



Munich Personal RePEc Archive

# **Optimal Monetary Policy in a Dual Labor Market Economy**

Lorenza Rossi and Fabrizio Mattesini

Universita Cattolica del Sacro Cuore, Milano

9. January 2007

Online at <http://mpa.ub.uni-muenchen.de/2827/>

MPRA Paper No. 2827, posted 19. April 2007

# Optimal Monetary Policy in Economies with Dual Labor Markets

Fabrizio Mattesini

*Università di Roma "Tor Vergata"  
Via Columbia 2, 00133 Rome, Italy*

Lorenza Rossi\*

*Istituto di Economia e Finanza,  
Università Cattolica del Sacro Cuore.  
Via Necchi 5, 20123 Milano, Italy*

March 2007

## Abstract

We analyze, in this paper, DSNK general equilibrium model with indivisible labor where firms may belong to two different final goods producing sectors: one where wages and employment are determined in competitive labor markets and the other where wages and employment are the result of a contractual process between unions and firms. The presence of monopoly unions introduces real wage rigidity in the model and this implies a trade-off between output stabilization and inflation stabilization i.e., as in Blanchard and Galí (2005), the so called "divine coincidence" does not hold. We show that the negative effect of a productivity shock on inflation and the positive effect of a cost-push shock is crucially determined by the proportion of firms that belong to the competitive sector. The larger is this number, the smaller are these effects. We derive a welfare based objective function as a second order Taylor approximation of the expected utility of the economy's representative agent and we analyze optimal monetary policy under discretion and under constrained commitment. We show that the larger is the number of firms that belong to the competitive sector, the smaller should be the response of the nominal interest rate to exogenous productivity and cost-push shocks. If we consider, however, an instrument rule where the interest rate must react to inflationary expectations, the rule is not affected by the structure of the labor market. The results of the model are consistent with a well known empirical regularity in macroeconomics, i.e. that employment volatility is larger than real wage volatility.

---

\*Corresponding author. Tel.: +39 02 72342696. E-mail Address: lorenza.rossi@unicatt.it

# 1 Introduction

One important difference between the U.S. and the European labor markets, is the role that trade unions play in determining wages and employment. While in the U.S., in the year 2002, only about 15% of workers were covered by collective contract agreements<sup>1</sup>, in countries such as France, Italy or Sweden the percentage of workers covered by collective contracts was above 84%.<sup>2</sup> This is by no means the only difference between the labor markets on the two sides of the Atlantic, since European economies have also other sources of rigidity like higher employment protection and greater unemployment benefits<sup>3</sup>; but nevertheless is a very striking difference which, in our opinion, deserves careful consideration, especially if one tries to derive policy prescriptions. What are the consequences of the different weight of trade unions for monetary policy? How should the Fed and the ECB behave in response to exogenous, persistent shocks? Should these two central banks follow different policy rules?

In order to answer these important questions we propose, in this paper, a Dynamic Stochastic General Equilibrium New Keynesian (DSGE-NK) model where firms may belong to two different final-goods producing sectors: one where wages and employment are determined in a competitive labor market and the other where wages and employment are the result of a contractual process between unions and firms. In order to evaluate movements of labor along the extensive margin, we assume, as in Hansen [22] and Rogerson and Wright [31], that labor supply is indivisible and that workers face a positive probability to remain unemployed; as in Maffezzoli [25], we assume that unions set wages according to the popular monopoly-union model introduced by Dunlop [12] and Oswald [28]. By doing this we depart, in this paper, from the recent literature, that has recently tried to improve on the "standard" DNK model by considering explicitly the role of labor market frictions for monetary policy. While these papers, among which we find Chéron and Langot [7], Walsh [35] [36], Trigari [33], [34], Moyen and Sahuc [27] and Andres et al. [2] and, more recently by Christoffel and Linzert [9] and Blanchard and Galí [4] [5], analyze monetary policy in search and matching models of the labor market à la Mortensen-Pissarides ([26]), we chose to introduce unemployment in the simpler Rogerson and Wright [31] framework and to concentrate on the fact that collective bargaining between unions and firms may give rise to real wage rigidity.<sup>4</sup>

---

<sup>1</sup>The concept of union coverage, i.e. workers covered by collective contracts as a percentage of total employment is a better indicator of the role of trade unions than the concept of union density, i.e. the percentage of workers that belong to a union. This is because in many countries collective contracts signed by unions and representative of firms are binding also for non-members.

<sup>2</sup>More precisely, the number of persons covered by collective agreements over total employment was 94.5% in France in 2003, 84.1% in Italy in the year 2000 and 85.1% in Sweden in the year 2000. For a complete set of data on union coverage on the various countries see Lawrence and Ishikawa [24].

<sup>3</sup>Also these characteristics of the European labor markets can be probably be reconduced to the crucial role that trade unions play in the social structure of European countries.

<sup>4</sup>Also Christoffel and Linzert [9] and Blanchard and Galí [4] [5] consider explicitly real wage rigidity, but in their models real wage rigidity is simply assumed and it is not derived

The fact that monopoly unions tend to keep real wage rigid, allows us to consider an economy composed of two sectors: one where real wages do not adjust in response to productivity shocks and one where real wages adjust; the relative weight of the unionized and competitive sectors can be taken as a measure of the extent of real wage rigidity in the economy. As was recently shown by Blanchard and Galì [4], real wage rigidity has a very important consequence for monetary policy since in this case what Blanchard and Galì define as the "divine coincidence" does not generally hold : for a central bank stabilizing output around the level that would prevail under flexible prices (natural output) is not equivalent to pursuing the efficient level of output, in which case a trade-off between inflation stabilization and output gap stabilization arises. The reason is simple: a productivity slowdown, i.e. a negative productivity shock, tends to lower efficient output but, since in the unionized sector real wages remain constant, the natural level of output that would prevail under price flexibility decreases even more, so that the difference between efficient output and "natural" output increases. In sticky price models inflation depends on marginal costs and, in turn, marginal costs depend on the difference between "natural" output and actual output; as a consequence a Phillips curve, correctly defined as depending on the gap between efficient output and actual output, will depend on productivity shocks. A central bank that tries to fully accommodate a negative productivity shock will have to accept higher inflation.

One important result of our model is that the trade-off between inflation stabilization and output stabilization that arises endogenously in the economy is significantly affected by the relative weight of the unionized and the competitive sectors: the larger is the fraction of firms that are able to set wages in a competitive labor market, the smaller is the trade-off they face in response to productivity shocks. This has significant consequences for optimal monetary policy, that in our model is derived, as in Woodford [37], from the maximization by the central bank of a second order approximation of agents' utility function. In an economy where unions are not very important the nominal interest rate should change much less in response to a productivity shock than in an economy where wages are largely set by collective bargaining between unions and firms. The larger is the fraction of firms that set wages in competitive labor markets, the smaller is the effect of productivity shocks on inflation, and therefore the smaller the need to increase interest rates to prevent an increase in the rate of inflation.

If we turn however to an instrument rule for monetary policy, i.e. to an interest rate rule defined in terms of inflation and the rate of interest that supports the efficient allocation, we find that the structure of labor markets should not influence the response of the nominal interest rate to inflationary expectations. In general the Taylor principle will apply, i.e. nominal interest rate should increase more than proportionately with respect to inflation, but the type of response will not be affected the fraction of firms that set wages in a competitive labor markets. Therefore, if we consider two countries hit

---

as a consequences of the institutional structure that characterizes the labor market.

by the same shocks and where the central bank behaves optimally, we will observe that in the country where the number of "walrasian" firms is larger the interest rate will vary much less than in the other country. This, however, will not be the consequences of differences in the reaction of the two central banks to a unit change in expected inflation; rather it will be caused by the fact that the economy where labor markets are more competitive will experience smaller inflationary tensions. It is interesting to observe that, if one looks at the monetary policy of the Fed and of the ECB from January 1999 (when the ECB started to operate) we find a much larger variability of the policy rate in the U.S. than in the Euro area; according to our model, this seems to indicate that either the two areas were not hit by similar shocks, or the two Central banks did not behave optimally.

Besides analyzing the response of the economy to productivity shock, our model provides also a convenient framework to address important normative issues such as, for example, the optimal behavior of central banks in periods characterized by labor market turmoil and exogenous wage shocks. In the framework we propose here, in fact, a policy trade-off for the central bank arises also in response to exogenous changes in the unions' reservation wage, that we interpret as cost push shocks. If the unions' reservation wage is subject to exogenous changes, and these changes tend to be persistent over time, then a welfare maximizing central bank must again face the problem of whether to accommodate these shocks with a easier monetary policy. As in the case of technology shocks, also in this case optimal monetary policy requires only partial accommodation and the response of the central bank is crucially determined by the fraction of firms that, in the economy, set wages in competitive labor markets.

One last, important result is that the model is able to account for a well known stylized fact in macroeconomics, i.e. the relatively smooth behavior of wages and the relatively volatile behavior of unemployment over the business cycle. When the level of unemployment that the economy achieves under an optimal discretionary policy is written as a function of the relevant shocks, an exogenous wage shock will in general induce a movement both in the real wage and in the rate of unemployment; a productivity shock, instead, will induce a movement in the rate of unemployment, but not in the real wage. An economy frequently hit by exogenous changes in technology will show, therefore, a strong variability in the rate of unemployment without experiencing, at the same time, significant movements in the real wage.<sup>5</sup>

The paper is organized as follows. In Section 2 we start by introducing indivisible labor in a DNK model. In Section 3 we develop the two-sectors labor market model. In Section 4 we study optimal monetary policy and, finally, in Section 4 we calibrate the model under the optimal rule and some simpler policy rules.

---

<sup>5</sup> Also Gertler and Trigari [19] propose a model where wages and unemployment move consistently with the observed data. They achieve this result, however, by introducing exogenous multiperiod wage contracts.

## 2 A model with indivisible labor

### 2.1 The Representative Household

We consider an economy populated by many identical, infinitely lived worker-households each of measure zero. Households demand a Dixit, Stiglitz [11] composite consumption bundle produced by a continuum of monopolistically competitive firms. In each period households sell labor services to the firms and each firm is endowed with a pool of households from which it can hire. As a matter of fact firms hire workers from a pool composed of infinitely many households so that the individual household member is again of measure zero. Since each household supplies its labor only to one firm, which can be clearly identified, workers try to extract some producer surplus by organizing themselves into a firm specific trade unions. As organizing in a union is costly, we assume that workers, each time, succeed in organizing in a union only in  $1 - q$  firms, while in the remaining  $q$  firms they do not succeed and labor markets remain competitive. Given the structure of the economy,  $q$  not only represents the number of firms that face a walrasian labor market but also the probability that a worker is assigned to the walrasian sector. Once a household is assigned to a firm specific sector, as in Hansen [22], Rogerson [31] and Rogerson and Wright [32], it has the alternative between working a fixed number of hours and not working at all. For the sake of simplicity we assume that  $q$  is constant.

Let us first consider the problem of an agent that supplies his labor to a firm in the walrasian sector, i.e. to a firm that faces a competitive labor market where firms and workers act as a price taker. We assume that households enter employment lotteries, i.e. sign with a firm a contract that commits them to work a fixed number of hours, that we normalize to one, with probability  $N_t^w$ . Since all households are identical, they will all choose the same contract, i.e. the same  $N_t^w$ . However, although households are ex-ante identical, they will differ ex-post depending on the outcome of the lottery: a fraction  $N_t^w$  of the continuum of households will work and the rest  $1 - N_t^w$  will remain unemployed. The allocation of individuals to work or leisure is determined completely at random by a lottery, and lottery outcomes are independent over time. Before the lottery draw, the expected intratemporal utility function is:

$$N_t^w [C_{0,t}^w v(0)]^{1-\sigma} + (1 - N_t^w) [C_{1,t}^w v(1)]^{1-\sigma} \quad (1)$$

where  $C_{0,t}^w$  is the consumption level of employed individuals. We denote by  $v(\cdot)$  the utility of leisure. Since the utility of leisure of employed individuals  $v(0)$  and the utility of leisure of unemployed individuals  $v(1)$  are positive constants, we assume  $v(0) = v_0$  and  $v(1) = v_1$ . As in King and Rebelo<sup>6</sup> [21], we assume  $v_0 < v_1$ .

Since they face a probability  $1 - N_t^w$  of not working at all, workers will try to acquire insurance against the risk of remaining unemployed. We assume that

---

<sup>6</sup>This depends on the fact that the utility of leisure  $\phi(1 - N_t)$  as usual, is an increasing function of the time spend in leisure. Given that the time spend in leisure is greater for unemployed agent than for employed agent this means that  $v(1) > v(0)$ .

asset markets are complete, so that employed and unemployed individuals are able to achieve perfect risk sharing, equating the marginal utility of consumption across states.

Let us now consider the case of a household that works in a unionized labor market. The unionized sector is populated by decentralized trade unions, so that each intermediate goods-producing firm negotiates with a single union  $i \in (0, 1)$ , which is too small to influence the outcome of the market. Unions negotiate the wage on behalf of their members. Once the wage rate is defined, firms choose the amount of labor that maximizes their profits. Similarly to what happens in the competitive case, labor is indivisible and workers participate to employment lotteries. As in the previous case, therefore, before the lottery draw, the expected intratemporal utility function of workers, who happen to belong to the unionized sector is

$$N_t^u [C_{0,t}^u v(0)]^{1-\sigma} + (1 - N_t^u) [C_{1,t}^u v(1)]^{1-\sigma} \quad (2)$$

where  $C_{0,t}^u$  is the consumption level of employed individuals. Again, we assume  $v(0) = v_0$  and  $v(1) = v_1$ .

Since they face a positive probability of being unemployed, risk averse workers will try to obtain insurance against the risk of being unemployed; access to complete asset markets will allow individuals to achieve perfect risk sharing. It is important to observe that, beside the risk of remaining unemployed, workers in this model face also another type of uncertainty since they do not know, a priori, whether they will participate to a competitive labor market or to a unionized one. We assume that, through complete asset markets, agents can also acquire insurance against the income fluctuations implied by this type of uncertainty. Recalling that  $q$  is the probability of belonging to the walrasian sector and  $1 - q$  is the probability of belonging to the unionized sector, before the lotteries are drawn and before learning in what sector they will happen to work, given (1) and (2) the expected intratemporal utility function of an household is:

$$\frac{1}{1-\sigma} \left\{ \begin{aligned} & q N_t^w [C_{0,t}^w v_0]^{1-\sigma} + q(1 - N_t^w) [C_{1,t}^w v_1]^{1-\sigma} + \\ & +(1 - q) N_t^u [C_{0,t}^u v_0]^{1-\sigma} + (1 - q)(1 - N_t^u) [C_{1,t}^u v_1]^{1-\sigma} \end{aligned} \right\} \quad (3)$$

Perfect risk sharing implies,

$$\begin{aligned} (C_{0,t}^w)^{-\sigma} v(0)^{1-\sigma} &= (C_{1,t}^w)^{-\sigma} v(1)^{1-\sigma} \\ (C_{0,t}^u)^{-\sigma} v(0)^{1-\sigma} &= (C_{1,t}^u)^{-\sigma} v(1)^{1-\sigma} \\ (C_{0,t}^w)^{-\sigma} v(0)^{1-\sigma} &= (C_{0,t}^u)^{-\sigma} v(0)^{1-\sigma} \\ (C_{1,t}^w)^{-\sigma} v(1)^{1-\sigma} &= (C_{1,t}^u)^{-\sigma} v(1)^{1-\sigma} \end{aligned} \quad (4)$$

which imply

$$\begin{aligned} C_{0,t}^u &= C_{0,t}^w = C_{0,t} \\ C_{1,t}^u &= C_{1,t}^w = C_{1,t} \end{aligned} \tag{5}$$

As we show in Appendix 1, this allows us to write the life-time expected intertemporal utility function of a representative household as:

$$U_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{1}{1-\sigma} [C_t \phi(N_t)]^{1-\sigma}, \tag{6}$$

where  $0 < \beta < 1$  is the subjective discount rate and where

$$N_t = qN_t^w + (1-q)N_t^u \tag{7}$$

is the probability to be employed.

The flow budget constraint of the representative household is given by:

$$P_t C_t + P_t^{-1} B_{t+1} \leq qW_t^w N_t^w + (1-q)W_t^u N_t^u + B_t + \Pi_t - T_t \tag{8}$$

where  $W_t^h$ ,  $h = w, u$  is the wage rate in the two sectors, and

$$P_t = (P_t^w)^q (P_t^u)^{1-q} \tag{9}$$

is the corresponding consumption price index (CPI) obtained<sup>7</sup> as in the standard setup, as the price that minimizes total consumption expenditure subject to  $C_t = 1$ . The purchase of consumption goods,  $C_t$ , is financed by labor income, profit income  $\Pi_t$ , and a lump-sum transfers  $T_t$  from the Government. We assume that agents can also have access to a financial market where nominal bonds are exchanged. We denote by  $B_{t+1}$  the holdings of a nominal bond carried over from period  $t$  that pays one unit of currency in period  $t + 1$ . In solving the maximization of (6) subject to (8) we should remember that the worker chooses the levels of consumption  $C_t$  and  $C_{t+1}$  and the supply of labor  $N_t^w$ , while  $N_t^u$  is taken as given, as it is determined by the union together with the firm. The first order conditions imply,

$$1 = \beta R_t E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{\phi(N_t)}{\phi(N_{t+1})} \right)^{1-\sigma} \frac{P_t}{P_{t+1}} \right] \tag{10}$$

$$\frac{W_t^w}{P_t} = -C_t \frac{\phi_{N^w}(N_t)}{\phi(N_t)} = -C_t q \frac{\phi_N(N_t)}{\phi(N_t)} \tag{11}$$

where equation (10) is the standard consumption Euler equation. Equation (11) holds only for households employed in the walrasian sector.

<sup>7</sup>This is derived in appendix A2.



For households hired by firms in the Unionized sector, Unions negotiate the wage on behalf of their members. In particular, we consider a version of the well-known monopoly union model introduced by Dunlop [12] and Oswald [28]. The union chooses the nominal wage rate that maximizes the following welfare function<sup>8</sup>:

$$N_t(i) \frac{W_t^u}{P_t} + (1 - N_t(i)) \frac{W_t^r}{P_t} \quad (12)$$

subject to the labor demand of the unionized sector.  $W_t^r$  is the reservation wage, which can be broadly interpreted as the disutility of labor as perceived by the union and represents both the value union members assign to leisure and any unemployment subsidy granted by the government. With equation (12) we assume that unions are risk neutral and maximize members average wage. We assume that the reservation wage, which represents the disutility of employment perceived by the unions, follows a stochastic process. Denoting by  $w_t^r$  the logarithm of  $W_t^r$  we assume that:

$$w_t^r = \rho_w w_{t-1}^r + \hat{w}_t^r \quad (13)$$

where  $\rho_w < 1$  and  $w_t^r$  is a normally distributed serially uncorrelated innovation with zero mean and standard deviation  $\sigma_w$ .

The employment rate and the wage rate are determined in a non-cooperative dynamic game between the unions and the firms. We restrict the attention to Markov strategies, so that in each period unions and firms solve a sequence of independent static games. Each union behaves as a Stackelberg leader and each firm as a Stackelberg follower. Once the wage has been chosen, each firm decides the employment rate along its labor demand function. Even if unions are large at the firm level, they are small at the economy level, and therefore they take the aggregate wage as given. The ex-ante probability of being employed is equal to the aggregate employment rate and the allocation of union members to work or leisure is completely random and independent over time.

From the first order conditions of the union's maximization problem with respect to  $W_t(i)$  we have:

$$\frac{W_t^u(i)}{P_t} = \frac{1}{\alpha} \frac{W_t^r}{P_t}. \quad (14)$$

Since  $\frac{1}{\alpha} > 1$ , this implies that the real wage rate is set always above the reservation wage.

Finally we assume that total consumption is a geometric average of consumption of the total consumption of the walrasian good,  $C_{w,t}$ , and of total consumption of the Unionized good  $C_{u,t}$ . Then,

$$C_t = \frac{(C_{w,t})^q (C_{u,t})^{1-q}}{q^q (1-q)^{1-q}}. \quad (15)$$

---

<sup>8</sup> A complete discussion of the problem of defining a union's objective function can be found in Farber [13]. In assuming this utility function we follow Maffezzoli [25]. The utility function above corresponds to the risk neutral analogue of the utilitarian utility function of Oswald [28]. Anderson and Deverux ([1]) and Pissarides ([29]) use a similar utility function.

## 2.2 The Two Representative Final Goods-Producing Firms

We begin describing the production of final goods. In each sector ( $h = w, u$ ) a perfectly competitive final good producer purchases a  $Y_t^h(j)$  units of each intermediate good  $j \in [0, 1]$  at a nominal price  $P_t^h(j)$  to produce  $Y_t^h$  units of the final good (labeled as  $w$  and  $u$ ) with the following constant returns to scale technology:

$$Y_t^w = \left[ \int_0^q \frac{1}{q} Y_t^w(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad \text{and} \quad Y_t^u = \left[ \int_q^1 \frac{1}{1-q} Y_t^u(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad (16)$$

where  $\theta > 1$  is the elasticity of substitution across intermediate goods, which is equal across the two sector. Profit maximization yields the following set of demands for intermediate goods:

$$Y_t^w(j) = \left( \frac{P_t^w(j)}{P_t^w} \right)^{-\theta} Y_t^w \quad \text{and} \quad Y_t^u(j) = \left( \frac{P_t^u(j)}{P_t^u} \right)^{-\theta} Y_t^u \quad (17)$$

for all  $i$ . In Appendix A2 we show that

$$P_t^w = \left[ \int_0^q \frac{1}{q} P_t^w(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \quad \text{and} \quad P_t^u = \left[ \int_q^1 \frac{1}{1-q} P_t^u(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}. \quad (18)$$

is the price indexes of the walrasian and unionized sectors.

## 2.3 The Two Representative Intermediate Goods-Producing Firms

We abstract from capital accumulation and assume that representative intermediate good-producing firm  $j$  in sector  $h$ , hires  $N_t^h$  units of labor from the household and produce  $Y_t^h(j)$  units of the intermediate good using the following technology:

$$Y_t^h(j) = A_t N_t^h(j)^\alpha \quad (19)$$

where  $A_t$  is an exogenous productivity shock common to all firms. We assume that the  $\ln A_t \equiv a_t$  follows the autoregressive process

$$a_t = \rho_a a_{t-1} + \hat{a}_t \quad (20)$$

where  $\rho_a < 1$  and  $\hat{a}_t$  is a normally distributed serially uncorrelated innovation with zero mean and standard deviation  $\sigma_a$ .

Before choosing the price of its goods, a firm chooses the level of  $N_t^h(j)$  which minimizes its total costs, solving the following standard costs minimization problem:

$$\min_{\{N_t\}} TC_t = (1 - \tau^h) \frac{W_t^h}{P_t^h} N_t^h(j) \quad (21)$$

subject to (19), where  $\tau^h$  represents an employment subsidy to the sector  $h$  firm, which is set such that the steady state equilibrium in both sectors coincides with the efficient one. The first order condition with respect to  $N_t^h(j)$  is given by:

$$(1 - \tau^h) \frac{W_t^h(j)}{P_t^h(j)} = \frac{MC_t^{n,h}(j)}{P_t^h(j)} \alpha \frac{Y_t^h(j)}{N_t^h(j)}, \quad (22)$$

where  $MC_t^{n,h}(j)$  represents the nominal marginal costs of firm  $j$  in sector  $h$  while  $MC_t^h(j)$  represents firm's  $j$  real marginal costs. Defining sector  $h$  aggregate real marginal costs as:

$$MC_t^h \equiv \frac{MC_t^{n,h}}{P_t^h} = \frac{(1 - \tau^h) W_t^h N_t^h}{\alpha P_t^h Y_t^h} \quad (23)$$

equation (22) implies,

$$MC_t^h = \frac{MC_t^{n,h}(j) Y_t^h(j) N_t^h}{P_t^h(j) N_t^h(j) Y_t^h}. \quad (24)$$

## 2.4 Market Clearing

Equilibrium in the goods market requires that the production of the final good be allocated to expenditure, as follows:

$$Y_t^h(j) = C_{h,t}(j) \quad h = w, u \quad (25)$$

where  $C_{h,t}$  is the total consumption of the good produced by sector  $h$ . The market clearing condition for the two final consumption goods output therefore are given by:

$$Y_t^w = \int_0^q \frac{1}{q} C_{w,t}(j) dj \quad \text{and} \quad Y_t^u = \int_q^1 \frac{1}{1-q} C_{w,t}(j) dj \quad (26)$$

considering (16) then

$$Y_t = C_t \quad (27)$$

which represents the aggregate economy's resource constraint. Defining by  $X$  the steady state value of a generic variable  $X_t$  and by  $x_t = \ln X_t - \ln X$  the log-deviation of the variable from its steady state value, then a linear first order approximation of the resource constrained around the steady state is given by:

$$y_t = c_t \quad (28)$$

Since the net supply of bonds, in equilibrium is zero, equilibrium in the bonds market, instead, implies

$$B_t = 0. \quad (29)$$

Labor market clearing implies

$$N_t^w = \int_0^q N_t(j)^w dj \quad \text{and} \quad N_t^u = \int_q^1 N_t(j)^u dj \quad (30)$$

which implies

$$N_t \equiv N_t^w + N_t^u = \int_0^1 N_t(j) dj. \quad (31)$$

From equations (17) and (19) we have

$$D_t^w Y_t^w = A_t (N_t^w)^\alpha \quad \text{and} \quad D_t^u Y_t^u = A_t (N_t^u)^\alpha \quad (32)$$

where

$$D_t^w = \left[ \int_0^q \frac{1}{q} \left( \frac{P_t^w(j)}{P_t^w} \right)^{-\frac{\theta}{\alpha}} dj \right]^\alpha \quad \text{and} \quad D_t^u = \left[ \int_q^1 \frac{1}{1-q} \left( \frac{P_t^u(j)}{P_t^u} \right)^{-\frac{\theta}{\alpha}} dj \right]^\alpha \quad (33)$$

are measures of price dispersion. Given the market clearing conditions and given equation (15) we have that,

$$Y_t = \left( \frac{Y_t^w}{q} \right)^q \left( \frac{Y_t^u}{1-q} \right)^{1-q} \quad (34)$$

the total amount of goods produced by the economy is a geometric average of the aggregate production of the two sectors. Given that in a neighborhood of a symmetric equilibrium and up to a first order approximation  $D_t^h \simeq 1$ , log-linearizing equation (32) and (34) we obtain

$$y_t^w = a_t + \alpha n_t^w \quad \text{and} \quad y_t^u = a_t + \alpha n_t^u \quad (35)$$

The log-linearization of (34) yealds,

$$y_t = q y_t^w + (1-q) y_t^u \quad (36)$$

while the log-linearization of (31) implies

$$n_t = q n_t^w + (1-q) n_t^u \quad (37)$$

Considering now equations (35), (36) and (37) we find the log-linearized aggregate output which is given by

$$y_t = a_t + \alpha n_t. \quad (38)$$

## 2.5 The Two Sectors Labor Market Equilibrium

### 2.5.1 The Walrasian Sector

Multiplying both sides of the walrasian sector labor demand (23) multiplied by  $\frac{P_t^w}{P_t}$  and considering the consumers labor supply, (11), equilibrium in the walrasian labor market is given by:

$$-C_t \frac{\phi_{N^w}(N_t)}{\phi(N_t)} = MC_t^w \frac{P_t^w}{P_t} \frac{\alpha}{(1-\tau^w)} \frac{Y_t^w}{N_t^w} \quad (39)$$

From the household intertemporal problem (derived in the technical appendix A4) we have that  $P_t^w C_{w,t} = q P_t C_t$ , since the market clearing condition implies  $C_{w,t} = Y_t^w$ , then

$$-\frac{\phi_{N^w}(N_t^w, N_t^u)}{\phi(N_t^w, N_t^u)} N_t^w = \frac{\alpha q}{(1 - \tau^w)} M C_t^w. \quad (40)$$

### 2.5.2 The Unionized Sector

Considering the wage schedule (12) and the labor demand (23), labor market equilibrium in the unionized sector is given by:

$$\frac{1}{\alpha} \frac{W_t^r}{P_t} = M C_t^u \frac{P_t^u}{P_t} \frac{\alpha}{(1 - \tau^u)} \frac{Y_t^u}{N_t^u} \quad (41)$$

which, given the definition of the aggregate price index, can be rewritten

$$\frac{1}{\alpha} \frac{W_t^r}{P_t} = M C_t^u \left( \frac{P_t^u}{P_t^w} \right)^q \frac{\alpha}{(1 - \tau^u)} \frac{Y_t^u}{N_t^u}. \quad (42)$$

Notice that, differently from what happens in the walrasian sector, equation (42) contains the relative price between goods produced in the walrasian and in the unionized sector. In the walrasian labor market relative price does not affect equilibrium, since movements in the relative price are corrected by movements in relative wage. In the unionized sector instead, because of real wage rigidity, a change in relative prices has a significant effect on equilibrium. Since from the intertemporal household problem we have that  $P_t^u C_{u,t} = (1 - q) C_t P_t$  and considering market clearing conditions, we have that,

$$\frac{1}{\alpha} \frac{W_t^r}{P_t} = \frac{\alpha}{(1 - \tau^w)} M C_t^u \left( \frac{1 - q}{q} \right)^q (Y_t^w)^q (Y_t^u)^{1-q} \frac{1}{N_t^u} \quad (43)$$

## 2.6 The First Best Level of Output

The efficient level of output can be obtained by solving the problem of a benevolent planner that maximizes the intertemporal utility of the representative household, subject to the resource constraint and the production function. This problem is analyzed in the Appendix, where we show that the efficient supply of labor, in our economy, is given by:

$$\frac{\phi_N(N_t)}{\phi(N_t)} N_t = -\alpha. \quad (44)$$

Log-linearizing (44), and considering the economy aggregate production function, we obtain<sup>9</sup>

$$y_t^{Eff} = a_t. \quad (45)$$

---

<sup>9</sup>See appendix A?.

## 2.7 The Flexible Price Equilibrium Output in the Walrasian Sector

Under flexible prices, all firms set their price equal to a constant markup over marginal cost. Assuming that firms mark-up,  $\mu_t^P$  is constant, under the flexible price-equilibrium firms real marginal costs are constant at their steady state level and therefore are given by:

$$MC_t^w = \frac{1}{1 + \mu^P}. \quad (46)$$

Considering now the log-linearization of (40) we obtain<sup>10</sup>,

$$mc_t^w = \frac{1}{\alpha} y_t^w - \frac{\sigma - \alpha(\sigma - 1)}{\alpha\sigma} a_t - \frac{(\sigma - 1)}{\sigma} y_t \quad (47)$$

Notice that real marginal costs in the walrasian sector are increasing in the output of the walrasian sector decreasing in the aggregate output. Considering that  $mc_t^w = 0$ , from the aggregate production function we have that under the flexible price equilibrium:

$$y_t^{wf} = \frac{\sigma - \alpha(\sigma - 1)}{\sigma} a_t + \frac{\alpha(\sigma - 1)}{\sigma} y_t, \quad (48)$$

flexible price equilibrium output in the walrasian sector is increasing in the productivity shock and aggregate output. Notice that when  $q = 1$ , (48) can be rewritten as  $y_t^{wf} = a_t$ , i.e., the flexible price equilibrium output coincides with the efficient one.

Given equations (47) and (48), real marginal costs can be rewritten in terms of the gap between actual output and its natural level, as follows:

$$mc_t^w = \frac{1}{\alpha} (y_t^w - y_t^{wf}). \quad (49)$$

## 2.8 The Flexible Price Equilibrium Output in the Unionized sector

Also in the unionized sector when prices are flexible all firms set their prices as a constant markup over marginal costs, given by (46). The log-linearization of (43) implies:

$$mc_t^u = \frac{1}{\alpha} y_t^u - y_t - \frac{1}{\alpha} a_t + w_t^r, \quad (50)$$

As in the walrasian sector real marginal costs are increasing in the output of the unionized sector and decreasing in the aggregate output. When  $mc_t^u = 0$  then

$$y_t^{uf} = \alpha y_t + a_t - \alpha w_t^r, \quad (51)$$

---

<sup>10</sup>See appendix A?

which implies that flexible price equilibrium output in the unionized sector is increasing in the productivity shock and aggregate output. Notice that when  $q = 0$ , (48) can be rewritten as  $y_t^{uf} = \frac{1}{1-\alpha}a_t - \frac{\alpha}{1-\alpha}w_t^r$ .

Given equations (50) and (51), real marginal costs can be rewritten in terms of the gap between actual output and its natural level, as follows:

$$mc_t^u = \frac{1}{\alpha} \left( y_t^u - y_t^{uf} \right). \quad (52)$$

## 2.9 The Aggregate Flexible Price Equilibrium Output

Since the aggregate flexible price equilibrium output is the weighted sum of equations (48) and (51), we obtain,

$$y_t^f = \frac{\sigma - q\alpha(\sigma - 1)}{\sigma(1 - \alpha) + \alpha q} a_t - \frac{\sigma(1 - q)\alpha}{\sigma(1 - \alpha) + \alpha q} w_t^r. \quad (53)$$

Given (45), the output gap with respect to the efficient equilibrium output is given by:

$$y_t^f - y_t^{Eff} = \frac{\alpha\sigma(1 - q)}{\sigma(1 - \alpha) + \alpha q} a_t - \frac{\sigma(1 - q)\alpha}{\sigma(1 - \alpha) + \alpha q} w_t^r. \quad (54)$$

What is important to notice, here, is that, unlike what happens in the walrasian model, the difference between flexible equilibrium output (natural output) and the efficient equilibrium output is not constant, but is a function of the relevant shocks that hit the economy. In this model therefore, as in Blanchard and Galì [4] stabilizing the output gap - the difference between actual and natural output - is not equivalent to stabilizing the *welfare relevant* output gap - the gap between actual and efficient output. In other words, what Blanchard and Galì call "the divine coincidence" will not hold, since any policy that brings the economy to its natural level is not necessarily an optimal policy.

Defining by  $\Upsilon = \frac{\sigma(1-q)}{\sigma(1-\alpha)+\alpha q}$  the response of the welfare relevant output gap to the relevant shocks (notice that the response of to a technology shock is identical, but with the opposite sign, to the response to a cost push shock), we immediately observe that

$$\frac{d\Upsilon}{dq} < 0. \quad (55)$$

As the number of walrasian firms increases, the difference between natural output and efficient output decreases, i.e. natural output tends to efficient output. The reason is quite intuitive: the smaller is the population of unionized firms the smaller is the importance of real wage rigidity in the economy and the technology and cost push shocks become less and less relevant.

## 2.10 The Aggregate Phillips Curve

Each intermediate good-producing firm has a monopolistic power in the production of its own variety and therefore has leverage in setting the price. In particular, firms choose  $P_t^h(j)$  in a staggered price setting à la Calvo-Yun [6] with a decreasing return to scale production function. As shown in the appendix A6, the solution of the firm's problem in this case is given by:

$$\pi_t^h = \beta E_t \pi_{t+1}^h + \lambda_\alpha m c_t^h \quad (56)$$

where  $\lambda_\alpha = \frac{(1-\psi)(1-\beta\psi)}{\psi} \frac{\alpha}{\alpha+\theta(1-\alpha)}$  and  $\psi$  is the probability with which firms reset prices. Given equations (49) and (50), the sector specific Phillips curves in the walrasian and unionized sectors are

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \lambda_\alpha \frac{1}{\alpha} (y_t^w - y_t^{wf}) \quad (57)$$

$$\pi_t^u = \beta E_t \pi_{t+1}^u + \lambda_\alpha \frac{1}{\alpha} (y_t^u - y_t^{uf}) \quad (58)$$

where  $y_t^{wf}$  and  $y_t^{uf}$  are defined as in (48) and (51). Since  $\pi_t = q\pi_t^w + (1-q)\pi_t^u$ , the Phillips curve for the aggregate equation can be written as

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_\alpha \frac{1}{\alpha} (y_t - y_t^f). \quad (59)$$

It is important to notice, at this point, that the output gap that is contained in equation (59) is not the relevant output gap. What is relevant, for an optimizing central bank, is not to minimize the distance between actual and natural (flexible price equilibrium) output, but rather the distance between actual and efficient output. If we now express the policy relevant output gap as  $x_t = y_t - y_t^{eff}$  and we substitute (54), (59) can be rewritten as

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_\alpha \frac{1}{\alpha} x_t - \lambda_\alpha \Upsilon a_t + \lambda_\alpha \Upsilon w_t^r. \quad (60)$$

where  $\Upsilon = \frac{\sigma(1-q)}{\sigma(1-\alpha)+\alpha q}$ . From equation (60) is quite clear that, for a central bank, achieving  $x_t = 0$  does not imply obtaining  $\pi_t = 0$ . We have therefore:

**Result 1.** *In a two sector labor market economy, because of the presence of unions, the "divine coincidence" does not hold, i.e., stabilizing inflation is not equivalent to stabilizing the output gap defined as the deviation of output from efficient output. A negative (positive) productivity shock has a positive (negative) effect on inflation, while a cost push shock has an effect of the same size but with the opposite sign on inflation.*

This result depends on the existence of a real distortion in the economy, beside the one induced by monopolistic competition, and the nominal distortion caused by firms' staggered price setting. When a productivity shock hits



the economy, efficient output, given by equation (45), increases by the same amount. Natural output instead (i.e., the level of output that would prevail in a flexible price equilibrium) increases more than proportionally so that the difference between efficient output and natural output decreases. This is due to the fact that in a unionized sector, following a productivity shock, real wages remain constant and therefore do not offset the effects of the shock on real marginal cost. Therefore, the natural level of output differs from the efficient level and this difference is not constant. As it is evident from equation (63), if the Central Bank stabilizes output around the efficient level, inflation will be completely vulnerable to productivity and cost-push shocks; in other words the output gap is no longer a sufficient statistics for the effect of real activity on inflation.

Given (55), we immediately observe that the response of inflation to the technology and the exogenous wage shocks decreases as the fraction of walrasian firms in the market increases. We can therefore state,

**Result 2.** *The response of inflation to a negative productivity shock and to a positive reservation wage shock decreases as the number  $q$  of walrasian firms increases.*

Another interesting aspect of this model is that we are able to express the Phillips curve in its more traditional form, i.e. in terms of unemployment. From equations (38), (45) and (54) we obtain in fact that

$$n_t = \frac{x_t}{\alpha} \quad (61)$$

Expressing the rate of unemployment as  $U_t = 1 - N_t$  and log linearizing around the steady state we obtain

$$u_t = -\frac{\eta}{\alpha} x_t, \quad (62)$$

where  $\eta = \frac{N}{1-N}$ . We can therefore rewrite the Phillips curve as

$$\pi_t = \beta E_t \pi_{t+1} - \frac{\lambda_a}{\eta} u_t - \lambda_a \frac{\sigma(1-q)}{\sigma(1-\alpha) + \alpha q} a_t + \lambda_a \frac{\sigma(1-q)}{\sigma(1-\alpha) + \alpha q} w_t^r. \quad (63)$$

The relationship between unemployment and the output gap allows us to consider, indifferently, the output gap and the unemployment rate as policy objectives for the central bank.

## 2.11 The Aggregate IS-Curve

In order to obtain the IS curve we start by log-linearizing around the steady state the Euler equation (10). Considering that in steady state the optimal

subsidy setting implies  $\frac{\phi_N(N)N}{\phi(N)} = -\alpha$ , the log-linearized Euler equation is given by:

$$c_t = E_t \{c_{t+1}\} + \frac{\alpha(1-\sigma)}{\sigma} E_t \{\Delta n_{t+1}\} - \frac{1}{\sigma} (\hat{r}_t - E_t \{\pi_{t+1}\}). \quad (64)$$

with  $\hat{r}_t = r_t - \varrho$ , where  $r_t = \ln R_t$  and  $\varrho = -\ln \beta$  which is the steady state interest rate all the variables without a subscript are taken at their steady state levels. Given the economy resource constraint (28) and the production function (38), the Euler equation (64) can be written as:

$$y_t = E_t \{y_{t+1}\} - (1-\sigma) E_t \{\Delta a_{t+1}\} - (r_t - E_t \{\pi_{t+1}\}) \quad (65)$$

which represents the IS equation of our simple economy.

Let us define, the relevant output gap as the difference between actual output and efficient output, i.e.,

$$x_t = y_t - y_t^{Eff}. \quad (66)$$

In this case the IS equation can be rewritten in terms of the output gap as,

$$x_t = E_t \{x_{t+1}\} - (r_t - E_t \{\pi_{t+1}\} - r_t^e). \quad (67)$$

where  $\hat{r}_t^e$  is the rate of interest that supports the efficient allocation, which can be expressed as:

$$\hat{r}_t^e = \sigma E_t \{\Delta a_{t+1}\} = \sigma E_t \{\Delta y_{t+1}^{Eff}\} = -\sigma(1-\rho_a) a_t. \quad (68)$$

Note that (67) expressed in terms of the gap between the actual and the efficient output relates the output gap to current and anticipated deviations of the real interest rate from its efficient counterpart.

### 3 Optimal Monetary Policy

In the appendix A7 we show that also for the non-separable preferences assumed in our framework, consumers' utility can be approximated up to the second order by a quadratic equation of the kind:

$$W_t = E_t \sum_{t=0}^{\infty} \beta^t \tilde{U}_{t+k} = -\frac{U_{Y,t}}{2} E_t \sum_{t=0}^{\infty} \left[ \pi_{t+k}^2 + \frac{\lambda_a}{\theta\sigma} x_{t+k}^2 \right] + \mathcal{O}(\|\alpha\|^3) \quad (69)$$

where  $\tilde{U}_{t+k} = U_{t+k} - \bar{U}_{t+k}$  is the deviation of consumers' utility from the level achievable in the frictionless equilibrium, and  $\theta$  is the elasticity of substitution between intermediate goods, which are used as input in the final good sector. Notice that, the relative weights assigned to inflation and to the output gap are linked to the structural parameters reflecting preferences and technology.

### 3.1 Discretion

If the Central Bank cannot credibly commit in advance to a future policy action or a sequence of future policy actions, then the optimal monetary policy is discretionary, in the sense that the policy makers choose in each period the value to assign to the policy instrument, that here we assume to be the short-term nominal interest rate  $\hat{r}_t$ . In order to do so, the Central Bank maximizes the welfare-based loss function (69), subject to the economy Phillips curve, taking all expectations as given. Therefore, the Central Bank chooses the level of inflation and output gap that maximize:

$$\tilde{W}_t = -\frac{U_{Y,t}}{2} \left( \pi_t^2 + \frac{\lambda_a}{\theta\sigma} x_t^2 \right) + H_t \quad (70)$$

subject to (63)

$$\text{where } H_t = -\frac{U_{Y,t}}{2} E_t \sum_{k=1}^{\infty} \left[ \frac{\theta}{\lambda_a} \pi_{t+k}^2 + \frac{1}{\sigma} x_{t+k}^2 \right].$$

The first order conditions imply:

$$x_t = -\frac{\theta\sigma}{\alpha} \pi_t. \quad (71)$$

Substituting into (63) we obtain:

$$\pi_t = \frac{1}{\Omega} (\beta E_t \pi_{t+1} - \lambda_a \Upsilon a_t + \lambda_a \Upsilon w_t^r) \quad (72)$$

where  $\Omega = 1 + \lambda_a \frac{\theta\sigma}{\alpha^2}$ . and  $\Upsilon = \frac{\sigma(1-q)}{\sigma(1-\alpha)+\alpha q}$

Iterating forward (72),

$$\pi_t = -\frac{\Upsilon\lambda_a}{\Omega} E_t \sum_{i=0}^{\infty} \left( \frac{\beta}{\Omega} \right)^i (a_{t+i} - w_{t+i}^r) \quad (73)$$

and

$$E_t \pi_{t+1} = -\frac{\Upsilon\lambda_a}{\Omega} E_t \sum_{i=0}^{\infty} \left( \frac{\beta}{\Omega} \right)^i (a_{t+i+1} - w_{t+i+1}^r). \quad (74)$$

Given that,  $E_t \{a_{t+i+1}\} = \rho_a^i a_t$ , and that  $E_t \{w_{t+i+1}^r\} = \rho_w^i w_t^r$ , (73) and (74) can be rewritten as,

$$\pi_t = -\frac{\Upsilon\lambda_a}{\Omega - \beta\rho_a} a_t + \frac{\Upsilon\lambda_a}{\Omega - \beta\rho_w} w_t^r \quad (75)$$

$$E_t \pi_{t+1} = -\frac{\Upsilon\lambda_a\rho_a}{\Omega - \beta\rho_a} a_t + \frac{\Upsilon\lambda_a\rho_w}{\Omega - \beta\rho_w} w_t^r \quad (76)$$

Notice that we can express current inflation as a function of the relevant shocks  $a_t$  and  $w_t^r$ . A positive productivity shock requires a decrease in inflation and a positive cost push shock requires an increase in inflation. Because of rational

expectations we have a similar result for expected inflation. Given equation (72) and (75) we can write the expression of output gap as a function of the exogenous shocks, which is given by

$$x_t = \frac{\theta\sigma\Upsilon\lambda_a}{\alpha(\Omega - \beta\rho_a)}a_t - \frac{\theta\sigma\Upsilon\lambda_a}{\alpha(\Omega - \beta\rho_a)}w_t^r \quad (77)$$

Using (75), (76) and (68) we can also rewrite expected inflation as:

$$E_t\pi_{t+1} = \rho_w\pi_t + \frac{(\rho_a - \rho_w)}{\sigma(1 - \rho_a)}\frac{\Upsilon\lambda_a}{\Omega - \beta\rho_a}\hat{r}_t^n \quad (78)$$

The optimal level of inflation can be implemented by the Central Bank by setting the nominal interest rate. The interest rate rule can be obtained by substituting (71), (75) and (76) into the IS curve (67), in which case we obtain:

$$\begin{aligned} \hat{r}_t^* = & - \left[ \left[ 1 + \left( \frac{1 - \rho_a}{\rho_a} \right) \frac{\theta\sigma}{\alpha} \right] \left( \frac{\Upsilon\lambda_a\rho_a}{\Omega - \beta\rho_a} \right) + \sigma(1 - \rho_a) \right] a_t + \\ & + \left[ 1 + \left( \frac{1 - \rho_w}{\rho_w} \right) \frac{\theta\sigma}{\alpha} \right] \left( \frac{\Upsilon\lambda_a\rho_w}{\Omega - \beta\rho_w} \right) w_t^r \end{aligned} \quad (79)$$

We can therefore state

**Result 3.** *Under discretion an optimal monetary policy requires a decrease in the nominal interest rate following a positive productivity shock and an increase in the nominal interest rate following a positive reservation wage shock. The response of the nominal interests rate to both shocks decreases as the fraction of walrasian firms  $q$  increases.*

Equations (75) and (77) can also be rewritten in terms of standard deviations, which allows us to derive the output-gap inflation volatility frontier. Since by assumption both shocks are iid. and therefore  $\sigma_{aw} = 0$ , we can express the volatility of inflation and the volatility of the output gap as a function of the volatility of the technology and reservation wage shocks. In particular we have:

$$\sigma_\pi = \left( \frac{\Upsilon\lambda_a}{\Omega - \beta\rho_a} \right) \sigma_a + \left( \frac{\Upsilon\lambda_a}{\Omega - \beta\rho_w} \right) \sigma_w \quad (80)$$

and

$$\sigma_x = \left( \frac{\theta\sigma\Upsilon\lambda_a}{\alpha(\Omega - \beta\rho_a)} \right) \sigma_a + \left( \frac{\theta\sigma\Upsilon\lambda}{\alpha(\Omega - \beta\rho_a)} \right) \sigma_w. \quad (81)$$

Notice that, as  $q \rightarrow 1$  then  $\Upsilon \equiv \frac{\sigma(1-q)}{\sigma(1-\alpha)+\alpha q} = 0$  and therefore  $\sigma_x = \sigma_\pi = 0$ . When instead  $q \rightarrow 0$ , then  $\Upsilon = \frac{\sigma}{\sigma(1-\alpha)+\alpha}$  and both  $\sigma_x$  and  $\sigma_\pi$  reach their maximum possible values.

An interest rate rule that implements the optimal policy, can be found using (78) and (76). In this case we obtain:

$$\hat{r}_t^* = \left[ 1 + \left( \frac{1 - \rho_w}{\rho_w} \right) \frac{\theta\sigma}{\alpha} \right] E_t\pi_{t+1} + \left[ 1 + \frac{(\rho_w - \rho_a)}{\alpha\rho_w(1 - \rho_a)} \frac{\Upsilon\lambda_a\theta}{\Omega - \beta\rho_a} \right] \hat{r}_t^n. \quad (82)$$

In Appendix A6 we show that under rule (82) equilibrium is determinate. Assuming, as a particular case  $\rho_a = \rho_w = \rho$ , equation (82) becomes

$$\hat{r}_t^* = \left[ 1 + \left( \frac{1-\rho}{\rho} \right) \frac{\theta\sigma}{\alpha} \right] E_t \pi_{t+1} + r_t^n \quad (83)$$

We can now state:

**Result 4.** *Optimal monetary policy under discretion requires a more than proportional increase in the nominal interest rate following an increase in the expected rate of inflation. However, an increase in the rate of interest supporting the efficient equilibrium implies a proportional increase in the nominal interest rate if and only if  $\rho_a = \rho_w = 1$ . Otherwise an increase in  $r_t^e$  implies a more than proportional increase in the nominal interest rate if  $\rho_w > \rho_a$  and a less than proportional increase if  $\rho_w < \rho_a$ .*

Also in our model therefore, as in the standard DNK model, optimality requires that the Central Bank respond to increasing inflationary expectations by raising nominal interest rates more than proportionally. In other words, also in a dualistic economy where part of the labor market is unionized, the Taylor principle applies. The optimal response of the nominal interest rate to an increase in the efficient rate of interest, instead, is different from the one that is usually obtained in the "standard" DSGE New Keynesian model. When wages are set only in a perfectly competitive labor market, an increase in the rate of interest supporting the efficient equilibrium affects only the IS curve, and this implies that when the efficient rate of interest increases, the nominal interest rate must be raised by the same amount. When wages are also set by monopoly unions, instead, the efficient rate of interest, which is basically determined by the productivity parameter that characterizes the economy, also affects directly the Phillips curve. This extra effect on the Phillips curve requires a further response by the central bank. Observing (82) we can easily see that

**Result 5.** *An increase in the fraction of walrasian firms does not affect the response of the nominal interest rate to expected inflation but affects the response of the nominal interest rate to its efficient level.*

While the ratio between walrasian and unionized firms is crucial in determining the effect of technology and cost-push shocks on inflation, it does not affect the amount by which the interest rate must be raised in response to a unit increase in expected inflation. In other words, the extent of real wage rigidity in the economy does not affect the response coefficient to expected inflation. Two economies hit by the same shocks but with a different  $q$  will experience different movements in the nominal interest rate not because the central bank responds differently, but because the impact of the economy to the shocks is different.

Log linearizing the demand for labor (23) in both sectors, we have that, for  $w = u, h$ ,

$$w_t^h = mc_t^h + y_t^h - \frac{1}{\alpha} y_t^h + \frac{1}{\alpha} a_t. \quad (84)$$

Since the aggregate wage is given by the weighted sum of wages in the two sectors, it can be expressed as:

$$w_t = qmc_t^w + (1-q)mc_t^u - \frac{1-\alpha}{\alpha} y_t + \frac{1}{\alpha} a_t$$

Given(49), (48), (51) and (52), we obtain

$$w_t = x_t - (\Upsilon - 1) a_t + \Upsilon w_t^r$$

which, given (62), becomes

$$w_t = -\frac{\alpha}{\eta} u_t - (\Upsilon - 1) a_t + \Upsilon w_t^r.$$

Notice that (63) and (62) together imply

$$u_t = -\eta \Upsilon \left( \frac{(1 - \beta \rho_a)}{(\Omega - \beta \rho_a)} + 1 \right) a_t + \eta \Upsilon \left( \frac{(1 - \beta \rho_w)}{(\Omega - \beta \rho_w)} + 1 \right) w_t^r. \quad (85)$$

then we can rewrite the aggregate real wage as

$$w_t = \alpha \Upsilon \left( \frac{(1 - \beta \rho_a)}{(\Omega - \beta \rho_a)} + 1 \right) a_t - \alpha \Upsilon \left( \frac{(1 - \beta \rho_w)}{(\Omega - \beta \rho_w)} + 1 \right) w_t^r + -(\Upsilon - 1) a_t + \Upsilon w_t^r$$

Notice that,

$$\begin{aligned} \sigma_u &= \left( \eta \Upsilon \left( \frac{(1 - \beta \rho_a)}{(\Omega - \beta \rho_a)} + 1 \right) \right) \sigma_a \\ \sigma_w &= \left( \alpha \left( \frac{(1 - \beta \rho_a)}{(\Omega - \beta \rho_a)} + 1 \right) \right) \sigma_a \end{aligned}$$

which implies

$$\sigma_u - \sigma_w = \left[ (\eta - \alpha) \Upsilon \frac{(1 - \beta \rho_a)}{(\Omega - \beta \rho_a)} + (\eta \Upsilon - \alpha) \right] \sigma_a.$$

We can therefore state

**Result 6.** *If  $(\eta - \alpha) \Upsilon \frac{(1 - \beta \rho_a)}{(\Omega - \beta \rho_a)} + (\eta \Upsilon - \alpha) > 0$ , under an optimal discretionary policy and in response to a productivity shock, unemployment is more volatile than the real wage. If  $(\eta - \alpha) > 0$  the difference between unemployment volatility and real wage volatility decreases as the number of walrasian firms increases.*

As we will show in the following section, for very plausible parameter values, we always have  $\sigma_u - \sigma_w > 0$  which implies that, in general, our model is consistent with a well known fact in macroeconomics, i.e. the relatively smooth behavior of wages along the business cycle together with the relatively volatile behavior of unemployment.

### 3.2 Constrained Commitment

Let us assume that the Central Bank follows a rule for the target variable  $x_t$  which depends on the fundamental shocks  $w_t^r$  and  $r_t^n$ . In order to obtain an analytical solution we assume the following feedback rule equation

$$x_t^c = -\omega (a_t - w_t^r) \quad \forall t \quad (86)$$

and we also assume

$$\rho_a = \rho_w = \rho \quad (87)$$

where  $\omega > 0$  is the coefficient of the feedback rule and the variable  $x_t^c$  is the value of  $x_t$  conditional on commitment to the policy.

Before solving the Central Bank problem under constrained commitment, we iterate forward the Phillips curve (63) and we obtain:

$$\pi_t^c = \left( \frac{\omega}{\alpha} - \Upsilon \right) \frac{\lambda_a}{1 - \beta\rho} (a_t - w_t^r) \quad (88)$$

which, considering equation (86), can be rewritten as:

$$\pi_t^c = -\frac{\lambda_a}{\alpha} \frac{1}{1 - \beta\rho} x_t^c - \frac{\lambda_a \Upsilon}{1 - \beta\rho} (a_t - w_t^r) \quad (89)$$

Notice that, in this case, a one percent contraction of  $x_t^c$  reduces  $\pi_t^c$  by the amount  $\frac{\lambda_a}{\alpha} \frac{1}{1 - \beta\rho}$ , while under discretion, reducing  $x_t$  by one percent only produces a fall in  $\pi_t$  of  $\frac{\lambda_a}{\alpha} < \frac{\lambda_a}{\alpha} \frac{1}{1 - \beta\rho}$ . As in the case analyzed by Clarida, Gali and Gertler [8], the Central bank will enjoy an improved trade off, due to the fact that commitment to a policy rule affects expectations on the future course of the output gap.

Given (86) and (88) we can now write the problem of the Central Bank under constrained commitment as follows:

$$W_t = E_t \sum_{k=0}^{\infty} \beta^k \tilde{U}_{t+k} = -\frac{U_{Y,t}}{2} \left[ (\pi_t^c)^2 + \frac{\lambda_a}{\theta\sigma} (x_t^c)^2 \right] E_t \sum_{i=0}^{\infty} \left( \frac{w_{t+i} - a_{t+i}}{w_t^r - a_t} \right)^2 \quad (90)$$

subject to equation (89). The first order conditions imply:

$$x_t^c = -\frac{\theta\sigma}{\alpha} \frac{1}{1 - \beta\rho} \pi_t^c \quad (91)$$

Since  $\frac{\theta\sigma}{\alpha} \frac{1}{1 - \beta\rho} < \frac{\theta\sigma}{\alpha}$  this implies that commitment to a rule makes it optimal, for the central bank, to induce a greater contraction of output in response to an increase in inflation. Substituting (91) into the Phillips curve and iterating forward we obtain:

$$\pi_t = -\frac{\lambda_a}{\Omega^c (1 - \beta\rho)} (a_t - w_t^r) \quad (92)$$

and

$$E_t \pi_{t+1} = -\frac{\lambda_a \rho}{\Omega^c (1 - \beta \rho)} (a_t - w_t^r) \quad (93)$$

where  $\Omega^c = 1 + \lambda \left( \frac{1}{\alpha} \frac{1}{1 - \beta \rho} \right)^2 \theta \sigma > \Omega$ . The interest rate rule can be obtained by substituting (91), (92) and (93) into the IS curve (67), in which case we obtain:

$$\begin{aligned} r_t^{c*} = & - \left[ \left[ 1 + \left( \frac{1 - \rho}{\rho} \right) \left( \frac{1}{\alpha} \right) \frac{\theta \sigma \Upsilon}{1 - \beta \rho} \right] \left( \frac{\lambda_a \rho}{\Omega^c (1 - \beta \rho)} \right) + \sigma (1 - \rho) \right] a_t + \\ & + \left[ 1 + \left( \frac{1 - \rho}{\rho} \right) \left( \frac{1}{\alpha} \right) \frac{\theta \sigma \Upsilon}{1 - \beta \rho} \right] \left( \frac{\lambda_a \rho}{\Omega^c (1 - \beta \rho)} \right) w_t^r \end{aligned} \quad (94)$$

Using equation (91), the one of the Phillips curve and the one of the IS-curve we find the following optimal instrument rule:

$$\hat{r}_t^{c*} = \left[ 1 + \left( \frac{1 - \rho}{\rho} \right) \frac{1}{\alpha} \frac{\theta \sigma}{1 - \beta \rho} \right] E_t \pi_{t+1} + \hat{r}_t^n. \quad (95)$$

Since  $\frac{1}{1 - \beta \rho} > 1$ , we have the following

**Result 7.** *Under commitment to a simple feedback rule, when  $\rho_a = \rho_w = \rho$ , an optimal interest rule requires that the nominal interest rate increases in response to a technology shock and decreases in response to a reservation wage shock. The response of the interest rate to these shocks decreases as  $q$  increases. If we instead consider the optimal instrument rule, in reacting to an increase in expected inflation the nominal interest rate must be increased more than in the case of discretion, but the size of the reaction is again independent of the relative proportion of walrasian firms.*

### 3.3 Conclusions

We have considered in this paper a DSGE New Keynesian model where labor is indivisible and where there are, at the same time, two types of labor markets: one where wages are set competitively and one where wages are the result of the bargaining between firms and monopoly unions. We found that, with respect to the standard DNK framework, our model gives a more satisfactory description of the reality of modern industrialized economies, since it is able to account for the existence of significant trade-offs between stabilizing inflation and stabilizing unemployment, in response to technology and exogenous wage shocks. Because of real wage rigidity which is induced by the presence of unions, an optimizing central bank must respond to negative (positive) technology shocks by increasing (decreasing) the interest rate and, similarly, must respond to exogenous increases in unions' reservation wage with an interest rate increase.

The effect of these shocks on inflation and the necessary interest rate movements set by an optimizing central bank depend on the size of the walrasian sector relative to the unionized sector. If a large part of wages are set in a



competitive market, technology and cost-push shocks will have little effect on inflation and will induce small interest rate movements, while an economy where large part of wages are set in unionized markets will experience larger inflation and interest rate movements. If we consider however an optimal instrument rule where the central bank reacts to expected inflation, the response of the nominal interest rate to an increase in expected inflation is not influenced by the dualistic structure of the labor market. The model is also capable of accounting for the greater volatility of unemployment relative to the wage volatility that is usually found in the data.

Even though, for the sake of simplicity, we concentrate on a rigid dualistic structure of the labor market and we abstract from other market imperfections like search and matching costs, and we provide therefore a rather crude representation of the labor market, we are able to single out, with this model, some of the challenges provided to monetary policy by different institutional settings in the labor market. The model, in particular, captures some important differences between the European and the U.S. economies and can represent, therefore, a useful benchmark to evaluate and compare the monetary policies enacted by the Fed and the ECB.

## References

- [1] Anderson S., and Devereux M. (1988). Trade Unions and the Choice of Capital Stock, *Scandinavian Journal of Economics* 90, 27–44.
- [2] Andres J. R., Domenech R., and Ferri J. (2006). Price Rigidity and the Volatility of Vacancies and Unemployment, mimeo, Universidad de Valencia.
- [3] Amato J. D., Laubach T. (2003). Estimation and Control of an Optimization-Based Model with Sticky Prices and Wages. *Journal of Economic Dynamics and Control* 27, 1181-1215.
- [4] Blanchard O. and Galì J. (2006). Real Wage Rigidities and the New Keynesian Model, *Journal of Money Credit and Banking*, forthcoming.
- [5] Blanchard O. and Galì J. (2006). A New Keynesian Model with Unemployment. mimeo.
- [6] Calvo G. A. (1983), Staggered Prices in a Utility-Maximizing Framework, *Journal of Monetary Economics* 12(3) 383-398.
- [7] Chéron A. and Langot F. (2000). The Phillips and Beveridge Curves Revisited, *Economics Letters* 69, 371-376.
- [8] Clarida R., Galì J., Gertler M. (1999). The Science of Monetary Policy: A New Keynesian Perspective. *Journal of Economic Literature*, 47 (4) 1661-1707.

- [9] Christoffel K., Linzert T. (2005). The Role of Real Wage Rigidities and Labor Markets Friction for Unemployment and Inflation Dynamics, ECB Working Paper Series n° 556.
- [10] Christoffel K., Kuester K., Linzert T. (2006). Identifying the Role of Labor Markets for Monetary Policy in an Estimated DSGE Model, ECB Working Paper Series n° 635.
- [11] Dixit A. K., Stiglitz J. E., (1977), Monopolistic Competition and Optimum Product Diversity, *American Economic Review*, 67, 297-308.
- [12] Dunlop J. T. (1944). Wage Determination under Trade Unions, London, Macmillan.
- [13] Farber H. S. (1986). The Analysis of Union Behavior, in Ashenfelter O., Card D., a cura di (1986), vol. II, 1039-1089.
- [14] Francis N. R., Owyang M. T., Theodorou A. T. (2005). What Explains the Varying Monetary Response to Technology Shocks in G-7 Countries? *International Journal of Central Banking*, 1 (3) 33-71.
- [15] Galí J. (2001), The Conduct of Monetary Policy in the Face of Technological Change: Theory and Postwar U.S. Evidence, in Stabilization and Monetary Policy: the International Experience, Banco de México.
- [16] Galí J., López-Salido J. D., Vallès J. (2003). Technology Shocks and Monetary Policy: assessing the Fed's Performance, *Journal of Monetary Economics*, 50 723-743.
- [17] Galí J., Rabanal P. (2004). Technology Shocks and Aggregate Fluctuations: How Well Does the RBS Model Fit Postwar U.S. Data? NBER Working Paper n° 10636.
- [18] Galí J., Monacelli T. (2005). Monetary Policy and Exchange Rate Volatility in a Small Open Economy, *Review of Economic Studies*, 72, pp. 917-946.
- [19] Gertler M., Trigari A. (2006). Unemployment Fluctuations with Staggered Nash Wage Bargaining, mimeo.
- [20] King R., Rebelo S. T. (2000). Resuscitating Real Business Cycles, Rochester Center for Economic Research, Working Paper No. 467.
- [21] King R., C. Plosser, and S. Rebelo. (1988a). Production, Growth, and Business Cycles I: The Basic Neoclassical Model. *Journal of Monetary Economics*, 21, 195-232.
- [22] Hansen G. (1985). Indivisible Labor and the Business Cycle, *Journal of Monetary Economics*, 16 309-328.
- [23] Ireland P. N. (2004). Technology Shocks in the New-Keynesian Model, *Review of Economics and Statistics*, 86(4), 923-936.

- [24] Lawrence S., Ishikawa J. (2005). Trade Union Membership and Collective Bargaining Coverage: Statistical Concepts, Methods and Findings, Working Paper n° 59, International Labor Office of Geneva.
- [25] Maffezzoli M. (2001.) Non-Walrasian Labor Markets and Real Business Cycles, *Review of Economics Dynamics* 4, 860-892.
- [26] Mortensen D.T., Pissarides C.A., (1994), Job Creation and Job Destruction in the Theory of Unemployment, *Review of Economic Studies*, vol. 61, pp. 397-416.
- [27] Moyen S. and Sahuc , J. G. (2005), Incorporating labour market frictions into an optimizing-based monetary policy model, *Economic Modelling* 22, 159-186.
- [28] Oswald A. (1982). The Microeconomic Theory of the Trade Union, *Economic Journal*, 92, 576-595.
- [29] Pissarides C. (2000). Equilibrium Unemployment Theory, MIT Press.
- [30] Pissarides, C. (1998). The Impact of Employment Tax Cuts on Unemployment and Wages: The Role of Unemployment Benefits and Tax Structure, *European Economic Review* 42, 155–183.
- [31] Rogerson, R. (1988). Indivisible Labor Lotteries and Equilibrium, *Journal of monetary Economics* 21, 3-16.
- [32] Rogerson R., and Wright R. (1988). Involuntary Unemployment in Economics with Efficient Risk Sharing, *Journal of Monetary Economics* 22, 501-515.
- [33] Trigari A. (2005). Equilibrium Unemployment, Job flows, and Inflation Dynamics, ECB WP#304.
- [34] Trigari A. (2006). The Role of Search Frictions and Bargaining of Inflation Dynamics, mimeo.
- [35] Walsh C. (2003). Labor Market Search and Monetary Shocks", in Elements of Dynamic Macroeconomic Analysis, S. Altug, J. Chadha and C. Nolan eds.
- [36] Walsh C. (2005). Labor market search, Sticky Prices, and Interest Rate Rules", *Review of Economic Dynamics* 8, 829-849.
- [37] Woodford M. (2003). Interest and Prices: Foundations of a Theory of Monetary Policy, Princeton University Press, Princeton.
- [38] Smets F., Wouters R. (2003). An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area, *Journal of the European Economic Association*, MIT Press, 1(5), 1123-1175.

- [39] Zanetti F. (2004). A Non-Walrasian Labor Market and the European Business Cycle, Boston College, Working Papers in Economics, n° 31.