Revisiting Indicators of Public Debt Sustainability: Capital Expenditure, Growth and Public Debt in India

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ABSTRACT

The paper tests whether there exists an inverse long-term relationship between productive government spending and public debt in developing countries by using a multivariate time series framework. The theoretical model is based on dynamic optimization of utility and productive expenditure with respect to government capital, private capital and debt. An inverse long-term relationship between capital expenditure ratio and public debt in India is found using the cointegration and error correction analysis. Further, a new indicator based on capital expenditure derived from the Government Inter-temporal budget constraint is suggested.
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INTRODUCTION

Public debt sustainability is vital for both developing and developed nations. While recent literature has laid great emphasis on analyzing this issue, the approach has usually been biased towards the revenue account of the government. This is due to the fact that fiscal policy has historically been related solely to the stabilization process even if, in practice, it can be used to promote growth and long run welfare of a country. However, recent revival of interest in growth theory has led to deeper research on the role of the elements of public expenditure that bear significant association with economic growth. Since traditional theories of macroeconomics stress on the fact that government must redirect expenditure towards sectors where they see an improvement in the long run without compromising on the existing needs of the country, public expenditure is divided into two major categories: capital and current.

A recent empirical analysis by Bose, Osborne, Haque (2003) on a belt of developed and developing nations suggests that the share of capital expenditure in GDP, is positively and significantly correlated with economic growth, while the growth effect of current expenditure is insignificant. This makes capital expenditure and its growth rate a very important factor in determining fiscal sustainability. In another recent study by Gupta, Clements, Granados (2005) the authors find that the composition of public outlays matters. Countries where spending is concentrated on wages tend to have lower growth, while those that allocate higher shares to capital and enjoy faster output expansion. However, Devarajan, Swaroop, Zou (1996) add a caveat to the
above by demonstrating that once the optimal level of capital expenditure has been reached, the composition of public expenditure needs to be revisited. The bottomline of the above analysis is that focus on capital expenditure until an optimal level is important for the economy. However, each country would reach its optimal capital expenditure level in a different time frame depending on the level of economic advancement. Hence, to understand the linkages between government expenditure components and growth, a specific literature review on developing countries is important. More specifically focus on a fast growing developing nation would be fruitful. India, is a good case in point, because despite of an explosive debt trajectory of both the Central and state government, the economy shows no signs of a debt crisis.

A number of economists have studied the debt sustainability issue for India. Buiter and Patel (2004) used the traditional stationarity tests developed by Phillips and Perron(1998) and KPSS(1992). The paper argues that while deficits in India are large, at least in the short run, the risk of a deficit-induced crisis is minimal. Jha and Sharma(2004) performed a more extensive analysis on this issue by testing for cointegration between public expenditure and revenue. They argue that if the two series are nonstationary in levels, yet stationary in first differences and cointegrated, Indian public debt is sustainable. Their empirical analysis, based on data spread over both the pre and post independence period 1871-1997 suggests that the revenue and expenditure series are I(1) and cointegrated with regime shifts. Thus, Indian public debt may not be unsustainable. While the above two studies dealt with the issue of debt sustainability solely for the Central Government, Goyal, Kundarapakam et.al(2005) analyzed the same issue for all the levels of government. They test for stationarity of public debt as was done by Buiter and Patel(2004) and employ the cointegration test developed by Gregory and Hansen to factor in structural breaks. By addressing the issue of regime shift, their paper finds that while the fiscal stance of the Central and the State Government at the individual level is unsustainable, it is weakly sustainable for the combined finances as it nets out inter-governmental financial flows. Thus, claims about sustainability of India’s public finance, made on the basis of the assessment of individual finances
and neglecting inter-governmental flows and the possibility of regime shifts seem exaggerated.

This paper aims at validating if debt sustainability should be measured in terms of a new parameters more suited for developing countries like India namely the change in the composition of public expenditure from current towards capital. Public expenditure is generally classified as consumption and investment expenditure.

We make two contributions to the theoretical literature on public debt sustainability in developing countries and one contribution to empirical literature on debt sustainability in India. First, we re-frame the dynamic optimization problem originally proposed by Devarajan, Swaroop, Zou (1996) of maximizing consumption with respect to private capital by introducing productive expenditure as a control variable and the law of motion of debt as another state variable. Second, even though it is acknowledged that capital expenditure does share a positive correlation with economic growth, literature on the long-run relationship between capital expenditure ratio in total expenditure and public debt in a cointegrating framework is scarce. We bridge this gap by testing for cointegration between the two variables and then representing the relationship in a VECM framework.

Third, having established the long run relationship we reformulate the Government Inter-temporal budget constraint originally proposed by Blanchard (1991) to derive the ‘capital expenditure gap’ indicator. While this is just a slight variant of the ‘tax gap indicator’ suggested by Blanchard it would be useful in analyzing the gap between the current capital expenditure and the optimal level. A more important suggestion of the paper is the ‘capital expenditure ratio’ indicator which could be useful for policy makers to forecast debt. Modeling debt taking advantage of the long-run relationship between capital expenditure and debt could improve the precision of forecasting debt for developing nations. At the same time, fiscal consolidation aimed at restructuring expenditure can also help in reducing debt levels.
1.1 Public Sector: Expenditure, Budget Constraint and Growth

1.1.1 Public Expenditure and Growth

Arrow and Kurz (1970) developed a model where consumers derive utility from private consumption as well as public capital stock. The literature on endogenous growth theories has further generated models linking public spending with economy’s long-term growth rate. Barro (1990, 1991) introduces the government in the utility function to be maximized along with the private sector and classifies the expenditure as consumption and investment expenditure. His empirical findings suggest that all non productive expenditures\(^2\) can have a negative effect on the growth rate of real GDP per capita in the long term. This would indeed lead to higher level of debt as growth rate will be reduced. However, a caveat in both these models is that public spending only affects the economy’s transitional growth rate, while the steady-state growth rate remains unaltered. Hence, these models cannot be used until the effect of public spending components on growth and debt respectively is accounted for as endogenous. Devarajan, Swaroop, Zou (1996) is an improvement on the earlier models as they relax the assumption of the exogenous public spending. They build an optimization problem with two types of expenditure, namely productive and unproductive. These are optimized with respect to capital stock in the economy to determine the relation between these expenditures and the growth rate of consumption.

Bose, Haque, Osborn (2003) examine the growth effects of government expenditure for a panel of 30 developing economies with a focus on sectoral expenditures during the 1970s and 80s. The findings of this study are noteworthy since they focus on developing countries. Their main empirical result is that the share of government capital expenditure in GDP is positively and significantly correlated with economic growth, while the growth effect of current expenditure is insignificant for a large group of countries. Gupta, Clements, Baldacci, Granados (2005) test the effects of fiscal consolidation and expenditure composition on economic growth in a sample of 39 low-income countries during the 1990s. The results of the study confirm that there is a strong link
between public expenditure reform and growth, as fiscal consolidations achieved through curtailing current expenditures are, in general, more conducive to growth. Additionally, they find that for the developed economies, more choice can be exercised over expenditure priorities and higher public spending even if of current expenditure form could not necessarily contract economic activity. Their simple correlation analysis shows that higher capital outlays are associated with more buoyant growth, while higher current expenditures and domestic financing of deficit are associated with less favorable economic performance. Empirical literature with similar results includes Landa u(1983) and Summers and Heston (1984).

1.1.2 Government Inter-temporal Budget Constraint

Blanchard (1990) proposed two indicators of fiscal sustainability, the primary gap indicator (PGI) and the Tax Gap Indicator (TGI). PGI calculates the adjustment in the primary balance needed to stabilize the outstanding of public debt ratio. Instead, the tax gap indicator answers a fundamental question about a desired tax rate to ensure sustainability and is derived from the inter-temporal budget constraint. Using a similar framework we can derive the desired productive expenditure to ensure sustainability and compare it with existing categories of public expenditure.

\[
\frac{dB}{ds} = G_1 + G_2 + H - T + rB = D + rB \tag{1}
\]

(1) represents the the law of motion of the dynamic budget constraint. Here \( \frac{dB}{ds} \) represents the law of motion of public debt, \( G_1 \), public productive expenditure, \( G_2 \), less productive expenditure, \( H \), total transfers, \( T \) the total taxes and \( r \) is the real interest rate. \( D \) represents the deficit as a whole.

Taking ratios to GDP for all variables in the equation we obtain (2). \( \theta \) represents the rate of growth of the economy.

\[
\frac{db}{ds} = g_1 + g_2 + h - t + (r - \theta)b \tag{2}
\]
Integrating this equation forward we get the final inter temporal budget constraint

$$\int d\text{exp} - (r - \theta)sd = -b_0 \quad (3)$$

Now we can derive the 'expenditure gap' indicator as follows by substituting $d = g_1 + g_2 + h - t$:

$$\int (g_1 + g_2 + h - t)\text{exp} - (r - \theta)sd = -b_0$$

$$\int (g_2 + h - t)\text{exp} - (r - \theta)sd + \int g_1\text{exp} - (r - \theta)sd = -b_0$$

$$\int (g_2 + h - t)\text{exp} - (r - \theta)sd + b_0 = -\int g_1\text{exp} - (r - \theta)sd$$

$$\int (g_2 + h - t)\text{exp} - (r - \theta)sd + b_0 = g_1\frac{\text{exp} - (r - \theta)s}{r - \theta}$$

$$g_1^* = (r - \theta)((\int (g_2 + h - t)\text{exp} - (r - \theta)sd + b_0)$$

$$g_1^* = (r - \theta)[(\int (g_2 + h - t + (r - \theta)b_0)(\text{exp} - (r - \theta)s)]] \quad (4)$$

The above expression (4) defines the threshold of the level of productive expenditure in the economy. Revisiting the composition of expenditure cannot directly decrease debt, however, focusing on productive spending can help in handling the debt situation in a better way. Thus, $g_1^*$ defines the optimal level of productive expenditure in the economy. In the following section an indicator is suggested based on this expenditure aspect of public debt.

**METHOD**

2.1 Optimization Model on Productive Expenditure

In this section we examine the relationship between composition of public expenditure, growth and public debt in a dynamic optimization framework. The model expresses the difference between productive and unproductive expenditures by how a shift in the mix between the two alters the economy’s long-term growth rate and public debt. The aggregate production function has capital stock, $k$, and two types of government spending, $g_1$ and $g_2$. $g_1$ represents expenditure
that contributes to future productivity of output and hence is part of capital accumulation by the
government while \( g_2 \) contributes solely to current output. \( g_2 \) enters the capital stock equation
indirectly. Analyzing the functional form to be CES (constant elasticity of substitution) then the
relationship can be expressed as

\[
y = f(k, g_1, g_2) = \left[ \alpha k^{-\xi} + \beta g_1^{-\xi} + \gamma g_2^{-\xi} \right]^{-\frac{1}{\xi}}
\]

(5)

where \( \alpha > 0, \beta \geq 0, 0 \leq \gamma < \beta, \alpha + \beta + \gamma = 1, \xi \geq -1 \)

The share, \( \lambda (0 \leq \lambda \leq 1) \), of total government expenditure on \( g_1 \) is given by
\( g_1 = \lambda g \) and \( g_2 = (1 - \lambda) g \)

Utility in this model is assumed to be in the iso-elastic form (7) and the representative agent
maximizes his welfare by choosing consumption, \( c \) based on the utility function.

\[
u(c) = \frac{c^{1-\sigma}}{1-\sigma}
\]

(6)

We consider an optimal control problem with (8) as the function to be maximized with two
state variables, \( k \) and \( b \), namely the capital stock and public debt with their equations of motion as
represented by (9) and (10) and two control variables, \( c \) and \( g_1 \), consumption and productive
expenditure respectively. (5), (6) and (7) are also constraints in the optimization problem. In this
model we refrain from discussion on transfers done by the government, consumer preferences are
iso elastic, government productive expenditure contributes to future productive capacity while less
productive expenditure contributes only to current output.

Maximize

\[
U = \int_{0}^{\infty} u(c)e^{-\rho t} dt
\]

(7)

subject to

\[
k = (1 - \tau)y - c + g_1
\]

(8)
Thus, we introduce government debt in the model in the form of the dynamic inter temporal budget constraint (2) where \( t \) represents the taxes collected by the government and \( r \) and \( \theta \) are the interest rate on debt and growth rate of output in the economy respectively.

The current value Hamiltonian takes the following form where \( \mu_k \) and \( \mu_b \) represent the shadow prices of \( k \) and \( b \) respectively.

\[
H(c, h) = c^{1-\sigma} - 1 + \mu_k \left( (1-\tau) \left\{ \alpha k^{-\xi} + \beta g_1^{-\xi} + \gamma g_2^{-\xi} \right\}^{-1/\xi} - c + g_1 \right) \\
+ \mu_b \left[ g_1 + g_2 - t + (r - \theta) b \right]
\]

Solving the model using optimal control theory we obtain the following first order conditions

\[
c^{-\sigma} = \mu_k \tag{10}
\]

\[
\mu_k \left[ (1-\tau) \beta g_1^{-\xi-1} \left\{ \alpha k^{-\xi} + \beta g_1^{-\xi} + \gamma g_2^{-\xi} \right\}^{-\left(1+\xi\right)/\xi} + 1 \right] + \mu_b = 0 \tag{11}
\]

\[
(1-\tau) \beta g_1^{-(\xi+1)} \left\{ \alpha k^{-\xi} + \beta g_1^{-\xi} + \gamma g_2^{-\xi} \right\}^{-\left(1+\xi\right)/\xi} + 1 = -\frac{\mu_b}{\mu_k} \tag{12}
\]

(12) represents the co state equation with productive expenditure \( g_1 \) and the shadow price of debt \( \mu_b \). Since the relationship between the two is inverse we can conclude that a positive movement towards productive expenditure helps in repayment of debt much more smoothly over time.

In addition to the costate equations defined in the problem statement, the maximum principle requires the following equations of motion for the co-state variables to be satisfied:
\[
\mu_k = \mu_k \left[ (1 - \tau)\alpha k^{-\xi} - 1 \left\{ \alpha k^{-\xi} + \beta g_1^{-\xi} + \gamma g_2^{-\xi} \right\}^{-(1+\xi)/\xi} \right] + \rho \mu_k \tag{13}
\]

\[
\mu_b = \mu_b (r - \theta) + \rho \mu_b \tag{14}
\]

Since there are four differential equations, the system cannot be analyzed with a phase diagram. But our main question of interest is to see the relationship between productive expenditure, growth rate and public debt. So how does the productive expenditure affect the shadow price of debt? What is the relation between the proportion of productive expenditure \( \lambda \) with respect to growth rate of the economy and growth rate of consumption?

The basic features of such a steady state is that all the state and control variables grow at the same rate. Hence, in steady state

\[
\mu_k = -\sigma c^{-\sigma - 1} c \tag{15}
\]

Using (11) we get

\[
\frac{\mu_k}{\mu_k} = -\sigma \frac{c}{c} \tag{16}
\]

We substitute (14) into (17) and obtain

\[
\frac{c}{c} = - \left[ (1 - \tau)\alpha k^{-\xi} - 1 \left\{ \alpha k^{-\xi} + \beta g_1^{-\xi} + \gamma g_2^{-\xi} \right\}^{-(1+\xi)/\xi} \left\{ \alpha + \frac{g_1}{k} (\beta \lambda^{-\xi} + \gamma (1 - \lambda)^{-\xi}) \right\}^{-(1+\xi)/\xi} \right] + \rho \tag{17}
\]

Using the other constraints and substituting them in (18), (19) is obtained.

\[
g_1 = \lambda g \text{ and } g_2 = (1 - \lambda) g, \text{ hence } c_\theta = \frac{c}{c}, \text{ the growth rate of consumption}
\]

\[
c_\theta = - \left[ (1 - \tau)\alpha \left\{ \left( \alpha + \frac{g_1}{k} \right) \left( \beta \lambda^{-\xi} + \gamma (1 - \lambda)^{-\xi} \right) \right\}^{-(1+\xi)/\xi} \right] + \rho \tag{18}
\]
\[ \frac{dc_\theta}{d\lambda} = \frac{\alpha(1 - \tau)(1 + \xi)(\xi/k)^{-\xi} \left[ \beta \lambda^{-1(1+\xi)} - \gamma(1 - \lambda)^{-1(1+\xi)} \right]}{\sigma \left\{ \left[ \alpha + (\xi/k)^{-\xi}(\beta \lambda^{-\xi} + \gamma(1 - \lambda)^{-\xi}) \right]^{-1(1+2\xi)/\xi} \right\} (18) \]

Expression (19) should be positive if \( \lambda \) should have a positive effect on growth. The right hand side this equation will be positive if \((1 + \xi) \left[ \beta \lambda^{-1(1+\xi)} - (1 - \lambda)^{-1(1+\xi)} \right] > 0 \). It follows that \( \xi \geq -1 \), hence \( \frac{d\theta}{d\xi} > 0 \) if \( \left( \frac{\beta}{\gamma} \right) \eta > \frac{\lambda}{1 - \lambda} \) where \( \eta = 1/(1+\xi) \) is the elasticity of substitution. Since \( \lambda \) is an increasing proportion until optimality is reached, the left side will always be bigger than the right hand side. A caveat, is that the increase in growth on account of capital expenditure apart from depending on \( \beta \) and \( \gamma \), depends also on \( \lambda \), which is the initial share of productive expenditure. \(^6\) Thus if initial \( \lambda \) is very high, \( \beta > \gamma \) may not necessarily raise the growth rate. However, such debates would be more relevant for developed countries where the productive threshold of capital/productive expenditure has already been reached.

Since \( \mu = \mu_k \mu_b \) characterizes steady state, we can equate

\[ \theta = (r - \left[ \alpha(1 - \tau) \left\{ \alpha + (\xi/k)^{-\xi}(\beta \lambda^{-\xi} + \gamma(1 - \lambda)^{-\xi}) \right\}^{-1(1+2\xi)/\xi} \right] ) \quad (19) \]

This equation shows the growth of output in the economy and its relationship with composition of public expenditure. The analytical condition obtained here is intuitively the same as that of the growth of consumption(18).

\[ \frac{d\theta}{d\lambda} = (1 - \tau)(1 + \xi)\alpha \left\{ \alpha + (\xi/k)^{-\xi}(\beta \lambda^{-\xi} + \gamma(1 - \lambda)^{-\xi}) \right\}^{-1(1+2\xi)/\xi} \quad (20) \]

\[ (\xi/k)^{-\xi} \left[ \beta \lambda^{-1(1+\xi)} + \gamma(1 - \lambda)^{-1(1+\xi)} \right] \]

Summing up, the two main results are given by (12) and (20). The first is that the relationship between productive expenditure and growth rate of the economy is governed by initial shares of expenditures as well as the current proportions. The ratio of initial shares would always be more than that of current proportion of productive expenditure. Thus there is a constraint on the amount
until which the investment expenditure can be increased in the economy. Additionally, an increase in productive expenditure decreases the utility denominated value of debt which means that the welfare cost of an increase in government debt falls. Further, part of government expenditure considered to be highly productive is used to generate productive capacity in the future reducing the burden of debt. This helps the government in choosing a consumption path which helps in reabsorbing the value of debt slowly and extends the repayment time period. Thus it would be worthwhile to develop an indicator of debt sustainability based on $\lambda$, the productive expenditure component.

### 2.2 Proposed Indicator : $\lambda$

We propose an indicator defined as Capital Expenditure Ratio to aggregate expenditure ($\lambda$) and mathematically can defined as

$$\lambda = \frac{g_{ca}}{(g_{ca} + g_c)}$$

In line with the growth theory outlined above this indicator measures the share of capital expenditure in total public expenditure. As $\lambda$ increases, it is expected that the public debt levels will react inversely for developing countries. This unique feature makes it more feasible to be used for debt sustainability simulations in comparison to the existing indicators. It would be noteworthy to understand how this indicators maps with the Government inter-temporal budget constraint for policy and simulation purposes.

Having understood that $\lambda$ affects growth positively and growth and debt share a negative relationship, we can infer a negative relationship between $\lambda$ and debt intuitively. Since the empirical results confirm this intuition we can represent this relationship in the form of an indicator that can be used as a predictor of debt dynamics.

$(2)$ provides us with a representation of the inter-temporal constraint with all variables in a ratio to GDP form. $(22)$ can be used to substitute for $(g_{ca} + g_c)$ back in $(2)$. The resulting equation
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is as follows:

\[
\int \frac{db}{ds} = \int \frac{g_{ca}}{\lambda} + h - t + (r - \theta)bsd
\]

(22)

\[
\int \frac{g_{ca}}{\lambda} + h - texp - (r - \theta)sds = -b_0
\]

\[
\int (h-t)exp - (r - \theta)sds + \int \frac{g_{ca}}{\lambda}exp - (r - \theta)sds = -b_0
\]

\[
\int (h-t)exp - (r - \theta)sds + b_0 = -\int \frac{g_{ca}}{\lambda}exp - (r - \theta)sds
\]

\[
\int (h-t)exp - (r - \theta)sds + b_0 = \frac{g_{ca} \exp(-(r-\theta)s)}{(r-\theta)}
\]

\[
\frac{g_{ca}}{\lambda} = (r - \theta)(\int (h-t)exp - (r - \theta)sds] + b_0)
\]

(23)

(24) represents the capital expenditure ratio indicator. While dynamic future simulations are not in the focus of the paper, the empirical analysis in the next part will aim to test the theoretical hypothesis of inverse relationship between \( \lambda \) and \( b_t \) with the use of multivariate time series analysis followed by static VECM based simulations. This is mandatory for understanding the precision of this indicator for forecasting purposes.

2.3 Empirical Test

The long-run relation between debt to GDP ratio\((b)\)and Capital expenditure ratio\((\lambda)\) can be expressed as in (24):

\[
b_t = \frac{\beta}{\lambda_t} - \gamma \alpha
\]

(24)

\(7\), where \( \gamma = 1 \) if \( r = \theta \) which means that the interest rate on debt equals the growth rate of the economy.

A unit coefficient \((\beta = 1)\) would imply that capital expenditure ratio does affect the debt to GDP level in a perfect market. However, in reality since we are in an imperfect market this parameter should exceed one.
We apply Johansen’s(1992,1995) multivariate method to estimate the long-run relation\(^8\) between debt to GDP ratio\((b_t)\) and Capital expenditure ratio\((\lambda_t)\). Under this approach, a system of \(n\) endogenous variables can be parametrized into a vector error correction model:

\[
\Delta X_t = \mu + \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \ldots + \Gamma_{k-1} \Delta X_{t-k+1} + \Pi X_{t-k} + \varphi D_t + u_t \quad (25)
\]

where \(X_t\) is an \((n,1)\) vector; \(\Gamma_i\) and \(\Pi\) are \((n,n)\) coefficient matrices; \(D_t\) are deterministic components, such as seasonal and impulse dummies; \(\mu\) is a constant term; \(k\) is the lag length; and \(u_t\) is a vector of normally and independently distributed error terms. In our system, \(X_t = [b_t, \lambda_t]\) is a \(2 \times 1\) vector, and \(\Gamma_i\) and \(\Pi\) are \((2 \times 2)\) coefficient matrices. A cointegrated system implies that \(\Pi = \alpha \beta'\) is reduced rank, \(r\), for \(r < n\).

To understand this in more detail, we can take a deeper look at a multivariate generalization of single equation dynamic models (VAR).

\[
\gamma_t = \begin{bmatrix} b_t \\ \lambda_t \end{bmatrix} = A_1 \begin{bmatrix} b_{t-1} \\ \lambda_{t-1} \end{bmatrix} + A_2 \begin{bmatrix} b_{t-2} \\ \lambda_{t-2} \end{bmatrix} + \ldots + A_n \begin{bmatrix} b_{t-n} \\ \lambda_{t-n} \end{bmatrix} + u_t \quad (26)
\]

Now representing this model in levels and first differences, we subtract \(\gamma_{t-1}\) from both sides of the VAR; we obtain

\[
\Delta \gamma_t = (A_1 - I) \gamma_{t-1} + A_2 \gamma_{t-2} + \ldots + A_n \gamma_{t-n} + u_t
\]

Further we subtract \((A_1 - I) \gamma_{t-2}\) from both sides until \(n-1\)

\[
\Delta \gamma_t = \Pi_1 \Delta \gamma_{t-1} + \Pi_1 \Delta \gamma_{t-2} + \ldots + \Pi \Delta \gamma_{t-n} + u_t \quad (27)
\]

\[
= \sum_{i=1}^{n-1} \Pi_i \Delta \gamma_{t-i} + \Pi \Delta \gamma_{t-n} + u_t \quad (28)
\]

where

\[
\Pi_i = (I - \sum_{j=1}^{i} A_j)
\]
The above equation (28) is the parametrization of the VAR model as a VECM. The next section throws more light on how this econometric framework can be tested with data, results observed and their interpretation.

RESULTS

The empirical analysis has been done on Indian data covering the time period 1980-2009 for all three levels of government namely Central, State and Consolidated General Government. There are two main reasons as to why India has been selected for the analysis. Firstly, India is a developing nation, with huge level of public debt and deficits at all levels of Government. The puzzle, however, is that even then it has escaped a debt crisis so far. Secondly, owing to a strong federal structure, there is a clear demarcation on expenditure prerogatives.

India’s federal structures are an important aspect of its political and economic system. The Indian Constitution, in its Seventh Schedule, assigns the powers and functions of the Center and the States. The schedule specifies the exclusive powers of the Center in the Union list; exclusive powers of the States in the State list; and those falling under the joint jurisdiction are placed in the Concurrent list. All residuary powers are assigned to the Center. The nature of the assignments is fairly typical of federal nations. The functions of the central government are those required to maintain macroeconomic stability, international trade and relations and those having implications for more than one state. The major subjects assigned to the states comprise public order, public health, agriculture, irrigation, land rights, fisheries and industries and minor minerals. The States also assume a significant role for subjects in the concurrent list like education and transportation, social security and social insurance. According to the Indian constitution, capital disbursements are the responsibility of the Central Government, while the State Government is assigned current
and social disbursements. The dynamic optimization model discussed in the previous section of this paper considers the government as a body that governs the country in entirety. However for specific country analysis such as the empirical analysis in this paper, each level of Government must be separately analyzed. Thus, the following empirical analysis bridges the gap between the theoretical model and existing federal structure. Essentially this means that empirically the effect of capital expenditure on debt should be more pronounced for the Central and Consolidated General Government, than for the State Governments.

As a precursor to the cointegration tests, we regress $\lambda$ and its counterpart on current expenditure on $b_t$. This is a common approach when short run relationships between two variables need to be established. The results would mainly aim at checking the sign of the coefficients, and not necessarily on their statistical significance. Additionally, they can also help explain which of the categories of expenditure are more productive, in consistence with equation (13). Broadly consistent with the earlier discussion on the inverse relationship between the two time series in question, we do obtain a negative coefficient for $\lambda$ and vice versa for current expenditure for all levels of Government. Additionally, we observe that the regression is significant for Central, consolidated Gen Government and insignificant for the States. Table 1 summarizes the key results of this regression.

### 3.1 Unit Root Tests

Before testing for cointegrating relations, univariate time-series properties of capital expenditure ratio and Public debt are examined using two unit root tests developed by KPSS(1992) and the augmented Dickey Fuller developed by Dickey and Fuller (1979). The KPSS tests the null of stationarity, whereas the ADF tests the null of the unit root. If the KPSS test rejects the null but the ADF test does not, both tests support the same conclusions; that is, the series in question is a unit root process. Results of the ADF and KPSS tests are reported in Table 2.  

In case of Consolidated General Government and the Central Government, the ADF tests
cannot reject the unit root null in any of the indexes (ratio/log level), whereas the KPSS tests reject the null of stationarity for all indexes. At the first differences, the ADF reject the unit root for $\lambda$, but for $b_t$ the significance can be weakly seen at 10% levels. It has already been seen in the past literature that debt series for India suffer from structural breaks. The KPSS test however, clears this doubt by not rejecting the null for stationarity. Additionally, correlogram analysis (represented in Table 3) show the $b_t$ and $\lambda$ to be non stationary ratios and stationary in first differences respectively. Correlograms show the Auto correlation and Partial Auto correlations for a particular time series. For a non stationary time series, the auto correlations are extremely high and p-values are low. Additionally, the AC coefficient starts at a very high value and then declines slowly. They also test the null of stationarity, hence when p-values are low we reject the null. Thus, ADF and KPSS tests and correlogram analysis, confirm that both DEBT and CAPRATIO are unit root processes and seem to be I(1) for the Central and Consolidated General Government.

In case of the state level analysis as was perceived, while the log levels of the variables are I(0), the first differences seem to be I(2). To endorse this further, we check for cointegration between $b_t$ and $\lambda$ for each levels of government. In case of the Centre and Consolidated General Government, the two variables share a common I(1) trend, and $b_t$ is unlikely to be second-order cointegrated. However, in case of the State level analysis, we do not find any cointegration between the two variables which shows that the relation is less pronounced for the States than for the Centre and Consolidated General government. Thus, reemphasizing the fact that capital expenditure is the responsibility of the Central Government. Further analysis and discussions on forecasting emit the State level Analysis.

### 3.2 Cointegration and VAR/VECM

The reason for using a VAR model here is that all time series variables are endogenous and there is cointegration between the two. We have 30 observations for each time series variables and including too many lagged terms will consume degrees of freedom, and also a probability of
multicollinearity could arise. Hence, to estimate the Vector autoregressive (VAR) models, identifying the order of the VAR is important. We identify the lag lengths following Sim’s (1980) like-likelihood (LR) tests and multivariate Akaike information criterion (AIC). Under the LR tests, we begin with a maximum lag Length(k-max) of 7 and sequentially test down, deleting one VAR lag at a time until the deleted lags are jointly significant. Information criteria normally choose a shorter lag Length, which is not always sufficient to flush serial correlation from the VAR residuals. However, it is important to render VAR residuals uncorrelated (Johansen 1992). To circumvent this, we restrict the AIC search between k-max=7 and k-min=1. The VAR lengths specified by both the methods are reported in Table 4. In our case the VAR length selection is uniform since both LR and AIC identify the same lengths. Hence, we adopt VAR length of 3, as represented in the last column of Table 4. 11

Table 5 shows the trace tests for the cointegration rank $r$, for the two variables. The trace test equation indicates 1 cointegrating equation at 0.05 level. This means that $b_t$ and $\lambda$ are cointegrated, suggesting a long-run relation between these two variables. On grounds that debt and capital expenditure bear a long run inverse relationship, an error correction representation of them can be used to assess whether capital expenditure ratio indicator would be useful for forecasting and fiscal consolidation policies. If the VECM model does indicate significant coefficients in the cointegration equation, this could be useful for policy makers because they can refer policies suited towards redesigning of expenditure in developing countries.

The VAR specification for the analysis is as represented in equation (7) and (8) below. However, in this case, since we have a cointegration between the two variables, we reparametrize the VAR into a vector error correction model. Table 6 below shows the results of the VECM representations. The cointegration equation coefficient for $b_t$ is statistically significant, and so is the constant. In addition, the second lag coefficient is also significant. The lag coefficients for $\lambda$ for the first lag is significant too. The coefficient in the cointegration equation for $\lambda$ is highly significant for the consolidated General government whereas for the Centre the level of significance
is not so high. Thus, the error correction lends further support to the hypothesis that $\lambda$ affects $b_t$. The not so significant coefficient of the $\lambda$ does not affect further analysis, because the forecast has to be done for DEBT only. The overall $R^2$ is 0.46 and 0.39 respectively for the Consolidated and Central Governments respectively which makes the regression non-spurious statistically.

3.3 VECM Forecasting Simulations

Having obtained significant coefficients in the VECM, we proceed with evaluation of the model for forecasting purposes for the Consolidated General Government and the Centre. The representations below define the VECM model, and can be used for static and dynamic forecasts. While dynamic future forecasting is out of the scope of the paper, we can use the VECM model to estimate/forecast for the period between 1980-2010 and compare the forecast with the baseline. A converging pattern would suggest high precision of the indicator. Figure (1) in the appendix shows the graphical comparison of forecasts with actuals. It can clearly be observed, that the forecasts do converge around the observed values for both types of government. The baseline trajectory represents the simulations while Actual refers to the values observed historically.
Figure 1. Simulations

- Con Gen Govt.
- Centre
DISCUSSIONS

We propose a theoretical framework for devising optimal productive public expenditure in an economy in relation with the capital stock in the economy and public debt of the government. Co state equation and steady state conditions help in deriving two analytical results. The first being the fact that an inverse relationship is seen between productive expenditure and shadow price of debt, allowing the government to smooth the path for repayment of its debt. Additionally the ratio of this expenditure in total expenditure should always be lower than the ratio of the initial shares of productive and less productive expenditures.

Further, we examine whether capital expenditure has a long run relationship with public debt. We use capital expenditure and public debt annual time series spanning 30 years from India for all levels of government. We find capital expenditure, capital expenditure ratio and Public debt are cointegrated, which implies a long-run relation between them. While the VECM might not be the best procedure for testing future simulations it does empirically help in adding weight to use the indicator in (24) for forecasting and policy purposes.

Our investigation of annual data at all levels of government, extends the strands of empirical literature. We provide a robust empirical analysis by formally testing for stationarity and cointegration of debt to GDP ratio and capital expenditure ratio. In both analysis we identify the two variables to be integrated of first order and bear a cointegration relationship. The application of error correction representation, improves the results of the VAR model. Furthermore, dynamic simulations increase the confidence in using the suggested indicator. Overall, our empirical findings suggest that for developing countries like India, the percentage of capital expenditure in total public expenditure bears an inverse long-run relationship with debt, and the suggested 'capital expenditure ratio gap' indicator could be used for forecasting purposes.
APPENDICES

A.1 Data

The data used in this study are obtained from *Handbook of Statistics on the Indian economy* (2009), *National Accounts Statistics* (CSO) and *Indian Public Finance Statistics*-various issues. All variables used in analysis are ratio to GDP. CAPRATIO specifically has been calculated as capital expenditure divided by total public expenditure. The sample covers 1980-2010, yielding 30 observations for each variables at each level of Government. The other variables used in the analysis are total public expenditure, public debt, GDP at Factor cost and current expenditure.

The figure (2) represents some graphs that show the trajectories of $\lambda$, CURATIO (current expenditure to GDP ratio) and debt for each level of government. It is evident that for India the debt trajectory is an increasing one with CURATIO higher than $\lambda$. However, from 2006 the $\lambda$ has been increasing at the Consolidated and State levels, while there is a decrease in the value of the same for the Centre. This increase and decrease has been stagnant since 2006. Policymakers could aim at increasing the $\lambda$ since it lies even below that of CURATIO for better public debt management.
Figure 2

Con Gen Govt.

Centre

State

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</table>
A.2 VECM(Long Run Properties) and Econometric Relationship

derivation between $\lambda_t$ and $b_t$

$$\lambda = \frac{g_{ca}}{(r-\theta)((\int (h-t) \exp-(r-\theta)sds)+b_0)}$$ represents the CAPRATIO indicator. Normally a long run linear relationship between two variables, is defined by $b_t = \alpha + \beta \lambda_t$. However in this case since the relationship does not seem linear, we would need to deduce the relationship by rearranging the equation as follows.

$$\lambda - \frac{g_{ca}}{(r-\theta)((\int (h-t) \exp-(r-\theta)sds)+b_0)} = 0$$

$$\lambda \{(r-\theta)((\int (h-t) \exp-(r-\theta)sds)+b_0)\} - g_{ca} = 0$$

For this expression to hold good, we would need the numerator to be zero. Equating the numerator as 0 and assuming $\alpha = \left[ \int (h-t) \exp-(r-\theta)sds \right]$, $\beta = g_{ca}$, $(r-\theta) = \gamma$, we get

$$g_{ca} = \lambda_t [(r-\theta)\alpha + b_t]$$

Final rearrangement and substitution yields

$$b_t = \frac{\beta}{\lambda_t} - \gamma \alpha$$

(29)

$\Pi$'s properties explain the long run properties of the VECM model.

rank($\Pi$) = 0, non stationary with no cointegration

rank($\Pi$) = 2, full rank, which means that the system is stationary as a whole even if individual series are not

rank($\Pi$) = 1, non stationary with 1 cointegrating relationships.

For the Johansen test, the rank of the matrix= no. of characteristic roots that differ from zero. In case of no cointegration rank of $\Pi$ is 0 and all characteristic roots equal zero. $1 - \lambda_1 = 0$.

If rank($\Pi$) = 1, which is the case in point here, $0 < \lambda_1 < 1$, we have the following model which can be represented as a VAR.
\[
\begin{bmatrix}
\frac{b_t}{\lambda_t} \\
\frac{b_{t-1}}{\lambda_{t-1}}
\end{bmatrix} = A_0 + A_1 \begin{bmatrix}
\frac{b_{t-1}}{\lambda_{t-1}} \\
\frac{b_{t-2}}{\lambda_{t-2}}
\end{bmatrix} + A_2 \begin{bmatrix}
\frac{b_{t-2}}{\lambda_{t-2}} \\
\frac{b_{t-3}}{\lambda_{t-3}}
\end{bmatrix} + \begin{bmatrix}
u_t
\end{bmatrix}
\]

The VECM form hence would be

\[
\begin{bmatrix}
\triangle b_t \\
\triangle \lambda_t
\end{bmatrix} = \Pi_0 + \Pi_1 \begin{bmatrix}
\frac{b_{t-1}}{\lambda_{t-1}} \\
\frac{b_{t-2}}{\lambda_{t-2}}
\end{bmatrix} + \Pi_2 \begin{bmatrix}
\triangle b_{t-1} \\
\triangle \lambda_{t-1}
\end{bmatrix} + \begin{bmatrix}
u_t
\end{bmatrix}
\]

\[\Pi = \alpha \beta \text{ and } \Pi = -\left(1 - \sum_{i=1}^{2} A_i\right)\]
REFERENCES


Blanchard, Oliver (1990), Suggestions for a New Set of Fiscal Indicators, OECD Economics Department Working Papers No. 79.


Bose Niloy, Emranul Haque and Denise Osborn(2003), Public expenditure and Economic Growth : A Disaggregated Analysis for Developing Countries, Working Paper Centre for Growth and Business Cycle Research, University of Manchester, USA.


Horne J. (1991), Indicators of Fiscal Sustainability, International Monetary Fund, Fiscal Affairs Department WP/91/5.


Economic Activity, vol. 1, pp. 257-286


AUTHOR NOTE

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indicators of debt sustainability: india

FOOTNOTES

1 Owing to statistically significant cointegration results discussed in detail in Section 4 of the paper.

2 While Barro calls this element non productive expenditure Landau(1983) calls these consumption expenditure and they have a close bearing to the definition of current expenditure used in this model. Expenditure that provides long term stimulus to growth and thus helps in reducing public debt

3 Correlation coefficients are significant for capital outlays

4 Capital expenditure, which has been empirically found to give high stimulus to growth up till a certain level

5 Current expenditure, which is considered to give less stimulus to growth

6 This result is in consistence with that of Devarajan et.al. In (20) we obtain such an analytical condition also for growth rate of the economy

7 Detailed Derivation in appendix in the section of VECM Properties

8 The parameters of equation(25)

9 Refer to appendix for details on long run properties of the VECM and links with cointegration methodology of Johansen(1995)

10 No mention of * indicates that the variable is significant at multiple levels

11 The VAR length specification is particularly important for CAPRATIO since its dependency has to be checked with that of DEBT.

12 Refer to Table 7

13 Already defined in section 3 of the paper

14 Significant at 12%, could be due to the small size of the sample, as all data tested for is annual
<table>
<thead>
<tr>
<th></th>
<th>Con Gen Govt.</th>
<th>Centre</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPRATIO</td>
<td>-16.5 (0.65)</td>
<td>-13.8 (0.45)</td>
<td>-11.0 (1.11)</td>
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<tr>
<td>CURATIO</td>
<td>16.6 (0.65)</td>
<td>11.1 (0.42)</td>
<td>11.07 (1.11)</td>
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<td></td>
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<tr>
<td>R²</td>
<td>0.71</td>
<td>0.86</td>
<td>0.13</td>
</tr>
<tr>
<td>P-value</td>
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<td>0.000</td>
<td>0.033</td>
</tr>
<tr>
<td>DW</td>
<td>0.29</td>
<td>1.04</td>
<td>0.06</td>
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</table>

Note: The L(debt) is used to avoid problems of autocorrelation and spurious regression. The value of R² is very high for the Consolidated and Central Government. For the States the value is very low which means that the relationship between CAPRATIO and DEBT is not very well explained on the basis of the data.
Table 2

Table 2. Augmented Dickey-Fuller and Kwiatkowski, Phillips, Schmidt, and Shin Tests

<table>
<thead>
<tr>
<th>Log Levels</th>
<th>First Differences</th>
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<tr>
<td>DEBT</td>
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<tr>
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<td><strong>ADF Const</strong></td>
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<tr>
<td>DEBT</td>
<td>3.00***</td>
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<tr>
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<td><strong>ADF Const</strong></td>
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<td>-2.04***</td>
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<tr>
<td>DEBT</td>
<td>2.22***</td>
</tr>
</tbody>
</table>

Note: ADF= augmented Dickey-Fuller(1979) ; KPSS= Kwiatkowski, Phillips, Schmidt, and Shin(1992) ; CAPRATIO= Capital expenditure component in total public expenditure ; DEBT R = Public Debt to GDP ratio. The ADF tests are conducted by setting a lag length (k) of 7 as explained in the test. The KPSS tests are reported on the automatic (k) selection of 4 since the sample is small. The ADF tests , ADF Const denotes the only constant term in the estimating equation, whereas Trend denotes both the constant term and linear time trend. For ADF Trend log values of variables have been used. Same notations are used for constant and trend in the KPSS model.

Critical Values:

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<th>ADFTrend</th>
<th>KPSSConst</th>
<th>KPSSTrend</th>
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<tr>
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<tr>
<td>5%</td>
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*** Significant at the 1% level
** Significant at the 5% level
* Significant at the 10% level
Table 3: Correlogram

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Correlogram (Con Gen Govt.)

Correlogram (Centre)

Correlogram (State)
Table 4

Centre Lag selection criteria

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<th>AIC</th>
<th>SC</th>
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Con Gen Govt. Lag selection criteria

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<th>AIC</th>
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<td>18.00568</td>
<td>17.34073</td>
</tr>
</tbody>
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* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion
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<td>0.49** [0.48] (0.49)</td>
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<table>
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</tr>
<tr>
<td>32.5 [0.0001] (17.23)</td>
<td>15.32* [0.0001] (15.32)</td>
</tr>
</tbody>
</table>

Note: P-values are reported in brackets for this test. The 5% critical values of the trace statistics for $H_0 = 0$ are 15.49 and for $H_0 \leq 1$ are 3.84 respectively. In case of Central Government 2 cointegrating vectors are observed. The lag lengths used are as per the last column of Table 4.

*** Significant at the 1% level
**  Significant at the 5% level
*   Significant at the 10% level
### Table 6. Error –Correction models

<table>
<thead>
<tr>
<th></th>
<th>Con Gen Govt.</th>
<th>Lag 1</th>
<th>Lag 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in DEBT</td>
<td>0.39(0.21)</td>
<td>-0.08(0.17)</td>
<td></td>
</tr>
<tr>
<td>Change in CAPRATIO</td>
<td>-0.72(0.20)</td>
<td>-0.32(0.21)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>10.389</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Centre</th>
<th>Lag 1</th>
<th>Lag 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in DEBT</td>
<td>0.30(0.21)</td>
<td>-0.11(0.18)</td>
<td></td>
</tr>
<tr>
<td>Change in CAPRATIO</td>
<td>0.12(0.17)</td>
<td>-0.49(0.17)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-0.33</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The cointegration equation for DEBTR is statistically significant with a value of 78% with a t-stat of -3.7 for the consolidate general government. The 10% critical value for the t-stat is 1.31 for n=24, where n is the number of degrees of freedom. The constant term is also significant.

***  Significant at the 1% level
**   Significant at the 5% level
*    Significant at the 10% level
<table>
<thead>
<tr>
<th></th>
<th>Con Gen Govt.</th>
<th>Centre</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Table 7</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>CAPRATIO</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CAPRATIO(1)</td>
<td>0.397849</td>
<td>0.564912</td>
</tr>
<tr>
<td></td>
<td>(0.19933)</td>
<td>(0.19850)</td>
</tr>
<tr>
<td></td>
<td>[1.21013]</td>
<td>[1.29518]</td>
</tr>
<tr>
<td>CAPRATIO(2)</td>
<td>0.499495</td>
<td>-0.425072</td>
</tr>
<tr>
<td></td>
<td>(0.19494)</td>
<td>(0.19530)</td>
</tr>
<tr>
<td></td>
<td>[2.51004]</td>
<td>[2.17681]</td>
</tr>
<tr>
<td><strong>LDEBT</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LDEBT(1)</td>
<td>0.013787</td>
<td>0.058777</td>
</tr>
<tr>
<td></td>
<td>(0.01346)</td>
<td>(0.01371)</td>
</tr>
<tr>
<td></td>
<td>[1.19123]</td>
<td>[1.06868]</td>
</tr>
<tr>
<td>LDEBT(2)</td>
<td>-0.069544</td>
<td>-0.109376</td>
</tr>
<tr>
<td></td>
<td>(0.06915)</td>
<td>(0.07376)</td>
</tr>
<tr>
<td></td>
<td>[-0.05046]</td>
<td>[-0.04479]</td>
</tr>
<tr>
<td><strong>C</strong></td>
<td>-0.446573</td>
<td>0.111117</td>
</tr>
<tr>
<td></td>
<td>(0.14359)</td>
<td>(0.24250)</td>
</tr>
<tr>
<td></td>
<td>[-0.32434]</td>
<td>[3.50925]</td>
</tr>
</tbody>
</table>

| **R-squared** | 0.889123 | 0.838971 |
|               | 0.868053 | 0.808485 |
| **Sum sq. resid** | 0.007465 | 0.006552 |
| **S.E. equation** | 0.010864 | 0.017664 |
| **F-statistic** | 42.09973 | 243.2588 |
| **Log likelihood** | 70.2589 | 70.2589 |
| **Akaike AIC** | -4.934047 | -4.934047 |
| **Schwarz SC** | -4.692105 | -4.692105 |
| **Mean dependent** | 0.181930 | 13.51147 |
| **S.D. dependent** | 0.051871 | 1.102186 |

| Determinant resid covariance (df adj.) | 1.10E-07 | 2.69E-07 |
| Determinant resid covariance | 7.21E-09 | 1.00E-09 |
| Log likelihood | 140.0075 | 152.5971 |
| Akaikes information criterion | -10.00959 | -10.08126 |
| Schwarz criterion | -6.516085 | -6.603139 |