Estimating the output gap in the Polish economy: the VECM approach

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Abstract

This article presents three estimates of the output gap, one using the production function method, and the other two by assessing the long-term product using cointegration relationships (based on the production function and on the hypothesis of permanent income). It also presents an analysis of time-relationships between the estimated output gaps and selected measures of inflation using the covariance of a VAR-type stochastic process. The methods employed yield different estimates of the output gap. The time paths of calculated gaps and the analysis of time relationships (conditional on the existence of relationship described by the Phillips curve and the possibility of using obtained gaps in it) allow the authors to conclude that there's no inflationary pressure from the aggregate demand in the Polish economy, at least till the end of 2003.

Key words: Output gap, VECM, production function, Permanent-Transitory Decomposition

JEL classification: E32, C32

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1. Introduction

The aim of the paper is to present the results of research on calculating the output gap in the Polish economy over the period 1996-2002. The analysis is based on selected methods.

The output gap, defined as a relative discrepancy between the actual GDP and the potential GDP (relative to the level of potential GDP), is an indicator of disequilibrium in the real economy. Thus, the concept of output gap is based on the definition of unobservable potential.

One can find various definitions of potential output in literature. From the monetary authority's point of view the definition proposed by Okun (1962) is still the most influential. He described potential GDP as the maximum quantity of output the economy can produce under the conditions of full employment (which is understood as the maximum level of employment not generating inflationary pressure[1]). Later refinements of the definition stressed alternative aspects of the above-mentioned qualification, e.g. the intensity of use of labour and capital (Artus (1977)). Output gap is stressed in the neo-keynesian models of the DSGE type (*Dynamic Stochastic General Equilibrium*) with sticky prices (cf. Clarida, Gali, Gertler (2000)). Here the concept of potential output is different – the equilibrium level reached without nominal rigidities, that is, with fully flexible prices and wages.

Due to the unobservable character of potential product, there is no fully accepted method of estimation. Even the selection of one definition of potential often implies few empirical methods of estimation of output gap. Methods based directly or indirectly on Okun's definition usually use the production function. On the other hand, when taking into consideration neo-keynesian theory, the potential product is achieved when there are no rigidities, which is true in the long run. Thus, the potential is assumed to be the long-run growth path of the product. Methodologies that decompose the product into permanent and transitory components make use of this interpretation of potential output, but without direct connections with neo-keynesian definition.

Apart from discrepancy of inflation from the inflation target, the output gap is a relevant component of the so called "Taylor rule" (Taylor (1993)), which is a premise of decisions made by monetary authorities. Moreover, the concept of output gap is used to

[1] The equilibrium unemployment, not generating inflationary pressure, is called NAIRU (Non-Accelerating Inflation Rate of Unemployment). With different definitions of labor market equilibrium, one can also distinguish the NAWRU (Non-Accelerating Wage rate of Unemployment) and the natural rate of unemployment (Kwiatkowski (2002), p.154). These rates of unemployment are of course different, but taking into consideration their non-observable character, they are often exchanged in applications.

decompose the budget deficit into cyclical and structural components. The lack of a commonly accepted estimation method of potential output implies a high uncertainty of macroeconomic policy. The uncertainty is also strengthened by errors connected with real-time estimations of economic aggregates or revisions of the business cycle phase and different interpretations of economic processes due to the inflow of new data (Orphanides, van Norden (2002)).

The authors decided to present the results of output gap calculations using two methods:

- a method based on a two-factor dynamic production function (estimated in the co-integrated VECM system, in which the potential GDP is calculated as the product resulting from maximum (in the Okun sense) level of production inputs (Chapter 2);
- the GDP Permanent-Transitory Decomposition, using long-term restrictions in the vector error correction model (VECM) imposed in an endogenous way by co-integrating relationships; Chapter 3 contains the results for two models: one based on the long-term production function and the other on the permanent income hypothesis.

The selected methods are connected by making use of cointegration analysis and the system estimation of Johansen. Their common feature is also the fact, that they have origins in economics, in contrast to the mechanical methods of potential estimation (e.g. deterministic trend, Hodrick-Prescott filter, Band-Pass filter[2]). This is why the applied methods are said to be structural (Chagny, Döpke (2001)), although the approximation of some variables with filters means that they should be called rather semistructural approaches.

On the basis of Phillips curve, research on the influence of obtained output gaps on inflation in the Polish economy was conducted, though the selected definitions of the potential output do not necessarily have to lead to such a relation. Section 4 analyses time relationship between the estimated time-series of output gaps and various inflation measures (CPI, PPI, GDP deflator and one of the core inflation indexes). This analysis was made using cross-correlations of the inflation and the gap, implied by the covariance generating function of the VAR stochastic process.

[2] One should also bear in mind that even pure mechanical methods imply some economic features of the filtered variables.

2. Production function approach

Estimating the production function

In order to estimate the potential GDP of the Polish economy, the dynamic Cobb-Douglas function was selected as the production function. It's one of the simplest ways of describing the transformation process of inputs into output. Regardless of the functional form, on the assumption that production inputs are paid proportionally to their marginal products, a constant share of their income in GDP implies that elasticity of substitution equals one. It suggests the choice (within the family of functions with constant elasticity of substitution – CES) of a Cobb-Douglas production function. Stability of factor income shares in GDP seems to hold in the Polish economy – during 1995-2001 the share of labour compensation of employees in gross added value oscillated between 50% and 53%, and it was almost identical in 1995 and 2001. Stationarity of this series was confirmed by the ADF-GLS test (at 10% significance level) and the complementary KPSS test (at 5% significance level). The reliability of these tests is low due to the short sample used in the analysis.

Assuming Cobb-Douglas technology, constant returns to scale and Hicks neutral technological progress, the production function can be written as follows (cf. Żółtowska (1997)):

$$Y_t = A(\alpha_t L_t)^{\alpha}(\beta_t K_t)^{1-\alpha},$$

where Y_t is the output (GDP), L_t and K_t are the inputs of labour and capital, while α_t and β_t are technical progress functions in the meaning of Harrod and Solow, respectively. As it is difficult to separate labour productivity growth from capital productivity growth, the production function is often presented as follows:

$$Y_t = TFP_t L_t^{\alpha} K_t^{1-\alpha},$$

where TFP_t is the total factor productivity and reflects technical progress increasing the productivity of both labour – by improving the qualifications of employees, and capital – with introduction of advanced technologies. Having this structure, the TFP_t variable makes it possible to introduce variations in the *A* factor, and thus take into account factors which cannot be explained by technical development. One of such factors is the effectiveness of social resistance to the introduction of new technologies (cf. Prescott (1997)).

The direct estimation of the production function using the OLS (in which the simultaneous-equation model is reduced to one-equation) does not seem to be a proper method for at least two reasons. Firstly, the output and inputs of labour and capital cannot be treated as independent, so the assumption that explanatory variables are exogenous does not hold (cf. Griliches, Mairess (1995)). Thus, the assumption of the exogenity of explanatory variables is violated. Secondly, according to economic theory, at least GDP and capital are generated by non-stationary stochastic processes (King, Plosser, Stock, Watson (1991)), so the use of OLS may lead to spurious regressions, which is the pure statistical relation between time series, without any economic meaning.

One of the methods allowing to avoid the above mentioned methodological errors is the multidimensional cointegration analysis. In this paper, the system was estimated in the form of a vector error correction model (VECM), according to the Johansen (1991) procedure. The cointegration relationship between production and inputs of labour and capital estimated in the above way can be considered as a well-estimated production function provided that the model has been correctly specified.

The quarterly data[3] covering the period 1995-2002 was used for empirical analysis. To eliminate the impact of seasonal factors on the results, all variables were seasonally adjusted using the multiplicative moving average method. The labour input (L_t) was assumed to be equal to the number of employed persons according to the Labour Force Survey (LFS). The variable describing the capital in the Polish economy (K_t) was assumed to be equal to the gross value of fixed assets in the national economy.[4] The data on real GDP (Y_t) is taken from publications of the Central Statistical Office.

The results of unit root testing based on the augmented Dickey-Fuller test with the generalised least square method (GLS) indicates that the logarithms of seasonally-adjusted Y_t, L_t and K_t variables are integrated of order one (Table 1). As the power of ADF-type tests is quite low, the complementary KPSS test was also conducted indicating the same integration order. Economic assumptions about the integration order of GDP and capital have been empirically confirmed. Thus, there are grounds for looking for cointegrating relationships between these variables.

[3] Cointegration analysis is generally combined with a long-run approach, based on annual data. Due to a short sample problem (the analysis covers 8 years), interpretation of obtained relations as long-run relations should be cautious.

Table 1. Results of the unit root test

Variable	ADF-GLS statistics	KPSS statistics	Conclusion at the 0.1 significance level
$\Delta\log(Y_t)$	-1.99**	0,09**	I(1)
$\Delta\log(L_t)$	-2.00**	0,08**	I(1)
$\Delta\log(K_t)$	-1.91*	0,16*	I(1)

* significant at the 0.1 significance level
** significant at the 0.05 significance level

Source: Own calculations

In the first stage of the estimation process it was assumed that the TFP_t variable can be approximated by the exponential trend, i.e. a linear trend after calculating the logarithm. The assumption was made in order to make use of standard Johansen (1991) procedure and calculate factor elasticises of the product.

Cointegration tests indicate that there is one cointegrating relationship between Y_t, L_t and K_t at the 1% significance level. This is so both in the case of the trace test and the test of maximum eigen-value (Table 2). Consequently, the VECM system was estimated with the assumption that there is one cointegrating relationship.

Table 2. Johansen cointegration test

Hypothesis: number of cointegrating relationships	Eigen-value	Trace statistic	Maximum eigen-value statistic
None	0.74	53.07***	37.38***
No more than one	0.28	15.68	9.06

*** hypothesis rejected at a 0.01 level of significance

Source: Own calculations

The estimation yields the following assessment of the long-term relationship between GDP, labour input and capital input:

$$\hat{Y}_t = L_t^{0,57} K_t^{0,43} e^{0,29+0,01t}$$

The estimated long-term relationship parameters have the expected signs and are statistically significant. The same can be said of the α adjustment matrix (for one cointegrating relationship it has the form of a vector), corresponding to the equations describing labour and capital dynamics. The adjustment parameters indicate that the equilibrium state is regained quicker through labour input (half of the adjustments lasted no more than 5 quarters) than through capital input (half of the adjustments after 5 years). This seems intuitively right and is consistent with observations of how the economy works.

[4] According to calculations done by Marcin Kolasa and Roman Sawiński, DAMS, NBP.

The adjustment parameter in the equation describing GDP dynamics is statistically not different from zero. This characteristic has important implications for the entire system and is related to the idea of weak exogeneity (cf. S. Johansen (1991)). The weak exogenous character of GDP with respect to the cointegrating relationship can be interpreted by saying that adjustments stem only from production factors, and not from the production itself[5].

For the estimation, two restrictions were imposed on the VECM system. The first has a normalising nature, the second is connected with the assumption of constant economies of scale. The quotient test of the likelihood ratio imposed on the restricted system indicates that there is no reason for rejecting the hypothesis of their legitimacy (with the probability of 0.52). The system describes the dynamics of endogenous variables well, and this is proven by R^2 (equal 0.42 for the GDP equation, 0.40 for the employment and 0.95 for the capital equation) and the tested lack of autocorrelation of the random component. The roots of the characteristic polynomial corresponding to the system indicate that it is stable.

The last stage in estimating the production function is to replace the exponential deterministic trend approximating the total factor productivity with the TFP_t^* estimation. For this purpose, after estimating the production function using the exponential trend, residual values of the production function without the trend were calculated and smoothed using the Hodrick-Prescott filter. At the end, the process of estimating the production function was repeated in an analogous VECM system, in which the dynamic structure and restrictions of the original system were maintained, but the exponential trend was replaced with the estimated TFP_t^*. The calculated elasticities of the product with respect to the input of production factors differ from those previously calculated only at the third decimal place. Consequently, the final estimation of the production function can be described by the following formula:

$$\hat{Y}_t = TFP_t^* L_t^{0,55} K_t^{0,45},$$

where TFP_t^* is the series of residuals after smoothing them with the HP40 filter.

The estimated production function was then used to calculate the potential output.

Potential labour input

According to the methods used by OECD (cf. C. Giorno et al. (1995)), the potential labour input used to calculate the potential output is obtained from the following formula:

[5] It could suggest the demand characted of observed GDP, but it's not the case when one takes into consideration the fact, that innovations in the VECM system are the combination of short- and long-run disturbances.

$$L^* = LF(1 - NAWRU),$$

where *LF* is the labour force, while *NAWRU* is the non-accelerating wage rate of unemployment. Hence L^* is the number of employed persons at the natural unemployment rate, consistent with the concept of the production function.

Labour force was assumed to be the number of professionally active persons according to LFS, but it is more difficult to estimate the *NAWRU*. The starting point was the method proposed by Elmeskov (1993), in which the change in the rate of the wage inflation is proportional to the difference between the actual unemployment rate and *NAWRU*, which can be expressed as follows

$$\Delta^2 \log W = -a(U - NAWRU),\ a > 0$$

where *W* – wage level, *U* – unemployment rate. The underlying methodology is thus consistent with the Phillips curve supplemented with adaptative expectations, according to which the expected wage inflation in the current period is equal to the rise in wages in the previous period (cf. Staiger, Stock, Watson (1996)). Assuming that the *NAWRU* is constant between any two consecutive quarters[6] allows one to calculate the *a* parameter for subsequent periods and, as a result, to calculate the *NAWRU* series.

The method proposed by Elmeskov is used for annual data, so it seems right to modify it for calculations based on quarterly data. In this study, the modification consists in replacing the current value of the *U* variable with a distribution of its lags, i.e.

$$\Delta^2 \log W = -a(\varphi(L)U - NAWRU),\ a > 0$$

where $\varphi(L)$ is a lag polynomial of order four, while *L* is the lag operator. A hypergeometrical distribution of $\varphi(L)$ polynomial coefficients was assumed to take into account the delay with which the labour market situation affects rises in wages. The resulting formula is:

$$NAWRU = \varphi(L)U - \frac{\Delta\varphi(L)U - \Delta^3 \log W}{\Delta^2 \log W}.$$

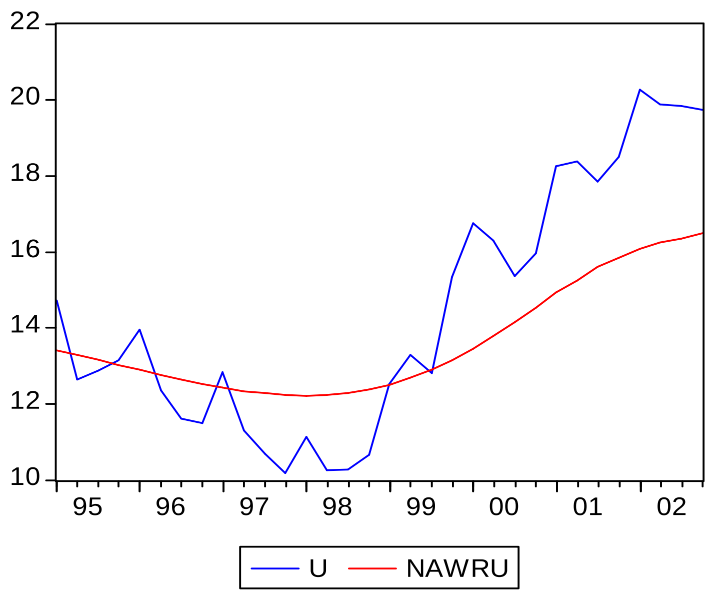
The drawback of the Elmeskov approach is that the short-term *NAWRU* generated by this method varies significantly with unemployment changes, which can be explained by the impact of not only the unemployment level, but also its changes, on wage inflation. This

[6] The assumption that NAWRU is constant between two consecutive quarters seems to be only technical. However, in many interpretations NAWRU is dependent on institutional and structural factors (por. Layard, Nickel, Jackman (1991)), which in the short run can be assumed constant. Hence, the assumption made is to some extent theory-driven.

problem is usually reduced by smoothing the series using the Hodrick-Prescott filter (cf. Giorno et al. (1995)).

The use of the HP filter gives rise to the generally known doubts connected with the choice of a right smoothing parameter and the end of sample bias. As there were no other premises, the smoothing parameter of 1600 was adopted, which is the same value as that chosen by Hodrick and Prescott (1980) and also a standard parameter used for quarterly data[7]. The problem of the beginning of the series can be easily eliminated by adding data from 1992 – 1994 to the *NAWRU* calculation. The end of the series bias is far more problematic. Thus the most recent elements of sample have been adjusted so that the average *NAWRU* level in 2002 is approximately 16%[8].

Chart 1. NAWRU in Poland, 1995-2002



Source: Own calculations.

Since the *NAWRU* value for the end of the sample is assumed arbitrarily, the estimate of the potential GDP and the output gap should also include a sensitivity analysis taking account of a possibly wrong estimate of the equilibrium unemployment. This analysis is presented later on in the text. The estimate obtained on the basis of the actual unemployment level according to BAEL is presented in Chart 1.

[7] Discussion of the effect of using smoothing parameters in the HP filter in: Canova (1993).

Estimating the potential production and the output gap

According to the methodology developed by OECD (cf. Giorno et al. (1995)), the following formula is used to calculate the potential GDP:

$$Y_t^* = f(K_t, L_t^*, TFP_t^*),$$

where Y_t^* is the potential GDP and f is the estimated production function.

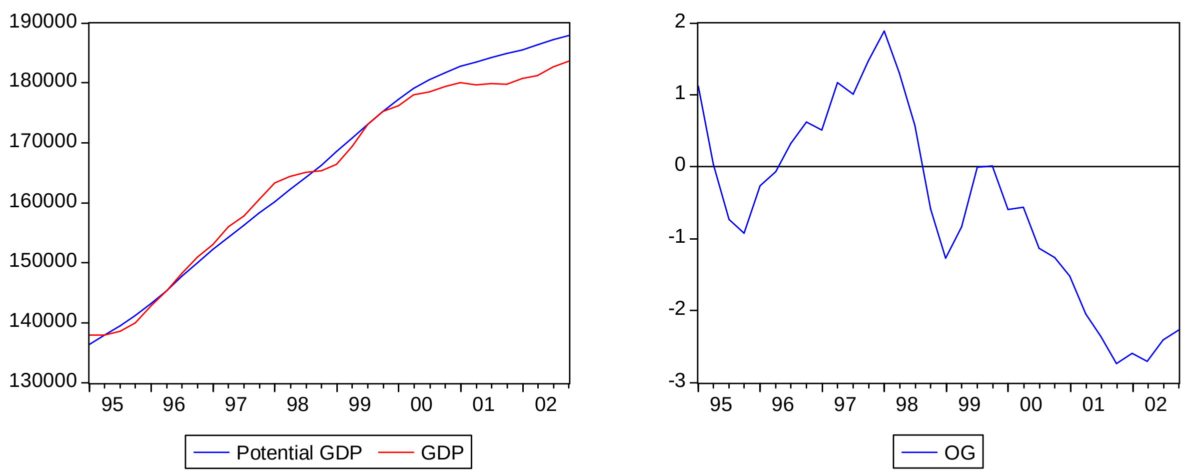
If the estimated Cobb-Douglas production function is used, the potential GDP for the Polish economy can be calculated using the formula below

$$Y_t^* = TFP_t^* L_t^{*0,57} K_t^{0,43}.$$

The estimated potential output can be used to calculate the contribution of particular factors to potential GDP growth. For a description, calculations and results see appendix A.

After the potential GDP has been estimated, it is possible to calculate the output gap as the difference between the actual and potential GDP level. The results for Poland are presented in chart 2.

Chart 2. GDP, potential GDP and the output gap as a percentage of the potential GDP in Poland (seasonally adjusted data)



Source: Own calculations.

The results of output gap estimation using a method based on the production function show that this gap was positive until the third quarter of 1998, and then it fell below 1%. In the second half of 1999, GDP was almost equal to its potential level. As from the beginning of

[8] In comparison, the NAIRU rate for Poland estimated using the SVAR method was 15% in mid-2002 and grew by about 1 percent annually. Cf. BRE Bank S.A. (2002).

2000, the output gap was negative and kept worsening until the end of 2001. Since 2002, one can observe a gradual decrease of output gap.

It has already been mentioned that the estimation of the potential GDP and the output gap depends on the estimation of the NAWRU unemployment rate. The elasticity of GDP in relation to labour input indicates that the underestimation (overestimation) of NAWRU by one percentage point decreases (increases) the potential GDP by 0.6%-0.7%. As a result, the output gap decreases (increases) by 0.6-0.7 of a percentage point.

3. Permanent-transitory decomposition of the GDP

An alternative approach to determining the potential output, used both in economic theory and empirical research, is to treat it as the long-term GDP trend. The disputed issue is the method of determining the trend, as a theoretical concept, on the basis of a time series of a relatively high frequency.

Econometric research based on the stochastic description of economic phenomena concentrates on the permanent-transitory (PT) decomposition of the product time series. In this approach, the potential product is treated as the permanent part of the output constructed by eliminating the effect of transitory disturbances.

The starting point of the analysis is a VAR-type dynamic econometric model composed of variables integrated of order one. There should be long-term relationships between the variables making up this system, consistent with the selected economic theory[9]. This condition imposes recursive cross-restrictions on the parameters on the moving average representation of the original model, leading to the representation of the vector error correction (VECM). The next step in the analysis is the PT decomposition of disturbances affecting the system. The decomposition itself and identification of both types of disturbances is made endogenously in the system, using the long-run restrictions imposed not implicitly on the VAR system by VECM. The last element of the identification process is the assumption that short- and long-run factors are independent (uncorrelated). The number and type of both sorts of disturbances is implied by the number of cointegrating relationships in the system

[9] In the econometric sense, the existence of long-term relationships means that there is a stationary linear combination of I(1) variables. A relationship like this is called a cointegrating relationship.

(permanent disturbances are called common trends). The mathematical details of the described decomposition, developed by Yang (1998), are presented in appendix B.

PT decomposition methods are often based on strong assumptions. Multivariate Beveridge-Nelson decompositions (Evans, Reichlin (1994)) or unobservable factor methods (usually solved using the Kalman filter) assume that the trend of product is a random walk process and ignore the adjustments of potential after the occurrence of a permanent shock. The potential is then an imagined level of the product achieved after all transitory adjustments have died out. However, research, using Real Business Cycle models, indicates that the transitory dynamics of permanent productivity shocks also influence potential output (Lippi, Reichlin (1994)). The method of trend determination used in this study takes into account not only the long-term impact of permanent disturbances, but also the accompanying transitory adjustments. Compared to other methods based on cointegrating relationships (incl. Cochrane (1994), Duspaquier, Guay, St-Amant (1999)), which take transitory adjustments into account, this methodology developed by Yang properly identifies the number of permanent disturbances affecting the system (which is equal to the number of common stochastic trends governing the behaviour of the whole system). In addition, this procedure does not require multiple estimations (as do the above mentioned methods), thus its relative efficiency.

The basis for the decomposition is the estimation of the VECM, in which the permanent relationship is advocated by a certain economic theory. A decision was made to estimate two systems:

- The PT-PF model, based on the long-term production function hypothesis;
- The PT-PIH model, based on the permanent income hypothesis.

The PT-PF model

The analysed dynamic system is composed of 3 variables: real GDP, the number of employed persons (data from LFS survey) and capital[10]. All variables are measured quarterly and were adjusted seasonally using the TRAMO/SEATS[11] method. The sample range is 1995-2002.

[10] Calculations made by Marcin Kolasa and Roman Sawinski, DAMS, NBP.
[11] The choice of different seasonal adjustment was an effect of the desire to get more smooth data without random deviations, which is important when using methods making use of long-run properties of time series. On the other hand, with the PF approach, the emphasis was on leaving the annual dynamics of the data intact. The different character of employed methodologies is thus the reason for estimating two production functions using different data sets.

The variables making up the system are integrated of order 1 (cf. table 1) and the economic theory defines the cointegration relationship in this system as the Cobb-Douglas production function (discussed in more detail in Chapter 2). This justifies estimating the system as a vector error correction mechanism (VECM).

A restriction of constant returns to scale was imposed on the cointegration relationship parameters (validated successfully by the likelihood ratio test at the probability level of around 0.82). The estimated GDP elasticity of labour input is 0.493 (with an error of 0.09), and product elasticity of capital input is 0.507 (with an error of 0.09). Johansen tests of rank of cointegration (trace and the maximum eigenvalue) confirm, just as the economic theory suggests, that there is one cointegrating vector for the described system (with a slope and trend in the cointegration relationship) – see table 3. The existence of one cointegration relationship was also confirmed by eigenvalues of the VAR(4) system corresponding to the analysed VECM system: two of them reached values close to 1, while the remaining eigenvalues were definitely lower, implying the existence of two common stochastic trends, which, in a three-dimensional system, means one cointegration relationship. The precise form of the long-run relationship is as follows:

$$\hat{Y}_t = L_t^{0,493} K_t^{0,507} e^{-0,055+0,009t}$$

Values of adjustment coefficients to the previous period's disequilibria (the error correction term) for labour and capital inputs have the appropriate signs, implying convergence to the long-run path, and the following respective values: –0.20 (meaning that 50% of adjustments are done after 6 quarters) and –0.05 (one half of adjustments take place in 6 years). The speed of adjustments of labour is greater than that of capital, which is consistent with the theory. Estimation results imply that the parameter measuring the strength of output adjustments to the long-run level is statistically insignificant, which is referred to as the weak exogenity of the output with respect to the cointegration vector, so the results are analogous to that obtained in the Chapter 2.

Table 3. Johansen cointegration test for the PT-PF model

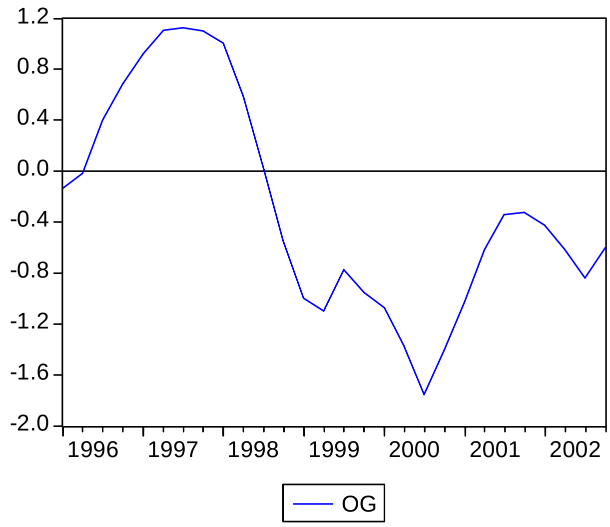
Hypothesis: number of cointegrating relationships	Eigenvalue	Trace statistic	Maximum eigen-value statistic
None	0.73	51.58***	37.06***
No more than one	0.33	14.52	11.36

*** hypothesis rejected at a 0.01 significance level

Source: Own calculations.

Because of the quarterly frequency of data used for estimation, the short-term dynamic is reflected by rates of growth to the 3rd order (this corresponds to a four lag VAR in levels). A zero restriction on lags of the 2nd order was imposed on the short-term dynamics (this was successfully validated by the Wald significance test of the regressor group). This restriction was imposed on the short-term dynamics to gain degrees of freedom while assuring the description of the seasonal pattern. All free (not implied by the restrictions of common trends) roots of the characteristic polynomial are located within a unit circle, which implies system stability. Lagrange multiplier tests of the auto-correlation indicate that there are no reasons for rejecting the hypothesis of the zero autocorrelation of disturbances up to the 12th order.

Chart 3. Output gap in the PT-PF model as a percentage of the potential



Source: own calculations

The course of the output gap in 1996-2002 is shown on chart 3. In accordance with the assumptions for the decomposition, the output gap is stationary in the econometric sense. Both the ADF non-stationary test and the complementary KPSS stationary test successfully validated the appropriate hypothesis at the 0.05 significance level.

In 1996-1998 the gap is positive and reaches a peak equal to 1.15% of the potential GDP, later the gap became negative, with a local minimum (–1.8%) in the third quarter of 2000. In 2001, the tendency of the economy to regain equilibrium was interrupted again, with another minimum at –0.8%. However, recent observations again show signs of the economy returning to equilibrium.

The comparison of systems based on production function (PF and PT-PF models)

Despite the different method of filtering data from seasonal factors, both estimated VECM systems, based on production function (PF and PT-PF models) have very similar properties. Statistical tests confirmed the equality of fundamental parameters of both models (see Table 4).

Table 4. Equality tests of models PF and PT-PF

Parameter	t statistics	Propability
Elasticity (α)	0.423	0.679
TFP dynamice	0.772	0.455
Employment equation ECT coefficient	0.001	0.999
Capital equation ECT coefficient	0.471	0.646

Source: own calculations.

Elasticity of production with respect to the labour input was 0.55 in the PF model and 0.49 in the PT-PF model. More importantly, obtained estimates are not significantly different form the share of labour costs in the gross value added (ranging from 0.5 and 0.53), used often as an approximation of this elasticity. Quarterly growth rates of TFP (assuming constant rates in order to make use of Johansen procedure), equal in both cases 1% and 0.9%, are not also statistically different. The same results arise from the analysis of the speed of equilibrium correction mechanism. In both systems product proved to be weakly exogenous.

In spite of the similarity of both systems the resulting output gaps (see chart 2 and 3) are different, especially in the period 2000-2002. This is of course the result of different methods and assumptions applied to calculate the potential. The full comparison of output gaps should be extended with another dimension – time relations with inflation, which will be shown in Chapter 4.

The PT-PIH model

The authors decided to make the Yang decomposition based on a model with different economic assumptions. The construction of this model and the character of cointegration restrictions are in line with the consumption theory, particularly with Permanent Income

Hypothesis (PIH), though the model is not an empirical verification of this hypothesis (especially in its strong version developed by Hall (1978)).

The system consists of three variables, measured quarterly (the sample range is 1995-2002) and seasonally adjusted using the TRAMO/SEATS method: the real GDP, consumption and the short-term real interest rate. These variables are integrated of order one (cf. Table 5). For GDP and consumption, the appearance of the unit root in time series is economically justified (cf. King, Plosser, Stock, Watson (1991)), but there is no clear agreement among economists as to the real interest rate.

Table 5. Results of the unit root test

Variable	DF-GLS statistic	KPSS statistics	Conclusion at 0.1 significance level
$\Delta\log(Y_t)$	-1,99**	0,09**	I(1)
$\Delta(r_t)$	-2,58**	0,25*	I(1)
$\Delta\log(C_t)$	-3,59** (trend)	0,12**	I(1)

** significant at a 0.05 significance level

Source: Own calculations

In this system, the long-term relationship is defined as a stationary consumption/GDP ratio, independent on the interest rate. Imposing a restriction on the system, that eliminates the interest rate from the long-run relationship (successfully validated by the Wald test at the 0.4 probability level), led to the estimated elasticity of consumption in relation to the product equal to 1.0076. This confirms the hypothesis of the stationary proportion of consumption to the GDP.

The results of the Johansen cointegration rank test (see Table 6) indicate that there is one cointegration relationship at the 0.08 significance level. The existence of one long-term relationship is proven by an analysis of the eigenvalues of a corresponding VAR. Two eigenvalues are close to one (equal in module to 0.96), while the other are lower, which implies that there are two common stochastic trends in the system, and hence one co-integrating relationship.

Table 6. The Johansen cointegration test for the PT-PIH model

Hypothesis: number of cointegrating relationships	Eigenvalue	Trace statistic	Maximum eigenvalue statistic
None	0.43	22.38*	15.802*
No more than one	0.207	6.58	6.51

* hypothesis rejected at a 0.1 significance level

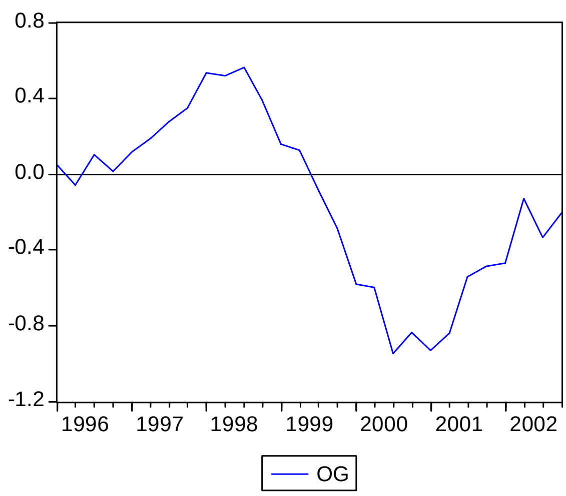
Source: Own calculations

The analysis of coefficients of the model at the error correction mechanism leads to the following conclusions:

- consumption adjusts very quickly to the equilibrium level (coefficient equal to –0.35) – more than half of the adjustments are finished after 2 quarters and 75% of the adjustments take place after 3 quarters;
- as in the previous system, the product is weakly exogenous (statistically insignificant in terms of adjustment coefficient);
- there are also strong movements in the interest rate, which does not directly participate in the error correction mechanism, but its effect is visible in the short-term dynamics of the system.

The short-term dynamics of the system were approximated by rates of growth of variables up to the 3rd order, which is consistent with the quarterly frequency of the data. On the basis of tests of the Lagrange multiplier of the joint residual auto-correlation, one can say that the adopted scheme of transitional dynamics correctly approximated the dynamics of this phenomenon, eliminating auto-correlation from disturbances (up to the 12th order). All free eigenvalues of the system are smaller than one, so the described system is stable (the variables in the system return to sustainable growth paths). The R^2 calculated for equations are as follows: 0,85 for the consumption equation, 0,58 for interest rate and 0,77 for GDP. Taking into account the fact, that the model describes dynamics of variables, one can say, that the fit is reasonable.

Chart 4. PT-PIH model output gap in percentages of the potential



Source: own calculations

The output gap generated by the PT-PIH model (see chart 4) is stationary (both the ADF and the KPSS tests at the 0.05 significance level successfully validate the hypothesis of the stationary nature of the gap). Its course in time is similar to the PF model (cf. chart 3), but the scale of imbalance is smaller (the maximum deviation of about 1% of the potential GDP). It is worth noting that the moment when the gap changes the sign is in the second quarter of 1999, it reaches minimum in 2000 and the break in the growth trend in 2002. Recent observations confirm that economy is heading towards an equilibrium path.

4. Links between the calculated output gaps and inflation

One of the applications of the output gap is the evaluation of inflationary pressure in the economy. Modelling the relationship between inflation and output gap can be based on the Phillips curve. This relation exists provided firms set prices in the Calvo (1983) style in the monopolistic competition environment, which has not been tested for the Polish economy. The following analysis assumes such a dependency exists. Moreover, definitions of the potential assumed in calculating output gaps are not theoretically the same as in the neo-keynesian Phillips curve (which is the product achieved without ridigities). The authors assumed that despite lack of strict theoretical consistency, the obtained output gap measures may influence inflation. Due to these remarks, one should treat the results carefully.

The impact of the output gap on inflation can be measured using several tools. One of them is direct estimation of the Phillips curve. However, it imposes *a priori* restrictions on the distribution of lags in the equation (in order to approximate the *forward-looking* part of the model). One way to avoid this problem is to calculate the cross-correlations between the current inflation and the lagged gap, ordered according to the increasing time-interval between the two series.

In the case of the short time series, an important limitation on using cross-correlations to assess the strength and time lag of the gap effect on inflation is the significant decrease in the precision of estimators as the delay grows. This problem can be solved by constructing a two-dimensional stochastic process generating output gap and inflation data, and then determining its covariance-generating function (cf. Hamilton (1994), pp. 261-268).

Normalisation of the elements of this function generates theoretical correlation relationships in time between the analysed time series, implied by the features of the stochastic process describing the joint development of these series.

In other words, one assumes that there exists a bivariate stochastic process, describing the dynamics of output gap and inflation (VAR type). In case of many stochastic processes, also in case of VAR process, one can analytically obtain moments of the process. The moments of the 2^{nd} order (off-diagonal elements of covariance matrix) are thus the theoretical counterparts of empirical cross-correlations

$$\Gamma_i = E[(\mathbf{y}_t - E(\mathbf{y}_t))(\mathbf{y}_{t-i} - E(\mathbf{y}_{t-i}))']\ ,$$

where E is an expectations operator and $\mathbf{y}_t$ is a vector containing output gap and inflation.

Charts 5, 6 and 7 (the horizontal axis shows the quarterly lag of the gap in comparison to the specific inflation rate) present the theoretical coefficients of correlation between the estimated output gaps and different price change measures, obtained by assuming a VAR-type data generation process. All estimated VAR systems are stable, disturbances are uncorrelated (up to 8^{th} order) and describe the variables with at least 90% fit.

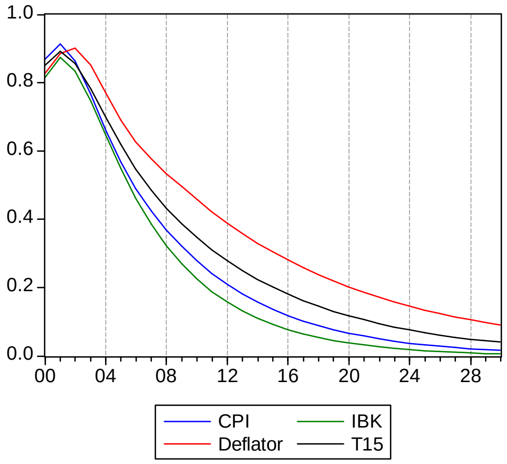
The following four measures of annual inflation were used in the analysis[12]:

- consumer price index (CPI),
- GDP deflator,
- core inflation (CPI excluded regulated prices - IBK);
- T15 – truncated distribution of prices according to volatility (15% of the most and the least volatile prices).

Theoretical correlations connected with PF model show strong connections with analysed inflation measures (see chart 5). The maximum of correlation is in the first quarter in case of CPI, IBK and T15 and at half year in case of the GDP deflator. A strong correlation (above 0,5) persists within a 1,5 year period, but in case of the GDP deflator it lasts about 2 years. This indicates a high persistence of demand factors generating inflationary pressure in the economy.

[12] One should mention that it is hard to establish empirically the measure of inflation consistent with the Phillips curve (in particular, some volatility of price indices is due to changes in relative process, which the Phillips curve does not describe).

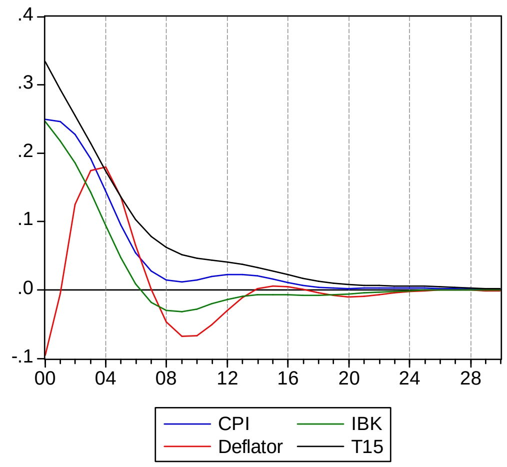
Chart 5. Correlations – the PF model



Source: own calculations

Interactions between the PT-PF output gap and inflation are weaker. The time structure of relations is also different (see chart 6). In case of inflation measured by T15 and core inflation IBK the maximum of correlation is in the current quarter and then gradually vanishes. The maximum influence on CPI is reached after one quarter and then quickly disappears. The time structure of the relationship between PT-PF output gap and the GDP deflator is different but the obtained correlations are quite small and probably statistically insignificant. It is worth noting that correlations with other measures are also quite low and inference based on this model could be inappropriate.

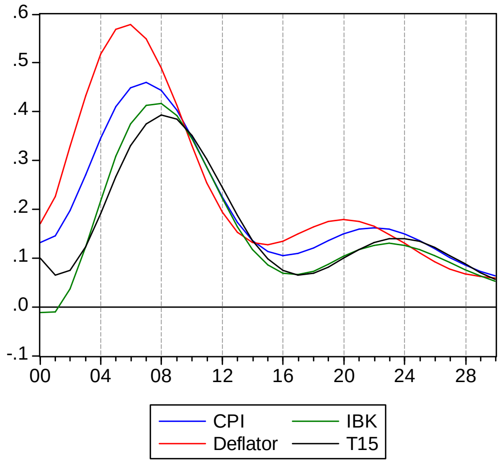
Chart 6. Correlations – the PT-PF model



Source: own calculations

In case of the PT-PIH model (see chart 7) the strength of correlation is moderate, but the time structure of relation is different. The strongest and fastest relation occurs with the GDP deflator. In all cases the maxima of correlation occurs after 5-8 quarters. The time path of correlations with the other three measures of inflation is similar, with the maximum of about 0,4.

Chart 7. Correlations – the PT-PIH model



Source: own calculations

The obtained lag structure in case of output gap calculated using PT-PIH decomposition is not in line with EU countries, in which the lag is not greater than one year (see Coenen, Wieland (2000)). The reason (mentioned before) could be that the definition of the potential is not in line with the potential used in Phillips curve. But when one assumes that such a great lag between output gap and inflation is correct, the changes in output gap resulting from the PT-PIH model influences inflation after over a year.

5. Summary

The alternative methods of estimating the output gap presented above differ with regard to the concept and method of calculation. The potential GDP estimated using the production function approach can be viewed as the supply-side of the economy, i.e. the GDP level corresponding to long-term inputs of production factors. On the other hand, methods based on the permanent-transitory decomposition of GDP use long-term relationships between macroeconomic aggregates and yield potential GDP which is a product of accumulated shocks.

Consequently, it is not surprising that these alternative methods lead to different estimates of the output gap. Moreover, the series obtained are not theoretically equivalent, hence the differences in their time relationships with inflation. The development of the gaps and the analysis of their impact on inflation (conditional on the existence of the relationship described by the Phillips curve and the possibility of using the gaps obtained in it) show the lack of any inflationary pressure from the demand side, which may be the case till the end of 2003. In view of relatively strong assumptions made during the estimation process and time relationships analysis, caution is recommended while drawing any conclusions.

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Appendix

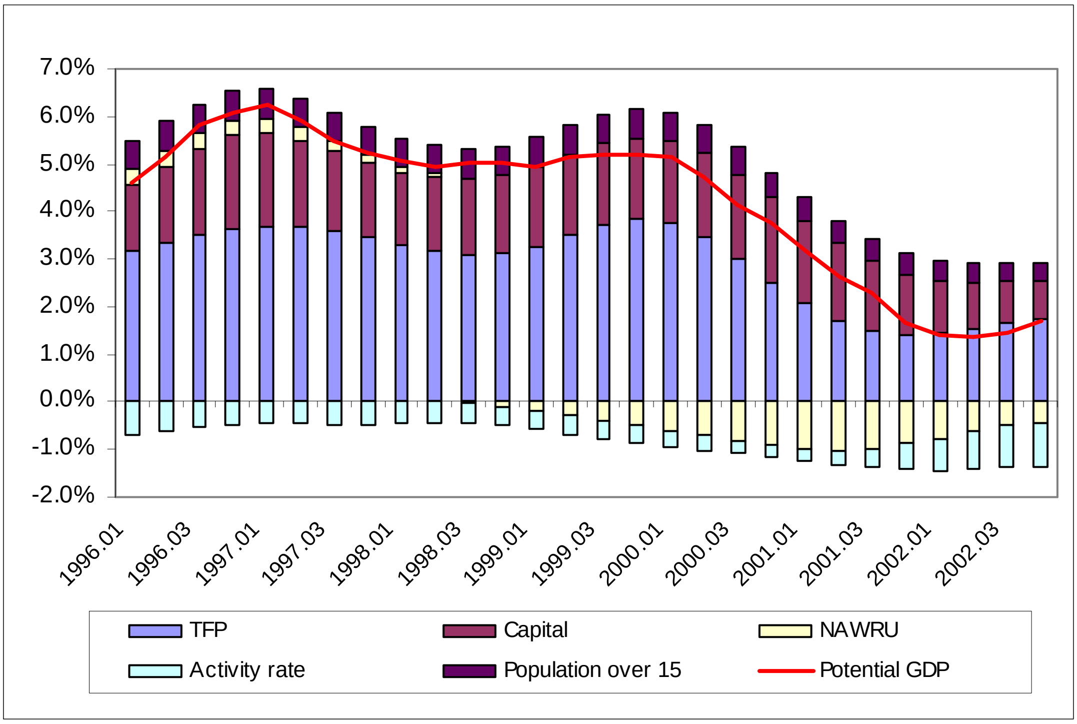
A. Decomposition of the potential GDP growth in Poland

One of the advantages of the method based on the production function is that it can be used to calculate the contribution of particular factors to potential GDP growth. If the professional activity rate is defined as the proportion of the labour force to the number of inhabitants older than 15 and if the estimated production function is used, the rate of potential GDP growth can be broken down into five factors using the following formula:

$$\Delta \log(PKB^*) = \Delta \log(TFP^*) + 0{,}57\Delta \log(\bar{L}) + 0{,}57\Delta \log(s) + 0{,}57\Delta \log(1 - NAWRU) + 0{,}43\Delta \log(K)$$

where: $\bar{L}$ is the number of inhabitants older than 15 and s is the professional activity rate defined above.

Chart 8. Decomposition of the potential GDP growth (year on year) in Poland



Source: Own studies.

Chart 8 shows that long-term GDP growth in Poland in 1995-2002 was mainly driven by the increasing total factor productivity. In every quarter of this period, the share of TFP in the potential GDP growth exceeded 60%, or even 90% in 2002. Between 1995-2002, the potential GDP increased on average by 2.9 percentage points a year due to TFP growth. Over the entire period analysed, the contribution of capital growth and growth of the population older than 15 were also positive, and amounted, respectively, to 1.6 and 0.6 of a percentage point. The breakdown reveals unfavourable trends on the Polish labour market. As a result of the decreasing professional activity rate, the long-term GDP growth rate was 0.5 percentage points lower every year. Since the end of 1998, the NAWRU unemployment level has been growing in Poland, slowing the potential GDP growth in this period by 0.6 percentage points a year on the average. The growing NAWRU can be interpreted as a symptom of unemployment hysteresis.

B. Permanent-transitory decomposition (Yang (1998))

Consider an *n*-dimensional stochastic process $\{x_t\}$ of integrated variables and its error correction (VECM) representation:

$$\Delta x_t = \delta + \theta x_{t-1} + \sum_{i=1}^{p} \Pi_i \Delta x_{t-i} + \varepsilon_t \tag{1}$$

where $\Delta = 1 - L$ is the difference operator and *L* is the lag operator, δ is deterministic part of the model, θ and Π_i are matrices of parameters. ε_t is a vector of random disturbances from individual equations with variance-covariance matrix Ω. The system (1) could be estimated by the ML procedure developed by Johansen (1988). In case of *r* cointegrating relations, tested by Johansen procedures, θ has a reduced rank and can be decomposed as $\theta = \alpha\beta'$ where *α* and *β* are full-rank $(n \times r)$ matrices. Columns of *β* are cointegrating relations between the variables of the system and elements of *α* are system reactions to previous period disequilibria. The system should converge monotonically to the long-run relationship $\beta' x_t$ with an adjustment rate given in α. Defining $\Pi(L) = I_n - \sum_{i=1}^{p} \Pi_i L^i$ and $A(L) = \Pi(L)(1-L) + \theta L$ one can convert VECM into a corresponding VAR representation: $A(L)x_t = \delta + \varepsilon_t$.

As elements of x_t are I(1), the Wold theorem assures that its first differences have an infinite Vector Moving Average representation (VMA), showing the way disturbances of previous periods affect the current value of variables:

$$\Delta x_t = C(L)(\delta + \varepsilon_t) = \mu + \varepsilon_t + C_1\varepsilon_{t-1} + C_2\varepsilon_{t-2} + \dots \tag{2}$$

where $\mu = C(1)\delta$ is a deterministic part, $C(1) = \sum_{i=0}^{\infty} C_i$ is a sum of all short-run multipliers and the matrix polynomial is of the form: $C(L) = \sum_{i=0}^{\infty} C_i L^i$ with normalization $C(0) = \mathrm{I}_n$.

Engle and Granger (1987) showed that by defining $C^*(L) = (C(L) - C(1))(1 - L)^{-1}$ equation (2) can be represented as:

$$\Delta x_t = C(L)(\delta + \varepsilon_t) = \mu + C(1)\varepsilon_t + C^*(L)\Delta\varepsilon_t \qquad (3)$$

In case of C(1) being of reduced rank *k* (*k* < *n*), there are *r*=*n* – *k* cointegrating relations. C(1) can be decomposed as: $C(1) = hg'$ where *h* and g are $(n \times k)$ matrices. Equation (3) shows the decomposition of the matrix polynomial C(*L*) into a permanent part C(1) and a transitory lag distribution $C^*(L)\Delta\varepsilon_t$. It is clear that there are only *k* linear combinations of disturbances permanently affecting x_t - they are of the form $g'\varepsilon_t$.

Johansen (1992) showed that $C(1) = \beta_+ (\alpha_+^{\mathrm{T}} (\mathrm{I} - \sum_{i=1}^{p} \Pi_i) \beta_+)^{-1} \alpha_+^{\mathrm{T}}$ where α_+ and β_+ are orthogonal complements[13] to corresponding matrices. Equation (3) can be then presented as:

$$\Delta x_t = \mu + \beta_+ (\alpha_+^{\mathrm{T}} (\mathrm{I}_{\mathrm{n}} - \sum_{i=1}^{p} \Pi_i) \beta_+)^{-1} \alpha_+^{\mathrm{T}} \varepsilon_t + C^*(L)\Delta\varepsilon_t \qquad (4)$$

with $\alpha_+^{\mathrm{T}} \varepsilon_t$ being a group of permanent shocks and matrix $\beta_+ (\alpha_+^{\mathrm{T}} (\mathrm{I} - \sum_{i=1}^{p} \Pi_i) \beta_+)^{-1}$ showing their long-run impact on the variables creating the system.

In order to obtain the components of x_t, that are created by permanent disturbances, taking into account not only their long-run, but also short-run impact, one should find a connection between long-run shocks $\alpha_+^{\mathrm{T}} \varepsilon_t$ and the MA representation of (2). The stochastic component $C(L)\varepsilon_t$ can be divided into 2 components, of which one is the permanent one $\alpha_+^{\mathrm{T}} \varepsilon_t$:

[13] Orthogonal complement of $(n \times r)$ matrix α is a $(n \times n - r)$ matrix α_+ that satisfy the relation $\alpha^{\mathrm{T}} \alpha_+ = 0$. It's a matrix which columns form a basis of subspace, which is orthogonal to the subspace generated by columns of α matrix.

$$C(L)\varepsilon_t = C(L)\overline{\alpha}_+ \cdot \alpha_+^{\mathrm{T}} \varepsilon_t + C(L)\overline{\gamma} \cdot \gamma^{\mathrm{T}} \varepsilon_t \tag{5}$$

where γ is any $(n \times r)$ matrix chosen to assure that $[\alpha_+ \quad \gamma]$ is invertible and satisfying the equation[14]: $[\overline{\alpha}_+ \quad \overline{\gamma}] = \{[\alpha_+ \quad \gamma]^{\mathrm{T}}\}^{-1}$. Representation showed in (5) assumes that multipliers of permanent and transitory disturbances are linear combinations of multipliers of model (3). The dynamic influence of shocks on variables forming x_t was then divided into disturbances coming from the long-run and short-run. To finish the decomposition one must choose matrix γ. Assuming that permanent disturbances $\alpha_+^{\mathrm{T}} \varepsilon_t$ and transitory ones $\gamma^{\mathrm{T}} \varepsilon_t$ are orthogonal (independent), which is desired in impulse-response analysis, one can get the following relation defining γ:

$$\gamma = \alpha - \alpha_+ (\alpha_+^{\mathrm{T}} \Omega \alpha_+)^{-1} \alpha_+^{\mathrm{T}} \Omega \alpha \tag{6}$$

Permanent components of vector x_t could be obtained as a deterministic part of equation (3) with the whole stochastic component of permanent disturbances:

$$x_t^* = \mu + (C(L)\overline{\alpha}_+)\alpha_+^{\mathrm{T}} \varepsilon_t$$

Thus x_t^* is defined to be that part of x_t ,that was generated by only permanent disturbances, so according to our definition, it could be treated as trends of elements of vector x_t.

[14] It is the solution to the equation of the form: $\overline{\alpha}_+ \cdot \alpha_+^T + \overline{\gamma} \cdot \gamma^T = I_n$