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The Nash-Equilibrium Requires Strong Cooperation

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Abstract

Contrary to the customary view that the celebrated Nash-equilibrium theorem in Game Theory is paradigmatic for non-cooperative games, it is shown that, in fact, it is essentially based on a particularly strong cooperation assumption. Furthermore, in practice, this cooperation assumption is simply unrealistic.

1. Introduction

One of the major divides in Game Theory is between *cooperative* and *non-cooperative* games. And as it happened along the development of that theory around the times of World War II, the idea emerged that, in certain important ways, non-cooperative games are more basic than the cooperative ones. In a few words, the thinking leading to that idea could be formulated as follows. A cooperative game between $n \geq 2$ players, if it is to be nontrivial, must leave each player some freedom, no matter which may be the binding agreements of the respective co-operation. And then, if one focuses on the respective freedom of each of the players, one may obtain a game which is like a non-cooperative one.

The difficulty with such a argument is that games, be they cooperative or non-cooperative, can be utterly complex. And to mention but one such aspect of complexity, let us recall that many games prove to be *algorithmically unsolvable*, Binmore [1-3].

A consequence of the significant complexities involved in games is that it may not always be so easy to separate or subtract the cooperative part of a game, in order to remain with a game which may be considered as being non-cooperative. In other words, the formula

$$game \;\; \equiv \;\; non - cooperative\ game \quad (modulo \;\; cooperation)$$

does *not* in general hold. Therefore, one is not justified in seeing non-cooperative games as being more basic than the cooperative ones.

Regardless of the above, one may note that the centrality in Game Theory of the Nash-equilibrium theorem about non-cooperative games is due not in a small measure to a certain tacit or lingering acceptance of the above incorrect formula.

The aim of this paper, however, is not so much about the clarification of the place of the Nash-equilibrium theorem with Game Theory, but rather to show that, no matter how much that theorem is seen as a *non-cooperative paradigm*, it does essentially depend on an assumption which requires such a *strong cooperation*, as to render it *unrealistic* from practical point of view.

Before going further, it may be useful to recall in short certain formative moments in the evolution of modern Game Theory, moments which may shed a light upon the mentioned views relating to the divide between cooperative and non-cooperative games.

Game theory, as initiated by John von Neumann, see von Neumann & Morgenstern, centers around the *individual* and aims to lead to a rational outcome when two or more individuals interact in well defined situations and do so, however, without the interference of an overall authority. The very development since the 1920s of the mathematical theory of games is in itself an act of rational behaviour, albeit on a certain *meta-level*, which involves both the levels of the interest in developing the general concepts, axioms, theorems, and so on, as well as the levels at which they are put to their effective uses in a variety

of relevant applications.
And yet, such a *meta-rational* behaviour appears to have mostly come to a halt when dealing with *cooperation*.
Instead, the effort has rather been focused upon non-cooperative contexts, and rationality got thus limited to them. A further aggravation of such a limitation upon rationality has come from the fact that cooperation is so often being associated with issues of ethics, morality, wisdom, philosophy, or on the contrary, politics, or mere expediency.

One of the few more prominent contributions to cooperation has been the 1984 study of Robert Axelrod, which however is limited to two person non zero sum games, and it centers around one of the simplest nontrivial such examples, namely, the celebrated game called the Prisoner's Dilemma, suggested in the early 1950s by Merrill Flood and Melvin Drescher, and formalized by Albert W Tucker, see Axelrod, Rasmusen.

However, even such simplest nontrivial examples can show that cooperation itself is often but a most natural matter of rationality, albeit manifested in more evolved forms. Indeed, in many non-cooperative situations what can be obtained by those involved proves to be *significantly less* than what may be available through suitable cooperation. Thus the choice of cooperation need not at all be seen as merely a matter which has to do with expediency, politics, wisdom, morality or ethics.

And then the issue is simply the following : do we limit rationality and stop it before considering cooperation in ways more adequate to its considerable depth and potential, or instead, are we ready to try to be rational all the way ?

And the fact is that very few situations are of a nature in which competition or conflict is total, and total to the extent that there simply cannot be any place whatsoever for one or another form of cooperation.
In game theory such a situation corresponds to extreme cases such as the *two person zero sum game*.
In rest, that is, in the vast majority of cases, the possibilities for coop-

eration and its considerable rewards may often be there. And then, all it takes is to *extend* and *deepen* rationality, and thus find and develop suitable ways of cooperation.

Needless to say, cooperative games are by no means less complex than the non-cooperative ones. Therefore, one can expect yet another reason why attention may focus more on the latter, and why one may try to reduce the former to them.
However, as follows from Binmore [1-3], this limitation of rationality mostly to the study of noncooperative games is still leading to complexities which are not algorithmically solvable.
During the late 1940s and early 1950s when game theory appears to have known its period of most massive development, there was not much awareness about the possibility of the presence of the type of deep difficulties which would more than three decades later be pointed out by Binmore.

In such a context, during the late 1940s and early 1950s, the so called "Nash Program" emerged which aimed to reduce cooperative games to non-cooperative ones, see Nash [1,3].

There is, however, a rather weighty reason, explanation, and maybe also excuse, for the fact that the Nash Program has known a certain popularity, and that its hoped for aim does not seem so easy to set aside. Namely, as is well known even from common everyday experience, cooperation will often involve considerable complications and difficulties. First of all, and already on its most basic *conceptual* levels, cooperation proves to be an extremely complex and rich phenomenon, which therefore cannot in any way be encompassed by a few general definitions and mathematical models. This fact is indeed in sharp contrast with the modelling of competition and conflict situations, where for instance in Game Theory, the so called non-cooperative games, see (2.1) and (2.5) below, describe quite well - and do so in spite of their manifest simplicity - a considerably large class of such situations. Second, one can only talk about cooperation if one can *rely* on the respective agreements undertaken by the autonomous agents involved. And the issue of such a reliance clearly depends on a variety of complex factors which can easily be outside of the realms of convenient mathematical modelling.

Be it as it may, one should not forget that, just like in the case of competition and conflict, the primary aspect of cooperation is *intent*, while the respective subsequent conceptualizations, models and actions are only specific instances of manifestation and expression of such an intent. Therefore, the primary issue is whether we intend to have competition or conflict in a context which may preferably include cooperation as well, or on the contrary, we intend, because of no matter what reasons, to relegate cooperation to a secondary role, or even exclude it altogether.

And if we do not a priori intend to exclude cooperation, then we should be careful not to allow that it is excluded merely by default, that is, simply due to the fact that it is in general not so easy to deal with it, be it conceptually, or practically, and thus it simply happens that we fail, avoid or decline to make use of it.
And a readiness to pursue rational behaviour beyond its non-coopertive limits will then suggest that the intent to cooperate, and even more importantly, *the intent to secure and keep up in the longer run a context suitable for cooperation* is but a clearly rational behaviour, even if on a certain meta-level.

The effect of the presence of such considerable difficulties related to cooperation has been that the modelling of cooperation has not received enough attention, see Axelrod.

As is well known, Nash himself tended to see Game Theory as being mainly moved by competition, conflict, and so on, rather than by cooperation, see Nasar.

On the other hand, the older and much more experienced John von Neumann, the originator of modern Game Theory, considered that there was a major and urgent need in economics, and other important human ventures involving strategic thinking, for the introduction of rational approaches to the respective variety of human interactions involved. And clearly, the very attempts to rationalize approaches to competition, conflict, and so on, rather tend to mollify, than prioritize them, see Neumann & Morgenstern.

As it happened, the first major result in Game Theory was obtained by John von Neumann in his 1928 paper. This is the famous Min-Max theorem about two player zero sum games. Such games involve the smallest possible nontrivial number of players, and the sharpest possible conflict among them, in which what one player wins, the other one must lose, thus the sum of what is won and lost is always zero. Clearly, in such a game there is no way for cooperation.

Here it is important to note the following. During the period around 1928, when von Neumann was only 25 years old, he was involved in at least two other major ventures, namely, the foundation of Set Theory, and the foundation of Quantum Mechanics, and in both of them he made most important and lasting contributions. In this way, von Neumann's involvement in Game Theory during that period can be seen as reflecting the special, if not in fact, fundamental importance he attributed to it. And indeed, he saw it as being the first ever systematic and rigorous theoretical approach to a *rational* management of conflict and competition between two or more conscious agents.

Problems of optimization had been considered earlier as well. After all, optimization appears as a rather permanent human concern in a most diverse range of activities. However, such problems could typically be seen as a game with one single conscious and rational player who was playing against Nature, or against everybody else.
But now, in von Neumann's view, the task was to be able to build an appropriate theory for conflicts and competitions between a number of conscious agents, assuming that they are firmly and reliably grounded in rationality.
It should be remembered in this regard that von Neumann happened to grow up in the Empire of Austria-Hungary, and did so during the disastrous years of World War I and its aftermath. And just like the well known philosopher Karl Popper, of the same generation and background, von Neumann was much influenced by the prevailing view during the post World War I years, a view according to which that war - called The Great War - had been the result of nothing else but systematic and catastrophic, even if rather trivial, failures of rational-

ity on the part of the leading elites of the Western powers.

The extend to which von Neumann gave a special priority to the development of Game Theory is further illustrated by his activity during the next one and a half decade, till the publishing in 1944, that is, during World War II, of his joint book with Morgenstern, entitled "Theory of Games and Economic Behaviour". Indeed, during the years of World War II, von Neumann was heavily involved in supporting the American war effort and doing so in a variety of ways. Consequently, at the time, he did very little theoretical research. And yet, he considered it important enough to dedicate time to Game Theory, and complete the mentioned book of over 600 pages, which is the first ever systematic and detailed presentation of that theory. It should also be mentioned that the theory in the book is due solely to von Neumann, and most of it, except for his Min-Max theorem of 1928, was developed by him during the years preceding its first publication in 1944. Morgenstern was an economist, and his contribution to the book consisted in the connections between game theory and economic behaviour. In this way, that book can in fact be seen as a research monograph in Game Theory.
As for the relevance of its content, a good deal of it still makes useful reading after more than six decades.

The importance attributed to Game Theory continued after World War II as well. And it was to a good extent due to the interest manifested in it by the RAND Corporation, a most influential California think tank at the time, which was heavily involved, among others, in strategic studies related to the just emerged Cold War.

As it happens, Emile Borel initiated in the early 1920s the study of certain well known card games which were related to the two person zero sum games. However, he did not obtain the respective major result, and in fact, he assumed that the Min-Max theorem was in general false. Later, in 1934, R A Fisher was also involved in a study of two person zero sum games, without however obtaining the major Min-Max theorem, see Luce & Raiffa.

Then starting in 1950, John Nash, who at the time was 22 years

old, published his fundamental papers Nash [1,3] on equilibrium in n-person non-cooperative games, and his main result was a significant extension of the Min-Max theorem of von Neumann.
Nash has also published important results in cooperative games, see Nash [2,4].

However, as it happened, the Nash-equilibrium theorem massively extended the von Neumann Min-Max theorem, and it does so in two ways. First, it is no longer restricted to two players, and instead, it can handle an arbitrary number of them. Second, it is no longer restricted to zero sum games, but it refers to arbitrary non zero sum ones.
Furthermore, the Nash-equilibrium theorem has ever since its inception remained the best known result of its kind, as it has not been further extended in any significant manner, when considered in its own terms of non-cooperation.
Consequently, that theorem has always been seen as paradigmatic for non-cooperative games.

To a certain extent, such an interpretation is not so surprising due to the following two facts. First, the Nash result on the existence of an equilibrium in mixed strategies is an obvious extension of the corresponding von Neumann Min-Max theorem, and the latter, as mentioned, is indeed about games which are outside of any possible cooperation. Second, as long as one is limited to the usual, and thus narrow concepts of cooperation, the result of Nash on equilibrium will be seen as falling outside of such concepts.

As we shall show in section 2, however, such an interpretation can only hold if alone the usual, and indeed narrow concepts of cooperation are considered. And as the very concept of equilibrium in the Nash result implies it, that result can have any practical meaning and value at all, and do so beyond its particular case when only two players are involved, only if the respective $n \geq 3$ players do accept - even if implicitly - certain *additional common rules* of behaviour. Thus in the case of $n \geq 3$ players, they *must* end up by cooperating, even if in ways other and more deep than those according to the usual views of cooperation. And in fact, the kind of cooperation needed in order to enable the Nash-equilibrium concept and result to function at all proves to be so *strong* as to be practically *unrealistic*.

Due to the reputation of von Neumann, the interest he showed in Game Theory led in the late 1940s and early 1950s to a considerable status for that theory among young mathematicians at Princeton, see Nasar. That status was further enhanced by results such as those of Nash and others.
There was also at the time a significant interest in Game Theory outside of academe. As mentioned, for instance, the well known RAND Corporation was conducting studies in political and military strategy which were modelled by a variety of games.

As it happened, however, soon after, certain major setbacks were experienced. First, and within game theory itself, was the fact that in the case of n-person games, even for $n \geq 3$ moderately large, there appeared to be serious *conceptual* difficulties related to reasonable concepts of solution. Indeed, too many such games proved not to have solutions in the sense of various such solution concepts, concepts which seemed to be natural, see Luce & Raiffa, Owen, Vorob'ev, Rasmusen. Later, the nature and depth of these conceptual difficulties got significantly clarified. For instance, in Binmore [1-3] it was shown that there are no Turing machines which could compute general enough games. In other words, solving games is not an algorithmically feasible problem.

The second major trouble came from outside of Game Theory, and it is not quite clear whether at the time it was soon enough appreciated by game theorists with respect to its possible implications about the fundamental difficulties in formulating appropriate concepts of solutions in games. Namely, Kenneth Arrow showed that a set of individual preferences cannot in general, and under reasonable conditions, be aggregated into one joint preference, unless there is a dictator who can impose such a joint preference. This result of Arrow was in fact extending and deepening the earlier known, so called, voter's paradox, mentioned by the Marquis de Condorcet, back in 1785, see Mirkin.

In subsequent years, developments in Game Theory lost much of their

momentum. In later years, applications of Game Theory gained the interest of economists, and led to a number of new developments in economic theory. An indication of such developments was the founding in 1989 of the journal Games and Economic Behavior.
At the same time certain studies of competition, conflict, and so on, were taken up by the developments following Arrow's fundamental paper, and led to social, collective or group choice theory, among others. Decision Theory got also involved in such studies involving certain specific instances of competition, conflict or cooperation, related to problems of optimization, see Rosinger [1-6].

In games, or in social, collective or group choice one has many autonomous players, participants or agents involved, each of them with one single objective, namely, to maximize his or her advantage which usually is defined by a scalar, real valued utility function. And by the early 1950s it became clear enough that such a situation would not be easy to handle rationally, even on a conceptual level.
On the other hand, a main objective of Decision Theory is to enable one single decision maker who happens to have several different, and usually, quite strongly conflicting objectives. And in view of Arrow's result, such a situation may at first appear to be more easy to deal with. The situation, however, proves to be quite contrary to such a first perception, see Rosinger [1-6].

As it turned out, games, social choice or decision making, each have their deeper structural limitations.

And as far as games are concerned, not even in the case of the celebrated and *paradigmatic non-cooperative* Nash-equilibrium theorem is it possible to escape the *paradox* of having the validity of that result essentially based on a *cooperative* assumption so strong as to make it unrealistic in practice.

2. The Nash-Equilibrium Theorem

The usual way an n-person non-cooperative game in terms of the players' *pure strategies* is defined is by

$$(2.1) \quad G \;=\; (P,\; (S_i \mid i \in P),\; (H_i \mid i \in P))$$

Here P is the set of $n \geq 2$ players, and for every player $i \in P$, the finite set S_i is the set of his or her pure strategies, while $H_i : S \longrightarrow \mathbb{R}$ is the *payoff* of that player. Here we denoted by

$$(2.2) \quad S \;=\; \prod_{i \in P} S_i$$

the set of all possible *aggregate pure strategies* $s = (s_i \mid i \in P) \in S$ generated by the independent and simultaneous individual strategy choices s_i of the players $i \in P$.

The game proceeds as follows. Each player $i \in P$ can freely choose an individual strategy $s_i \in S_i$, thus leading to an aggregate strategy $s = (s_i \mid i \in P) \in S$. At that point, each player $i \in P$ receives the payoff $H_i(s)$, and the game is ended. We assume that each player tries to maximize his or her payoff.

Remark 1

The usual reason the games in (2.1) are seen as non-cooperative is as follows. Each of the $n \geq 2$ players $i \in P$ can completely independently of any other player in P choose any of his or her available strategies $s_i \in S_i$. And the only interaction with other players happens on the level of payoffs, since the payoff function H_i of the player $i \in P$ is defined on the set S of aggregated strategies, thus it can depend on the strategy choices of the other players.
However, as we shall see in Remarks 2 - 4 below, in the case of $n \geq 3$ players, this independence of the players is only apparent, when seen in the framework the concept of Nash-equilibrium, and the corresponding celebrated Nash theorem.

$\square$

Before considering certain concepts of equilibrium, it is useful to introduce some notation. Given an aggregate strategy $s = (s_i \mid i \in P) \in S$ and a player $j \in P$, we denote by s_{-j} what remains from s when we delete s_j. In other words $s_{-j} = (s_i \mid i \in P \setminus \{j\})$. Given now any $s'_j \in S_j$, we denote by (s_{-j}, s'_j) the aggregate strategy $(t_i \mid i \in P) \in S$, where $t_i = s_i$, for $i \in P \setminus \{j\}$, and $t_j = s'_j$, for $i = j$.

For every given player $j \in P$, an obvious concept of *best strategy* $s^*_j \in S_j$ is one which has the *equilibrium* property that

$$(2.3) \quad H_j(s_{-j}, s^*_j) \ \geq \ H_j(s_{-j}, s_j), \quad \text{for all} \ \ s \in S, \ s_j \in S_j$$

Indeed, it is obvious that any given player $j \in P$ becomes completely independent of all the other players, if he or she chooses such a best strategy. However, as it turns out, and is well known, Rasmusen, very few games of interest have such strategies.
Consequently, each of the players is in general *vulnerable* to the other players, and therefore must try to figure out the consequences of all the possible actions of all the other players.

Furthermore, even when such strategies exist, it can easily happen that they lead to payoffs which are *significantly lower* than those that may be obtained by suitable cooperation. A good example in this regard is given by game called the Prisoner's Dilemma, see section 3.

Remark 2

It is precisely due to the mentioned vulnerability of players, which is typically present in most of the games in (2.1), that there may arise an interest in *cooperation* between the players. A further argument for cooperation comes from the larger payoff individual players may consequently obtain. A formulation of such a cooperation, however, must then come in *addition* to the simple and general structure present in (2.1), since it is obviously *not* already contained explicitly in that structure.

□

Being obliged to give up in practice on the concept of best strategy in (2.3), Nash suggested the following alternative concept which obviously is much *weaker*.

Definition (Nash)

An aggregate strategy $s^* = (s_i^* \mid i \in P) \in S$ is called a *Nash-equilibrium*, if no single player $j \in P$ has the incentive to change *all alone* his or her strategy $s_j^* \in S_j$, in other words, if

$$(2.4) \quad H_j(s^*) \geq H_j(s^*_{-j}, s_j), \quad \text{for all } j \in P,\ s_j \in S_j$$

Remark 3

Clearly, the Nash-equilibrium only considers the situation when *never more than one single* player does at any given time deviate from his or her respective strategy. Therefore, the Nash-equilibrium concept is *not* able to deal with the situation when there are $n \geq 3$ players, and at some moment, more than one of them deviates from his or her Nash-equilibrium strategy.
Needless to say, this fact renders the concept of Nash-equilibrium *unrealistically particular*, and as such, also *unstable* or *fragile*.

Furthermore, that assumption has a manifestly, even if somewhat implicitly and subtly, *cooperative* nature.

Above all, however, the larger the number $n \geq 3$ of players, the *less realistic* is that assumption in practical cases.

It is obvious, on the other hand, that when there are $n \geq 3$ players, in case at least two players change their Nash-equilibrium strategies, the game may open up to a large variety of other possibilities in which some of the players may happen to increase their payoffs.

Therefore, when constrained within the context of the Nash-equilibrium concept, the game becomes *cooperative* by *necessity*, since the following *dichotomy* opens up inevitably :

Either

- (C1) All the players agree that never more than one single player may change his or her Nash-equilibrium strategy,

Or

- (C2) Two or more players can set up one or more coalitions, and some of them may change their Nash-equilibrium strategies in order to increase their payoffs.

Consequently, what is usually seen as the essentially non-cooperative nature of the game (2.1), turns out, when seen within the framework of the Nash-equilibrium concept, to be based - even if tacitly and implicitly - on the *very strong cooperative* assumption (C1) in the above dichotomy.
On the other hand, in case (C1) is rejected, then the game falls out of the Nash-equilibrium framework, and thus it opens up to the wealth of possibilities under (C2), which among others, can contain a large variety of possible ways of cooperation.

In this way, both the Nash-equilibrium concept and the Nash theorem on the existence of the respective equilibrium in mixed strategies are highly *unstable* or *fragile* when there are 3 or more players involved.

Also, similar with the best strategies in (2.3), with the Nash-equilibrium strategies as well it can happen that they lead to payoffs which are significantly lower than those that may be obtained by suitable cooperation.

□

As in the particular case of (2.1) which gives the von Neumann Min-Max theorem on two person zero sum games, so with the weakened concept of Nash-equilibrium in (2.4), such an equilibrium will in general not exist, unless one embeds the *pure strategy* game (2.1) into its extension given by the following *mixed strategy* game

$$(2.5) \quad \mu G \ = \ (P, \ (\mu S_i \mid i \in P), \ (\mu H_i \mid i \in P))$$

Here, for $i \in P$, the set μS_i has as elements all the probability distributions $\sigma_i : S_i \longrightarrow [0,1]$, thus with $\Sigma_{s_i \in S_i} \ \sigma_i(s_i) = 1$. Let us now denote

$$(2.6) \quad \mu S \ = \ \prod_{i \in P} \ \mu S_i$$

Then for $i \in P$ we have the payoff function $\mu H_i : \mu S \longrightarrow \mathbb{R}$ given by

$$(2.7) \quad \mu H_i(\sigma) \ = \ \Sigma_{s \in S} \ \sigma(s) H_i(s)$$

where for $\sigma = (\sigma_i \mid i \in P) \in \mu S$ and $s = (s_i \mid i \in P) \in S$, we define $\sigma(s) = \prod_{i \in P} \ \sigma_i(s_i)$.

Now the definition (2.4) of Nash-equilibrium for pure strategy games (2.1) extends in an obvious manner to the mixed strategy games (2.5), and then, with the above we have, see Vorob'ev

Theorem (Nash)

The mixed strategy extension $\mu G \ = \ (P, \ (\mu S_i \mid i \in P), \ (\mu H_i \mid i \in P))$ of every pure strategy game $G \ = \ (P, \ (S_i \mid i \in P), \ (H_i \mid i \in P))$, has at least one Nash-equilibrium strategy.

Remark 4

Obviously, what was mentioned in Remarks 2 and 3 related to the inevitability of cooperation when the *pure strategy* games (2.1) are considered within the framework Nash-equilibrium, will also hold for the *mixed strategy* games (2.5), and thus as well for the above theorem of Nash.

Remark 5

The idea behind the Nash Program to reduce cooperative games to noncooperative ones seems at first quite natural. Indeed, in its very essence, a game means that, no matter what the rules of the game are, each player has a certain *freedom* to act within those rules, and can do so *independently* of all the other players. Therefore, it may appear that if we only concentrate on that freedom and independence, then within that context one can see the game as noncooperative, that being one of the usual ways to understand the very meaning of freedom and independence.
Furthermore, even if one cooperates, one is still supposed to be left in a game with a certain freedom and independence. Thus it may still appear that, after subtracting all what is due to the rules of the game and to one's possible cooperation, one is still supposed to remain with a certain freedom and independence.

According to Nash himself, it could be possible to express all communication and bargaining in a cooperative game in a formal manner, thus turn the resulting freedom and independence of the players into moves in an *extended* noncooperative game, in which the payoffs are also extended accordingly. Since such a program has never been fully implemented in all its details and only its ideas were presented, its criticism must unavoidably remain on the same level of ideas. However, a certain relevant and well tested objection can be made nevertheless, see McKinsey [p. 359] :

" It is extremely difficult in practice to introduce into the cooperative games the moves corresponding to negotiations in a way which will reflect all the infinite variety permissible in the cooperative game, and to do this without giving one player an artificial advantage (because of his having the first chance to make an offer, let us say)."

What is lost, however, in such a view as the Nash Program is that an *appropriate voluntary and mutual limitation* of one's freedom and independence, in order to implement a *cooperation* can *significantly change* the payoffs, and thus it can offer to players an increase in their payoffs, an increase which simply *cannot* be attained in any other non-cooperative way. And this is precisely the point in *cooperation*.

On the other hand, precisely to the extent that the above objection in McKinsey is valid related to the Nash Program, and all subsequent experience points to its validity, the very same objection touches essentially on any attempt to reconsider cooperation, and do so in more formal ways.

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