Modeling the volatility of FTSE All Share Index Returns

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1. Introduction

It is well known fact that time series have their own frequency behaviour. This is a very common phenomenon in practise, especially in financial time series data. As Mandelbrot points out that, "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes". This feature is known as volatility clustering. Modeling the volatility of stock returns is an essential key for pricing financial assets and derivatives. Observations of volatility clustering in time series has given a way to the use of ARCH and GARCH models in financial forecasting and asset and derivatives pricing.

Time-varying volatility was firstly introduced by Engle (1982) as an autoregressive conditioal heteroskedasticity (ARCH) model. A volatility model can be referred as a mean and variance equation.

Mean equation: $y_t = \sigma_t \varepsilon_t$

Variance equation: $\sigma_t^2 = \mu + \Phi y_{t-1}^2$

Where y_t is the asset returns and σ_t^2 is the volatility of these returns. Volatility can be described as a measure of risk on returns. Each observed data point y_t has a standard deviation σ_t and the error term is Gaussian $\varepsilon_t \approx iidN(0,1)$.

This model of Engle was extended by Bollerslev (1982) to become a generalized autoregressive conditional heteroskedasticity (GARCH) model.

$y_t = \sigma_t \varepsilon_t$

$\sigma_t^2 = \mu + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2$

Since the introduction of the ARCH model, there has been a massive amount of studies conducted on volatility modeling. As Bollerslev, Chou and Kroner (1992) state that more than 100 papers exist on this subject. In the various forms of GARCH models, the volatility is not only a deterministic function of the squares of past return but also squares of past volatility. A GARCH model also captures part of the excess kurtosis of the financial time series along with volatility clustering. These models have been widely used in finance literature examining the various types of financial data such as stock return data, interest rate data, foreign exchange data etc.

In this paper, I will examine the different volatility models and their ability to deliver volatility forecasts. The different aspects of volatility models such as GARCH, EGARCH and TGARCH are useful not only for modeling the historical volatility but also provide us multi-period future forecasts.

The rest of this paper is organized as follows, data used and methodology are briefly discussed in section 2, section 3 deals with tests and empirical results, and conclusion is drawn in section 4.

2. Methodology

The data used in this paper are the monthly data of the FTSE All Share Index traded on London Stock Exchange from January 1965 to November 2002. During this period average monthly return was 0.64% with a maximum of 42% and a minimum of -32%. Standard deviation of returns during the period was 5.7%. The historical returns of the index will be plotted and examined graphically in order to have some general idea about the structure of the series.Data are also tested in order to see whether it presents the January effect or not. The model designed to test this calender effect is a test of the average January effect in the returns of FTSE All Share Index. The model for testing the January effect can be expressed as,

$$R_{it} = \beta_0 + \beta_1 M_{it} + \varepsilon_{it}$$

Where

R_{it} = return on stocks in month i in year t and

M_{it} = 1 if the month is January, 0 otherwise

PcGive is employed to conduct the OLS regression to test the January effect.

In order to estimate goodness of fit of the model Akaike's Information Criterion (AIC) which was developed by Hirotsgu Akaike in 1971 is choosen. AIC has been employed by using PcGive to determine the correct lag lenght for the estimation.

For testing the misspecification of the conditional mean, error autocorrelation and the Durbin-Watson tests are employed. Durbin-Watson test is the simplest form of test that used to identify the presence of autocorrelation. When DW is close to zero, it implies positive autocorrelation, when DW is close to 4, there is a negative autocorrelation and if it is close to 2, there is no autocorrelation.

The effects of ARCH errors on the performance of lag length selection criteria are also tested. The most important outcome of this test is to demonstrate the relevance of the lag length selection criterion. We have to test if the criterion applicable to autoregressive process that exhibits ARCH effects.

Finally, various GARCH models in terms of their performance on volatility clustering are evaluated. Their robustness and forecasting abilities are also presented.

3. Empirical Results

This section briefly discusses the some empirical results associated with volatility clustering models. Graph 1 exhibits the historical tendency of the stock returns during the period of February 1965 and October 2002. As clearly seen in the graph that stock returns have the structure of volatility clustering small changes tend to be followed by small changes and large changes come after large changes. There are two big shocks in the series one in 1975 and second is in 1987. These large changes in the graph reflect the positive and negative effects of the market. For example, negative movement in 1987 is a product of the stock crash in October 1987 known as Black Monday.

Table 1 shows the normality test and descriptive statistics of the data during the sample period. During this period average of the monthly returns is 0.64% with a maximum of 42.13% and a minimum of -32.71%. These maximum and minimum returns are the results for big negative and positive effects in 1975 and in 1987. The standard deviations of these returns during the period is 5.77%. The skewness of the sample period is 0.159. Skewness measure the asymmetry of the probabiity distribution of the random variable. The positive skewness means the mass of the distribution is laid on the left side of the distribution which is called “right skewed”. High level of skewness can cause a skewness risk. Skewness risk indicates that the if the variables are too skewed the student t-test is not an appopriate method in testing hypothesis. The excess kurtosis of the series is 8.0985. Kurtosis describes the peakedness of the series. Positive kurtosis indicates a ‘peaked’ distribution and negative kurtosis indicates a ‘flat’ distribution.

Substantial evidence of a January effect in the stock market has well documented eveidences in the financial literature (Wilson and Jones 1990). According to the financial literature, stocks show consistently higher average returns in January, although this effect seems to be generally related to the small firms effect. The purpose of this test is to test specifically for a January effect in the returns of FTSE All Share Index. By regressing stock return series on dummy variable which is 1 in January and 0 in other months, we were able to show whether there is a January effect on our sample data during the sample period. Table 2 exhibits the results from regression for January effect. The average monthly return for months other than January (the constant) is 0.5167%, and premium for January over other months is 1.61%. The calculated t-value and r^2 for the period are 1.63 and 0.0059 respectively. The t-value for the period suggests the acceptance of the null hypothesis that there is no January effect on stock returns for our sample period.

The key element in the model is to determine the correct lag length. Several studies in this area demonstrate the importance of selecting a correct lag length. Estimates of the model would be inconsistent if selected lag length is different than the true lag length. Selecting a higher order lag length than the true one increases the forecasting errors and selecting a lower lag length usually generates autocorrelation errors. Therefore, accuracy of forecasts heavily depends on selecting the true lag lengths. There are several statistical methods that help us to select a lag length. Akakike’s Information Criterion (AIC) is considered to be nearly unbiased estimator of the selecting lag order. Therefore, AIC has been chooen to determine the correct lag length. In this paper, OLS regression is run with using different lag orders starting from 10 to 1. The table 3 shows the results from the progress of 10 equations. The equation which has the minimum AIC is determined as correct lag length for our model. The values of the

AIC from the table suggest that equation 8 which has 3 lags has the lowest AIC of -2.8741, therefore correct lag length appropriate for our model is 3.

In order the test misspecification of the conditional mean autocorrelation test are needed. Firstly, Durbin-Watson which is a simplest form of autocorrelation test of first-order is applied. DW is a test for autocorrelated residuals and can be calculated as,

$$DW = \frac{\Sigma_{t=2}^{n}(\hat{u}_t - \hat{u}_{t-1})^2}{\Sigma_{t=1}^{n}\hat{u}_t^2}$$

Where $\hat{u}$ t are the OLS residuals, $\hat{u}_t = y_t - X_t'\hat{\beta}$

In large samples,

$$DW \approx 2(1-\hat{\rho}), \hat{\rho} = \frac{\Sigma_{t=2}^{n}\hat{u}_t u_{t-1}}{\Sigma_{t=1}^{n}}$$

Since $-1 \leq \hat{\rho} \leq 1$, then $0 \leq DW \leq 4$. If DW is closer to zero , there is evidence of positive autocorrelation, if it is closer 4, there is a evidence of negative autocorrelation, and DW is closer to 2 there is zero autocorrelation. Table 4 shows the results from OLS regression at lag length 3. As seen from the table that result for the DW test is 1.99 which indicates that there is no autocorrelation. Although significance of DW is widely accepted in the literature, it can be biased towards 2 if the model includes a lagged dependent variable. Therefore, it is essential to conduct another error autocorrelation test for misspecification of the conditional mean. Table 5 shows the results of the error autocorrelation test performed by using PcGive. From the results *Chi^2(3) = 3.4197 [0.3313] and F-form F(3,443) = 1.1308 [0.3362]* we can conclude that the null hypothesis of there is no autocorrelation is accepted at both significance levels.

Table 6 presents the results of ARCH test. The ARCH test is conducted at lag order 3 in order to test ARCH effects of the regression. The results of F-form of the test *ARCH 1-3 test: F(3,440) = 3.9032 [0.0090]*** indicate that null hypothesis H=0 of there is no ARCH effect has been rejected at both significance levels of 1% and 5%.

In this part of the assignment, different asymmetric and symmetric volatility models are estimated. These models are respectively GARCH, TGARCH, EGARCH, and

AGARCH. Their ability to capture the volatility clustering and forecasting future volatility is determined. Misspecification tests for the volatility models are also presented.

Firstly, we begin by evaluating the traditional GARCH model first introduced by Bollerslev (1986) and have the following specification $h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$. The results of the estimation are presented in Table 7 suggest a consistent volatility presence with extremely significant t-statistics. The results are for the most part as expected with $\alpha_0 = 0.000243602, \alpha_1 = 0.107739, \beta_1 = 0.823097$ positive and $\alpha_1 + \beta_1$ less than one which means the process is covariance stationary. As $\alpha_1 + \beta_1$ is close to one (but not equal) which indicates that the volatility process might be integrated. Table 11 presents the some diagnostic information about the estimation. While, the standard deviation of the residuals is close 1 as expected, other descriptive statistics demonstrate some of the weaknesses of the GARCH model. Even statistically insignificant, the mean of the residuals is negative. Also residuals have statistically significant negative skewness and excess kurtosis.

Now, we move on to examine other models of volatility process. Three of the most popular specifications of the volatility process are explored. The first one is exponential GARCH (EGARCH) which was initially proposed by Nelson (1991) which parameterizes the volatility process as $\ln(h_t) = \alpha_0 + \alpha_1 \lceil \eta_{t-1} \rceil + \psi_1 \eta_{t-1} + \beta_1 \ln(h_{t-1})$ where $\eta_t = \varepsilon_t / \sqrt{h_t}$ represents the normalized error process. This specification has two main advantages. First, it allows h_t responding asymptotically good news and bad news. Second, because of the logarithmic form there are no non-negativity constraints of the parameters.

Secondly, we analyze the asymmetric GARCH (AGARCH) model of Engle and Ng (1993). The volatility equation is $h_t = \alpha_0 + \alpha_1 h_{t-1} + \beta_1 (\varepsilon_{t-1} + \psi_1)^2$. "The parameter ψ_1 is typically negative and thus AGARCH model also allows for asymmetric response of volatility to positive and negative shocks" (Goyal, 2000). Finally, threshold GARCH (TGARCH) model is explored. This model is similar to GJRGARCH model which volatility is measured by the conditional variance.

The estimation results of these three models of volatility are given in tables 8,9, and 10 respectively. In Table 8, **eps[-1]** is -0.0355325 and **|eps[-1]|** is significantly positive with a value of 0.192435. Moreover, the likelihood value is 680.891084 which is higher than that of GARCH model. These findings indicate that there is an obvious asymmetric response of shocks to volatility and the EGARCH model has been successful of capturing this asymmetry. On the other hand AGARCH model is not proved as expected with a positive asymmetric value and lower likelihood value than EGARCH model. TGARCH model presents some surprising results. The coefficient **threshold** is lower and close to zero suggests that negative shocks have more impact on volatility than the positive ones. Table 11 presents some diagnostics about all 4 models. We see again that all models produce negatively skewed residuals and positive excess kurtosis. EGARCH model seems to be superior to the other models in terms of log likelihood value.

4. Conclusion

In this study, different variations of volatility models have been analyzed. Their ability to capture volatility clustering, responding negative and positive shocks of the market and delivering adequate future forecasts of volatility has been tested. We have been tested and compared these models by using monthly returns of the FTSE All Share Index. Generally, GARCH models have been tested successful on modelling volatility clustering. But, frequency of data used for testing the models is a vital problem at this stage. Volatility estimated from daily data could be more precise than GARCH volatility estimated from monthly data because of the higher frequency of daily data.

Another question for this paper is that if GARCH forecasts are not fully capture the whole aspects of volatility forecasting, which alternative methods can be used? An extended study on simpler ARMA models or implied volatility embedded on option prices would help us to predict future volatility better.

References

Bollerslev, T. (1986) "Generalized Autoregressive Conditional Heteroskedassticity" *Journal of Econometrics,* 31, 307-327

Bollerslev, T., Chou, R. S., and Kroner, K. F. (1992). "ARCH modelling in finance - A review of the theory and empirical evidence" *Journal of Econometrics*, 52, 5-59.

Engle, R.F. (1982) "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation" *Econometrica,* 50, 987-1007

Engle, R.F., and Ng, V.K. (1993) "Measuring and Testing the Impact of News on Volatility" *Journal of Financ**e***, 48, 1749-1778

Goyal, A. (2000) " Predictability of Stock Return Volatility from GARCH Models" *Anderson Graduate School of Management, UCLA* Working Paper

Lee, B.J. (1992) "A Heteroskedasticity Test Robust to Conditional Mean Misspecification" *Econometrica,* Vol.60, No.1, 159-171

Nelson, D.B. (1991) "Conditional Heteroskedasticity in Asset Returns: A New Approach" *Econometrica,* 59,347-370

Liew, Venus Khim−Sen and Terence Tai−leung Chong, (2005) "Autoregressive Lag Length Selection Criteria in the Presence of ARCH Errors." *Economics Bulletin,* Vol. 3, No. 19 pp. 1−5

Wilson, J.W., and Jones, C.P. (1990) "Is There a January Effect in Corporate Bond and Paper Returns?" *The Financial Review,* Vol.25, No

Appendix: Tables And Graphs

Graph 1: Stock Returns

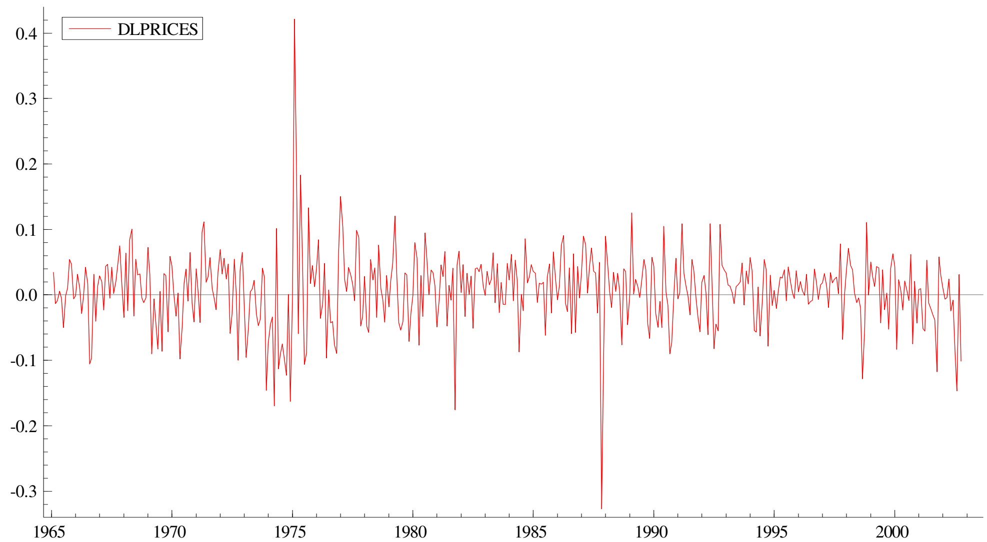

Table 1: Normality tests and descriptive statistics

Normality test for DLPRICES	
Observations	453
Mean	0.0064842
Std.Devn.	0.057702
Skewness	0.15934
Excess Kurtosis	8.0985
Minimum	-0.32711
Maximum	0.42133
Asymptotic test: Chi^2(2) =	1239.8 [0.0000]**
Normality test: Chi^2(2) =	377.99 [0.0000]**

Table 2: January Effect

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
Constant	0.00516693	0.002827	1.83	0.068	0.0074
Dummy_Var	0.0161274	0.009892	1.63	0.104	0.0059

sigma	0.0576601	RSS	1.49943324
R^2	000585928	F(1,451) =	2.658 [0.104]
log-likelihood	650.718	DW	1.78
no. of observations	453	no. of parameters	2
mean(DLPRICES)	0.00648418	var(DLPRICES)	0.00332952

Table 3: Specification for Conditional Mean

Progress	to date						
Model	T	p		log-likelihood	SC	HQ	AIC
EQ(1)	443	11	OLS	642.05641	-2.7474	-2.8089	-2.849
EQ(2)	444	10	OLS	643.79725	-2.7627	-2.8186	-2.8549
EQ(3)	445	9	OLS	643.4544	-2.7686	-2.8188	-2.8515
EQ(4)	446	8	OLS	645.38159	-2.7847	-2.8292	-2.8582
EQ(5)	447	7	OLS	647.22941	-2.8003	-2.8392	-2.8646
EQ(6)	448	6	OLS	648.68402	-2.8142	-2.8475	-2.8691
EQ(7)	449	5	OLS	648.79561	-2.822	-2.8497	-2.8677
EQ(8)	**450**	**4**	**OLS**	**650.67227**	**-2.8376**	**-2.8597**	**-2.8741**
EQ(9)	451	3	OLS	650.67789	-2.8448	-2.8614	-2.8722
EQ(10)	452	2	OLS	650.69118	-2.8521	-2.8631	-2.8703

Table 4: OLS regression at lag 3

	Coefficient	Std.Error	t-value	t-prob Part.R^2
DLPRICES_1	0.138186	0.0474	2.92	0.004 0.0187
DLPRICES_2	-0.102951	0.04759	-2.16	0.031 0.0104
DLPRICES_3	0.0934248	0.04779	1.95	0.051 0.0085
Constant	0.00559993	0.00275	2.04	0.042 0.0092
sigma	0.0572451		RSS	1.46154451
R^2	0.0300758		F(3,446) =	4.61 [0.003]**
log-likelihood	650.672		**DW**	**1.99**

Table 5: Error Autocorrelation Test

```
Error autocorrelation coefficients in auxiliary regression:
  Lag Coefficient  Std.Error
    1      0.6011     0.7062
    2      1.0002     0.6828
    3     0.12721     0.4922
RSS = 1.45044  sigma = 0.00327413

Testing for error autocorrelation from lags 1 to 3
 Chi^2(3) =   3.4197 [0.3313]   and F-form F(3,443) =   1.1308 [0.3362]
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Table 6: ARCH Effects

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ARCH coefficients:
  Lag Coefficient  Std.Error
    1   0.062731     0.0476
    2   0.11586      0.04739
    3   0.0709       0.04772
RSS = 0.0456707  sigma = 0.0101881

Testing for error ARCH from lags 1 to 3
ARCH 1-3 test:    F(3,440) =   3.9032 [0.0090]**
```

Table 7: GARCH Results

		Coefficient	Std.Error	robust-SE	t-value	t-prob
DLPRICES_1	Y	0.0541023	0.05362	0.05356	1.01	0.313
DLPRICES_2	Y	-0.118102	0.05319	0.06564	-1.8	0.073
DLPRICES_3	Y	0.0104494	0.05284	0.05545	0.188	0.851
Constant	X	0.00730119	0.002509	0.003312	2.2	0.028
alpha_0	H	0.000243602	0.0001065	9.61E-05	2.54	0.012
alpha_1	H	0.107739	0.0369	0.04616	2.33	0.02
beta_1	H	0.823097	0.05434	0.04314	19.1	0

log-likelihood	674.7844	**HMSE**	6.98908
mean(h_t)	0.003353	**var(h_t)**	6.82E-06
no of observations	450	**no. of parameters**	7
AIC.T	-1335.56	**AIC**	-2.9679308
mean(DLPRICES)	0.006501	**var(DLPRICES)**	0.00334859
alpha(1)+beta(1)	0.930836	**alpha_i+beta_i>=0,**	alpha(1)+beta(1)<1

Table 8: EGARCH Results

		Coefficient	Std.Error	robust-SE	t-value	t-prob
DLPRICES_1	Y	0.0659065	0.01898	0.00708	9.31	0
DLPRICES_2	Y	-0.104656	0.01524	0.005892	-17.8	0
DLPRICES_3	Y	0.0314604	0.01276	0.003882	8.1	0
Constant	X	0.00711716	0.001703	0.001143	6.23	0
alpha_0	H	-0.291188	0.1377	0.1824	-1.6	0.111
eps[-1]	H	-0.0355325	0.02667	0.05558	-0.639	0.523
|eps[-1]|	H	0.192435	0.05204	0.06447	2.98	0.003
beta_1	H	0.948291	0.02357	0.02956	32.1	0

log-likelihood	680.891084	**HMSE**	6.64097
mean(h_t)	0.0031957	**var(h_t)**	3.54E-06
observations	450	**no. of parameters**	8
AIC.T	-1345.78217	**AIC**	-2.99062704
mean(DLPRICES)	0.00650123	**var(DLPRICES)**	0.00334859

Table 9: AGARCH Results

		Coefficient	Std.Error	robust-SE	t-value	t-prob
DLPRICES_1	Y	0.0575385	0.05562	0.05227	1.1	0.272
DLPRICES_2	Y	-0.107753	0.05176	0.06595	-1.63	0.103
DLPRICES_3	Y	0.0362125	0.05349	0.06248	0.58	0.563
Constant	X	0.00625743	0.002698	0.00293	2.14	0.033
alpha_0	H	0.000613839	0.0002278	0.0003481	1.76	0.078
alpha_1	H	0.124965	0.05881	0.08268	1.51	0.131
beta_1	H	0.644969	0.1152	0.1112	5.8	0
asymmetry	H	0.0319304	0.02391	0.04656	0.686	0.493

log-likelihood	676.232528	**HMSE**	8.85607
mean(h_t)	0.00321014	**var(h_t)**	4.23E-06
observations	450	**no. of parameters**	8
AIC.T	-1336.46506	**AIC**	-2.96992235
mean(DLPRICES)	0.00650123	**var(DLPRICES)**	0.00334859
alpha(1)+beta(1)	0.769934	**alpha_i+beta_i>=0,**	alpha(1)+beta(1)<1

Table 10: TGARCH Results

		Coefficient	Std.Error	robust-SE	t-value	t-prob
DLPRICES_1	Y	0.0685472	0.05653	0.05321	1.29	0.198
DLPRICES_2	Y	-0.106473	0.05224	0.05982	-1.78	0.076
DLPRICES_3	Y	0.0444284	0.0556	0.05658	0.785	0.433
Constant	X	0.00651058	0.00265	0.002799	2.33	0.02
alpha_0	H	0.00066836	0.0003157	0.000442	1.51	0.131
alpha_1	H	0.0524821	0.04595	0.04955	1.06	0.29
beta_1	H	0.64541	0.1514	0.1875	3.44	0.001
threshold	H	0.183297	0.1371	0.1664	1.1	0.271

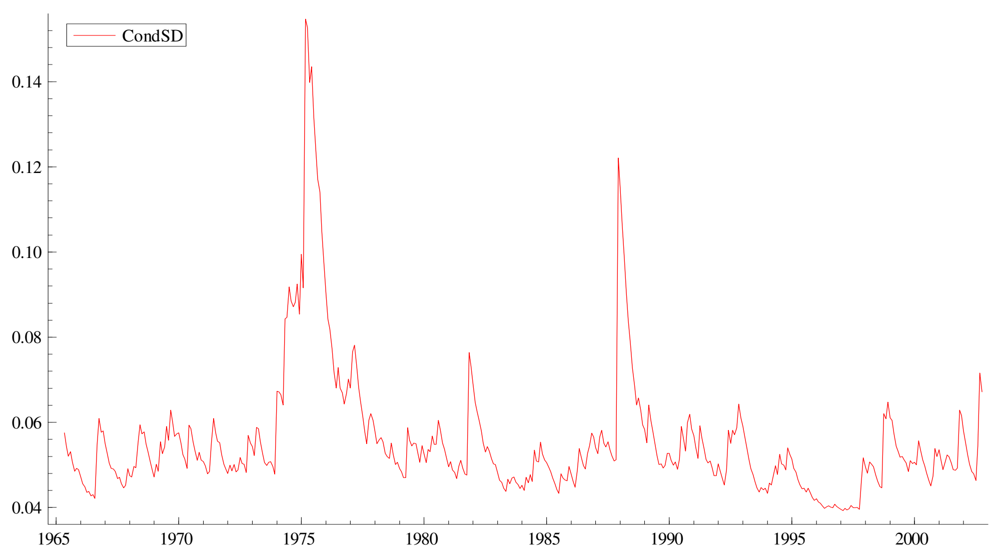
log-likelihood	676.804515	**HMSE**	8.0371
mean(h_t)	0.00324166	**var(h_t)**	5.35E-06
no of observations	450	**no. of parameters**	8
AIC.T	-1337.60903	**AIC**	-2.97246451
mean(DLPRICES)	0.00650123	**var(DLPRICES)**	0.00334859
alpha(1)+beta(1)	0.697892	**alpha_i+beta_i>=0,**	alpha(1)+beta(1)<1

Table 11: Diagnostic Tests

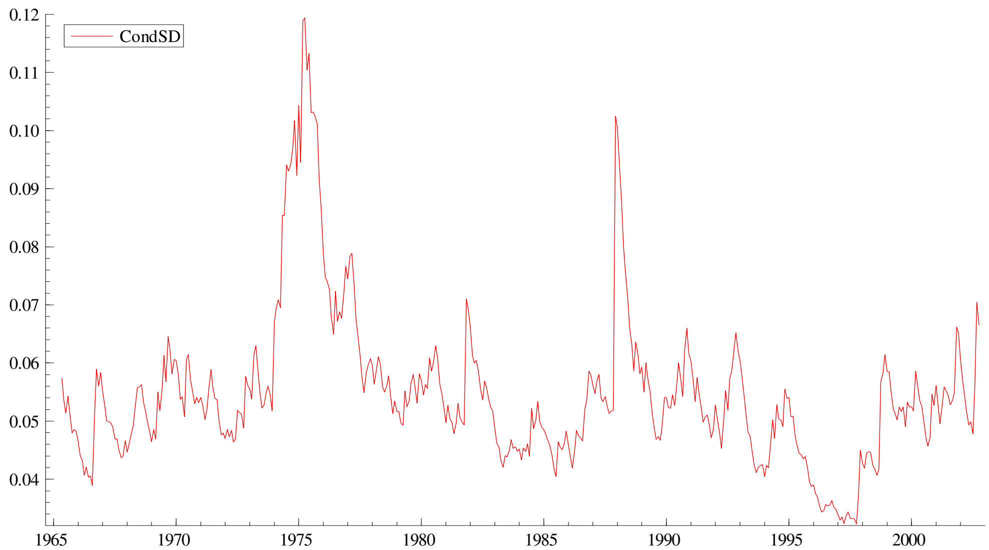
GARCH			
Asymptotic test:	Chi^2(2) =	539.8	[0.0000]**
Normality test:	Chi^2(2) =	106.4	[0.0000]**
TGARCH			
Asymptotic test:	Chi^2(2) =	766.45	[0.0000]**
Normality test:	Chi^2(2) =	131.36	[0.0000]**
EGARCH			
Asymptotic test:	Chi^2(2) =	471.93	[0.0000]**
Normality test:	Chi^2(2) =	99.769	[0.0000]**
AGARCH			
Asymptotic test:	Chi^2(2) =	975.26	[0.0000]**
Normality test:	Chi^2(2) =	161.65	[0.0000]**

	GARCH	**TGARCH**	**EGARCH**	**AGARCH**
Mean	-0.014646	-0.0047027	-0.013801	0.0014857
Std.Devn.	0.99612	0.99823	0.99587	0.99831
Skewness	-0.91491	-0.99607	-0.87408	-1.012
Excess Kurtosis	5.0439	6.0752	4.7025	6.9223
Minimum	-6.6554	-7.117	-6.5853	-7.304
Maximum	4.3095	4.2083	4.1995	4.6279

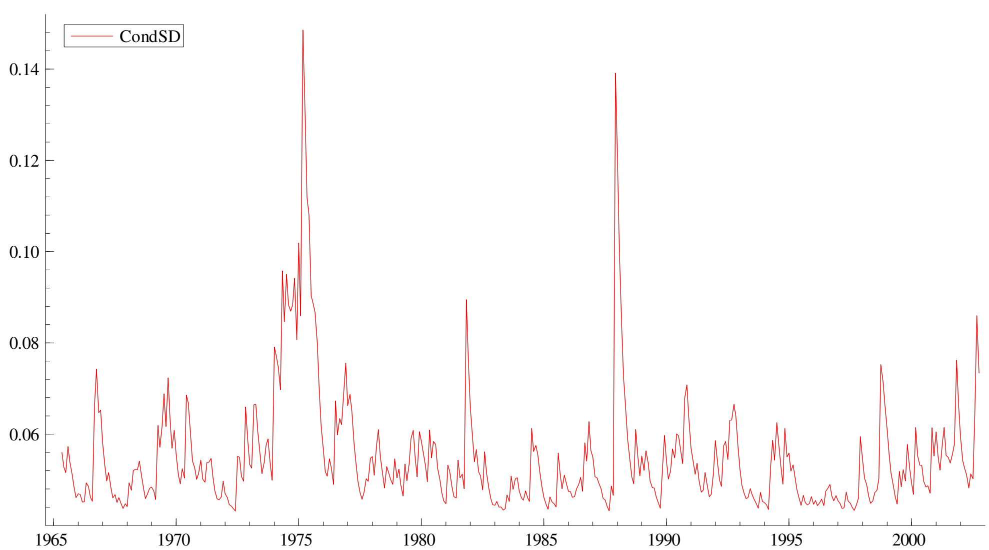
Graph 2: GARCH Conditional Standard Deviation



Graph 3: EGARCH Conditional Standard Deviation



Graph 4: AGARCH Conditional Standard Deviation



Graph 5: TGARCH Conditional Standard Deviation