumption equals labor income, $c(t) = \theta^w f_L(K(t), L)L$, as there are neither insertions nor withdrawals ($a^Q = b^Q = 0$). Here the continuity of the state variable $K(t)$ and the partial derivative f_L implies the continuity of $c(t)$.

The rest of the proof is given by contradiction. Assume a point of discontinuity at the junction point of two phases at time $\bar{t}$, say $\bar{c}$. Let $\underline{c} < \bar{c}$ be the (without loss of generality) left-hand limit of $c(t)$ as t goes to $\bar{t}$. A marginal decrease of consumption at $\bar{t}$ could be transferred into a marginal increase of consumption in the amount of $e^{-\int_{\underline{t}}^{\bar{t}} r(t)dt}$ at time $\underline{t} < \bar{t}$. This accounts for the fact that deferred consumption can be used for interest-bearing investment. On the other hand, the (marginal) decrease in consumption at $\bar{t}$ has to be discounted by the factor $e^{-\rho(\bar{t}-\underline{t})}$, in order to be comparable with the increase in $\underline{t}$. Put together the shift of a marginal unit of consumption from $\bar{t}$ to $\underline{t}$ leads to an overall effect in the amount of:

$$\Delta U(\underline{t}) = U'(c(\underline{t}))e^{-\int_{\underline{t}}^{\bar{t}} r(t)dt} - U'(\bar{c})e^{-\rho(\bar{t}-\underline{t})} \tag{93}$$

In the limiting case we have

$$\lim_{\underline{t} \to \bar{t}} \Delta U(\underline{t}) = U'(\underline{c}) - U'(\bar{c}) > 0 \tag{94}$$

as $U'' < 0$. That is, the assumed marginal shift of consumption from $\bar{t}$ to $\underline{t}$ increases utility, which contradicts the assumption that $c(t)$ is an optimal control. Observe that $\underline{c} > \bar{c}$ analogously leads to $U'(\underline{c}) - U'(\bar{c}) < 0$, i.e. a marginal deferral of consumption would lead to an increase in utility. Thus the optimum path $c(t)$ has to be continuous.

□

Proof of Proposition 11: To ensure $\dot{K} = 0$,

$$f(K, L) - \delta K - c = 0 \tag{95}$$

must hold in a steady state. During phase II all capital income (including economic rents, if existent) is reinvested ($c = \theta^w wL$). Phase I is characterized by $b^Q > 0$, i.e. less than labor income is consumed ($c < \theta^w wL$). Both phases would be in line with a steady state only if

$$f(K, L) - \delta K - \theta^w wL \leq 0 \tag{96}$$

This implies $rK \leq 0$ and, as K is positive, $r \leq 0$. Because of equation (62), μ^Q then grows at least at rate ρ, with $U'(c)$ remaining constant, as $\dot{c} = 0$ must hold as well. But this contradicts both $\mu^Q \leq U'(c)$ from equation (57) – as b^Q is nonnegative – and the transversality condition (17).

□

Proof of Proposition 12: As μ^Q is continuous this would imply a jump in $U'(c)$ to compensate for the arising wedge imposed by the tax on withdrawals, τ^c. This is not in line with the continuity of $c(t)$.

□

Proof of Proposition 13: During a phase of type II the optimal paths of $c(t)$ and $K(t)$ are characterized by $c(t) = \theta^w f_L(K(t), L)\, L$. Denote this part of the optimum path in a phase diagram in the (c, K) space by $c^{II}(K)$ (see Figure 3). Analogously define $c^I(K)$ and $c^{III}(K)$ to be the parts of the optimal path c in the (c, K) space in phases of type I and III, respectively.

As $c(t)$ has to be continuous, a succeeding phase III has to start at some point (c, K) in phase II. If phases of type II and III alternate, a type III phase also ends at some point in phase II. At such junction points phase III always reaches phase II from above, as $c^I \geq wL$ must hold. At a starting point, the slope of $c^{III}(K)$ exceeds that of $c^{II}(K)$ and vice versa at an endpoint.

Analogously, if phases of type I and II alternate, phase I reaches phase II from below, as $c^I \leq wL$ must hold. At an endpoint, the slope of $c^I(K)$ exceeds that of $c^{II}(K)$ and vice versa at a starting point.

Obviously, those phases do not alternate if the slopes of $c^I(K)$ and $c^{III}(K)$ are steeper than the slope of $c^{II}(K)$ at every possible junction point. The slope of $c^{II}(K)$ is given by

$$\frac{dc^{II}(K)}{dK} = \theta^w f_{LK}(K, L)\, L \tag{97}$$

Given initial values of c and K, phases I and III follow the same dynamic system. Thus the curves $c^I(K)$ and $c^{III}(K)$ exhibit identical slopes at a given point in the c-K space:

$$\frac{dc^I(K)}{dK} = \frac{dc^{III}(K)}{dK} = \frac{\dot{c}^I}{\dot{K}} = \frac{(f_K(K, L) - \delta - \rho)c/\eta}{f(K, L) - \delta K - c} \tag{98}$$

As $c = \theta^w wL = \theta^w f_L(K(t), L)\, L$ holds at possible junctions with phase II, $c^I(K)$ and $c^{III}(K)$ are steeper than $c^I(K)$ at such points if

$$\frac{f_K(K, L) - \delta - \rho}{f(K, L) - \delta K - \theta^w f_L(K, L)\, L} > \eta\, \frac{f_{LK}(K, L)}{f_L(K, L)} \tag{99}$$

At least the starting point of the – necessarily existing – final phase III of a convergent path, the inequality must hold.

□

B Explanation to Figure 2

Figure 2 illustrates graphically, how the non-negativity constraints potentially affect the combination of the three different ways of financing in the optimum. The x-axes depicts the amount of new equity inserted into the firm, b^K. The y-axes shows the amount of new debt, d. For the purpose of the figure, net investments, $\dot{K} > 0$, are exogenously given. A remaining gap, $\dot{K} - b^K - d$, is filled by retained earnings, p.

The thick border lines of the trapeze shown in the figure describe combinations of only two financial instruments employed to finance net investments. The upper thick line with slope -1 represents combinations of new equity and debt finance in the case of no retained profits being used. These combinations are optimal, if $\gamma > 0$ necessarily holds.

The vertical thick line represents all combinations of d and p with $b^K = 0$. Optimum finance is restricted to these combinations, if the FOC require $\beta^K > 0$. In the case of $\sigma > 0$ following necessarily from the FOC, the horizontal thick line is authoritative, i. e $d = 0$. In that case no equity will be employed to finance net investments.

The lower thick line with slope -1 represents the restriction $a^K \geq 0$, which ensures that retained profits do not exceed the firm's annual profits. Due to (2) this is equivalent to the requirement of non-negative dividends a^K. If the restriction is binding, i. e. $\alpha^K > 0$ necessarily holds, all profits are retained in optimum. For the purpose of Figure 2 we implicitly assume net investments $\dot{K}$ to exceed profits π, as otherwise retained profits would suffice to finance net investments.

Points in the interior of the trapeze represent combinations of all financial instruments employed at a positive amount ($b^K > 0$, $p > 0$ and $d > 0$ with $a^K > 0$). If $\alpha^K = \beta^K = \gamma = \sigma = 0$ is in line with the FOC, all feasible combinations are optimal including the ones represented by a point on the thick border lines. If, on the contrary, a tax system distorts the financial decision, optimum financing is represented by points which necessarily lie on a border line.

C Is the Transversality Condition Kept?

We prove the transversality condition concerning μ^Q as part of the necessary conditions of an optimum. Assume $Q^*(t)$ to meet the other first order conditions. The two control variables $a^Q \geq 0$ and $b^Q \geq 0$ make it possible to find admissible smooth paths Q^1 and Q^2, such that $Q^1(t) > Q^*(t)$ and $Q^2(t) < Q^*(t)$, for all $t > t_1$, for some $t_1 \geq 0$, and $Q^1(t) = Q^2(t) = Q^*(t)$, for all $t \leq t_1$. Of course, paths between Q^1 and Q^2 are admissible, too. Thus we can be sure to find a smooth function $\nu(t)$, $\nu(t) = 0$ $\forall t \leq t_1$ and $\nu(t) > 0$ $\forall t > t_1$, such that

$$Q(t, \xi) := Q^*(t) + \xi \nu(t) \tag{100}$$

is admissible in a neighborhood of $\xi = 0$ and for which $Q(t, 0) := Q^*(t)$ holds for all t. By construction (100) defines an admissible variation of Q(t), which neither varies $K(t)$ nor $D(t)$, as these paths are not affected by a^Q and b^Q. Thus we found a function

$$\boldsymbol{\nu} := (\nu, 0, 0) \tag{101}$$

and a one-parameter family

$$(Q, K, D)(t, \xi) = (Q^*, K^*, D^*)(t) + \xi \boldsymbol{\nu} \tag{102}$$

with X^* denoting a path of the state variable $X = Q, K, D$, which meets the other first order conditions.

We can now apply a general formulation of transversality conditions for infinite horizon problems with free boundaries for each state variable, which states that the product of the vector of costate variables (in present values) and the vector of any admissible variation in the state variables, that is, any admissible ν, must vanish as time goes to infinity (Hadley and Kemp, 1973, p. 225, 292). In our special case we thus have

$$\lim_{t \to \infty} (\mu^Q, \mu^K, \mu^D) e^{-\rho t} \boldsymbol{\nu} = \lim_{t \to \infty} \mu^Q e^{-\rho t} \nu = 0 \tag{103}$$

As ν does not vanish at infinity, $\mu^Q e^{-\rho t}$ has to.

Observe that an analogous way to prove the convergence to zero of the other costate variables must fail, as $p \geq 0$, $d \geq 0$ and $b^K \geq 0$ together with $\dot{D} = d$ and $\dot{K} = p + d + b^K$ do not allow for admissible paths in a full neighborhood of D^* and K^*.

D Figures

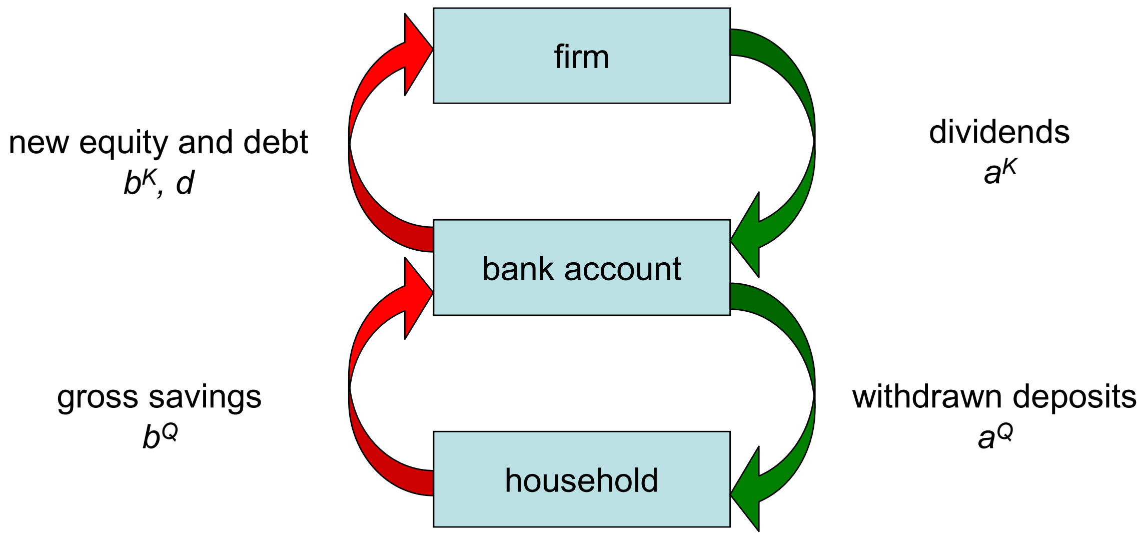

Figure 1: Basic structure of the model

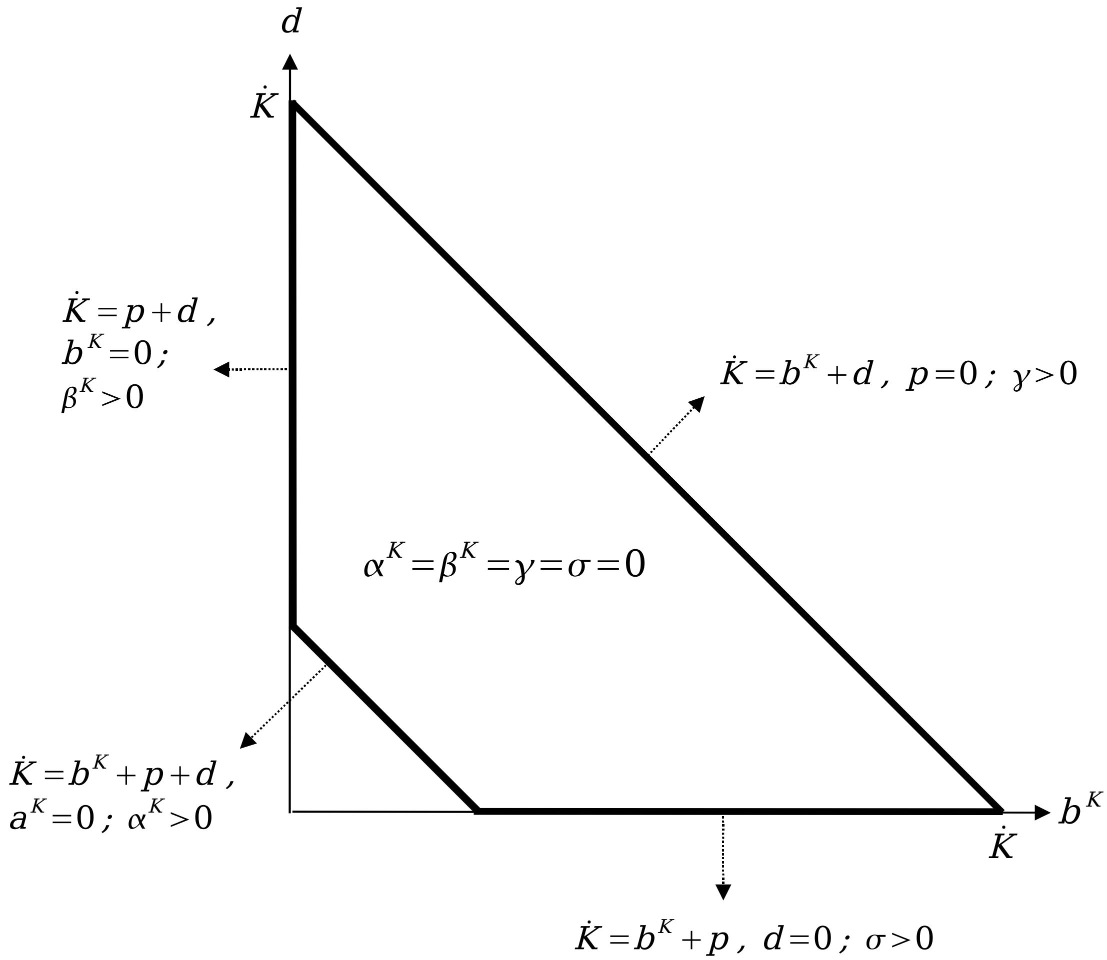

Figure 2: Restrictions on the financing of net investments

(Source: own representation at basis of Sinn (1987, p. 74))

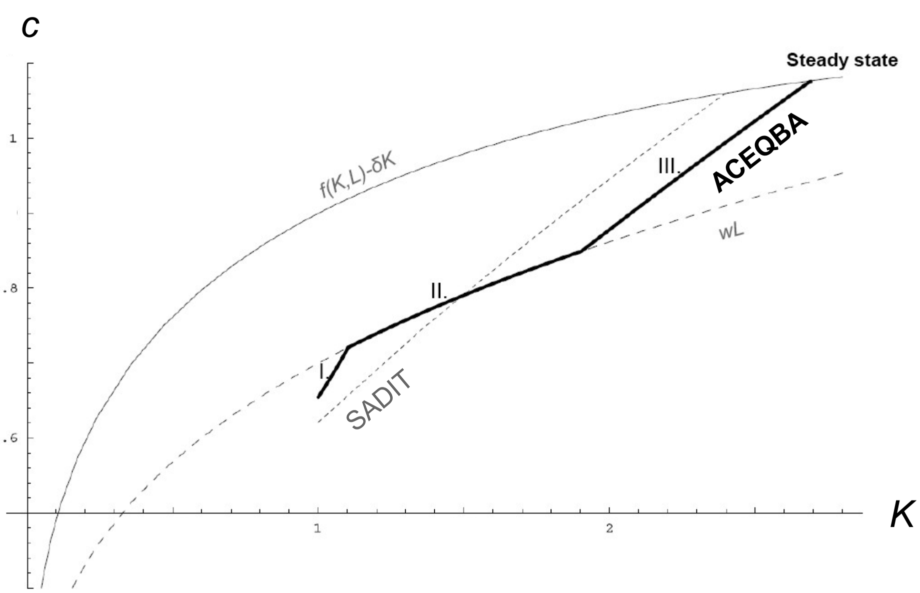

Figure 3: Convergent path in the ACE/QBA system in comparison to the SADIT with $\alpha_1 = 0$ and $\tau^a_{SADIT} = \tau^a_{ACE-QBA}$.