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The Production of Science

The job market for scientists and firms' research productivity

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Abstract

Empirical evidence shows that the average return on R&D spending in scientific research decreases with firm size. I provide an explanation to this fact by developing a model of science production where heterogeneous researchers are endogenously allocated to different firms. The main assumption is that firms invest in research to increase their *absorptive capacity*: the ability to use and understand scientific findings produced elsewhere. Firms create absorptive capacity by building labs and hiring researchers in a competitive market. Because of externalities, firms underinvest in labs. More interestingly, researchers and labs are substitutes in the revenue function, even though they are complements in the research production function. As a consequence, the greater the investment in science, the lower the productivity of the researcher working for the firm. This generates a novel form of inefficiency: for any given investment, the allocation of researchers to firms is non optimal.
KEYWORDS: Productivity of Scientific Research, Organization of Scientific Research, Externality, Absorptive Capacity, Matching with Investment.
JEL NUMBERS: D21, H23, L22, O31, O32, O38.

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1 Introduction

If firms invest in research to benefit from the scientific findings produced internally, big firms should be more productive than small firms in their research effort. Big firms gain more than small firms from increasing their scientific output, since they capture a greater share of the total benefit (private benefit plus social benefit) derived from the extra science produced. It follows that big firms should always manage to outbid small firms in order to purchase any factor relevant to the production of science. Big firms should hire the most productive researchers, purchase the best equipment, be located in the best spots and, ultimately, be more productive than small firms. However several sources document that the average return on research spending decreases with firm size. For example, Halperin and Chakrabarti (1987) find that the number of papers produced per dollar of R&D spending is negatively correlated with firms size and with total R&D spending.[1]

In this paper I build a model of science production where heterogeneous researchers are hired by firms in order to work in their labs. The ability of each researcher determines the scientific productivity of the firm's lab. In order to explain the empirical puzzle discussed above, I assume that firms carry out research in order to increase their *absorptive capacity*: the ability to use *outside* science. Research provides "a ticket of admission to an information network":[2] it allows firms to be always up to date with the science produced by other firms and universities. Also, science is difficult: only scientists that are actively engaged in research can read and understand several papers in a timely fashion. In other words, using publicly available science can be costly to firms; this cost is lower when firms produce more in-house research.

Intuitively, absorptive capacity implies that firms produce science so that their in-house researchers can be part of the scientific community. Once firms reach the required scientific output, doing additional research generates little extra benefit: firms do not want to produce Nobel-prize-winning research. In order to achieve this goal, firms can either invest in labs or hire a very productive researcher: researchers and labs can be substitutes in the revenue function. This implies that the competitive market generates a misallocation of researchers to labs. In my stylized model, this takes the form of Negative Assortative Matching (NAM) rule: the worst researcher works with the biggest lab.

Therefore, if absorptive capacity is the only determinant of the investment in research, the model predicts a negative correlation between size of the investment and scientific productivity. Also, if bigger firms have lower cost of investing, there is a negative correlation

[1] See also Scherer (1965), Acs and Audretsch (1987), Cohen and Klepper (1996) who review the empirical evidence. However Halperin and Chakrabarti (1987) are the most relevant here because they looks explicitly at scientific output.

[2] Rosenberg (1990), p.170

between firms size and scientific productivity. This is consistent with the empirical evidence showing that productive R&D workers are more likely than unproductive R&D workers to be hired by small firms.[3]

At the same time I assume that researchers and labs are complements in the research production function; the total science produced is maximized under a Positive Assortative Matching (PAM) rule assigning the best researchers to the biggest labs. Therefore, in the allocation of researchers to labs, there is a trade-off between producing science and using science. Since firms aim at using science, for any given investment in labs the private sector minimizes the amount of science produced. The decentralized allocation of researchers to labs is inefficient. This inefficiency is novel and arises in addition to the usual underinvestment in science. I show that an appropriate set of taxes/subsidies to the amount of science produced by each firm can solve the inefficiency by inducing the first-best investment and the first-best allocation of researcher to firms. However, subsidies to the investment in labs cannot restore efficiency since they do not affect the job-market for researchers.

In the second part of the paper I enrich the model by introducing universities. I assume that their mission is to produce science and that academic scientists can work as consultants for the private sector. The job of a consultant is to help a firm using the available stock of science. Scientists endogenously sort between the university sector and the private sector. Under these assumption, I show that the best researchers are hired by universities, and within universities researchers are allocated according to PAM: better researchers get to work with bigger labs. These researchers consults for the small firms, while large and productive firms will hire scientists. Therefore, within the university sector the model predicts a positive correlation between size of the investment and research productivity.

Finally, I extend the model by assuming that researchers care about reputation, which is built by producing science. I show that, if reputation concerns are strong enough, the equilibrium in the private sector may switch from NAM to PAM. Intuitively, researchers are willing to receive lower wages in order to work in firms with big labs. In addition, for a given lab, productive researchers are willing to forfeit a bigger portion of their wages than unproductive researchers. In the new competitive equilibrium, productive researchers work in big labs, but may be paid less than unproductive researchers because they receive a higher reputation reward. Therefore reputation affects the production of science not by changing the researchers' incentives (as in Dasgupta and David (1985)) but by affecting the job market for scientists. The prediction of the model is that scientific sectors where

[3] See Zenger (1994) and Elfenbein, Hamilton, and Zenger (2010), the latter being the most relevant since it looks at scientists. The explanations offered in the literature rely on the assumption that small firms offer tighter performance-contingent contracts than big firms, and therefore attract more productive agents. With respect to the job market for scientists, my paper can be interpreted as an alternative explanation, having a very different implication with respect to the market efficiency (see the next paragraph).

reputation concerns are stronger are more likely to display a positive correlation between size of the investment in scientific research and research productivity.

1.1 Relevant Literature.

There are several pieces of evidence showing that, because of local spillover, the presence of very productive scientists has a positive impact on the productivity of firms[4]. However, the efficiency properties of the the job market for researchers have not been discussed before, despite headline-grabbing stories about elite scientists leaving one country for another country, or leaving one firm for another firm.[5]With this respect, the contribution of this paper is to show that the allocation of scientists across firms is not a zero-sum game: the decentralized job market for scientists can be inefficient. If the social planner could reallocate some scientists, social welfare would increase.

The fact that the allocation of talented agents across sectors and occupations can have important aggregate welfare consequences is not new. However, the argument is usually that, by joining different sectors, productive agents will be doing different things (see, for example, Baumol (1996) and Murphy, Shleifer, and Vishny (1991)), or they will be subject to a different set of incentives (in the context of science and scientific research, see Aghion, Dewatripont, and Stein (2008)). With respect to these works, the contribution of my paper is to show that, even if scientists are always doing research, according to the same production function, it does matters whether a given researcher joins the for-profit research sector or the university sector, because resources are organized differently in different sectors.

It has long been observed that sometimes firms carry out scientific research to be more effective at using outside science. This idea was first brought forward by Tilton (1971), who analyzes the semiconductor industry during the '50s and '60s. Tilton observes that, for these firms, investing in research was a form of insurance: they were always guaranteed to be up to date with the latest scientific breakthrough. The term *absorptive capacity* was introduced by Cohen and Levinthal (1989), who provide both the first theoretical model of this concept and its first empirical test. Other important empirical works are Cockburn and Henderson (1998), Gambardella (1992) and Griffith, Redding, and Reenen (2004). On the theory side, several researchers explored the strategic implications of absorptive capacity (see, for example, Hammerschmidt (2006), Kamien and Zang (2000) and Leahy and Neary (2007)). In particular, Leahy and Neary (2007) derive some policy implications by showing that research joint ventures may decrease the amount of research carried out by firms. The

[4] See the literature on knowledge spillover and geography of innovation reviewed in Audretsch and Feldman (2004).

[5] For example Liu, M. (2009, November 14). Steal This Scientist. *Newsweek*; or Climbing Mount Publishable: the old scientific powers are starting to lose their grip. (2010, November 11). *The Economist.*

reason is that firms invest in research partly to be able to use outside science. When the access to science is made easier by the creation of a joint venture, there is no need to carry out much research anymore.

My paper relies on the assumption that, because of absorptive capacity, there is a strong form of decreasing returns in science. This particular point is supported by Gittelman and Kogut (2003). The goal of their paper is to establish whether valuable science leads to valuable patents. The authors measure the quality of the scientific output by counting the number of citations received by papers produced within a given firm. Similarly, they measure patent quality by adding all the citations received by patents produced by the same firm. They find that "scientific knowledge and patents are related, but good publications and good patents are not."[6] In other words, producing some science deliver some benefit, but producing a lot of science does not (actually, in some of their specifications, the relationship between valuable patents and valuable science is negative).

Finally, the existing empirical investigations on the allocation of resources to researchers deal exclusively with specific public institutions. For example, Arora, David, and Gambardella (1998) analyze the funding allocation decisions of the Italian CNR (equivalent to the NSF) and show that the reputation (past publication record) is the main explanatory variable. I am not aware of any study looking at the determinants of the allocation of resources to researchers working in the private sector.

In the next section, I describe the model. In the second section, I characterize the equilibrium for a given distribution of labs. In the third section, I derive the distribution of labs, formally define the equilibrium, and prove its existence. In the fourth section I discuss the normative aspects of the model. I introduce universities in the fifth section, and reputation in the sixth section. In the last section I conclude by discussing possible empirical tests, policy implications, and extensions.

2 The Model

The economy is populated by a continuum of firms and a continuum of researchers. Firms differ in their size s, continuously distributed over $S = [0, \bar{s}]$. Researchers differ in their ability a, continuously distributed over $A = [0, \bar{a}]$. All agents have the same outside option assumed to be zero. The economy runs for three periods.

2.1 Investing in Labs.

In period $t = 0$ firms build labs. If a firm s sets up labs of size L it bears a cost $c(s, L)$ continuous, positive, with continuous first and second derivative, increasing in L, decreasing

[6] Gittelman and Kogut (2003), p. 380.

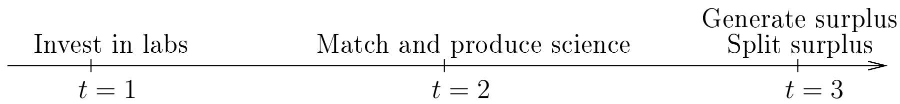

Fig. 1: Timeline

in s, with $\frac{\partial^2 c(s,L)}{\partial L^2} \geq 0$, $\frac{\partial^2 c(s,L)}{\partial L \partial s} < 0$ and $c(s, 0) = 0 \, \forall s$.

2.2 Producing Science.

In period $t = 1$, each researcher is hired by one firm and works in the firm's lab. The amount of research produced within each match is:

$$R(a, L) = af(L)$$

where $f(L) \geq 0$, $f'(L) > 0$, and $f''(L) \leq 0$. Note that the two inputs are complements in the research production function. This implies that, for given distribution of labs, the allocation of researchers to labs that maximizes the production of science is Positive Assortative Matching (PAM): the most productive researcher should work in the biggest lab.

The reader should interpret the lab size L as everything that can increase the chance of a discovery for given researcher's ability. This include physical machines (a bigger telescope, a more powerful microscope, a state of the art DNA sequencing machine), as well as the number of technicians and post-docs. The fact that some of these inputs do not require an investment ex-ante but can be purchased after hiring the researcher will turn out to be irrelevant. In the next section I will show that, in equilibrium, firms invest taken as given the researcher allocated to them. This implies that the timing could be reversed with no effect on the equilibrium investment.

Finally, in real life, researchers work in team. This can be incorporated into the model by defining a as the research team's average quality. A previous matching stage determines how researchers form research teams, and how from a distribution of individual ability we can derive the distribution of a. In order to keep the model as simple as possible, I will not pursue this interpretation further.

2.3 The Private Benefit of Research.

At the beginning of the last period ($t = 2$) there is a stock of new science available in the economy. Call its expected commercial value V, and interpret it as the value of all the patents that can be produced out of the available science. The private surplus generated by a match between a researcher and a firm during period $t = 1$ depends on the amount of research carried out in house and the aggregate science V. I assume that the private surplus has an additive form:

$$\Phi(a, L, s) = sV - g(af(L))$$

where $g()$ is continuous and differentiable, $g'() < 0$ and $g''() > 0$. The surplus produced is then split between researcher and firm. Finally, V is taken as given by firms and researchers but will be determined endogenously.

Note two things. First, firms do not compete with each other on the product market. The reader should imagine a scientific field where many small firms produce patents out of the same scientific base. For example, firms may belong to the bio-tech sector, some developing DNA sequencing machines, some developing drugs, others developing bacteria that can produce bio-fuel out of garbage. Some firms will compete, some will not compete, some other will complement each others. For this reason I abstract from competition issues. Second, firm's size affects the benefit of producing science: the benefit of a new patent are greater for bigger firms. It follows that size matters in two ways: directly and through the investment L.[7]

The interpretation of the above specification is that, because of absorptive capacity, firms carry out in-house research in order to decrease the cost of using the public stock of science. However, the function $g(x)$ could be everywhere negative, implying that science is always carried out for a direct benefit. Therefore, nothing in the mathematical formulation presented so far contains the absorptive capacity hypothesis. The following two assumptions formally introduce it into the model:

Assumption 1. *It is impossible to understand a new piece of science if no research is carried out in house:* $\lim_{x \to 0} g(x) = \infty$.

Remember that V represents the new science that will be introduced tomorrow. Under the above assumption, firms need to produce some in-house science today if they want to be active in the market and exploit the new aggregate science V. Note that this does not imply that the science produced in-house should be enough to lead to any publication or scientific discovery, neither it implies that all the firms active in the market invest in labs (it will depend on the specific functional form of $f(L)$), but it does mean that all the firms active in the market hire a researcher.

Assumption 2. *The marginal benefit of producing science is decreasing rapidly:* $g'''(x) > 0$.

Assumption 2 captures the following consideration. Absorptive capacity implies that firms produce science so that their in-house researchers can be part of the scientific community. Let's say that this is achieved by attending conferences. It follows that a firm will

[7] In general, the private surplus could be $\Phi(a, L, s) = \eta(s)V - g(af(L))$ with $\eta(s)$ strictly increasing. To save on notation, I assume that $\eta(s) = s$.

want to produce enough science so that its researcher can attend conferences, but producing even more science provides little extra value. Therefore, the marginal benefit a firm's enjoy from doing research is decreasing rapidly.

Proposition 3. *Under assumptions 1 and 2, from the private sector's point of view the two inputs are always substitutes:*

$$\frac{\partial^2\Phi(a,L,s)}{\partial a\partial L} < 0\, \textit{for every } a, L \in \mathbb{R}^+$$

The proof of proposition 3 is based on the fact that, when both assumptions 1 and 2 hold, the curvature of the cost function $g()$ is given by:

$$-\frac{g''(af(L))}{g'(af(L))} > \frac{\frac{\partial^2 R}{\partial a \partial L}}{\frac{\partial R}{\partial a}\frac{\partial R}{\partial L}} = \frac{1}{af(L)}$$

This curvature implies substitutability.

Are assumptions 1 and 2 strong? Are they realistic? To have an intuitive grasp of what is going on, assume for a moment that $g()$ is an isoelastic function. Assumptions 1 and 2 imply that $g()$ is bounded below. This is quite natural if there is no production motive and $g()$ only represents the cost of using the public science. In this case, firms invest in labs to reduce their cost. It follows that the benefit a firm receives from carrying out research is never above V. This assumptions can also accommodate the case where there is a direct benefit of producing science, in the sense that $g()$ can be negative, as long as this benefit has an upper bound.[8] However, it may be restrictive if the production motive is particularly strong.

In what follows, I will assume that absorptive capacity is the main reason why firms carry out research in the sense that assumptions 1 and 2 are satisfied. In subsection 3.1 I will discuss more in depth the consequences of relaxing these two assumptions.

2.4 Endogenous Science.

The value of science is taken as given by firms but it is determined endogenously aggregating all the research carried out in the economy. Call ν the expected commercial value of a unit of research and $h(L)$ the *p.d.f* of L. The expected value of the stock of science is given by:

$$V = \nu \int m(L)f(L)h(L)dL \tag{1}$$

[8] It is possible to show that boundedness implies local substitutability for large enough $af(L)$. However to have global substitutability one needs to assume 1 and 2: boundedness and assumption 1 or boundedness and assumption 2 are not enough.

where the function $m(L) : \mathbb{R}^+ \to \{A, \emptyset\}$ assigns labs to researchers, with the convention that $m(L) = \emptyset$ represents an unmatched firm. The function $m(L)$ is determined in equilibrium.

3 The Equilibrium for Given Investment in Labs and for Given Aggregate Science.

In this section, I derive the equilibrium arising in period $t = 1$, when firms have already invested in labs. I analyze the problem taking as given the total amount of science produced in the economy V, and the investment made by each firm.

Let's introduce the following notation:

- $i(s) : S \to \mathbb{R}^+$, the equilibrium investment in labs made by a firm s.
- $\tilde{m}(s) \equiv m(i(s)) : S \to A$, the matching rule on the equilibrium path (for investment performed by some firms) mapping firms to researchers.
- $x(s, L) : S \times \mathbb{R}^+ \to \mathbb{R}^+$, the payoff of a firm of size s and with lab L.
- $\tilde{x}(s) \equiv x(s, i(s)) : S \to \mathbb{R}^+$, the payoff of firms on the equilibrium path.
- $w(a) : A \to \mathbb{R}^+$, the payoff of a researcher with ability a.

I conjecture that the function $i(s)$ is strictly increasing. This conjecture will be proven in the next section.

Definition 4. For given V, the job market for researchers is in equilibrium if:

- Feasibility: $\tilde{x}(s) + w(\tilde{m}(s))) \leq \Phi(\tilde{m}(s)), i(s), s)\ \forall s.$
- Stability: $\tilde{x}(s) + w(\tilde{m}(s'))) \geq \Phi(\tilde{m}(s')), i(s), s)\ \forall s, s'.$

The existence of a unique equilibrium for given V is a standard result in matching theory (see, for example, Kamecke (1992)).

Proposition 5. *Negative assortative matching (NAM) in the job market for researchers: the most productive researchers work in the smallest labs and the least productive researchers work in the biggest labs. Similarly, the most productive researchers work in the smallest firms and the least productive researchers work in the biggest firm.*

Proof. For given s, the two inputs L and a are global substitutes. It follows that, for given s, the equilibrium matching between a and L is NAM. The result follow from the fact that $i(s)$ is increasing in s, since s enters linearly in the private surplus function. $\square$

From the firms' point of view, researchers and labs are substitutes. Since the private sector allocates researchers to labs so to maximize their marginal product, it follows that, in equilibrium, the most productive researchers will work in the smallest labs. However, labs and researchers' ability are complements in the research production function. The matching rule maximizing the total stock of science is PAM: the best researcher should work in the biggest lab. Therefore, the private sector, for a given distribution of labs, is minimizing the value of science V. There is a trade-off between maximizing science and maximizing the use of science. Since the private sector only considers the latter, the decentralized equilibrium is inefficient.

Proposition 6. *For given distribution of labs, if ν is high enough, the matching pattern emerging in the private sector is inefficient.*[9]

Proof. See appendix. □

3.1 Discussion.

Proposition 6 shows that the competitive equilibrium allocation of researchers to labs is inefficient so that, for given distribution of labs, the production of science is inefficient. This result is robust to several modification of the baseline assumptions, although the model may become impossible to solve.

First of all, the fact that science enters additively in the private-surplus function is not relevant. Consider a generic $\Phi(a, L, s)$. If assumption 1 and assumption 2 hold (with the appropriate modifications) then the two inputs will remain substitutes. Assuming that the social-welfare function has some range of complementarity, the private sector equilibrium allocation is, again, inefficient.

Suppose now that assumptions 1 and 2 do not hold. Proposition 6 shows that if over some range with positive mass of researchers and labs the social-welfare function is supermodular while the private-surplus function is submodular the private sector allocation is inefficient. The reason is that, over that specific range, the equilibrium matching will be NAM, but welfare can be improved by implementing PAM. Therefore, even in situations where assumptions 1 and 2 do not hold, it is possible for the private sector matching pattern to be inefficient. However, if the function $\Phi(a, L, s)$ is not globally submodular in a and L, the exact allocation of labs to researchers arising in the market can only be determined numerically.

It is also interesting to check what happen when the two inputs are global complements in the private-surplus function, so that there is no inefficiency in the matching stage. Lemma

[9] The equilibrium concept used in this model is called F-core, and the type of externality is called widespread externality. For a theoretical analysis of the inefficiencies of an F-core economy with widespread externalities see Hammond, Kaneko, and Wooders (1989) and Hammond (1995).

8 in the next section will show that firms underinvest in labs because they do not fully appropriate the benefit of new science. Therefore, if labs and researchers are global complements, the model collapses back to a standard model of science production where the only source of inefficiency is the firms' underinvesment.

4 The Ex-Ante Equilibrium

The definition of equilibrium I use is similar to the one in Cole, Mailath, and Postlewaite (2001). The differences are that, here, only one of the two sides invests, and the firm's type affect the total surplus not only through the investment, but also directly.

Definition 7. The quadruple $\{i(.), m(.), x(.), w(.)\}$ constitutes an equilibrium if:

1. The investment is optimal:

$$i(s) = \arg\max_{L \geq 0}\{x(s,L) - c(s,L)\}$$

2. Ex post, the matching $\{i(.), \tilde{m}(.), \tilde{x}(.), w(.)\}$ is feasible and stable:

   - Feasibility: $\tilde{x}(s) + w(\tilde{m}(s)) \leq \Phi(\tilde{m}(s), i(s), s)\ \forall s \in S$.[10]
   - Stability: $\tilde{x}(s) + w(\tilde{m}(s')) \geq \Phi(\tilde{m}(s'), i(s), s)\ \forall s, s' \in S$.

3. For any firm s, the payoff from investing is

$$x(s,L) = \max_a \{\Phi(a,L,s) - w(a)\}$$

To understand the definition, assume that there is an equilibrium, and consider deviations made by a single firm. Since we are in a large economy, any action this firm may take has no impact on the equilibrium $w(a)$. Therefore, whatever the investment, this firm can match with any researcher a provided that it pays $w(a)$.

Lemma 8. *In equilibrium, for $L \geq 0$:*

$$\frac{\partial x(s,L)}{\partial L} = \frac{\partial \Phi(a,L,s)}{\partial L}|_{a=m(L)}$$

Proof. From point 3 of the definition of equilibrium. □

[10] The general definition of feasibility is more complicated (see Cole et al. (2001)). However, in the cases I consider here it is possible to use this simpler version.

Lemma 8 implies that firms' investment solves:

$$\frac{\partial c(s,L)}{\partial L} = \frac{\partial \Phi(a,L,s)}{\partial L}|_{a=m(L)} \tag{2}$$

In other words, firms maximize surplus taking V and the researchers they will be matched with as given. Since the social planner would take into account the impact of the individual investment on the total stock of science, lemma 8 implies that the investment is inefficient. Finally, note that the matching pattern expected to emerge in the following period affects the investment decisions. It follows, for example, that any policy attempting to change the allocation of researchers to labs will affect the investment and may turn out to be counter-productive.[11] Also, any subsidy to the investment in labs may reduce the underinvestment, but it is unable to affect the inefficiency in the matching between labs and researchers. In the next section I will show that the only way to reach the first best in this economy is using a set of taxes and subsidies to the amount of science produced by each firm.

Lemma 9. *In equilibrium the biggest firm hires the least productive researcher:*

$$\tilde{m}'(s) < 0$$

Proof. See appendix □

Before showing the existence of an equilibrium, let's introduce a new piece of notation. Let's call $l(a) \equiv i(\tilde{m}^{-1}(a))$ the lab a researcher of ability a receives in equilibrium.

Proposition 10. *An equilibrium with zero research always exists. If the commercial value of research ν is high enough, there are also equilibria where a positive amount of science is produced. In these equilibria, researchers belonging to the set $[\underline{a}, \overline{a}]$ match with firms investing $l(a)$, where:*

$$l(a) = max\left\{\left\{L \in \mathbb{R}^+ : \frac{\partial \Phi}{\partial L} = \frac{\partial c}{\partial L}\right\}, 0\right\} \tag{3}$$

$$\underline{a} : \nu \int_{\underline{a}}^{\overline{a}} af(l(a))z(a)da = \frac{P(\underline{a}) + g(\underline{a}f(l(\underline{a})))}{\tilde{m}^{-1}(\underline{a})} \tag{4}$$

$$P(\underline{a}) = \int_{\tilde{m}^{-1}(\overline{a})}^{\tilde{m}^{-1}(\underline{a})} \frac{\partial c(s, i(s))}{\partial L} \gamma(s)ds \tag{5}$$

$z(a)$ is the p.d.f. of a, and $\gamma(s)$ is the p.d.f of s.

Proof. See appendix. □

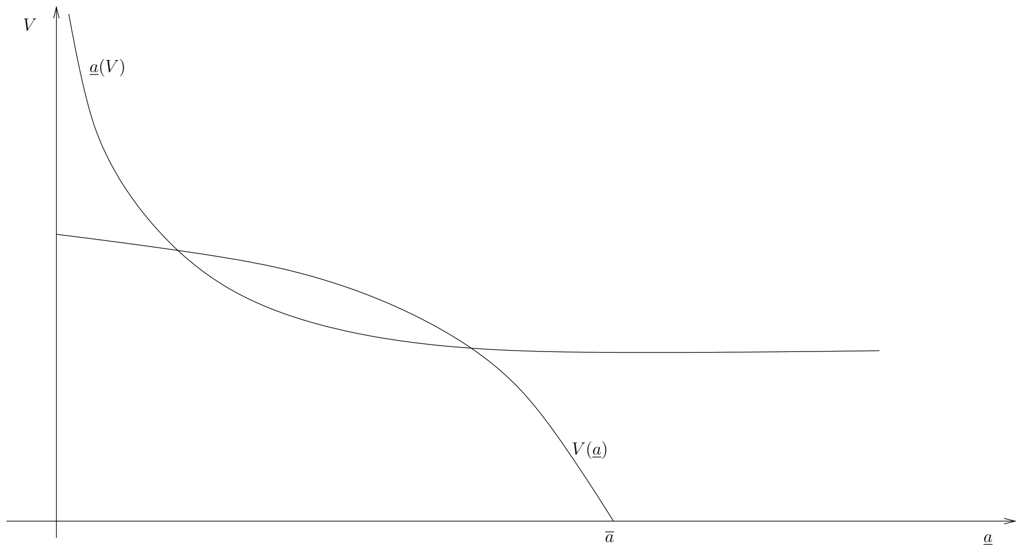

Fig. 2: Equilibrium $\underline{a}$ and V.

Figure 2 illustrates the case of two positive investment equilibria, given by the intersection of $V(\underline{a})$ and $\underline{a}(V)$, where $V(\underline{a})$ represents the aggregate science produced as a function of the measure of researchers employed, and $\underline{a}(V)$ represents the worst researcher employed in the economy for given aggregate science V. Of the two equilibria represented in figure 2, one can be considered stable (the high V, low $\underline{a}$ one) and the other unstable.

By focusing on the stable equilibrium, it is possible to make a few comparative static exercises. If the value of a discovery ν increases, $V(\underline{a})$ moves upward: more researchers are matched and more research is produced. It is also possible to introduce an exogenous stock of science V^f, science produced, for example, by a foreign country. The graph should be modified by writing on the vertical axes V^h instead of V, and by shifting $\underline{a}(V^h)$ downward: home country is producing more research as well. Obviously, all the comparative statics are reversed if we consider the unstable equilibrium.

5 The First Best

The social welfare generated within each match is:

$$SW(a, L) = s\nu af(L) - g(af(L))$$

[11] Gall, Legros, and Newman (2009) analyze this problem in a different context.

This function is neither globally supermodular nor globally submodular. It follows that the optimal allocation of researchers to labs can only be derived numerically, and it may involve implementing PAM over some range, and NAM over some other range. Intuitively, the social planner may, over some range, give priority to the production of science, and over some other to the use of science.

However, we know that the social planner problem has a unique solution. This implies that the first best allocation can be easily implemented if transfers based on the amount of science produced by each firm are feasible.[12]

Proposition 11. *The first best is implementable announcing the following rule: every firm producing some science receives a transfer equal to its size times the value of the science produced by that firm minus V.*

Since there is a mass 1 of firms, V is the value of the average amount of science produced. Therefore, firms producing more than the average receive a subsidy, while the others are taxed. However, even if scientific output is observable, it is usually non contractible and, therefore, non taxable. For this reason, the first-best implementation has little practical interest.

6 The University Research Sector

Given the technical difficulties in dealing with the first best, from now on I switch to a positive analysis. I will introduce into the model new elements: universities, the government, and reputation concerns for researchers. I will then describe how they interact with the private sector and the decentralized equilibrium, and I will show that these policies and institutions play an important role in determining how resources are allocated to researchers. To start, I will introduce into the model the sector that, in most countries, produces the vast majority of new science: universities.

6.1 Consultants.

As before, let's start analyzing the problem taking the distribution of labs as given. Universities are made up of labs. If a researcher a works in a university, he receives a lab of size $l^u(a)$. Researchers working in a university in period $t = 1$ can then work as consultants in period $t = 2$.

This assumption is motivated by the literature on *star scientists.* Zucker, Darby, and Brewer (1998) show that the birth of the biotechnology industry during the 1970s in a particular region can be explained by the presence of star scientists: researchers with an

[12] See Hammond (1995).

outstanding research track in genetics. These scientists worked in academia, and, at the same time, were active as consultants, were part of the board of companies, and sometimes even created their own start-ups. Doing so, they brought into these private labs the public science they contributed to create.[13] For simplicity, I will refer to all these activities as simply consulting.

If an academic researcher works as consultant, the disutility the researcher has to incur in period $t = 2$ is equal to $g(R(a, l^u(a)))$, so that the total surplus created by a match between a firm and a researcher/consultant is:

$$\Phi = sV - g(R(a, l^u(a)))$$

Therefore, researchers (and firms) prefer the researcher to work in the university sector if $l^u(a) \geq l(a)$: in the university sector the researcher works in a lab bigger than the one she would work in if she had stayed in the private sector. Because of NAM in the private sector, for any $l^u(a)$, the most productive researchers are willing to join the university sector. These researchers will then consult for the smallest firms (that, anticipating this, will not invest), while big firms will invest in lab and hire their own researchers.

6.2 University Labs and Subsidies.

In order to derive the size of the university sector endogenously, I introduce into the model a government, and I assume that its objective is to maximize the total stock of science under an exogenous resource constraint.[14] It is important to stress I'm performing a positive analysis and not a normative one. In most countries, the government plays a crucial role in determining the amount of research carried out within the economy. My goal is to introduce it into the model in the most reasonable way and to analyze the impact of its policies on the overall production of science.

I assume that the government can employ its resources either to subsidize the production of labs, or to build a university research sector. Subsidies are cheaper than financing universities since they build on top of what firms are already investing. However, subsidies have no impact on the matching phase. Instead, building universities, although more expensive, allows the government to choose the optimal allocation of researcher to labs. Note that in the standard public good model of science there is little difference between direct provision of science or subsidies to private research. Here these two policies achieve different goals

[13] Note how both the star scientists literature and the absorptive capacity literature focus on sectors where, for firms, it is crucial to be up to date with the latest scientific discovery. For example, in biotechnology, once a piece of science reaches a textbook and becomes accessible without any absorptive capacity building or star scientist help, this very same piece of science is typically useless to firms.

[14] In the model, the government is uniquely characterized by its objective function. Readers may safely substitute the word "government" with, for example, "foundations."

at different costs: depending on the conditions, the government will use one, the other or both.

The introduction of subsidies and universities changes the private sector equilibrium only marginally. Before the investment phase begins, the government announces $l^u(a)$, the lab a given researcher will receive if he joins the university sector. If a firm expects to be matched with a researcher that, by moving to the university sector, would work in a lab bigger than the one the firm owns, this researcher should work in a university lab and then act as a consultant. In the anticipation of this event such a firm does not invest at all.

Finally, suppose that each firm receives from the government a transfer $\tau(L)$, continuous and differentiable. The private surplus function is now $\Phi(a,L,s)+\tau(L)$. By lemma 8 in equilibrium $\frac{\partial\Phi(a,L,s)}{\partial L}+\frac{\partial\tau}{\partial L}=\frac{\partial c}{\partial L}$. In the same way, the constrained efficient investment equilibrium exists and the worst researcher matched is given by $\Phi(\underline{a},l(\underline{a}),\tilde{m}^{-1}(\underline{a}))+\tau(l(\underline{a}))-P(\underline{a})=c(l(\underline{a}))$. As far as $\tau(l(\underline{a}))=0$, finding the equilibrium V and $\underline{a}$ is analogous to the problem solved in the previous section.

The government problem can be formalized in the following way:

$$\max_{L^u(a),\tau(l(a))}\{\nu\int_{\underline{a}}^{\overline{a}} af(\hat{l}(a))z(a)da\} \tag{6}$$

$$\text{s.t.}\begin{cases}\hat{l}(a)=\max\{l(a),l^u(a)\} & \text{(I)}\\ G=\int_{\underline{a}}^{\overline{a}}\left(\tau(l(a))+l^u(a)\right)z(a)da & \text{(II)}\\ l(a)=\left\{L:\frac{\partial\Phi(a,L,s)}{\partial L}|_{a=m(L)}+\frac{\partial\tau}{\partial L}=\frac{\partial c}{\partial L}\right\} & \text{(III)}\\ \frac{\partial l(a)}{\partial a}\leq 0 & \text{(IV)}\\ \underline{a}:\nu\int_{\underline{a}}^{\overline{a}}af(l(a))z(a)da=c(l(\underline{a}))+g(\underline{a}f(l(\underline{a})))+P(\underline{a}) & \text{(V)}\\ \tau(L)\geq 0 & \text{(VI)}\end{cases}$$

where $\hat{l}(a)$ are the labs in use in the economy, some of which are private $l(a)$ and some of which belong to universities $l^u(a)$. The first constraint says that whenever researchers can choose between universities and private labs, they will work in the biggest lab. The second line is the government budget constraint. The following three say that the investment in labs induced by the government by means of subsidies is an equilibrium. The last line restricts $\tau(L)$ to be a subsidy rather than a tax.

It is possible to characterize the solution to the government problem.

Proposition 12. *In the university research sector, better researchers work in bigger labs.*

Proof. In building university labs, the only constraint that matters is constraint (II). Therefore, the government will set:

$$f'(l^u(a)) = \left(\frac{a'}{a}\right) f'(l^u(a'))$$

for all a and a' working in the university sector. □

Proposition 13. *All firms receiving subsidies invest the same amount.*

Proof. The allocation of labs in the university research sector is not achievable using subsidies because of constraint (IV). Therefore if the government uses subsidies, constraint (IV) is binding:

$$l(a) = \bar{l}$$

for all $l(a)$ receiving a positive subsidy. □

Proposition 14. *University labs are bigger than subsidized private labs.*

Proof. If this were not the case, the government could save money by turning some university labs into subsidized private labs. It also implies that the government will allocate the best researchers to the university sector. □

Figure 3 provides a careful illustration of the problem. In the top graph, the shaded area represents the cost borne by the government. In the bottom graph, the shaded area represents the increase in V due to government intervention.

The government problem is too complicated to be solved analytically. Therefore, I resort to numerical methods in order to determine when the government should subsidize, build universities or do both (the details of the simulation are in the appendix). The results are reported in figures 4 and 5.

In figure 4 different quadrants report the optimal distribution of labs for different values of $\bar{a}$ and G (G increases going from left to right, and $\bar{a}$ increases going from the top down). Figure 5 summarizes the results of the same exercise for a wider range of $\bar{a}$ and G. In both figures it is evident that, if the quality of the best researcher increases, the government is more likely to build university labs. When a researcher is very productive, the lab that he would work with in the private sector is very small: building universities allows the government to allocate more resources on the most productive researchers. Finally, figure 5 shows that when the government has more resources, it is more likely to use a mix of university labs and subsidies, rather than only one of the two policies.

The government's policies increase the equilibrium V. Compared to the economy without a government, now more entrepreneurs invest and more researchers are matched. This is represented in the bottom graph of figure 3 by a decrease in $\underline{a}$ from $\underline{a}(V')$ (where V' is the

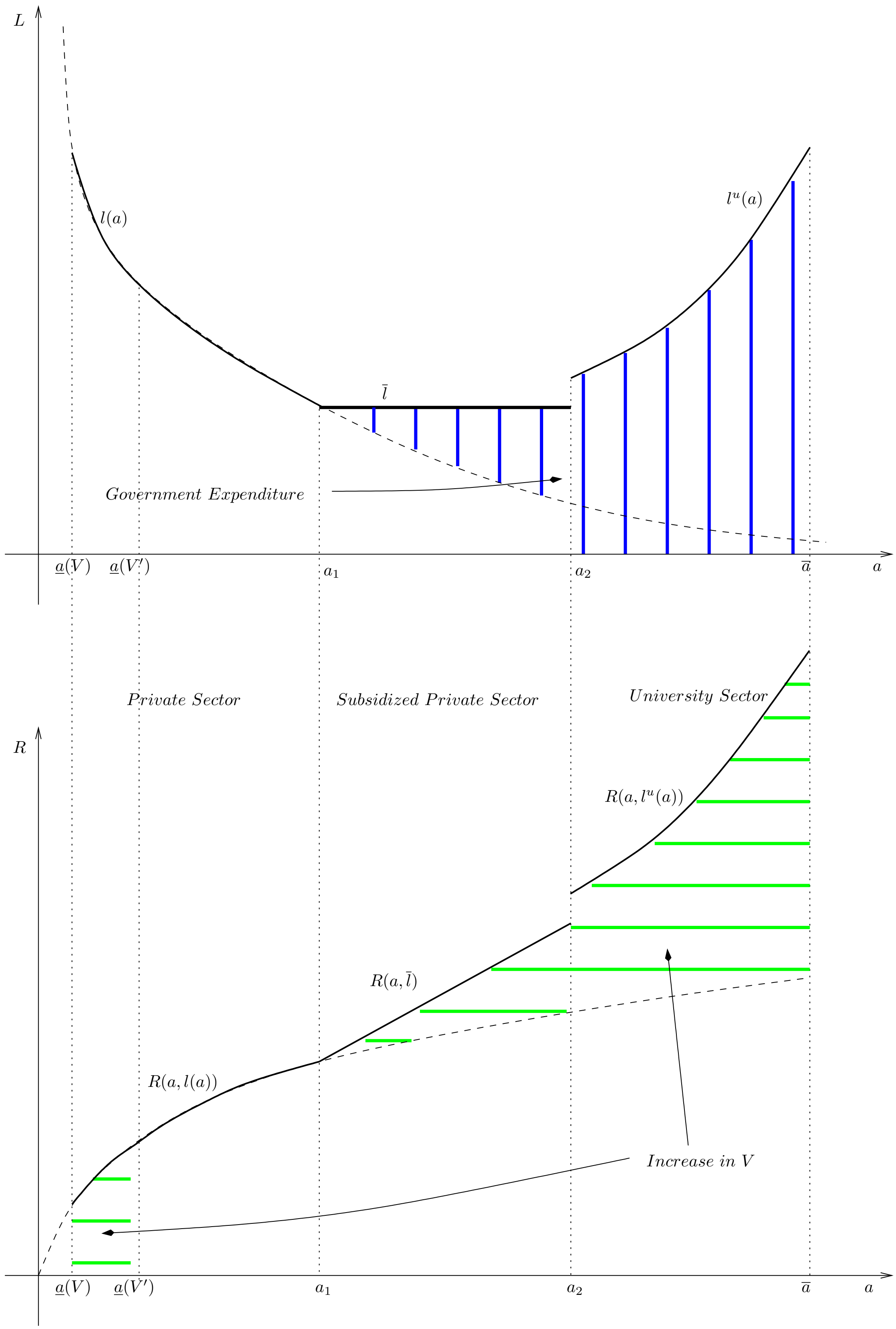

Fig. 3: Cost and Benefit of Government Intervention.

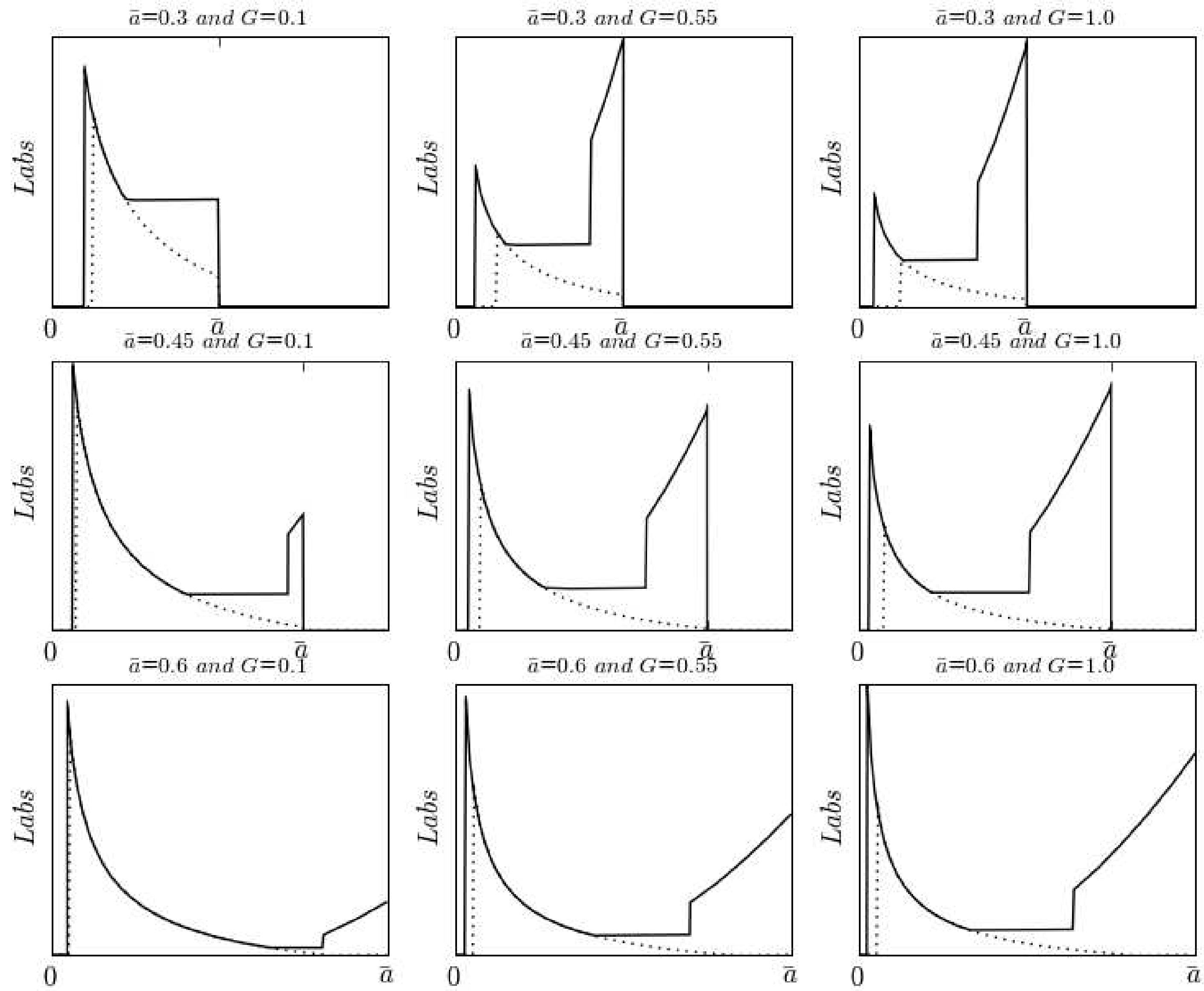

Fig. 4: Optimal Distribution of Labs (dotted line, no government; solid line, with government).

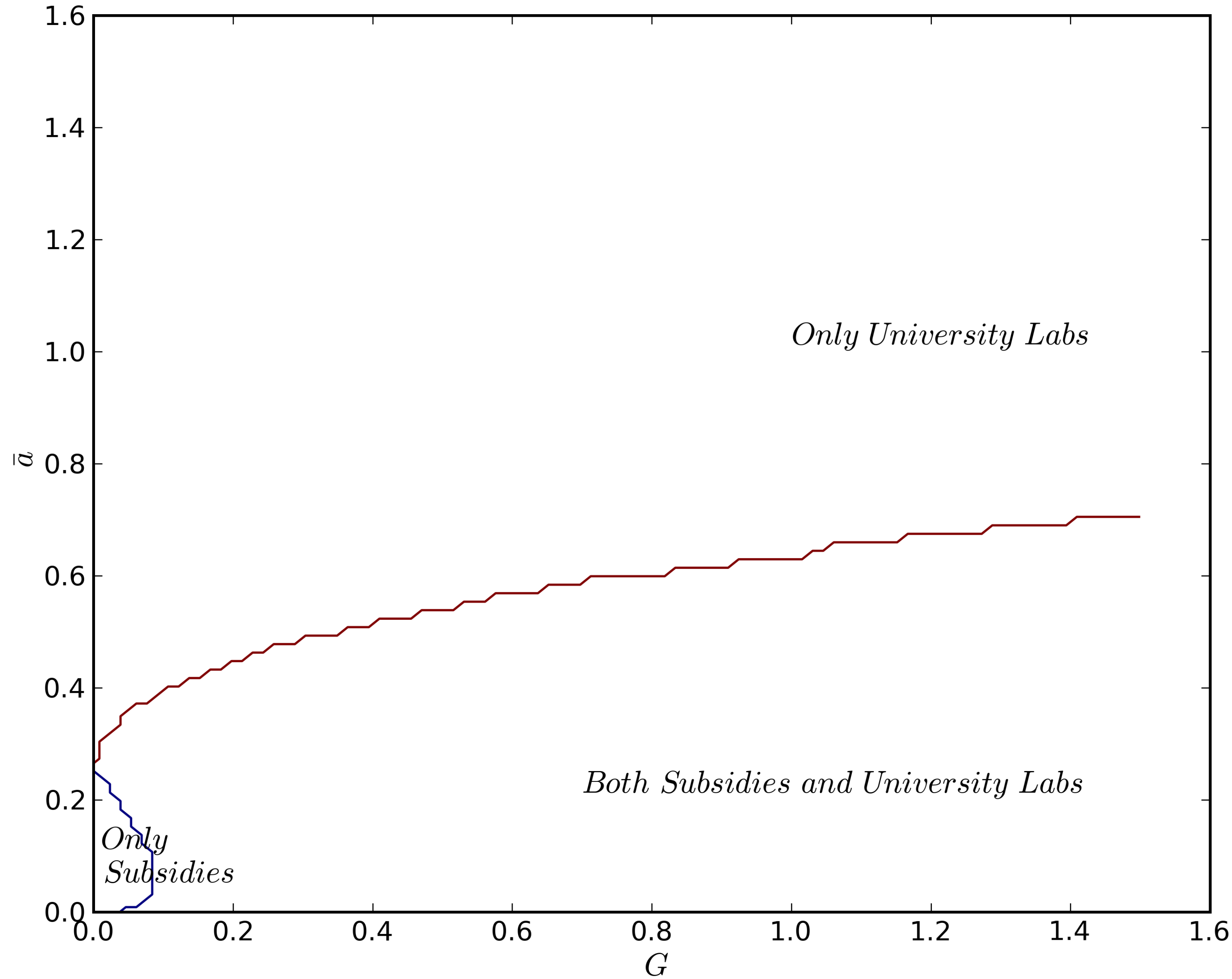

Fig. 5: Optimal Policy.

stock of knowledge before government intervention) to $\underline{a}(V)$. Whether university research is a complement or a substitute to private research depends on the number of new firms investing in research compared to the number of firms that stop investing because of the creation of university labs. In figure 4, the researchers joining the university sector would work with small labs in the private sector, so there is little decrease in private investment if the government increases its expenditure. Simulations (not reported) carried out for several parameters values always found private and university research to be complements. These findings are consistent with the empirical literature. David, Hall, and Toole (2000) review the existing econometric evidence trying to establish if university and private research are substitutes or complements. They report that most of the papers looking at aggregate measures find a complementarity effect, while, at the single firm level, there is evidence of a substitution effect.

7 Reputation

Since the work of Merton (1957), it is well known that researchers care about reputation. Merton calls it the *race for priority*: scientists want to be recognized as the first to discover something. The role of reputation in science has already been explored in the economic literature by Dasgupta and David (1985). The general conclusion is that, on the one hand, reputation motivates researchers. This is very important because an incentive scheme based exclusively on the quality of scientific output would be very hard to implement. Second, it fosters openness. This guarantees the circulation of ideas and generates a faster pace of scientific progress. Here I will show that reputation may have an additional effect. If researchers care about science, they may be willing to accept a lower payment to work in a firm with a big lab. In equilibrium, good researchers may outbid bad researchers for the right to work in a given firm, therefore changing the matching pattern in the private sector.

Let's assume that the researchers' utility is:

$$U(a) = w(a) + \rho(R(a, l(a)))$$

where $w(a)$ is the net payment received working for the firm, and $\rho()$ is the *reputation concern*: the utility derived from doing science. Researchers may care about science because their future earning depend on it (through the reputation they build today), or simply because they like science. The following lemma shows that if reputation concerns are strong enough the equilibrium allocation of researchers to firms will change.

Lemma 15. *Assume that an equilibrium with positive investment exists. If:*

$$\rho'(x) \geq 0 \, \forall x \tag{7}$$

$$\rho''(x) = g''(x) \, \forall x \tag{8}$$

the equilibrium is PAM between labs and researchers.

Proof. See appendix. □

Intuitively, researchers are willing to give up part of their payment in order to work in a firm with a bigger lab. Because of the complementarity between labs and researchers, a productive researcher is always willing to give up more than an unproductive researcher for the right to work in a firm with a given lab. Therefore, the final allocation of researchers to labs depends on the first derivative of $\rho()$: how fast the utility grows with the amount of research produced. Note also that a similar conclusion will be true even if condition 8 is not satisfied. In this case $\rho'(x)$ should be greater than a very complicated expression involving both $g''(x)$ and $\rho''(x)$ (see the appendix for more details).

To conclude, I show that, for any $\rho()$ that satisfies lemma 15 there exists an equilibrium.

Proposition 16. *Consider a $\rho()$ that satisfies lemma 15. An equilibrium with zero research always exists. If the commercial value of research ν is high enough, there are also equilibria where a positive amount of science is produced.*

Proof. See appendix. □

It is possible to characterize the net payment schedule that should emerge in the market when reputation concerns have the form described in lemma 15.

Lemma 17. *Consider a $\rho()$ that satisfies lemma 15. If reputation concerns are strong, good researchers will receive a lower net payment than unproductive researchers. In other words, if $\rho'(R(a, L))$ is large enough, $w'(a) < 0$.*

If reputation concerns are strong, good researchers receive a high reputation reward $\rho(R(a, L))$. Since, when the allocation is PAM, the disutility $g()$ is decreasing in ability, this implies that the equilibrium gross payment (the wage) can be decreasing in a.

Therefore, the model is consistent with Stern (2004). In his paper "Do Scientists Pay to be Scientists?" the author collects data on job offers received by a sample of biology Ph.D. job market candidates. He finds that firms engaged in science offer wages 25% lower than firms that are not engaged in science. The author interprets his results against the absorptive capacity hypothesis: firms giving a positive value to the production of science should pay researchers that are involved in science more. The alternative explanation is based on reputation concerns: firms do science as a way to reward scientists by letting them build their reputation. Lemma 17 shows that the two explanations can coexist.

Finally, it is possible to sketch what happen in a model with reputation concerns, universities, and subsidies. Clearly, if reputation concerns satisfy lemma 15, there is no need for universities and the government can spend all its resources in subsidies. However, if the lemma does not hold, the private sector allocation will be NAM over some range and PAM over some other. Universities may still be necessary to make sure the best researchers receive the biggest labs.

8 Conclusions

There are several reasons for firms to invest in research. The one proposed most often in the economic literature is *production*: firms invest in research because they want to increase the stock of science. This explanation imply that bigger firms should be more productive in their R&D effort than small firms. Big firms gain more than small firms from any extra science produced, therefore they should always be able to hire the most productive researchers, have access to the most productive machines, locate themselves in the best locations. However, there is empirical evidence that the productivity of R&D investment decreases with firms' size.

A second explanation to why firms invest in research has been recently proposed. Using outside science is costly to firms. This cost is lower if firms produce science. Therefore, firms invest in research to enhance their *absorptive capacity*, which is the ability to use the publicly available stock of science. In this paper I show that absorptive capacity can explain the negative correlation between firms' size and research productivity.

I build a model where firms build absorptive capacity in order to use outside science. I show that the private sector allocation is inefficient. In the model, there are researchers of different ability levels and firms owning labs of different sizes. The private sector allocates researchers and firms according to NAM: the best researcher works in the smallest lab. However, this matching pattern minimizes the total research produced in the economy.

I modify the baseline model in two ways. First, I introduce universities. I show that the best researchers work in university labs, and that, within the university sector, better researchers work with bigger labs in order to maximize the total amount of research produced.

Finally, I explore the effect of reputation. If researchers care about doing research, the market allocation of researchers to firms may change. In particular, I show that if the reputation concerns are strong enough, the matching pattern emerging in the private sector is PAM: good researchers work in big labs.

The model can be tested empirically in several ways. For example, it should be possible to check whether labs and researchers are substitutes in the private sector. Substitability implies that the increase in revenues following an increase in expenditure in research facilities should be greater in firms with researchers that are less productive. Alternatively, one could check the market allocation of researchers to firms. In this case, however, the test should take into consideration the strength of the reputation concerns. Without reputation, the model predicts NAM. If reputation concerns exist and have the features I derived, we should observe PAM. For example, assuming that old researchers are less sensitive to reputation than young ones, the model predicts that productive young researchers should work in big labs and unproductive young researcher should work in small labs, while productive old researchers should work in small labs and unproductive old researchers should work in big labs.

Introducing absorptive capacity opens interesting policy questions. For example, in this context access to science is a policy instrument. Suppose that firms can learn about new discoveries only by sending their researchers to conferences. A rule that allows researchers from the private sector to participate in conferences only if their scientific contribution is above a certain threshold, may increase the amount of research carried out by the private sector.[15] Also, the way researchers are rewarded is an important determinant of the amount of science produced. It should be possible to transform all the different prizes and awards a researcher may receive during his career into a coherent policy instrument.

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[15] Leahy and Neary (2007) address exactly this point, but in a different context.

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A Appendix

Proof of Proposition 3.

It is straightforward to check that substitutability at a given $\hat{a}, \hat{L}$ is equivalent to:

$$\frac{g''(\hat{a}f(\hat{L}))\hat{a}f(\hat{L})}{-g'(\hat{a}f(\hat{L}))} > 0$$

the proof of the proposition requires two steps:

1. *Show that under assumption 2* $r(x) \equiv \left[\frac{g''(x)x}{-g'(x)}\right]$ *is increasing in* x.
   Compute $r'(x)$
   $$r'(x) = \frac{g''(x)}{-g'(x)} + \frac{g'''(x)x}{-g'(x)} + \frac{g''(x)x}{(g'(x))^2}$$
   that is increasing if $g'''(x) > 0$.

2. *Show that under assumption 1,* $lim_{x\to 0} r(x) \geq 1$
   suppose not: $\exists \epsilon > 0$ arbitrarily close to zero such that $g''(\epsilon)\epsilon < -g'(\epsilon)$. Take an arbitrary $\sigma > 0$ and define:
   $$K_{\epsilon,\sigma}(x) \equiv a_{\epsilon,\sigma}\left(\frac{x^{1-\sigma}}{1-\sigma}\right) + b_{\epsilon,\sigma}$$
   where $a_{\epsilon,\sigma}$ and $b_{\epsilon,\sigma}$ are such that:
   $$K_{\epsilon,\sigma}(\epsilon) \equiv a_{\epsilon,\sigma}\left(\frac{\epsilon^{1-\sigma}}{1-\sigma}\right) + b_{\epsilon,\sigma} = g(\epsilon)$$
   $$K'_{\epsilon,\sigma}(\epsilon) \equiv a_{\epsilon,\sigma}\epsilon^{-\sigma} = g'(\epsilon)$$
   since we assumed that $g''(\epsilon)\epsilon < -g'(\epsilon)$, it follows that:
   $$g''(\epsilon) < \frac{-g'(\epsilon)}{\epsilon} = a_{\epsilon,\sigma}\epsilon^{-\sigma-1}$$
   because of the strict inequality, it is always possible to take a $\sigma < 1$, arbitrarily close to one, such that:
   $$g''(\epsilon) < a_{\epsilon,\sigma}\sigma\epsilon^{-\sigma-1} = K''_{\epsilon,\sigma}(\epsilon)$$
   this implies that, in a neighbour of ϵ, $g(x) < K_{\epsilon,\sigma}(x)$. Finally, note that $x = 0$ is in a neighbour of ϵ and at the same time $K_{\epsilon,\sigma}(0)$ is well defined for $\sigma < 1$. Therefore $g(0)$ is well defined and finite. This is a contradiction.

Point 2 alone implies that the inputs are substitutes for small enough $af(L)$. Point 1 and point 2 imply that the two inputs are always substitutes.

Proof of Proposition 6.

The social welfare generated within each match is equal to:

$$SW(a, L) = s\nu af(L) - g(af(L))$$

one obvious difference between the first best allocation and the private sector allocation is in who is matched. In the private sector, researchers and labs are matched if $sV \geq g(af(L))$. Note that V is determined endogenously, and that there are multiple equilibria. However, the private sector condition for being matched is, in general, different than the social optimal one.

Going back to the matching pattern, note that NAM is inefficient only under some conditions on ν. To see this, imagine that the economy is so unproductive (low ν) that both from the social point of view and from the private point of view, nobody should be matched. In this case any matching pattern will lead to the same welfare (zero) so that NAM is trivially efficient.

It is easy to show that $SW_{12} > 0$ if:

$$s\nu > g'(x) + xg''(x)$$

Given this, we can be in one out of three possible situations. The first one is illustrated in figure 6a. In this case there is no complementarity in the relevant range of the social welfare function and NAM is efficient. Imagine now to increase ν. The area of complementarity expands, and eventually we reach the situation illustrated in figure 6b. In this case, it is possible for the social planner to reallocate some researchers and some labs in order to have an area of PAM. However, this leaves some unmatched agents, that should be re-matched somehow. Whether this deviation increases social welfare or not is left to be determined in future works. If ν is even higher, eventually the economy will reach the situation depicted in figure 6c. In this case it is possible to rematch researchers between a^1 and a^2 with labs from L^1 and L^2 according to PAM and increase the social welfare.

Proof of Lemma 9.

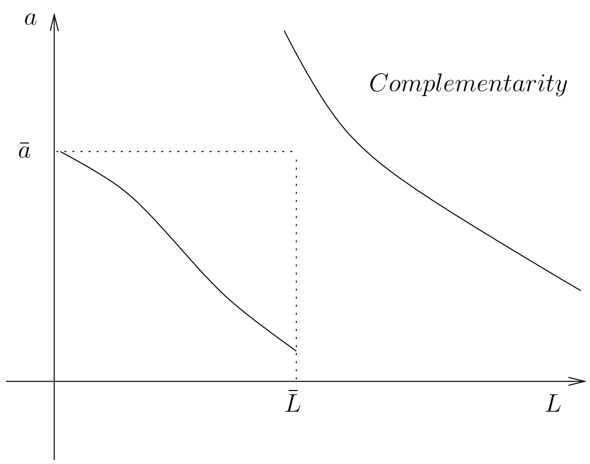
By point 1 in the definition of equilibrium and using lemma 8 we get:

$$i'(s) = \frac{c_{sL}}{\Phi_{LL} - c_{LL}}$$

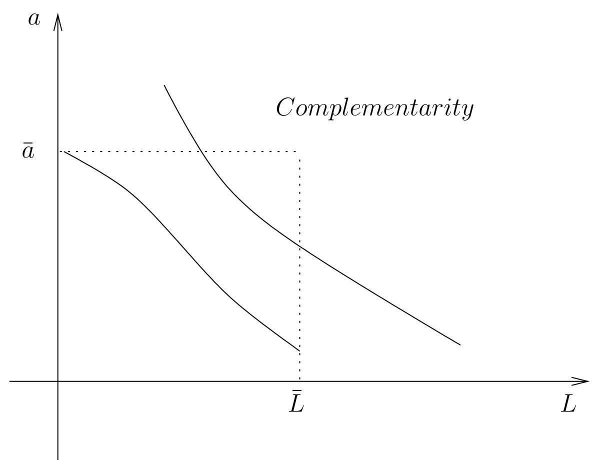
note that $c_{sL} < 0$, $c_{LL} > 0$, and

$$\Phi_{LL} = -\left[\underset{+}{g''(af(L))} \underset{+}{(af'(L))^2} + \underset{-}{g'(af(L))} \underset{-}{af''(L)} \right] < 0$$

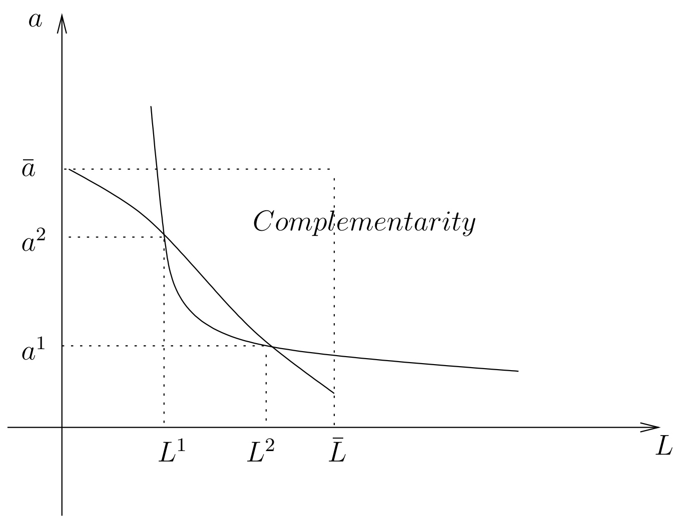
so that $i'(s) > 0$: biggest firms invest the most. By NAM between researchers and labs, this implies that biggest firms hire the least productive researcher.



(a) No Inefficiency.



(b) Conjecture: inefficient matching.



(c) Inefficient matching.

Fig. 6: Complementarity range and matching function.

Proof of Proposition 10.

For the first part, note that if firms expect $V = 0$, they have no reason to invest in research. Therefore, the total science produced will be zero.

Consider an equilibrium with positive investment. In general, if all the researchers and all the entrepreneurs in the economy were matched, the worst member of each group could enjoy a strictly positive payoff. In our case, since the worst researcher in the economy is $a = 0$ and $\lim_{a \to 0} \Phi(a, L) = -\infty$, on both sides there is always someone that is not matched. Consider the match between the firm that invested the most and the worst researcher. The researcher receive a payoff equal to zero, while the firm receives

$$\tilde{x}(\overline{s}) = \int_{\tilde{m}^{-1}(\overline{a})}^{\tilde{m}^{-1}(\underline{a})} \frac{\partial \Phi(\tilde{m}(s), i(s), s)}{\partial L} \gamma(s) ds = \int_{\tilde{m}^{-1}(\overline{a})}^{\tilde{m}^{-1}(\underline{a})} \frac{\partial c(s, i(s))}{\partial L} \gamma(s) ds \equiv P(\underline{a})$$

where $\gamma(s)$ is the p.d.f. of s, and the second equality follows from lemma 8 and equation 2. In other words, the payoff received by the most productive firm depends on the productivity of the worst researcher matched. The equilibrium $\underline{a}$ and V are the solutions to:

$$\underline{a} = \left\{ a : \Phi(a, l(a), \tilde{m}^{-1}(a)) = P(a) \right\} \tag{9}$$

and:

$$V = \nu \int_{\underline{a}}^{\overline{a}} a f(L(a)) z(a) da \tag{10}$$

The equilibrium with positive investment exists if there is a $\{\underline{a}, V\}$ solution to equations 9 and 10.

Note that equation 10 has a finite value at $\underline{a} = 0$, is equal to zero at $\underline{a} = \overline{a}$, and is strictly decreasing. Finally, equation 9 can be rewritten as:

$$V = \frac{P(\underline{a}) + g(\underline{a} f(L(\underline{a})))}{\tilde{m}^{-1}(\underline{a})} \tag{11}$$

Because of assumption 1, if $\underline{a} \to 0$ the solution to 11 diverges to infinity, has finite values for $\underline{a} \in (o, \overline{a}]$, and is continuous. Therefore, if ν is high enough, equations 9 and 10 will cross.

Proof of Proposition 11.

The social welfare generated in each match is equal to:

$$SW(a, L) = s\nu a f(L) - g(a f(L))$$

the private surplus is:

$$\Phi(a, L) = sV - g(af(L))$$

clearly, a transfer like the one described transforms the private surplus into the social welfare function. Finally, because of lemma 8, when firms invest they equate marginal cost to marginal benefit. In this case, it implies that firms' investment is efficient.

Details of the Simulation.

I choose the following functional forms:

- $c(s, L) = (1 + r)L$
- $R(a, L) = af(L) = a(1 + L)^{\frac{1}{2}}$
- $g(R(a, L)) = \frac{1}{a(1+L)^{\frac{1}{2}}}$

and I assume that $\tau(l(\underline{a})) = 0$: the firm matched with the worst researcher receives no subsidy. This can be seen as a restriction on the amount of resources the government has. Note that all firms are identical.

The government problem can be written as:

$$\max_{l^u(\overline{a}),\bar{l},a_1,a_2} \{\int_{a_1}^{a_2} a(1+\bar{l})^{\frac{1}{2}} da + \int_{a_2}^{\overline{a}} a^2(1+l^u(\overline{a}))^{\frac{1}{2}} da - \int_{a_1}^{a_2} a(1+l(a))^{\frac{1}{2}} da\} \tag{12}$$

$$\text{s.t.} \begin{cases} \bar{l} = \left(\frac{1}{2(1+r)a_1}\right)^{\frac{2}{3}} - 1 & (1) \\ a_1 \leq a_2 \leq \overline{a} & (2) \\ \left(\frac{a_2}{\overline{a}}\right)(1+l^u(\overline{a})) - 1 \geq \bar{l} & (3) \\ \int_{a_1}^{a_2} \left(\bar{l} - \max\left\{\left(\frac{1}{2(1+r)a}\right)^{\frac{2}{3}} - 1, 0\right\}\right) da + \int_{a_2}^{\overline{a}} \left[\left(\frac{a}{\overline{a}}\right)^2 (1+l^g(\overline{a})) - 1\right] da = G & (4) \end{cases}$$

Figure 3 on page 18 represents it graphically. The objective function is the extra research produced thanks to the policy in place (the shaded area in the lower axes) at a cost summarized by constraint (4) and represented by the shaded area in the upper axis. Note that the increase in research at the bottom of the distribution of labs (between $\underline{a}(V)$ and $\underline{a}(V')$) can be safely ignored since it is an increasing function of the extra research V produced by the rest of the economy.

The simulation simply compares values of the objective function at different a_2 and $\bar{l}$. The aim is not to determine the exact optimal policy, but to check whether there is an interior solution (both subsidies and university labs) or one of the two corner solutions (only subsidies, only university labs).

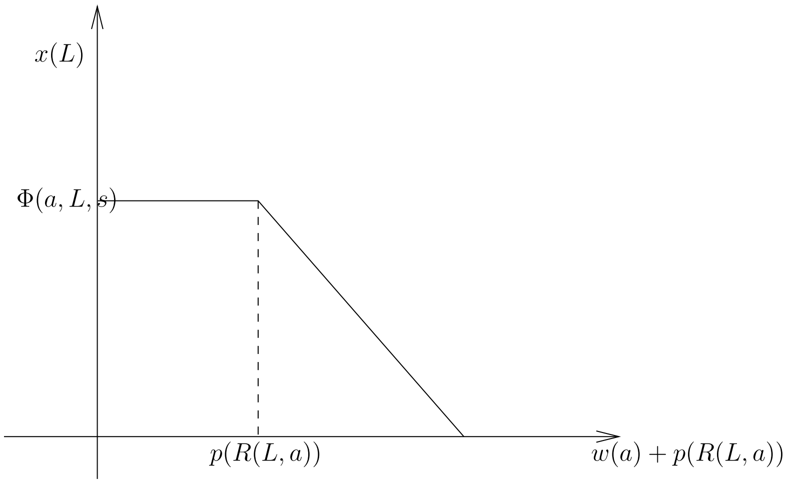


Fig. 7: Utility Possibility Frontier.

I construct a grid $\{0, ..., \overline{a}\}$ containing all possible values of a_2. For every value of a_2, I construct a grid of possible value of $\bar{l} \in \left\{ \left(\frac{1}{2(1+r)a_2} \right)^{\frac{2}{3}} - 1, ..., \tilde{l} \right\}$ where $\tilde{l}$ is an appropriate large number. For every a_2 and $\bar{l}$ I compute $l^u(\overline{a})$ using constraint (4) of 12. I consider the pair a_2 and $\bar{l}$ admissible if $l^u(a_2) = \left(\frac{a_2}{\overline{a}}\right)(1 + l^u(\overline{a})) - 1 \geq \bar{l}$. Finally, I compute the value of the objective function. The final solution is the admissible pair $\{a_2, \bar{l}\}$ returning the highest value.

Finally, in the standard simulation, the value for r is 0.01 and for ν is 100. When checking for the complementarity or substitability of private and university research, the parameters I tried are: $\underline{a} \in [0, 5]$, $G \in [0, 5]$, $r \in \{0.01, 0.1, 1\}$ and $\nu \in \{75, 100, 150\}$; technical reasons restricted the choice of ν; the other parameters were picked arbitrarily.

Proof of Lemma 15.

Figure 7 represents the utility possibility frontier of a match. For a given distribution of labs, whenever the equilibrium payoffs lie on the 45 degrees part, under lemma 15 the equilibrium matching is PAM. The reason is that the total surplus function $\Phi(a, L, s) + \rho(R(a, L))$ (transferable between researchers and firms) is supermodular: firms with bigger labs are better off by matching with more productive researchers, and vice versa.

However, since the wage cannot be negative, the utility possibility frontier has a kink. At the kink, researchers receive $\rho(R(a, L))$ and firms receive $\Phi(a, L, s)$. Again, for given s, the payoff of each side is increasing in the other side's type. This implies that the equilibrium is PAM for these agents as well.

Finally, note that both sides prefer to be matched with a high type than with a low type, even when it means switching from the kink region to the the 45 degree region. This implies that the equilibrium is PAM overall.

Proof of Proposition 16.

Since lemma 15 imposes restrictions only on the slope of $\rho()$ I can normalize $\rho(R(\underline{a}, l(\underline{a}))) = 0$. This implies that, as before, the worst researcher matched is given by:

$$\Phi(\underline{a}, l(\underline{a}), s) = c(l(\underline{a})) + P(\underline{a})$$

and the value of the total stock of science in the economy is given by:

$$V = \nu \int_{\underline{a}}^{\overline{a}} af(l(a))z(a)da$$

This problem is identical to the one solved in the proof of proposition 10.

Proof of Lemma 17.

By stability, whenever $w(a) > 0$:

$$\Phi(a, i(s), s) + \rho(R(a, i(s))) - \tilde{x}(s)) \geq \Phi(a, i(s'), s') + \rho(R(a, i(s')) - \tilde{x}(s')$$

Write the same condition for a', and take limits for $a' \to a$:

$$\tilde{x}'(s) = \Phi_L i'(s) + \Phi_s + \rho' R_L i'(s)$$

note that $i'(s) > 0$ since we are considering only the transferable-utility part of the utility possibility fronteer. By feasibility:

$$\Phi(m(i(s)), i(s), s) = \tilde{x}(s) + w(i(s))$$

Differentiate both sides with respect to s. By simple algebra:

$$w'(a) = \Phi_a m'(a) + \Phi_L + [i'(s)]^{-1} [\Phi_s - \Phi_a - \rho']$$

that is negative if ρ' is big enough.