A Public Firm's R&D Policy and Trade Liberalization

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Abstract

This paper studies a public firm's incentive to raise its productive efficiency by undertaking cost-reducing R&D investment when it competes against a foreign private firm. Our focus is to ravel out how a decrease in an importing tariff levied on foreign goods affects this investment level inter alia. We show that when the government imposes non-negative tariffs, a tariff reduction lowers the R&D investment, irrespective of whether the public firm has downward or upward sloping reaction curve. Namely, R&D investment conducted by the public firm is substitutable to an importing tariff. Furthermore, under a linear demand assumption, it is concluded that a tariff reduction necessarily enhances world welfare if both R&D investment and tariffs are set to domestic welfare-maximizing levels. More strict assumptions on marginal cost and R&D cost function make complete trade liberalization desirable from the viewpoint of world welfare.

JEL Classification: L13; L32; F13

Keywords: Mixed Oligopoly; R&D; Trade Liberalization

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1 Introduction

This paper investigates the relationship between a public firm's R&D policy and an importing tariff. In particular, we clarify how a tariff reduction influences the cost-reducing R&D investment which the public firm determines to maximize its objective. More precisely, this paper endeavors to reveal whether the current worldwide movement toward trade liberalization enhances the productive efficiency of public enterprises. Fruthermore, this paper also discusses an influence of such trade liberalization movement on world welfare.

The productive inefficiency of public enterprises has been one of controversial issues. Empirical studies reveal equivocal results (see Stiglitz 1988, and Bös 1991), but many recent works substantiate that private firms produce at lower costs (see Megginson and Netter 2001, and many papers cited by them). At the same time, there are some theoretical studies investigating whether public enterprises could produce less efficiently than private rivals in the associated markets. Nishimori and Ogawa (2002) compare the levels of cost-reducing R&D investment conducted by a public firm in public monopoly and mixed oligopoly with both public and private firms.[1] From this comparison, they show that the level of R&D investment is lower in mixed oligopoly. Tomaru (2007) shows that this result hold even though the public firm competes against foreign private firms. Moreover, Matsumura and Matsushima (2004), using a Hotelling model with public and private firms' choices on price, location and R&D investment, prove that the marginal cost of the public firm is higher than that of the private firm in equilibrium.[2]

All these theoretical works indicate public firms' disincentives to reduce its production costs. Given these disincentives, this paper goes further over these works. The shortcoming of them

[1]Assuming that public enterprises are welfare-maximizers and compete against private firms, De Fraja and Delbono (1989), a harbinger in mixed oligopoly theory, show that privatization of the public firms leads to an improvement if the relevant market is competitive. Following their research, recently, many studies on mixed oligopoly have been proliferated. For recent studies, see Ishida and Matsushima (2009), Long and Stähler (2009), Kamijo and Nakamura (2009), and Ino and Matsumura (2010).

[2]The mixed oligopoly literature contains many studies on cost-reducing R&D. Cato (2008a) proves Nishimori and Ogawa's (2002) result in a more general model, Cato (2008b) takes spillover into account, Poyago-Theotoky (1998) investigates the role of a public firm in the free-rider problem when imitation is easy, Ishibashi and Matsumura (2006) consider a patent race model where each firm chooses not only R&D expenditure but also its innovation size, and etc.

lies in an ignorance of correlations between public firm's R&D policy and other governmental intervention. The exception is Poyago-Theotoky, Gil-Molto, and Zikos (2010). They investigate incentives of both public and private firms to undertake R&D investment when these firms receive subsidies per unit of R&D outputs. Namely, their focus is on the relationship between domestic industrial policy and firms' R&D. The focus of this paper differs from their research, and it is on that between trade policy and public firm's R&D. In particular, we concentrate on the effect of trade liberalization on this R&D activity, or on public firm's productive efficiency.

As frequently observed in many countries or regions such as EU and ASEAN countries, trade liberalization usually goes with capital liberalization and deregulation. This influences market outcomes in various industries, even in mixed oligopolistic industries. For instance, the Swedish market have been liberalized and linked up with other countries such as Finland and Norway. As a result of liberalization and internationalization, Vattenfall, which is wholly owned by the Swedish government, is now facing fierce competition with various foreign firms such as EDF IVO, and Statkraft. As for such competition through trade liberalization and deregulation, it might be widely thought a tariff reduction or trade liberalization contributes to an improvement in public firm's productivity, since it accelerates competition between public and foreign firms. Is this really true? This paper is devoted to answer this question.

Then, we start with an international mixed duopoly model with one inefficient public firm undertaking the cost-reducing R&D investment and one efficient foreign private firm not undertaking such investment. In this model, the domestic government levies an importing tariff on foreign goods. Using this setting, we show that a tariff reduction leads to an increase in public firm's investment. In other words, trade liberalization enhances the productive efficiency of public firm. The intuition behind this result is intelligible. A tariff reduction leads to the terms of trade for the domestic country and a decrease in the exploited rents from the private firm. To make up for these welfare losses, the public firm attempts to offset a deterioration in the terms of trade by expanding its output through R&D investment. More surprisingly, a rise in R&D investment level via a tariff reduction does not depend on whether the public firm has upward or downward sloping reaction curve under the output-setting stage.

Furthermore, we explore the optimal tariff determined by the domestic government. We obtain three new results. First, the domestic government has an incentive to impose a positive tariff. This does not depend on the level of R&D investment. Second, if both R&D and trade policies are utilized, the public firm always produces more than the private firm. Seemingly, the relationship should be reversed, since production cost can be saved by replacing public firm's production with private firm's one through keeping both an importing tariff rate and R&D investment level lower. However, our result indicates that it is not the case. The government adjusts both policy instruments to bolster the aggressive public firm's behavior. Finally, with linear demand, a tariff reduction under the optimization with respect to both policies enhances the world welfare. Furthermore, with more strong restriction (i.e.,linear demand, linearly decreasing marginal cost, and quadratic R&D cost), the whole world can enjoy higher benefits from complete trade liberalization, that is, no tariffs.

The remainder of this paper is organized as follows. Section 2 presents our basic model and elaborates on an influence of a tariff reduction on public firm's R&D investment level. In section 3, we discuss the optimal tariff and both public and private firms' behaviors under this tariff. On the basis of the results from sections 2 and 3, section 4 is devoted to reveal whether trade liberalization contributes to an improvement in world welfare. Finally, section 5 concludes this paper.

2 The model

Consider one country wherein one public firm is established and competes against one foreign private firm. The demand in this country is given by $P = P(Q)$, where P is the price, $Q = q_0 + q_1$ is total outputs, q_0 is the output of the public firm, and q_1 is the output of the foreign private firm. As usual in the existing works on mixed oligopoly such as De Fraja and Delbono (1989), we assume that the public firm maximizes domestic welfare, whereas the foreign private firm maximizes its profits.

Each firm is assumed to have a constant marginal cost technology. As stated in the introduction, our main purpose is to investigate public firm's inefficiency and influences of tariff reduction on this inefficiency. To this end, in this paper, the public firm is assumed to conduct cost reducing R&D, and its marginal cost is represented as $C(x_0)$ where x_0 is the level of R&D investment. In addition, the public firm incurs investment costs $f(x_0)$. On the other hand, we do not assume that the foreign private firm undertakes cost-reducing R&D.[3] Then, we denote the marginal cost of the foreign private firm by c.

The profits of public and private firms are given by

$$\Pi_0(q_0, q_1, x_0) := [P(Q) - C(x_0)]\, q_0 - f(x_0),$$
$$\Pi_1(q_0, q_1, t) := [P(Q) - c - t]\, q_1,$$

respectively. t is a tariff rate levied by the domestic government. The domestic welfare is given by

$$W(q_0, q_1, x_0, t) := \int_0^Q P(z)dz - P(Q)q_1 - C(x_0)q_0 - f(x_0) + tq_1.$$

For our subsequent analysis, we assume that:

Assumption 1. *$P(Q)$ is twice continuously differentiable with $P'(Q) < 0$ for all $Q \geq 0$ such that*

[3]The reason for this assumption is germane to private firm's activeness in the market. As seen in the succeeding analysis, the welfare-maximizing public firm acts more aggressively than private firms, regardless of rival firms' nationality. Such a public firm's aggressive behavior would repel private firms from the market unless private firms are more efficient than the public firm. Especially, when $t = 0$ and firms have constant marginal costs, cost-superiority or parity of the public firm to private firms makes private firms inactive. For detailed discussion, see Matsumura and Kanda (2005) and Tomaru and Kiyono (2010). In addition, under the assumption, our model can be interpreted as an analysis on the world economy where the foreign firm technologically mature whereas the domestic public firm endeavors to catch up. For this justification, see Konishi (1999).

$P(Q) > 0$. *Furthermore, it satisfies*

$$\varepsilon(Q) \in (-1, 1) \quad \text{and} \quad \varepsilon'(Q) = 0, \quad \text{where} \quad \varepsilon(Q) := \frac{QP''(Q)}{P'(Q)}.$$

Assumption 2. *$C(x_0)$ is twice continuously differentiable and satisfies $C(x_0) > 0$, $C'(x_0) < 0$, and $C''(x_0) > 0$ for all $x_0 \geq 0$.*

Assumption 3. *$f(x_0)$ is twice continuously differentiable and satisfies $f(x_0) \geq 0$ for all $x_0 \geq 0$ with $f(x_0) = 0$ if and only if $x_0 = 0$. Furthermore, it satisfies $f'(x_0) > 0$ and $f''(x_0) \geq 0$ for all $x_0 > 0$.*

For tractability, we assume that $c = 0$.[4] Coupled with this assumption and Assumption 2, the public firm cannot produce at the lower cost than the private firm, even though it engages in R&D to reduce its marginal cost. Consequently, the private firm always has an incentive to serve the good to the domestic market, unless t is prohibitive. Assumption 3 indicates that R&D investment is subject to non-increasing return to scale.[5] On the contrary to Assumptions 2 and 3, it might seem that Assumption 1 is somewhat strong assumption. As described in the introduction, the aim of this paper is to investigate how the optimal R&D investment level of the public firm is affected by a tariff reduction, and on the basis of this investigation, to unlock whether trade liberalization improves world welfare. This exploration involves the intricate multi-stage game and moreover, calls for a close examination and perplexing calculation. Inevitably, therefore, the results are recondite and equivocal in highly general models. To avoid such ambiguity of results and to obtain the definite policy implications as well, we impose Assumption 1.[6]

[4]If $\lim_{x_0 \to \infty} C_0(x_0) > c$ is added into Assumption 2, a lack of assumption $c = 0$ leaves the results, which will be obtained subsequently, intact.

[5]As observed later, Assumption 3 plays an important role in ensuring the second-order condition in the R&D setting stage. d'Aspremont and Jacquemin (1988), Poyago-Theotoky (1996), Petit and Sanna-Randaccio (2000), among others assume the strong convexity of f to ensure the second-order conditions of R&D-conducting firms. On the other hand, Suzumura (1992), Neary and Ulph (1997), Konishi (1999), among others assume that f is constant, but take function C satisfying those conditions. Our assumptions include both of them.

[6]Okuno-Fujiwara and Suzumura (1993) shall vindicate this assumption. Similar to our analysis, they utilize it to accomplish an ambitious objective; an investigation of whether the number of private firms engaging in both R&D and quantity competition is excessive from the viewpoint of social welfare.

We consider the following two-stage game. In the first stage, the domestic public firm determines its cost-reducing R&D investment level x_0. Observing this investment level, both domestic public and foreign private firms simultaneously select their outputs q_i $(i = 0, 1)$ in the second stage. As usual, we apply backward induction in solving this game.

Now let us proceed to an analysis on the second stage. The foreign firm selects its output to maximize its profits, whereas the public firm selects its output to maximize domestic welfare, which results in the following first-order conditions:

$$\frac{\partial W}{\partial q_0} = P(Q) - P'(Q)q_1 - C(x_0) = 0, \tag{1}$$
$$\frac{\partial \Pi_1}{\partial q_1} = P(Q) + P'(Q)q_1 - t = 0.$$

As observed straightforwardly, the second-order conditions are satisfied under Assumption 1. It is well-known, in the mixed oligopoly literature, that a public enterprise competing against only domestic private firms sets its output such that the price is equal to its marginal cost. However, the first-order condition Eq.(1) states that its marginal cost exceed the price if the public firm faces competition with only foreign private firm.[7] Namely, competition with foreign firms makes the domestic public firm more aggressive. This is attributable to an improvement in the terms of trade though the domestic public firms' aggressive behaviors.

From the first-order conditions, the reaction function of each firms is derived; $R_0(q_1, x_0)$ and $R_1(q_0, t)$. Under Assumption 1, the private firm's reaction curve is downward sloping and its slope is less than unity in absolute value. Thus, its strategy is strategic substitute. On the other hand, unfortunately, this assumption does not warrant the strategic substitutability in the case of the

[7]The mixed oligopoly literature includes a wide variety of studies on foreign competition. Fjell and Pal (1996) show that a public firm increases its output as the number of foreign private firms becomes large. Pal and White (1998) investigate a relationship between privatization and strategic trade policy. For other works, see Matsushima and Matsumura (2006), Mukherjee and Suetrong (2009), and Matsumura, Matsushima and Ishibashi (2009).

public firm. Indeed,

$$\frac{\partial R_0}{\partial q_1} = \frac{P''(Q)q_1}{P'(Q) - P''(Q)q_1} = \frac{\varepsilon\theta}{1-\varepsilon\theta}, \quad \text{where} \quad \theta := \frac{q_1}{Q}.$$

This implies that the public firm's strategy is substitute (resp. complement) if and only $\varepsilon < 0$ (resp. > 0). Nonetheless, $|\partial R_0/\partial q_1|$ falls down in $(0, 1)$. Hence, irrespective of whether its strategy is substitute or complement, the second-stage equilibrium is unique and stable if this equilibrium exists. Then, we assume that interior solutions always exist in the second stage as long as t and/or x_0 are not extremely large. Additionally, we preclude such t and x_0 from the target from our investigation. Henceforth, we, without any notice, refer to *all* x_0 and/or *all* t as all x_0 and t except for those which deter the interior solution.

Let us denote $q_i^*(x_0, t)$ be the second-stage Cournot Nash equilibrium ($i = 0, 1$). For succeeding analysis, we ascertain how these Cournot equilibrium outputs are affected by x_0. Straightforward computation asserts the following:

Lemma 1. *Define $Q^*(x_0, t) := q_0^*(x_0, t) + q_1^*(x_0, t)$ and $\theta^*(x_0, t) := q_1^*(x_0, t)/Q^*(x_0, t)$. For each x_0 and t,*

$$\frac{\partial q_0^*}{\partial x_0} = \frac{[2+\varepsilon\theta^*(x_0,t)]\,C'(x_0)}{2P'(Q^*(x_0,t))} > 0, \quad \frac{\partial q_1^*}{\partial x_0} = -\frac{[1+\varepsilon\theta^*(x_0,t)]\,C'(x_0)}{2P'(Q^*(x_0,t))} < 0,$$
$$\frac{\partial q_0^*}{\partial t} = \frac{\varepsilon\theta^*(x_0,t)}{2P'(Q^*(x_0,t))}, \quad \frac{\partial q_1^*}{\partial t} = \frac{1-\varepsilon\theta^*(x_0,t)}{2P'(Q^*(x_0,t))} < 0.$$

Proof: See Appendix.

An increase in x_0 discourages the private firm's production and expedites the public firm's production. On the other hand, a rise in t contracts the output of the private firm, while the effect of tariffs on that of the public firm depends on the sign of ε, in other words, on the public firm's best response to the private firm's strategy. When the public firm's strategy is substitute (resp. complement), a rise in t decreases (resp. increases) q_0^*. The behaviors of $\theta^*(x_0, t)$ in response to x_0 and t are also double-edged. All we can say is that an increase in x_0 always lowers θ^* if $\varepsilon \geq 0$, whereas a rise in t always decreases θ^* if $\varepsilon \leq 0$.

Now we go on to the first stage. The payoff function which the government confronts in this stage is given by

$$W^*(x_0, t) := W(q_0^*(x_0, t), q_1^*(x_0, t), x_0, t).$$

We assume that the second-order condition is satisfied. The optimal R&D investment level $x_0^*(t)$ satisfies

$$\begin{aligned} 0 = \frac{\partial W^*(x_0^*(t), t)}{\partial x_0} &= \frac{\partial W}{\partial q_0} \cdot \frac{\partial q_0^*}{\partial x_0} + \frac{\partial W}{\partial x_0}, \\ &= \left[t - P'(Q^*(x_0^*(t), t)) q_1^*(x_0^*(t), t)\right] \cdot \frac{\partial q_1^*}{\partial x_0} - \left[C'(x_0^*(t)) q_0^*(x_0^*(t), t) + f'(x_0^*(t))\right]. \quad (2) \end{aligned}$$

The effect of an increase in x_0 on domestic welfare is decomposed into the following two. One is *strategic effect* which is the first term in the right-hand side. This effect emanates from a change in q_1 via strategic substitution. Especially, this is definitely negative when $t \geq 0$. The other is *cost-reducing effect* which refers to the second term. This is a direct effect of investment. It is ramified into two effects; a decrease in marginal cost of the public firm and an increase in R&D cost. The sign of this effect is double-edged. The public firm attempts to balance the strategic effect and the cost-reducing effect in choosing x_0.

Here, as a clue to understand what level of investment the public firm selects, we compare $x_0^*(t)$ with the cost-minimization levels of investment. For this purpose, we take up two types of cost-minimizing R&D investment level. One is that in public monopoly. Let x_0^m denote this investment level. The other is that in international mixed duopoly which is denoted by $x_0^d(t)$. From Assumptions 2 and 3, x_0^m evidently meets $C'(x_0^m)q_0^m + f'(x_0^m) = 0$ where q^m is a welfare-maximizing output in public monopoly. As for $x_0^d(t)$, assuming that $\partial^2 F(x_0, t)/\partial x_0^2 > 0$ for all $x_0 \geq 0$, where $F(x_0, t) := C(x_0)q_0^*(x_0, t) + f(x_0)$, we find that $\partial F(x_0^d(t), t)/\partial x_0 = 0$ holds. Comparisons among $x^*(t)$, x^m and $x_0^d(t)$ derive the following result.

Lemma 2. *For all $t \geq 0$, (i) $x_0^d(t) < x_0^*(t)$ and (ii) $x_0^*(t) < x_0^m$ if either $\varepsilon \leq 0$ or*

$$\frac{q_0^m - q_0^*(x_0^m, t)}{q_1^*(x_0^m, t)} - \frac{1 + \varepsilon\theta^*(x_0^m, t)}{2} < 0. \tag{3}$$

Proof: See Appendix.

Lemma 2 states that $x_0^d(t) < x_0^*(t) < x_0^m$ if the public firm has an downward sloping reaction curve, or if the line, which goes through both public monopoly and international mixed duopoly equilibria in (q_0, q_1) space, is not steep. In other words, when the public firm's reaction to private firm's strategy is not strong complement, the optimal investment level is excessive compared to the cost-minimization level in international mixed duopoly, but it falls short of that in public monopoly.

To expatiate this result, let us start from the situation wherein the public firm chooses $x_0^d(t)$ instead of $x_0^*(t)$. When the domestic government levies a positive tariff, the strategic effect is negative whereas the cost-reducing effect is positive. As explained, the aggressive behavior of the public firm improves the terms of trade and thus enhances domestic welfare. Since this terms-of-trade improvement effect is strong, the cost-reducing effect dominates the strategic effect. Therefore, it is concluded that $x_0^d(t) < x_0^*(t)$. Next, we explain the intuition behind Lemma 2 (ii). Under the assumption of either $\varepsilon \geq 0$ or Eq.(3), with $x_0 = x_0^*(t)$ remaining, a move from mixed duopoly to public monopoly equilibrium leads to an increase or a small decrease in q_0. In this regard, the public firm can enhance domestic welfare by producing more efficiently and by increasing its output. This is because the public firm is a sole producer and cannot have the private firm incur production cost by replacing its production with private one. Hence, R&D investment level is raised to x_0^m.

One of our main aims is to explore the relationship between the optimal R&D investment $x^*(t)$ and t. As for this issue, highly complicated calculation proves the following:

Proposition 1. *For each $\varepsilon \in [-\frac{1}{3}, 1)$ and $t \geq 0$, $x_0^{*\prime}(t) < 0$.*

Proof: See Appendix.

Given that the public firm selects x_0 to maximize domestic welfare under a certain tariff rate $t \geq 0$, a gradual trade liberalization is prone to boost its R&D investment level. In other words, Proposition 1 indicates that the current worldwide movement toward trade liberalization enhances the productive efficiency of the domestic public firm. It also demonstrates that the public firm's R&D and an importing tariff are substitutable policies for the domestic government. Some works on mixed oligopoly have explored the substitutability between domestic and trade policies. Chao and Yu (2006) show that partial privatization of raises the optimal tariff. Matsumura and Tomaru (2010) show that the optimal subsidy rate provided to both public and private firms decreases with a progress in gradual capital liberalization if the relevant market is not so competitive. Most studies on international mixed oligopoly, including these two studies, assume that the public firm's reaction function is either upward sloping or horizontal (or, $P''(Q) \leq 0$). This assumption corresponds to $\varepsilon \geq 0$ in our model. Surprisingly, Proposition 1 states that it does not matter whether the public firm's response is strategic substitute to, complement to, or independent from the private firm's strategy. Namely, the strategic attitude of the public firm is not a clincher of substitutability between domestic and trade policies.

The intuition behind Proposition 1 is as follows. A tariff reduction worsens the terms of trade for the domestic country. Furthermore, this reduction decreases the efficient foreign firm's rent which the domestic government can exploit. In order to compensate such a domestic welfare loss, the public firm attempts to offset a deterioration in the terms of trade by raising the R&D investment level. Accordingly, we can assert that $x^{*\prime}(t) < 0$. However, this is not true when ε is negative and $|\varepsilon|$ is large. In this case, the public firm has a steeply downward-sloping reaction curve. A tariff reduction replaces the public firm's production with the private firm's. In addition, when the public firm refrains from its investment, the private firm's output increases. All these increases in the private firm's production saves the domestic production costs. Therefore, if a welfare improvement due to this cost saving dominate that due to an improvement in the terms of trade, a tariff reduction decreases it, far from raising R&D investment level.

The optimal tariff

In the previous section, we have discussed the relationship between R&D conducted by a public firm and an importing tariff. Although the conclusion of substitutability between them is important and suggestive, we should not ignore the fact that the derivation of the conclusion is based on a non-negative tariff assumption. In this section, we derive the optimal tariff to examine the plausibility of this assumption. Previously we differentiated W^* with respect to t. Instead, let us differentiate W^* with respect to t, which result in

$$0 = \frac{\partial W^*}{\partial t} = [t - P'(Q^*)q_1^*]\,\frac{\partial q_1^*}{\partial t} + q_1^*. \tag{4}$$

As observed straightforwardly, the effects of t on domestic welfare is twofold, as in the case of optimization for x_0. One is strategic effect and the other is *tariff revenue effect*. This is a direct effect of a tariff change on tariff revenue. Similar to the effects of x_0, strategic effect goes either way of positive or negative. Of course, it is negative when $t \geq 0$. On the other hand, the tariff revenue effect is definitely positive. The government selects an importing tariff (or an importing subsidy) so as to offset these two effects. Let us define the optimal tariff, which satisfies Eq.(4), by $t^*(x_0)$. Then, we can establish the following.

Proposition 2. *For all* $\varepsilon \in (-1, 1)$ *and* $x_0 \geq 0$*,* $t^*(x_0) > 0$*.*

Proof: See Appendix.

Proposition 2 states that the domestic government levies a positive tariff on foreign goods. Instead, suppose that the domestic government does not impose any tariffs. In this case, tariff revenue effect dominates strategic effect. In fact,

$$\begin{aligned}\left.\frac{\partial W^*}{\partial t}\right|_{t=0} &= -P'(Q^*(x_0,t))q_1^*(x_0,t)\cdot\left.\frac{\partial q_1^*}{\partial t}\right|_{t=0} + q_1^*(x_0,0),\\ &= \frac{[1+\varepsilon\theta^*(x_0,0)]\,q_1^*(x_0,0)}{2},\\ &> 0.\end{aligned}$$

This implies that the government levies a positive tariff in order to garner tariff revenues, expecting this positive welfare impact to exceed the negative impact through strategic effect. Note that the tariff revenue itself is increasing in t around $t = t^*(x_0)$. Defining $T(x_0,t) := tq_1^*(x_0,t)$, welfare decomposition Eq.(4) can be rearranged as follows:

$$0 = \frac{\partial W^*}{\partial t} = -P'(Q^*)q_1^*\cdot\frac{\partial q_1^*}{\partial t} + \frac{\partial T}{\partial t}.$$

Since the first term in the right-hand side is negative, the second term is positive. Therefore, a decrease in t lessens tariff revenue at $t = t^*(x_0)$.

To close this section, we consider optimization for both R&D and tariff. Let us denote the R&D investment and tariff, which satisfy both Eqs.(2) and (4), by x^e and t^e, respectively. Using these, we can establish the following interesting result:

Proposition 3. *Provided that $\varepsilon \geq 0$, the domestic public firm produces more than the foreign private firm for x_0^e and t^e.*

Proof: See Appendix.

As described in Section 2, for certain x_0 and t, the public firm behaves aggressively. Therefore, the claim in Proposition 3 seems nonsense and feckless. Of course, this is reasonable if both public and private firms possess the same technology or the public firm has superiority in its technology. However, given there is a cost differential between them even though the public firm engages in R&D, we cannot assert that the output of the public firm exceeds that of the private firm. Indeed, we can present some examples in which $q_0 < q_1$ when only one of policies is applied out of R&D and tariff. Suppose that $P(Q) = a - Q$, $C(x_0) = c_0 - x_0$, and $f(x_0) = 4x_0^2$. In the case wherein t is controlled, simple calculation reveals that $t^*(0) = c_0/3$, $q_0^*(0, t^*(0)) = a - c_0$, and $q_1^*(0, t^*(0)) = c_0/3$. Evidently, $q_1^*(0, t^*(0)) \geq q_0^*(0, t^*(0)) > 0$ if $3a/4 \leq c_0 < a$. Besides, consider that x_0 is a sole control variable. Equilibrium outcomes are given by $x_0^*(0) = (4a - 5c_0)/27$, $q_0^*(x_0^*(0), 0) = (31a - 32c)/27$, and $q_1^*(x_0^*(0), 0) = 2(8c - a)/27$. It follows from them that $q_1^*(x_0^*(0), 0) \geq q_0^*(x_0^*(0), 0)$ if $11a/16 \leq c_0 < 4a/5$. These mathematical results demonstrate that with the government having just one policy at hand, the aggressive public firm would produce less than the foreign private firm, when the initial technology of the public firm c_0 is in the middle range. However, Proposition 3 shows that the relationship of q_0 and q_1 depends not on demand size or initial technology, but on whether the public firm's reaction curve is upward sloping or downward sloping.

3 Trade liberalization and world welfare

This section aims to gauge an influence of trade liberalization on world welfare. For this purpose, we define world welfare W_T as the sum of domestic welfare W and foreign private firm's profits Π_1.

$$\begin{aligned} W_T(q_0, q_1, x_0) &:= W(q_0, q_1, x_0, t) + \Pi_1(q_0, q_1, t), \\ &= \int_0^Q P(z)dz - C(x_0)q_0 - f(x_0). \end{aligned}$$

Our focus in this section is on the effect of trade liberalization on world welfare, given both R&D investment level and tariff are selected to maximize the domestic welfare. Then, we need W_T which is evaluated at $(q_0^*(x_0, t), q_1^*(x_0, t))$. Then, we redefine the world welfare, which is presented as

$$W_T^*(x_0, t) := W_T(q_0^*(x_0, t), q_1^*(x_0, t), x_0). \tag{5}$$

Before turning to the analysis, it would be useful to ascertain whether the R&D investment level maximizing world welfare is larger or smaller than $x^*(t)$. Differentiating Eq.(5) with respect to x_0,

$$\begin{aligned} 0 = \frac{\partial W_T^*}{\partial x_0} &= [P(Q^*) - C(x_0)] \frac{\partial q_0^*}{\partial x_0} + P(Q^*)\frac{\partial q_1^*}{\partial x_0} - C'(x_0)q_0^* - f'(x_0), \\ &= q_1^* P'(Q^*)\frac{\partial q_0^*}{\partial x_0} + [t - P'(Q^*)q_1^*] \frac{\partial q_1^*}{\partial x_0} - C'(x_0)q_0^* - f'(x_0) \end{aligned} \tag{6}$$

(from both firms' first-order conditions)

Different from domestic welfare maximization, one more effect is added when we probe world welfare maximization. This is the first term in the right-hand side. Let us designate this effect as *overproduction effect*. Although $P(Q^*) = C(x_0)$ is desirable from the viewpoint of world welfare, the domestic public firm overproduces as observed from the first-order condition (1). This overproduction exacerbates world welfare, and thereby overproduction effect is negative. Alternatively, this negativity can be explained by the fact that an increase in q_0 though a rise in x_0 deteriorates the terms of trade for the foreign country and this leads to serious world welfare loss.

Let $x_0^{**}(t)$ be the R&D investment level satisfying Eq.(6). Then, the following result is established:

Lemma 3. *The domestic public firm undertakes the excessive investment from the viewpoint of world welfare.*

Proof: See Appendix.

R&D investment encourages the public firm to expand its production. This induces it to overproduce and the private firm to underproduce, from the viewpoint of world welfare. Such an inefficient allocation can be removed by lowering R&D investment level. Thus, we have $x_0^*(t) > x_0^{**}(t)$.

We now get into the discussion on trade liberalization. For analysis, define $\widehat{W}_T(t)$ as follows:

$$\widehat{W}_T(t) := W_T^*(x_0^*(t), t).$$

We look into a change in world welfare in response to trade liberalization by using this function. Defining $\Pi_1^*(x_0, t) := \Pi_1(q_0^*(x_0, t), q_1^*(x_0, t), t)$, the derivative of this function is given by

$$\begin{aligned}
\widehat{W}_T'(t) &= \left(\frac{\partial W^*}{\partial x_0} + \frac{\partial \Pi_1^*}{\partial x_0}\right) x^{*\prime}(t) + \frac{\partial \Pi_1^*}{\partial t} + \frac{\partial W^*}{\partial t}, \\
&= \frac{\partial \Pi_1^*}{\partial x_0} \cdot x_0^{*\prime}(t) + \frac{\partial \Pi_1^*}{\partial t} + \frac{\partial W^*}{\partial t}, \qquad \text{(from Eq.(2))}.
\end{aligned}$$

The first term in the right-hand side is positive for all $\varepsilon \in [\frac{1}{3}, 1)$, and the second term is negative. On the other hand, the sign of the third term is dependent on t, but it is zero for $t = t^e$. Consequently, even though $\widehat{W}_T'(t)$ is measured at $t = t^e$, the sign of $\widehat{W}_T'(t)$ is indetermined, depending on which effect is predominant, the first or second. With linear demand and an additional condition, the following definite result can be concluded.

Proposition 4. *Suppose that the domestic government sets the R&D investment level and tariff at x^e and t^e, respectively. Moreover, suppose that $\varepsilon = 0$. Gradual trade liberalization always enhances the world welfare.*

Proof: See Appendix.

To elucidate the intuition behind this proposition, we present some remarks when $\varepsilon = 0$. First, defining $\theta^e(t) = \theta^*(x_0^*(t), t)$, it follows that a tariff reduction along the optimal schedule of x_0 raises the private firm's market share, i.e., $\theta^{e\prime}(t) < 0$. In fact,

$$\begin{aligned}
\theta^{e\prime}(t) &= \frac{\partial \theta^*}{\partial x_0} \cdot x_0^{*\prime}(t) + \frac{\partial \theta^*}{\partial t}, \\
&= \frac{x_0^{*\prime}(t)}{Q^*(x_0(t), t)} \left(\frac{\partial q_0^*}{\partial x_0} - \theta^e(t) \cdot \frac{\partial Q^*}{\partial x_0} \right) + \frac{\partial \theta^*}{\partial t}, \\
&= \frac{(2 - \theta^e(t)) C'(x_0^*(t)) x_0^{*\prime}(t)}{2P'(Q^*(x_0(t), t)) Q^*(x_0(t), t)} + \frac{\partial \theta^*}{\partial t}, \qquad \text{(from Lemma 1 and } \varepsilon = 0).
\end{aligned}$$

Recall that $\partial\theta^*/\partial t < 0$ if $\varepsilon \leq 0$, as shown below Lemma 1. Therefore, this equation is negative. Second, a tariff reduction along the optimal schedule of x_0 decreases total outputs, i.e., $Q^{e\prime}(t) < 0$ where $Q^e(t) := Q^*(x_0^*(t), t)$. Combined with the first and second remarks, augmenting profits of the private firm can be concluded. The final remark is that a tariff reduction lowers production by the public firm. This implies that overproduction is resolved. In sum, these two effects—an increase in private firms' profits and removal of overproduction—improves world welfare.

Remark 1. *Complete trade liberalization (i.e., $t = 0$) enhances the world welfare in the following model (linear demand, linearly decreasing cost, quadratic R&D cost): $P(Q) = 1 - Q$, $C(x_0) = c_0 - x_0$, and $f(x_0) = 4x_0^2$, where $1/8 < c_0 < 3/4$ which ensures the interior solution.*

Simple computation apprise us that in such an environment,

$$\begin{aligned}
&x_0^*(t) = \frac{4 - 5c_0 - t}{27}, \quad q_0^*(x^*(t), t) = \frac{31 - 32c_0 - t}{27}, \quad q_1^*(x^*(t), t) = \frac{-2 + 16c_0 - 13t}{27}, \\
&W^*(x_0^*(t), t) = \frac{-20t^2 + 2(8c_0 - 1)t + 40c_0^2 - 64c_0 + 31}{54}, \\
&W_T^*(x_0^*, t) = \frac{-202t^2 + 50(1 - 8c_0)t + 1592c_0^2 - 1856c_0 + 845}{1458}.
\end{aligned}$$

As ascertained straightforwardly, world welfare maximization calls for an importing *subsidy*, while the domestic government has an incentive to levy a positive tariff. Since both $W^*(x_0^*(t), t)$ and $W_T^*(x_0^*, t)$ are quadratic functions with respect to t, we can verify that the removal of tariffs always raises the world welfare.

4 Concluding remarks

We have explored a public firm's incentive to conduct cost-reducing R&D investment when it competes against a private firm and the domestic government levies an importing tariff on the foreign goods. In particular, our focus is on how the optimal R&D investment level is affected by an importing tariff. We showed that a tariff reduction facilitates more R&D investment. Surprisingly, this result is not dependent on whether the public firm's reaction curve is upward or downward sloping in the output-setting stage. In most of studies on international mixed oligopoly, it is assumed that public firms have either horizontal or upward sloping reaction curves, as a sufficient condition to ensure the second-order conditions for their and private firms' optimization. On the contrary to these studies, without such an assumption, we proved the substitutability between an R&D policy and an importing tariff.

Furthermore, we investigated the domestic government's optimization of tariffs. This investigation revealed that it always has an incentive to impose a positive tariff. In addition, when both R&D policy and tariff are controlled so as to maximize the domestic welfare, the public firm produces more than the private firm. This result is surprising, since the cost-reducing R&D investment does not allow the public firm to have more efficient technology than the private firm, though the welfare-maximizing public firm behaves aggressively.

Finally, on the basis of analyses described above, we looked into the relationship between trade liberalization and world welfare. With linear demand, or the horizontal reaction curve of the public firm, we found that a small tariff reduction contributes to an improvement in world welfare. Both an increase in private firm's profits and removal of overproduction by the public firm account for this result. Moreover, complete trade liberalization can be concluded if additional assumptions; (i) the public firm's marginal cost is linearly decreasing in installation and (ii) R&D cost function is quadratic.

We finish this paper by make two remarks on the future researches. One is related to the competition and markets we considered. We ignored the presence of domestic private firms. As pointed out by Nishimori and Ogawa (2002), if domestic private firms are taken into account, the public firm attempts to decrease its R&D investment level to replace its production with production by more efficient private firms. This illustrates that in order to compensate a decrease in domestic private firms' outputs due to a tariff reduction, the public firm could lower its investment. In other words, it is possible that the optimal R&D level is increasing in tariffs. Such a possibility should be explored in detail. The other remark is relevant to a world wide wave of liberalization and deregulation. All over the world, many public firms have been privatized corresponding to such a wave. In this paper, unfortunately, we did not investigate privatization of public firms. The literature on mixed oligopoly have explored correlation among priavatization and other policies such as industrial policies, strategic trade policies, and so on. In line with these streams, we should attempt to unlock the effect of privatization in international mixed oligopoly with public firms' R&D policy.

Appendix

Proof of Lemma 1

The third-stage equilibrium is defined as $q_0^*(x_0, t) = R_0(q_1^*(x_0, t), x_0)$ and $q_1^*(x_0, t) = R_1(q_0^*(x_0, t), t)$. Differetiating them with respect to x_0,

$$\begin{pmatrix} 1 & -\dfrac{\partial R_0}{\partial q_1} \\ -\dfrac{\partial R_1}{\partial q_0} & 1 \end{pmatrix} \begin{pmatrix} \dfrac{\partial q_0^*}{\partial x_0} \\ \dfrac{\partial q_1^*}{\partial x_0} \end{pmatrix} = \begin{pmatrix} \dfrac{\partial R_0}{\partial x_0} \\ \\ 0 \end{pmatrix}.$$

Solving the equation system, we obtain $\partial q_i^*/\partial x_0 = \Delta_i^x/\Delta$ $(i = 0, 1)$, where

$$\begin{aligned}
\Delta &:= 1 - \frac{\partial R_0}{\partial q_1} \cdot \frac{\partial R_1}{\partial q_0} = \frac{2}{[2 + \varepsilon\theta^*(x_0, t)]\,[1 - \varepsilon\theta^*(x_0, t)]} > 0, \\
\Delta_0^x &:= \frac{\partial R_0}{\partial x_0} = \frac{C'(x_0)}{[1 - \varepsilon\theta^*(x_0, t)]\,P'(Q^*(x_0, t))} > 0, \\
\Delta_1^x &:= \frac{\partial R_1}{\partial q_0} \cdot \frac{\partial R_0}{\partial x_0} = -\frac{[1 + \varepsilon\theta^*(x_0, t)]\,C'(x_0)}{[1 - \varepsilon\theta^*(x_0, t)]\,[2 + \varepsilon\theta^*(x_0, t)]\,P'(Q^*(x_0, t))} < 0.
\end{aligned}$$

Thus, one of the desired results is acquired. Through the same procedure, we find that $\partial q_i^*/\partial t = \Delta_i^t/\Delta$ $(i = 0, 1)$, where

$$\begin{aligned}
\Delta_0^t &:= \frac{\partial R_0}{\partial q_1} \cdot \frac{\partial R_1}{\partial t} = \frac{\varepsilon\theta^*(x_0, t)}{[2 + \varepsilon\theta^*(x_0, t)]\,[1 - \varepsilon\theta^*(x_0, t)]\,P'(Q^*(x_0, t))}, \\
\Delta_1^t &:= \frac{\partial R_1}{\partial t} = \frac{1}{[2 + \varepsilon\theta^*(x_0, t)]\,P'(Q^*(x_0, t))} < 0.
\end{aligned}$$

Therefore, all the results are proved. ■

Proof of Lemma 2

First, we prove (i) by evaluating Eq.(2) at $x = x_0^d(t)$. Noticing that $\partial F(x_0^d(t), t)/\partial x_0 = C'(x_0^d(t))q_0^*(x_0^d(t), t) + C(x_0^d(t))(\partial q_0^*/\partial x_0) + f'(x_0^d(t)) = 0$, we have

$$\begin{aligned}
\left.\frac{\partial W^*}{\partial x_0}\right|_{x_0 = x_0^d(t)} &= P(Q^*(x_0^d(t), t))\left.\frac{\partial q_1^*}{\partial x_0}\right|_{x_0 = x_0^d(t)} + C(x_0^d(t))\left.\frac{\partial q_0^*}{\partial x_0}\right|_{x_0 = x_0^d(t)}, \\
&\qquad \text{(from definition of } x_0^d(t) \text{ and Eq.(1))} \\
&= -\frac{C'(x_0^d(t))}{2P'(Q^*(x_0^d(t), t))}\left\{-P(Q^*(x_0^d(t), t)) + \left[2 + \varepsilon\theta^*(x_0^d(t), t)\right]P'(Q^*(x_0^d(t), t))q_1^*(x_0^d(t), t)\right\}, \\
&\qquad \text{(from Lemma 1 and Eq.(2))} \\
&> 0.
\end{aligned}$$

Thus, by the second-order condition, we obtain $x_0^d(t) < x_0^*(t)$. Next, we proceed to the proof of (ii). Note that in public monopoly, the public firm selects x_0^m and q_0^m which satisfy $P(q_0^m) - C(x_0^m) = 0$ and $C'(x_0^m)q_0^m + f'(x_0^m) = 0$. Similar to the proof of (i), we evaluate Eq.(2) at $x = x^m$. Abbreviating augment x_0^m for saving pages, we have

$$\begin{aligned}
\left.\frac{\partial W^*}{\partial x_0}\right|_{x_0=x_0^m} &= [t - P'(Q^*)q_1^*]\frac{\partial q_1^*}{\partial x_0} - C'(x_0^m)q_0^* + C'(x_0^m)q_0^m, \\
&\quad \text{(from the definition of } x_0^m\text{)} \\
&= t \cdot \frac{\partial q_1^*}{\partial x_0} + C'(x_0^m)\left[\left(\frac{1+\varepsilon\theta^*}{2}\right)q_1^* + q_0^m - q_0^*\right], \qquad (7) \\
&\quad \text{(from Lemma 1).}
\end{aligned}$$

Noting that $R_0(0, x_0^m) = q_0^m$ and $\operatorname{sgn}(\partial R_0/\partial q_1) = \operatorname{sgn}(\varepsilon)$, we find that $q_0^m \gtreqless q_0^*(x_0^m, t)$ if and only if $\varepsilon \lesseqgtr 0$. Then, suppose that $\varepsilon \leq 0$. In this case, Eq.(7) is negative and thus, $x_0^m > x_0^*(t)$. Next, suppose that $\varepsilon > 0$. The presumption stated in Lemma 2, Eq.(3), ensures that Eq.(7) is positive. Therefore we obtain $x_0^m > x_0^*(t)$. ■

Proof of Proposition 1

Step 1. We start from deriving the second-order derivatives of R_0 and R_1. Differentiating the first-order condition for the private firm with respect to q_0 and arranging the equation leads to

$$P'(q_0 + R_1(q_0,t))\left[1 + 2\frac{\partial R_1}{\partial q_0} + \left(1 + \frac{\partial R_1}{\partial q_0}\right)\varepsilon\hat{\theta}\right] = 0,$$

where $\hat{\theta} = R_1(q_0,t)/(q_0 + R_1(q_0,t))$. Differentiate both sides of this equation with respect to q_0 once again, we find that

$$(2+\varepsilon\hat{\theta})\frac{\partial^2 R_1}{\partial q_0^2} + \frac{\varepsilon}{q_0 + R_1(q_0,t)}\left(1 + \frac{\partial R_1}{\partial q_0}\right)\left[\frac{\partial R_1}{\partial q_0} - \left(1 + \frac{\partial R_1}{\partial q_0}\right)\hat{\theta}\right] = 0,$$

where use is made of the envelope theorem. Thus, it follows that

$$\begin{aligned}
\frac{\partial^2 R_0}{\partial q_0^2} &= -\frac{\frac{\varepsilon}{Q}\left(1+\frac{\partial R_1}{\partial q_0}\right)\left[\frac{\partial R_1}{\partial q_0}-\left(1+\frac{\partial R_1}{\partial q_0}\right)\hat{\theta}\right]}{2+\varepsilon\hat{\theta}}, \\
&= -\frac{\frac{\varepsilon}{2+\varepsilon\hat{\theta}}\left[-\frac{1+\varepsilon\hat{\theta}}{2+\varepsilon\hat{\theta}}-\frac{\hat{\theta}}{2+\varepsilon\hat{\theta}}\right]}{Q(2+\varepsilon\hat{\theta})}, \qquad \text{(from Proof of Lemma 1)}, \\
&= \frac{\varepsilon\left[1+\hat{\theta}(1+\varepsilon)\right]}{Q(2+\varepsilon\hat{\theta})^3}.
\end{aligned}$$

Analogously, the cross second-order derivative of R_1 can be derived as follows:

$$\frac{\partial^2 R_1}{\partial t \partial q_0} = -\frac{\varepsilon(1-\hat{\theta})}{QP'(Q)(2+\varepsilon\hat{\theta})^3}.$$

Next, we proceed to derivation of the second-order derivatives of R_0. To this end, differentiating the first order condition with respect to q_1 and rearranging it result in

$$P'(R_0(q_1, x_0)+q_1)\left[\frac{\partial R_0}{\partial q_1}-\varepsilon\tilde{\theta}\left(1+\frac{\partial R_0}{\partial q_1}\right)\right]=0.$$

where $\tilde{\theta} := q_1/(R_0(q_1, x_0)+q_1)$. By differentiating this equation with respect to q_1 and x_0, we find that

$$\frac{\partial^2 R_0}{\partial q_1^2} = \frac{\varepsilon\left[1-\tilde{\theta}(1+\varepsilon)\right]}{Q(1-\varepsilon\tilde{\theta})^3}, \qquad \frac{\partial^2 R_0}{\partial x_0 \partial q_1} = -\frac{\varepsilon\tilde{\theta}C'(x_0)}{QP'(Q)(1-\varepsilon\tilde{\theta})^3}.$$

Step 2. We calculate $\partial^2 q_i^*/\partial t \partial x_0$ $(i = 0, 1)$. From differentiating the second-stage equilibrium conditions with respect to x_0 and then to t,

$$\begin{pmatrix} 1 & -\dfrac{\partial R_0}{\partial q_1} \\ -\dfrac{\partial R_1}{\partial q_0} & 1 \end{pmatrix} \begin{pmatrix} \dfrac{\partial^2 q_0^*}{\partial t \partial x_0} \\ \dfrac{\partial^2 q_1^*}{\partial t \partial x_0} \end{pmatrix} = \begin{pmatrix} A_1 \\ \\ A_2 \end{pmatrix},$$

where

$$\begin{aligned} A_1 &= \frac{\partial q_1^*}{\partial t}\left(\frac{\partial q_1^*}{\partial x_0} \cdot \frac{\partial^2 R_0}{\partial q_1^2} + \frac{\partial^2 R_0}{\partial q_1 \partial x_0}\right) = -\frac{\varepsilon(1-\varepsilon\theta^*)\left[-1+3\theta^*+\varepsilon(1+\varepsilon)\theta^{*2}\right]C'(x_0)}{4P'(Q^*)^2 Q^*(1-\varepsilon\theta^*)^3}, \\ A_2 &= \frac{\partial q_0^*}{\partial x_0}\left(\frac{\partial q_0^*}{\partial t} \cdot \frac{\partial^2 R_1}{\partial q_0^2} + \frac{\partial^2 R_1}{\partial t \partial q_0}\right) = \frac{\varepsilon(-1+\theta^*+\varepsilon\theta^*)C'(x_0)}{4P'(Q^*)(2+\varepsilon\theta^*)}. \end{aligned}$$

Therefore, we get $\partial^2 q_i^*/\partial t \partial x_0 = \Delta_i^{tx}/\Delta$ $(i = 0, 1)$, where

$$\begin{aligned} \Delta_0^{tx} &:= A_1 + \frac{\partial R_0}{\partial q_1} A_2 = -\frac{\varepsilon\left[-1+3\theta^*+2\varepsilon\theta^{*2}+\varepsilon^2(1+\varepsilon)\theta^{*3}\right]C'(x_0)}{2Q^* P'(Q^*)(2+\varepsilon\theta^*)(1-\varepsilon\theta^*)^2}, \\ \Delta_1^{tx} &:= A_2 + \frac{\partial R_1}{\partial q_0} A_1 = \frac{\varepsilon\left[-1+(2+\varepsilon)\theta^*+(1-\varepsilon)\varepsilon\theta^{*2}+\varepsilon^2(1+\varepsilon)\theta^{*3}\right]}{2Q^* P'(Q^*)(2+\varepsilon\theta^*)(1-\varepsilon\theta^*)^2}. \end{aligned}$$

Step 3. By virtue of the first-order condition in the first-stage, applying the implicit function theorem, we obtain

$$x^{*\prime}(t) = -\frac{\partial^2 W^*(x_0^*(t), t)/\partial t \partial x_0}{\partial^2 W^*(x_0^*(t), t)/\partial x_0^2}.$$

Coupled with the second-order condition, this implies that $\mathrm{sgn}(x^{*\prime}(t)) = \mathrm{sgn}(\partial^2 W^*(x_0^*(t), t)/\partial t \partial x_0)$. Simple calculation reveals that $\partial^2 W^*/\partial t \partial x_0$ is decomposed into three parts as follows:

$$\frac{\partial^2 W^*}{\partial t \partial x_0} = \left[1 - P'(Q^*)\frac{\partial q_1^*}{\partial t} - q_1^* P''(Q^*)\frac{\partial Q^*}{\partial t}\right]\frac{\partial q_1^*}{\partial x_0} + \left[t - P'(Q^*)q_1^*\right]\frac{\partial^2 q_1^*}{\partial t \partial x_0} - C'(x_0)\frac{\partial q_0^*}{\partial t}. \quad (8)$$

Invoking Lemma 1, the first term in the right-hand side can be reduced to

$$\left[1 - P'(Q^*)\frac{\partial q_1^*}{\partial t} - q_1^* P''(Q^*)\frac{\partial Q^*}{\partial t}\right]\frac{\partial q_1^*}{\partial x_0} = \frac{1}{2}\cdot\frac{\partial q_1^*}{\partial x_0}.$$

Suppose that $\partial^2 q_1^*/\partial t\partial x_0 < 0$. Then, Eq.(8) is rewritten by

$$\begin{aligned}
\frac{\partial^2 W^*}{\partial t\partial x_0} &= \frac{1}{2}\cdot\frac{\partial q_1^*}{\partial x_0} + [t - P'(Q^*)q_1^*]\frac{\partial^2 q_1^*}{\partial t\partial x_0} - C'(x_0)\frac{\partial q_0^*}{\partial t},\\
&= [t - P'(Q^*)q_1^*]\frac{\partial^2 q_1^*}{\partial t\partial x_0} + \frac{1}{2}\left[\frac{(1+\varepsilon\theta^*)C'(x_0)}{2P'(Q^*)}\right] - \frac{\varepsilon\theta^* C'(x_0)}{2P'(Q^*)},\\
&= [t - P'(Q^*)q_1^*]\frac{\partial^2 q_1^*}{\partial t\partial x_0} - \frac{(1+3\varepsilon\theta^*)C'(x_0)}{4P'(Q^*)}.
\end{aligned}$$

Therefore, we have $x^{*\prime}(t) < 0$ for $\varepsilon \in [-\frac{1}{3}, 1]$. Next, on the contrary, suppose that $\partial^2 q_1^*/\partial t\partial x_0 \geq 0$. By virtue of Eq.(2) and Lemma 1, Eq.(8) evaluated at $x_0 = x_0^*(t)$ can be rearranged as follows.

$$\begin{aligned}
\frac{\partial^2 W^*(x_0^*(t), t)}{\partial t\partial x_0} &= \frac{1}{2}\cdot\frac{\partial q_1^*}{\partial x_0} - C'(x_0^*)\frac{\partial q_0^*}{\partial t} + \frac{C'(x_0^*)q_0^* + f'(x_0^*)}{\partial q_1^*/\partial x_0}\cdot\frac{\partial^2 q_1^*}{\partial t\partial x_0},\\
&= -\frac{(1+\varepsilon\theta^*)C'(x_0^*)}{4P'(Q^*)} - \frac{(1-\varepsilon\theta^*)C'(x_0^*)}{2P'(Q^*)} - \frac{2P'(Q^*)}{(1+\varepsilon\theta^*)C'(x_0^*)}\cdot\frac{\partial^2 q_1^*}{\partial t\partial x_0}\left[C'(x_0^*)q_0^* + f'(x_0^*)\right].
\end{aligned}$$

Furthermore, invoking the result of Step 2 ($\partial^2 q_1^*/\partial t\partial x_0 = \Delta_1^{tx}/\Delta$), this can be rewritten by

$$\frac{\partial^2 W^*(x_0^*(t), t)}{\partial t\partial x_0} = -\frac{C'(x_0)B}{4P'(Q^*)(1+\varepsilon\theta^*)(1-\varepsilon\theta^*)} - \frac{2P'(Q^*)f'(x_0^*)}{(1+\varepsilon\theta^*)C'(x_0^*)}\cdot\frac{\partial^2 q_1^*}{\partial t\partial x_0}, \tag{9}$$

where

$$\begin{aligned}
B &= (1+\varepsilon\theta^*)(1-\varepsilon\theta^*)(3-\varepsilon\theta^*) + 2(1-\theta^*)\varepsilon\left[-1 + (2+\varepsilon)\theta^* + (1-\varepsilon)\varepsilon\theta^{*2} + \varepsilon^2(1+\varepsilon)\theta^{*3}\right],\\
&> (1-\theta^*)(3-\varepsilon\theta^* - 2\varepsilon) + 2(1-\theta^*)\varepsilon\left[(2+\varepsilon)\theta^* + (1-\varepsilon)\varepsilon\theta^{*2} + \varepsilon^2(1+\varepsilon)\theta^{*3}\right],\\
&= (1-\theta^*)\left[3 - 2\varepsilon(1-\theta^*) + 2\varepsilon^4\theta^* + 2\varepsilon^2\theta^*(1+\theta^*) - 2\varepsilon^3\theta^{*2}(1-\theta^{*2})\right],\\
&> 0.
\end{aligned}$$

Since both the first and second terms in the right-hand side of Eq.(9) are negative, it follows that $x^{*\prime}(t) < 0$. ■

Proof of Proposition 2

Solving Eq.(4) with respect to $t^*(x_0)$, we have

$$\begin{aligned} t^*(x_0) &= P'(Q^*(x_0, t^*(x_0))) - \frac{q_1^*(x_0, t^*(x_0))}{\partial q_1^*/\partial t}, \\ &= -\frac{(1 + \varepsilon\theta^*(x_0, t^*(x_0)))q_1^*(x_0, t^*(x_0))P'(Q^*(x_0, t^*(x_0)))}{1 - \varepsilon\theta^*(x_0, t^*(x_0))}, \quad \text{(from Lemma 1)}, \\ &> 0. \end{aligned}$$

Thus, we obtained the desired result. ■

Proof of Proposition 3

Consider Eqs.(2) and (4) evaluated at x_0^e and t^e. By multiplying both sides of Eq.(4) by $C'(x_0^e)$ and subtracting the multiplied equation from Eq.(2), it follows that

$$[t^e - P'(Q^*(x_0^e, t^e))q_1^*(x_0^e, t^e)]\left[\frac{\partial q_1^*}{\partial x_0} - \frac{\partial q_1^*}{\partial t}C'(x_0^e)\right] - C'(x_0^e)Q^*(x_0^e, t^e) - f'(x_0^e) = 0.$$

Moreover, this can be simplified as

$$Q^*(x_0^e, t^e) = -\frac{t^e - P'(Q^*(x_0^e, t^e))q_1^*(x_0^e, t^e)}{P'(Q^*(x_0^e, t^e))} - \frac{f'(x_0^e)}{C'(x_0^e)}.$$

where use is made of Lemma 1. In addition, using Eq.(4) again, we have

$$q_1^*(x_0^e, t^e) = -[t - P'(Q^*(x_0^e, t^e))q_1^*(x_0^e, t^e)]\frac{\partial q_1^*}{\partial t} = -\frac{\left[1 - \varepsilon\theta^*(x_0^e, t^e)\right]\left[t - P'(Q^*(x_0^e, t^e))q_1^*(x_0^e, t^e)\right]}{2P'(Q^*(x_0^e, t^e))}.$$

These equations show that

$$
\begin{aligned}
\frac{q_1^*(x_0^e, t^e)}{Q^*(x_0^e, t^e)} &= \frac{\dfrac{\left[1-\varepsilon\theta^*(x_0^e, t^e)\right]\left[t - P'(Q^*(x_0^e, t^e))q_1^*(x_0^e, t^e)\right]}{2P'(Q^*(x_0^e, t^e))}}{\dfrac{t^e - P'(Q^*(x_0^e, t^e))q_1^*(x_0^e, t^e)}{P'(Q^*(x_0^e, t^e))} + \dfrac{f'(x_0^e)}{C'(x_0^e)}}, \\
&= \frac{1-\varepsilon\theta^*(x_0^e, t^e)}{2\left\{1 + \dfrac{P'(Q^*(x_0^e, t^e))f'(x_0^e)}{C'(x_0^e)\left[t - P'(Q^*(x_0^e, t^e))q_1^*(x_0^e, t^e)\right]}\right\}}, \\
&< \frac{1}{2} \qquad \text{(if } \varepsilon \geq 0\text{)}.
\end{aligned}
$$

This directly purports that $q_0^*(x_0^e, t^e) > q_1^*(x_0^e, t^e)$. ■

Proof of Lemma 3

Evaluating Eq.(5) at $x_0^*(t)$,

$$
\frac{\partial W_T^*(x_0^*(t), t)}{\partial x_0} = P'(Q^*(x_0^*(t), t))q_1^*(x_0^*(t), t) \cdot \frac{\partial q_0^*}{\partial x_0} < 0.
$$

From the second-order condition for optimization of W_T^* with respect to x_0, it is concluded that $x_0^*(t) > x_0^{**}(t)$. ■

Proof of Proposition 4

To prove this proposition, let us arrange $\widehat{W}_T'(t^e)$.

$$
\begin{aligned}
\widehat{W}_T'(t^e) &= \frac{\partial \Pi_1}{\partial q_0} \cdot \frac{\partial q_0^*}{\partial x_0} \cdot x_0^{*\prime}(t^e) + \frac{\partial \Pi_1}{\partial q_0} \cdot \frac{\partial q_0^*}{\partial t} + \frac{\partial \Pi}{\partial t}, \\
&= P'(Q^*)q_1^* \cdot \frac{2C'(x_0^*(t^e))}{2P'(Q^*)} \cdot x^{*\prime}(t^e) - q_1^*, \quad \text{(from Lemma 1 and } \varepsilon = 0\text{)}, \\
&= -\frac{q_1^*}{\partial^2 W^*/\partial x_0^2}\left[C'(x_0^*(t^e)) \cdot \frac{\partial^2 W^*}{\partial t \partial x_0} + \frac{\partial^2 W^*}{\partial x_0^2}\right]
\end{aligned}
$$

Complicated computation leads to the quantity in the brackets as follows:

$$
\begin{aligned}
&C'(x_0^*(t^e))\left(\frac{\partial q_1^*}{\partial x_0} - \frac{\partial q_0^*}{\partial x_0}\right) - C''(x_0^*(t^e))q_0^* - f''(x_0^*(t^e)) \\
&= -\frac{3C'(x_0^*(t^e))^2}{2P'(Q^*)} - C''(x_0^*(t^e))q_0^* - f''(x_0^*(t^e))
\end{aligned}
$$

The second-order condition of optimization for world welfare, with $\varepsilon = 0$, is given by

$$
\begin{aligned}
\frac{\partial^2 W_T^*}{\partial x_0^2} &= \frac{\partial^2 W^*}{\partial x_0^2} + \frac{\partial^2 \Pi_1^*}{\partial x_0^2}, \\
&= \left[\frac{C'(x_0)}{2}\cdot\frac{\partial q_1^*}{\partial x_0} - C''(x_0)q_0^* - C'(x_0)\cdot\frac{\partial q_0^*}{\partial x_0} - f''(x_0)\right] + \left[C''(x_0)q_1^* + C'(x_0)\cdot\frac{\partial q_1^*}{\partial x_0}\right], \\
&= -\frac{7C'(x_0)^2}{4P'(Q^*)} + C''(x_0)q_1^* - C''(x_0)q_0^* - f''(x_0), \\
&< 0.
\end{aligned}
$$

Since this condition must hold for all $x_0 \geq 0$ as long as interior solutions exist, we find that

$$
\begin{aligned}
0 > \frac{\partial^2 W^*(x_0^*(t^e), t^e)}{\partial x_0^2} &= -\frac{7C'(x_0^*(t^e))^2}{4P'(Q^*)} + C''(x_0^*(t^e))q_1^* - C''(x_0^*(t^e))q_0^* - f''(x_0^*(t^e)), \\
&> -\frac{7C'(x_0)^2}{4P'(Q^*)} - C''(x_0)q_0^* - f''(x_0), \quad \text{(from Assumption 2)}, \\
&> -\frac{3C'(x_0^*(t^e))^2}{2P'(Q^*)} - C''(x_0^*(t^e))q_0^* - f''(x_0^*(t^e)), \quad \text{(from Assumption 1)}.
\end{aligned}
$$

Thus, it follows that $\widehat{W}_T'(t^e) < 0$. ■

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