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Abstract

This paper addresses the question of why the price of nontradables relative to tradables is positively correlated with income per-worker. I construct a two-sector model in which agents differ with respect to managerial ability. Agents sort themselves by choosing to become a worker, a manager in nontradables, or a manager in tradables. A fixed cost of exporting places the most productive managers in the tradable sector, and the magnitude of the fixed cost determines the extent of this margin. Fixed costs together with trade costs determine the amount of competition across sectors which in turn determines prices across sectors. The calibrated model explains more than 60% of the cross-country differences in the relative price of nontradables, due to the presence of larger fixed costs in poor countries combined with nontrivial import costs.

Keywords: relative prices; PPP; tradables; nontradables; competition.
JEL Classification: F10, F12, F16.

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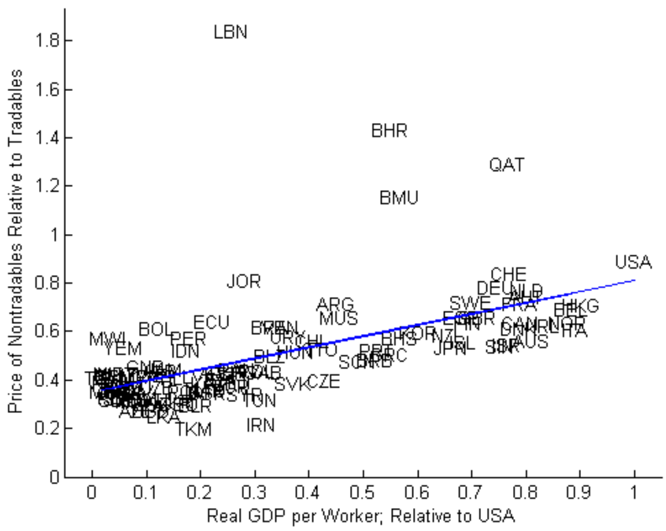
1 Introduction

This paper addresses the question of why the price of nontradables relative to tradables is positively correlated with income per-worker. This is an important question because prices reveal information about, among other things, productivity. Since all countries allocate a sizeable fraction of resources into production of both tradables and nontradables, then gaining an understanding of how productivity arises at the sectoral level will shed light on the sources of cross-country differences in aggregate productivity.

The relationship between relative prices and income is clear by inspection of Figure 1, where the relative price of nontradables is on the vertical axis, and real income per-worker is on the horizontal axis. Excellent sources for further empirical documentations of this fact include Kravis and Lipsey (1988) as well as Heston, Nuxoll and Summers (1994). To explain this puzzle, I propose a theory in which prices at the sectoral level depend on differences in competition across sectors, and competition in turn depends on trade barriers.

Figure 1: Cross-country relative prices.



All else equal, sectors with more competition have lower prices. In the model, competition within each sector is determined endogenously through sorting, the process by which heterogenous agents select a sector in which to operate. In particular, each agent chooses to be a worker or a manager, and if one becomes a manager, he chooses which sector to manage in: nontradables or tradables. The sorting outcome results in the highest ability agents choosing to be managers in the tradable sector, the next highest ability agents choosing to be managers in the nontradable sector, and the lowest ability agents choosing to be workers. Therefore, productivity at the sectoral level is endogenous and is a function of competition.

I focus on two types of barriers which determine competition at the sectoral level: *i*) fixed costs of exporting and *ii*) import costs which are modeled as iceberg costs. The sorting outcome within a given country is uniquely determined by the domestic fixed cost; a higher fixed cost means there will be fewer managers in the tradables sector, i.e., less domestic competition. Trade costs, which do not affect the domestic pattern of sorting, control the amount of competition in tradables that comes from abroad. An important property of the model is that changes in trade costs affect competition, but equilibrium effects from sorting produce a wage effect which leaves the allocation of agents across sectors unchanged; that is, prices adjust so that agents do not reallocate across sectors. The same feature is present regarding changes in TFP as well. This novel feature allows me to isolate the wage effect from the competition effect and in turn quantify the individual importance of fixed costs, trade costs, and TFP. I can then measure how important each one is in explaining relative price differences.

I implement the theory quantitatively by calibrating the model to data on income per-worker, bilateral trade flows, and average size of manufacturing establishments. I restrict the calibration to a two country world, where the developed country consists of a group of rich countries, and the developing country consists of a group of poor countries. The two main quantitative findings are that the fixed cost is larger in the developing country, and the import cost is also larger in the developing country. Given the predicted asymmetries in both fixed costs and import costs, the model explains over 60% of the observed difference in the relative price of nontradables. More interestingly, almost all of this variation is due to differences in competition at the sectoral level.

The model constructed in this paper encompasses two main theories of relative prices. The first theory is based on productivity differences, pioneered by Balassa (1964) and Samuelson (1964), which has come to be known as the Balassa-Samuelson hypothesis. This hypothesis states that there are larger cross-country productivity differences in tradable goods than in nontradable goods. The second theory is based on endowment differences. Bhagwati (1984) provides the first general equilibrium explanation in this context. The idea is that developing countries have a larger endowment of labor, relative to, say, capital, which leads to a lower wage rate. This pushes down prices of nontradable goods while the law of one price equalizes the price of tradable goods. Finally, my model includes another dimension in which differences in competition at the sectoral level determine prices. The calibration exercise allows the data to determine to what extent each theory is responsible for differences in relative prices.

In the model, productivity differences across sectors are endogenous. Therefore, by construction, I do not directly impose the Balassa-Samuelson effect in order to generate differences in relative prices via, say, sectoral specific TFP differences. I find that if productivity is measured as the marginal product of labor, then the model is consistent with the Balassa-Samuelson hypothesis. However, the marginal product of labor in the tradables sector is a function of both domestic and foreign productivity. Moreover, this result is a direct consequence of the sorting outcome, and therefore not independent of the competition effect. On the other hand, if productivity is measured as output per-worker, then there is no evidence of the Balassa-Samuelson effect. For this reason I explain the theory through measures of competition as there is no discrepancy. The model allows for exogenous differences in relative endowments, but counterfactuals show that differences in endowments explain relatively little of the relative price differences. This leaves essentially all of the burden of explaining relative prices on differences in competition, which stem from differences in barriers to trade. My results are consistent with a conjecture in Rogoff (1996) which says that trade barriers are likely a large determinant of relative price differences.

2 Model

There are K countries, indexed by $k = 1, \ldots, K$. Each country produces a continuum of varieties. Each variety belongs to one of two sectors: tradables, denoted by s_T, or nontradables, denoted by s_N. Varieties being produced in tradables can be exchanged across countries, while varieties being produced in nontradables can not.

Within each sector, varieties are exchanged on markets characterized by monopolistic competition. In particular, tradable varieties compete against all other tradable varieties regardless of where they were produced, while nontradable varieties compete only against other domestic nontradable varieties. Next I describe the primitives of the model, and unless otherwise mentioned, I construct objects from country k's point of view.

2.1 Environment

2.1.1 Population

There is a continuum of agents in each country indexed by $z \in Z \subseteq \mathbb{R}_+$, and from now on I set $Z = [0, 1]$. The agent's type z represents his managerial ability, and this ability is distributed according to the atomless distribution G which is common across countries. Conditions will be placed on this distribution below.

Each agent selects an occupation: a worker, a manager in nontradables, or a manager in tradables. There are two sectors in which goods are produced: tradables s_T and nontradables s_N. I denote the working sector by s_0. This is not a sector per-se, but it denotes the occupation of a worker. Given this, I denote the set of occupations available for selection by $S = \{s_0, s_N, s_T\}$, and define an occupation selection function as follows.

Definition 1: *An occupation selection function (OSF) for country k is a measurable function $\sigma_k : Z \to S$ that assigns to each agent, an occupation.*

If an agent is a manager, he produces a unique variety which will be indexed by his ability z. The set of agents in a particular sector in country k is therefore equivalent to the set of varieties produced in that sector in country k, and is denoted by $\zeta_k(s_i) = \sigma_k^{-1}(s_i)$. To this end, agents sort themselves across occupations, and this sorting determines the set of varieties each country produces.

The concept of sorting is based on Lucas (1978), which I extend to multiple sectors by introducing a fixed cost. The result is that the highest ability agents are managers in the tradable sector, next highest are managers in the nontradable sector, and the lowest ability agents are workers.

2.1.2 Preferences

Preferences are represented by a two-tiered utility function. In the outer tier, preferences are defined over two composite goods: a nontradable composite good and a tradable composite good, with a unit elasticity of substitution between the two:

$$U_k = \sum_{i \in \{N,T\}} \delta_k(s_i) \log C_k(s_i)$$

where $C_k(s_i)$ is the country k's consumption of the sector s_i composite good.

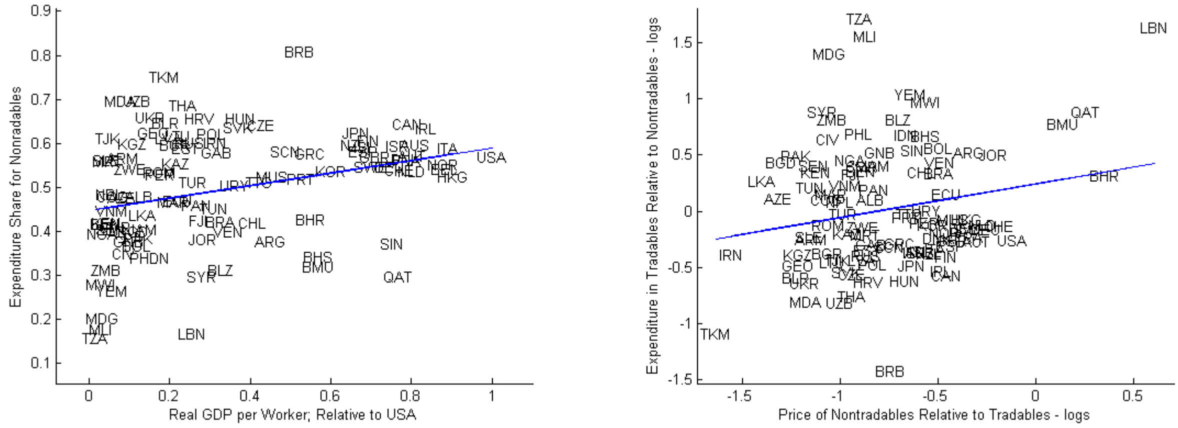
In the second tier, each composite good is a CES aggregation over each variety available in that sector, while each variety enters symmetrically within its respective composite bundle. Each aggregation is taken with a constant elasticity of substitution $\eta > 1$, which is common to all countries. Specifically,

$$\begin{aligned} C_k(s_N) &= \left[\int\limits_{z \in \zeta_k(s_N)} c_{kk}(s_N, z)^{\frac{\eta-1}{\eta}} dG \right]^{\frac{\eta}{\eta-1}} \\ C_k(s_T) &= \left[\sum_{l=1}^{K} \int\limits_{z \in \zeta_l(s_T)} c_{kl}(s_T, z)^{\frac{\eta-1}{\eta}} dG \right]^{\frac{\eta}{\eta-1}} \end{aligned}$$

The parameter $\delta_k(s_i)$ determines country k's expenditure share on each sector $\{s_N, s_T\}$. The notation $c_{kl}(s_i, z)$ denotes the quantity consumed in country k of a sector s_i variety z good, which was produced in country l.

I allow for expenditure shares to differ across countries since the data show that rich countries spend a slightly larger fraction of their income on nontradables; see the left panel of Figure 2. I impose a unit elasticity of substitution between the nontradable and tradable composite goods although the data reveals a weak relationship between relative expenditures (across sectors) and relative prices; see the right panel of Figure 2. Qualitatively, all of my results are robust to allowing for more general preferences such as a CES aggregation over the two composite goods, and I discuss the quantitative consequences of relaxing the log specification in section 6.1. For the body of the paper, I stick with the log specification as it provides superior tractability while offering clear insights into the mechanisms at work.

Figure 2: Cross-country Patterns of Expenditure Shares.



To guarantee that the measure of competing varieties will be finite, even if all agents select the same occupation, I impose the following condition.

Assumption 1: *The ability distribution G satisfies $\int_Z z^{\eta-1}dG < \infty$.*

2.1.3 Budget Constraint

The household faces a budget constraint given by

$$\sum_{i\in\{N,T\}} P_k(s_i)C_k(s_i) = Y_k,$$

where Y_k is aggregate income in country k and $P_k(s_i)$ is the country k ideal price index for the sector s_i composite good.

Due to the log-specification in preferences over the two composite goods, k's total expenditures on nontradables is given by

$$P_k(s_N)C_k(s_N) = \int_{z\in\zeta_k(s_N)} p_{kk}(s_N,z)\,c_{kk}(s_N,z)\,dG = \delta_k(s_N)Y_k,$$

while total expenditures on tradables is given by

$$P_k(s_T)C_k(s_T) = \sum_{l=1}^{K} \int_{z\in\zeta_l(s_T)} p_{kl}(s_T, z)\, c_{kl}(s_T, z)\, dG = \delta_k(s_T)Y_k,$$

The term $p_{kl}(s_i, z)$ is the price in country k of a sector s_i variety z good which was produced in country l.

2.1.4 Technology

Production of each variety requires two inputs: a manager, and workers. Since each manager produces a unique variety, I index varieties by the ability of the manager producing it. The manager decides how much of his variety to produce, how many workers to hire, and what price to charge for his variety.

The quantity of variety z produced in country k in sector s_i is

$$q_k(s_i, z) = z\, A_k\, (l_k(s_i, z) - f_k(s_i))\,, \tag{1}$$

where $l_k(s_i, z)$ is the labor hired by manager z, and $f_k(s_i)$ is the sector specific fixed cost which is paid in units of labor. The term A_k is country specific, but neutral across sectors. I refer to this term as TFP for the remainder of the paper. I want to emphasize that since I have not explicitly modeled capital, and since each country's endowment of labor is equal, the TFP term also captures differences in relative endowments across countries.

In sum, if a manager decides to operate in sector s_i, he hires workers, pays the relevant fixed cost, and the remaining workers are assigned to production. Finally, the manager's ability augments this residual labor to produce the desired quantity, and keeps all rents generated from production.

I assume that there is no fixed cost in the nontradable sector; that is $f_k(s_N) = 0$ for $k = 1, \ldots, K$. For the main results that follow, all that is required is that this fixed cost be smaller in nontradables than in tradables. The reason is that when an agent chooses between managing in the two sectors, he only cares about the difference in fixed costs, not the levels. Moreover, this normalization gives the useful interpretation of fixed costs being associated with exporting, which has been a staple in the trade literature since Melitz (2003).

2.1.5 Trade Costs

There is a cost of trading varieties between countries. This cost is modeled as an iceberg cost where the amount of the sector s_T variety that must be shipped from country l, in order for one unit to arrive in country k, is $\tau_{kl} \geq 1$. I allow for asymmetry between and across countries so that τ_{kl} may or may not be equal to τ_{lk} for $k \neq l$. I do, however, assume that $\tau_{kk} = 1$ for $k = 1, \ldots, K$.

2.1.6 Solution Procedure

The economy is modeled as a two-stage game. In stage one, agents select an occupation to maximize their income. In stage two, production, trade, and consumption take place. Equilibrium in the sub-game (stage two) consists of the following: *i*) managers choose pricing and production plans to maximize rents given the residual demand and taking wages as given, *ii*) households maximize utility given prices and income, *iii*) markets clear, and *iv*) trade balances. Equilibrium in stage one – the occupation selection stage – requires each agent to simultaneously select an occupation which maximizes his income, conditional on behaving optimally in the second stage, given the decisions of all other agents. The remainder of this section works out this solution using backward induction. First, equilibrium in stage 2 yields a value function for each agent which tells him the income he would earn in each occupation. Next, taking these value functions back to the first stage, an equilibrium is reached when every agent has chosen an occupation to maximize his value function, given all other agents occupational choice.

2.2 Stage 2: Production, Trade, and Consumption

Given an arbitrary collection of OSFs, equilibrium in stage two is standard: *i*) managers set prices to maximize rents given the residual demand curve they face for their variety, by hiring labor at a competitive wage, *ii*) households optimize given prices, *iii*) markets clear, and *iv*) trade balances. Given the primitives of the model, I will express all objects in terms of the OSFs.

In this stage, each agent, taking the decisions of other agents as given, obtains a value of each occupation which is a result of the equilibrium. As I will show in section 2.3, occupation selection will be made in such a way that we need only know certain thresholds $\underline{z}_k$ and $\bar{z}_k$. That is, all $z \in [0, \underline{z}_k]$ are workers in country k, all $z \in (\underline{z}_k, \bar{z}_k]$ are managers in k's nontradable sector, and all $z > \bar{z}_k$ are managers in k's tradable sector. Moreover, each interval will be nontrivial. For now take this as given to simplify the following exposition.

2.2.1 Demand and Prices

Demand for the sector s_i composite good in country k is $C_k(s_i) = \frac{\delta_k(s_i)\, Y_k}{P_k(s_i)}$. Demand in country k for a sector s_i variety z good that was produced in country l is $c_{kl}(s_i, z) = \delta(s_i) Y_k P_k(s_i)^{\eta-1} p_{kl}(s_i, z)^{-\eta}$. Prices are set to maximize profits given residual demand. Therefore, the optimal pricing policy for manager z from country l is a constant markup above his marginal cost of serving country k: $p_{kl}(s_i, z) = \left(\frac{\eta}{\eta-1}\right)\left(\frac{w_l\, \tau_{kl}}{z\, A_l}\right)$.

2.2.2 Market Clearing

The market clearing condition for each variety in the tradable sector is

$$q_k(s_T, z) = \sum_{l=1}^{K} \tau_{lk}\, c_{lk}(s_T, z),$$

and for the nontradable sector it is

$$q_k(s_N, z) = c_{kk}(s_N, z).$$

2.2.3 Price Indices and Competition

Using standard Dixit-Stiglitz type algebra and the prices derived above, the ideal price index in country k is given by

$$\begin{aligned} P_k(s_i) &= \left[\sum_{l=1}^{K} \int\limits_{z\in\zeta_l(s_i)} p_{kl}(s_i,z)^{1-\eta} dG\right]^{\frac{1}{1-\eta}} \\ &= \left(\frac{\eta}{\eta-1}\right)\left(\frac{w_k}{A_k}\right)\left[\sum_{l=1}^{K} \Psi_{kl}(s_i)^{\eta-1}\right]^{\frac{1}{1-\eta}} \\ &= \left(\frac{\eta}{\eta-1}\right)\left(\frac{w_k}{A_k}\right)\Phi_k(s_i)^{-1}. \end{aligned}$$

The term $\Psi_{kk}(s_i) = \left[\int\limits_{z\in\zeta_k(s_i)} z^{\eta-1} dG\right]^{\frac{1}{\eta-1}}$ is the ability of the average manager from country k who is operating in sector s_i; that is, if this sector admitted a representative manager, his ability would be precisely this in order to generate the same aggregate outcome. The term $\Psi_{kl}(s_i) = \frac{A_l}{A_k}\frac{w_k}{w_l}\frac{1}{\tau_{kl}}\Psi_{ll}(s_i)$ is what the representative manager from country l would look like operating in country k. That is, his effective ability is distorted by factors that affect his marginal costs of selling in country k. Finally, $\Phi_k(s_i) = \left[\sum_{l=1}^{K} \Psi_{kl}(s_i)^{\eta-1}\right]^{\frac{1}{\eta-1}}$ is the average ability of all sellers, independent of their origin, in the tradable market in country k. Note that this is just an average over each country's representative manager, after taking into account that they are selling in country k's market. Therefore, I interpret these as measures of productivity on the intensive margin, i.e., they are simply the marginal product of labor at the sectoral level.

If there were a representative manager in sector s_i in country k with ability (and hence marginal product of labor) equal to $z = \Phi_k(s_i)$, he would set a price equal to this sectors' price index. This is why this is equivalent to the marginal product of labor at the sectoral level. However, this marginal product has both domestic and foreign components in the tradable sector, and therefore does not truly reflect the domestic marginal product of labor. This distinction is important because it means that measurements of productivity at the sectoral level which rely on domestic price data, such as Herrendorf and Valentinyi (2010), are not truly reflecting the productivity of the domestic country.

Due to fixed costs, output per-worker is different from the marginal product of labor.

However, the notion of marginal productivity is isomorphic to measures of competition in this model, and therefore, I appeal to a different interpretation of the aforementioned objects. The term $\Psi_{kl}(s_i)^{\eta-1}$ is the effective measure of competing varieties of sector s_i goods that are produced in country l and consumed in country k. Absent equilibrium effects, large import costs decrease the measure of competing varieties coming from abroad. Equivalently, $\Phi_k(s_i)^{\eta-1}$ is the total measure of competing varieties in sector s_i in country k. Note that in the nontradable sector, $\Phi_k(s_N) = \Psi_{kk}(s_N)$ since $\Psi_{kl}(s_N) = 0$ for $k \neq l$. From this point of view, there is a one-to-one link between marginal productivity and the measure of competition at the sectoral level.

The following propositions summarize the relationship between prices and competition within a country, as well as the relationship between prices across countries.

Proposition 2.1: *For any country k, the price index for nontradables relative to the price index for tradables is uniquely determined by the inverse ratio of the respective measures of competing varieties: $P_k(s_N)/P_k(s_T) = \Phi_k(s_T)/\Phi_k(s_N)$.*

Proposition 2.2: Law of One Price *If trade is free then the price indices for the tradable composite good are equal across countries.*

Proof:

$$\begin{aligned} P_k(s_T) &= \left(\frac{\eta}{\eta-1}\right)\left(\frac{w_k}{A_k}\right)\left[\sum_{l=1}^{K}\Psi_{kl}(s_T)^{\eta-1}\right]^{\frac{1}{1-\eta}} \\ &= \left(\frac{\eta}{\eta-1}\right)\left[\left(\frac{w_k}{A_k}\right)^{1-\eta}\sum_{l=1}^{K}\left(\frac{A_l}{A_k}\frac{w_k}{w_l}\frac{1}{\tau_{kl}}\Psi_{ll}(s_T)\right)^{\eta-1}\right]^{\frac{1}{1-\eta}} \\ &= \left(\frac{\eta}{\eta-1}\right)\left[\sum_{l=1}^{K}\left(\frac{A_l}{w_l}\Psi_{ll}(s_T)\right)^{\eta-1}\right]^{\frac{1}{1-\eta}}. \end{aligned}$$

■

The law of one price holds regardless of what fixed costs look like. So even if one country produces less tradables domestically, since trade is free, the remaining varieties can be imported with no additional frictions to construct the desired composite good.

2.2.4 Income and Aggregation

I now derive aggregate income as a function of wages and thresholds. Agents who choose to become workers earn the wage rate w_k. Agents who become managers earn rents denoted by $\pi_k(s_i, z)$:

$$\pi_k(s_i, z) = \frac{\Omega_k(s_i)^{1-\eta}}{\eta} z^{\eta-1} - w_k f_k(s_i); \quad i \in \{N, T\}, \tag{2}$$

where $\Omega_k(s_N)^{1-\eta} = \delta_k(s_N) Y_k \Psi_{kk}(s_N)^{1-\eta}$ is expenditure per-competing variety in the non-tradable sector, and $\Omega_k(s_T)^{1-\eta} = \sum_{l=1}^{K} \delta_l(s_T) Y_l \left(\frac{A_l}{A_k} \frac{w_k}{w_l} \tau_{lk} \Phi_l(s_T) \right)^{1-\eta}$ is total expenditure per-competing variety on the international market for tradables that a manager from k must compete against. Each term under the summation is country l's expenditure per-competing variety available on the international market. In sum, a managers pay-off depends positively on the expenditure per-competing variety faced in a given sector, positively on his ability, and negatively on fixed costs.

In order to aggregate income, define $\psi_{lk}(s_i) = \Psi_{lk}(s_i)/\Phi_l(s_i)$. First note that $\psi_{kl}(s_i)^{\eta-1} = \frac{\Psi_{kl}(s_i)^{\eta-1}}{\sum_{l=1}^{K} \Psi_{kl}(s_i)^{\eta-1}} \in (0,1)$ for all k, l, and $\sum_l \psi_{kl}(s_i)^{\eta-1} = 1$ for all k. In fact, $\psi_{kl}(s_T)^{\eta-1}$ is the fraction of country k's aggregate expenditure on tradables which were purchased from country l, and $\psi_{kl}(s_N) = 1$, if $k = l$, and zero otherwise. Then the aggregate profits in country k generated in the tradable sector s_T are given by

$$\Pi_k(s_T) = \frac{1}{\eta} \left[\sum_{l=1}^{K} \psi_{lk}(s_T)^{\eta-1} \delta_l(s_T) Y_l \right] - w_k\, f_k(s_T)(G(1) - G(\bar{z}_k)),$$

while for the nontradable sector they are

$$\Pi_k(s_N) = \frac{\delta_k(s_N) Y_k}{\eta}.$$

Aggregate income is the sum of all such profits over all sectors, plus wage payments.

$$Y_k = w_k\, G(\underline{z}_k) + \sum_{i \in \{N,T\}} \Pi_k(s_i) \tag{3}$$

Proposition 2.3: *Given a vector of wages $w \gg 0$, there is a unique vector of aggregate incomes $Y \gg 0$.*

Proof: See appendix B. ■

To complete the analysis of the second stage, the next step is to find such a wage vector by imposing trade balance, and therefore have expressed all equilibrium objects as functions of only the thresholds, which are in turn determined in the first stage.

2.2.5 Trade Balance

In the appendix I show that total imports in k coming from l is $\delta_k(s_T)Y_k\psi_{kl}(s_T)^{\eta-1}$. The trade balance condition then becomes

$$\delta_k(s_T)Y_k = \sum_{l=1}^{K} \delta_l(s_T)Y_l\psi_{lk}(s_T)^{\eta-1}, \tag{4}$$

where the left-hand side is k's total spending on tradables and the right-hand side is k's receipts from sales tradables.

Proposition 2.4: *Given an arbitrary assignment of occupations such that there are enough workers to cover fixed costs, there exists a wage vector w that solves equilibrium in stage 2.*

Proof: See appendix C.1. ■

The condition in Proposition 2.4, which requires that there are enough workers to cover fixed costs, guarantees that the income vector $Y \geq 0$. I have to impose this at this point since I have taken occupational assignments as given, but I do show in the appendix that an equilibrium in occupation selections will satisfy this.

Now all equilibrium objects are functions of only the thresholds, given that Y_k and $\psi_{kl}(s_i)^{\eta-1}, (l, k = 1, \ldots, K)$ are all functions of the wage, which in turn is determined by trade balance. Next I show how agents use this information in assigning values to each occupation.

2.2.6 Value Functions (IPPs)

The resulting equilibrium in stage two provides the information that agents will use in determining their occupations. It turns out that there are only a few summary statistics that each agent needs to know from the subgame: total expenditures by sector, total measures of competing varieties by sector, and the domestic wage. Using this information, each agent will learn the value of each occupation, in terms of the income he earns, which depends on his own ability.

Definition 2: *An Income Possibilities Profile (IPP) for each agent z in country k is a function $\bar{\pi}_k(\cdot, z) : S \to \mathbb{R} \cup \{-\infty, \infty\}$ given by*

$$\overline{\pi}_k(s_i, z) = \begin{cases} w_k & i = 0 \\ \pi_k(s_i, z) & i \in \{N, T\} \end{cases}$$

where the profit functions $\pi_k(\cdot)$ are given in equation (2). The completes the analysis of the stage two.

2.3 Stage 1: Occupation Selection Game

Now that all variables are in terms of arbitrary pairs of thresholds, agents can observe their IPPs and decide which occupation maximizes their income, given the actions of all other agents. I use the Nash equilibrium solution concept to solve this.

Before defining and proving existence of an equilibrium, I will first establish important conditions that an equilibrium must satisfy, each based on the fact that all sectors must have a positive measure of agents who select to go there.

To see why all occupations have a positive measure of agents, first, suppose that the nontradable sector, s_N, has a measure zero of managers. Then the competition in that sector is zero since goods can not be imported, and therefore, IPP's tend toward infinity for each agent. This would make all agents want to manage in this sector, so clearly this can not be an equilibrium.

Now suppose that the tradable sector s_T in country k has a measure zero of managers. Then no varieties in k will be produced in this sector. But no varieties can be imported either because of trade balance. Then there would be zero competition in that sector and again, IPP's will go off without bound. For a similar reason as the last case, this can not be an equilibrium.

Finally, since there is a positive measure of managers in each sector, and each manager will hire a some workers, then the measure of workers also needs to be positive. This result is summarized by the following Lemma, and will be the foundation for a series of results which follow.

Lemma 2.5: *In equilibrium, for every country $k = 1, \ldots, K$ and for each sector $s_i \in S$, there is a strictly positive measure of agents, so that $\mu\big(\zeta(s_i)\big) > 0, i \in \{0, N, T\}$ where μ is the Lebesgue measure defined on subsets of Z.*

2.3.1 Choosing an Occupation

When deciding between an occupation in either sector s_i or sector s_j, agent z in country k strictly prefers s_i to s_j if and only if $\bar{\pi}_k(s_i, z) > \bar{\pi}_k(s_j, z)$. When $i, j \neq 0$, agent z strictly prefers s_i to s_j if and only if

$$z > (w_k\, \eta)^{\frac{1}{\eta-1}} \left(\frac{f_k(s_i) - f_k(s_j)}{\Omega_k(s_i)^{1-\eta} - \Omega_k(s_j)^{1-\eta}} \right)^{\frac{1}{\eta-1}} \tag{5}$$

If there were no fixed costs, in particular, if fixed costs did not differ across sectors, then in equilibrium, all managers would be indifferent between managing in either sector, and sorting would not occur. When comparing working versus managing, an agent becomes a worker if and only if for each $i \neq 0$,

$$z < (w_k\, \eta[1 + f(s_i)])^{\frac{1}{\eta-1}}\, \Omega_k(s_i). \tag{6}$$

2.3.2 Properties of Sorting

I will now prove and discuss some results about sorting in equilibrium. First, to simplify notation, define $d_k(s_i, s_j) = [f_k(s_i) - f_k(s_j)]/[\Omega_k(s_i)^{1-\eta} - \Omega_k(s_j)^{1-\eta}]$.

Proposition 2.6: *In equilibrium, for each country $k = 1, \ldots, K$, expenditure per-competing variety is larger in their tradable sector than in their nontradable sector.*

***Proof*:** Recall that expenditure per-competing variety in sector s_i is $\Omega_k(s_i)^{1-\eta}$, so it suffices to show that $\Omega_k(s_T) < \Omega_k(s_N)$ since $\eta > 1$. Suppose not. But since by assumption $f_k(s_T) > f_k(s_N)$, then $d_k(s_T, s_N) < 0$ and (5) implies that $\bar{\pi}_k(s_T, z) > \bar{\pi}_k(s_N, z)$ for all z. Therefore, no manager produces in sector s_N which contradicts Lemma 2.5. ■

Intuitively, agents prefer sectors with more expenditure per-competing variety and lower fixed costs. So Lemma 2.6 says that in equilibrium, there must be a trade off between the two. The next result establishes a monotonicity property of sorting with respect to managerial ability.

Lemma 2.7: *Suppose an agent with ability z weakly prefers sector s_T to sector s_N. Then in equilibrium, any agent with ability $z' > z$ strictly prefers sector s_T to sector s_N. Similarly, if an agent with ability $\hat{z}$ weekly prefers sector s_N to sector s_0, then any agent with ability $\hat{z}' > \hat{z}$ strictly prefers sector s_N to sector s_0.*

***Proof*:** The latter is trivial so consider the former. From Lemma 2.6, $d_k(s_T, s_N) > 0$ and therefore, $z' > z \geq (w_k \eta d(s_T, s_N))^{\frac{1}{\eta-1}}$. ■

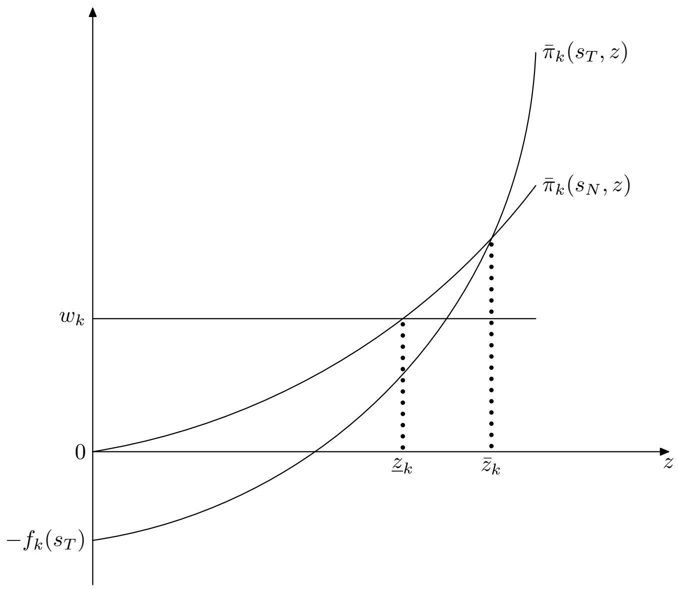
This guarantees that you will never see a manager with a higher ability in a sector with a lower fixed cost. Moreover, If there is a manager with ability z managing in some sector s_i for $i \neq 0$, then an agent with ability $z' > z$ will also be a manager, i.e., not a worker. The next result summarizes all of the properties of sorting.

Proposition 2.8: *In equilibrium, for every country $k = 1, \ldots, K$, the correspondence $\zeta_k : S \twoheadrightarrow Z$ is convex valued. Moreover, if $z \in \zeta_k(s_i)$ and $z' \in \zeta_k(s_j)$ with $i \neq j$, then $i < j$ if and only if $z < z'$, where I appeal to the ordering $\{0 < N < T\}$.*

***Proof*:** This result follows immediately from Lemma 2.5 and Lemma 2.7. ■

So this establishes that sectors consist of intervals of abilities with the highest ability agents being managers in the sectors with higher fixed costs, and the lowest ability managers being workers. This means that the OSF will be an increasing and onto step function in equilibrium, and justifies the use of thresholds. The intuition is that the highest ability agents, pay larger fixed costs, and in turn enjoy larger profits by means of larger market shares due to fewer competing varieties per unit of expenditure.

Figure 3: The pattern of sorting.



Notes: The horizontal axis is the ability dimension. The vertical axis is the income an agent receives. This income is w_k if he is a worker, $\bar{\pi}(s_N, \cdot)$ for managing in nontradables, and $\bar{\pi}(s_T, \cdot)$ for managing in tradables. The thresholds are denoted by $\underline{z}_k$ and $\bar{z}_k$.

Figure 6 displays how sorting takes place in equilibrium. Workers earn the market wage independent of their managerial ability. The slopes of the other two curves are determined by expenditure per-competing variety in their respective sectors. Since the tradable sector has a larger fixed cost, managers who select to go there must enjoy less competition, i.e., more expenditure per-competing variety; this is reflected by a larger slope.

2.4 Equilibrium

In equilibrium, each agent selects an occupation which yield the highest income, conditional on behaving optimally within that occupation. The IPP's are the value functions that agents look at when deciding which occupation is optimal.

Definition 3: *An equilibrium in the occupation selection game is an OSF* $\sigma_k : Z \to S$, *for each* $k = 1, \ldots, K$, *such that for each* i *and for all* $z \in \zeta_k(s_i)$, $\bar{\pi}_k(s_i, z) \geq \bar{\pi}_k(s_j, z)$ *for all* $s_j \in S$, *where* $\zeta_k(s_i) = \sigma_k^{-1}(s_i)$.

The occupation selection problem is conveniently modeled as a noncooperative anonymous game with a continuum of players with a nonatomic measure and a finite action space. The solution concept I resort to is that of a Nash equilibrium. A paper by Codognato and Ghosal (2003) establishes sufficient conditions for a Nash equilibrium to exist in games of this sort but rely on the payoff functions being bounded. However, the game I construct does not adhere to this condition because out of equilibrium IPP's explode as the measure of competition goes to zero. However, it is possible to patch up these discontinuities and obtain an existence result.

Proposition 2.9: *There exists a Nash equilibrium in the occupation selection game.*

Proof: See appendix C.2. ■

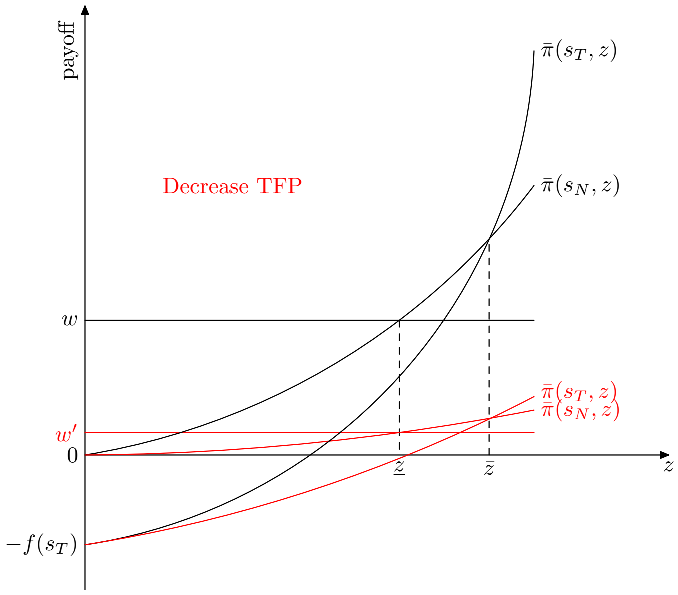
Proposition 2.9 guarantees that there is a Nash equilibrium in mixed strategies. However, it is straightforward to show that there is a pure strategy equilibrium as well. Recall that Proposition 2.8 says that the OSF σ_k is an increasing and onto step function. In particular, for any two adjacent sectors, say s_i and s_{i-1}, there is exactly one z for which $\bar{\pi}_k(s_{i-1}, z) = \bar{\pi}_k(s_i, z)$. Therefore, there are only a finite number of agents using mixed strategies. I can thus impose that an agent will go to the "lower" sector when indifferent. This clearly does not affect the income for these agents, and since this occurs on a set of measure zero, it does not affect the income of any other agent.

Corollary 2.10: *There exists a pure strategy Nash equilibrium in the occupation selection game.*

3 Comparative Statics

Before taking the model to the data, I perform comparative static exercises to analyze the effect on domestic sorting from changes in the following: domestic TFP, domestic import costs, and domestic fixed costs. Unfortunately, there is no way to obtain analytical results along these dimensions so I will perform these numerically. Each exercise was performed over a large range of parameter values and were qualitatively robust under each specification. One novel result, which I will describe below, is that thresholds do not depend on the TFP terms A_k or the trade costs τ_{kl}. Therefore, any changes in these parameters generate only a wage effect.

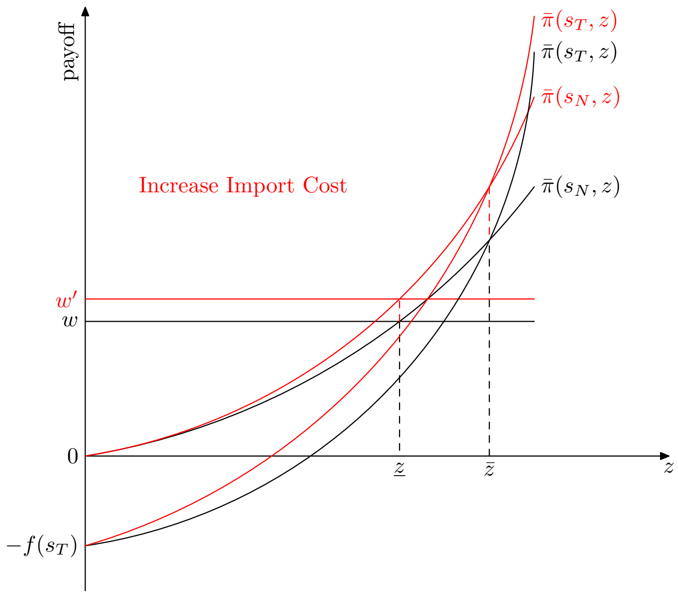
Figure 4: The effects of changes in aggregate productivity on sorting.



Notes: The black lines describe the sorting outocme before a change. The red lines describe the domestic sorting outcome after an increase in domestic TFP.

Changes in TFP Consider a decrease in A_k and the resulting implications for sorting in country k. At current prices and occupations, managers in k's tradable sector are suddenly less competitive on the international market. Managers in tradables will want to move into nontradables. This inflow of competition in nontradables makes managers in nontradables want to become workers. This shifts the supply of workers to the right and demand for workers to the left. These shifts offset on the quantity of labor dimension and result in a lower wage. The lower wage makes marginal costs lower for all managers and the original managers in tradables are now competitive again; hence there is no change in thresholds. This is sustained as an equilibrium since aggregate income in k is lower which lowers the slope of the IPPs over the ability dimension; see Figure 4.

Figure 5: The effects of changes in import costs on sorting.

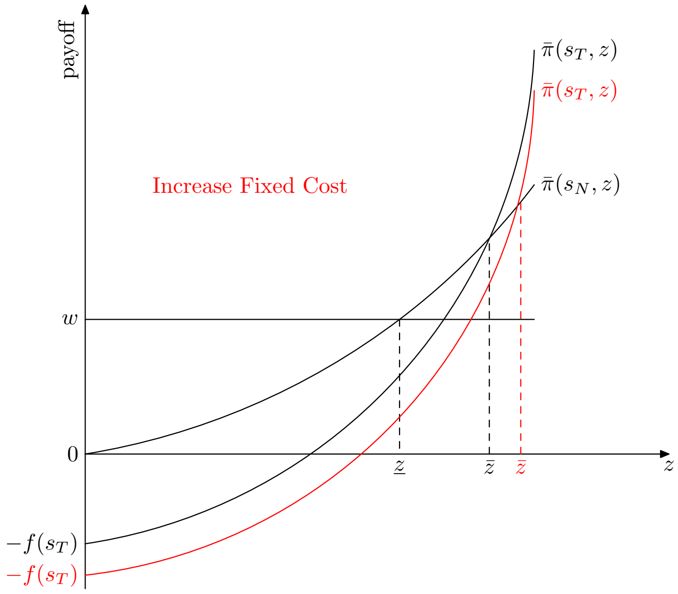


Notes: The black lines describe the sorting outocme before a change. The red lines describe the domestic sorting outcome after a decrease in domestic import cost.

Changes in trade costs Consider now an increase in the import cost τ_{kl} for country k importing from $l \neq k$. This reduces the measure of competing varieties coming from abroad. At current expenditures, prices, and occupations, managers in tradables are making more profits; there is more expenditure per-competing variety in tradables. Managers from nontradables want to move to tradables. This reduces competition in nontradables and workers will want to become managers in nontradables. This shifts demand for workers to the right and supply of workers to the left. These shifts offset on the quantity of labor dimension and result in a higher wage. The higher wage makes marginal costs higher for all managers and takes away any incentive for agents to switch occupations; hence there is no change in thresholds. This is sustained as an equilibrium since aggregate income in k is higher which raises the slope of the IPPs over the ability dimension; see Figure 5.

Changes in fixed costs Finally, consider an increase in the fixed cost $f_k(s_T)$. Managers in tradables now have to pay a larger fixed cost. Since there is no change in the nontradables fixed cost (normalized to zero) some managers will want to move to nontradables. Due to the trade-off between fixed costs and expenditure per-competing

Figure 6: The effects of changes in fixed costs on sorting.



Notes: The black lines describe the sorting outocme before a change. The red lines describe the domestic sorting outcome after an increase in domestic fixed cost.

variety summarized in Proposition (2.6), there will be more expenditure per-competing variety in tradables in the new equilibrium. This means that the threshold determining the nontradable-tradable manager cutoff will increase, i.e., fewer varieties will be produced in the tradable sector . This is reflected by a higher slope and a lower intercept for the IPP in tradables; see Figure 6. The other threshold may change slightly, as well as the wage, but I abstract from those in the current exposition in order to focus on the important channel.

4 Calibration

I assume there are two countries: North and South. The North consist of a group of developed countries, and the South consists of a group of developing countries. I describe in the appendix which countries I use. First, since I will be calibrating parameters to bilateral trade flows, this assumption allows me to avoid dealing with zeros in the bilateral trade data. Second, since I am interested in differences in relative prices between developed and developing countries, this assumption allows me isolate direct cross-country effects. The functional form for the ability density I impose is $g(z) = 200(1 - z)$. The coefficient 200 normalizes the population mass to 100. This form is mostly for computational efficiency as the term $\int z^{\eta-1} g(z)$ will be evaluated routinely, and finding thresholds in $[0, 1]$ is much easier than say if the support was unbounded. Adding curvature to the density does not affect quantitative results dramatically[1].

4.1 Data and Targets

The model year is 1996. I take the expenditure shares $\delta_k(s_i), k \in \{No, So\}$, and $i \in \{N, T\}$ directly from the data. The remaining parameters are jointed calibrated by solving the model and matching relevant targets in the data as described below.

Elasticity of substitution between varieties η This parameter determines the degree of substitutability across varieties, hence market power, and therefore has implications for the fraction of the population that are workers as opposed to managers. Using 1997 census data, Guner, Ventura and Yi (2008) argue that a literal interpretation of managers, that would be consistent with the model in my paper as well, suggests that around 95% of the US population are workers. Similarly, using PSID data, Chang (2000) claims that a lower bound for the fraction of labor force that are workers is 85%. So my target for the fraction of employees that are workers in the North is 90% (the quantitative results are robust over this entire range).

TFP terms $A_k, k \in \{No, So\}$ I normalize $A_{No} = 1$ which is without loss of generality. I target the ratio of real income per-worker (South to North) in the data which has a value of 0.116 to discipline A_{So}. Real income per-worker in the data is computed from the Penn World Tables version 6.1; see Heston, Summers and Aten (2002). To compute the real income per-worker in the North for instance, I need two numbers; total real GDP in the North, and the total workforce in the North. Total real GDP in the North is the

[1] I introduced curvature by setting $g(z) = 200(1 - z)^{\alpha-1}$. This implies that $z^{\eta-1}g(z)$ is proportional to a beta density which MATLAB can integrate pretty efficiently. Curvature does strongly affect how many workers there are in each country, but only weakly affects the sorting of managers across tradables and nontradables, the dimension of first-order importance of this paper.

sum of each of its members' real GDP. Workforce for a given member is total real GDP divided by real GDP per-worker. The total workforce in the North is then the sum of each of its members' workforce. This aggregation is taken as opposed to averages to allow for a more realistic interpretation of treating the North as indeed one country. The same procedure is applied to the South.

Iceberg costs to trade τ_{kl} I target bilateral trade shares, or import shares, which are constructed as in Bernard, Eaton, Jenson and Kortum (2003). To construct these I need the following statistics: North's exports to the rest of the world, South's exports to the rest of the world and total flows between North and South. To map trade flows into the unit interval I need to compute absorbtion figures which requires South's total manufacturing production, as well as North's. Data on bilateral trade flows is compiled from Feenstra, Lipsey, Deng, Ma and Mo (2005) using the year 1996, which is available on the web page of Robert Feenstra: http://cid.econ.ucdavis.edu/. Manufacturing output data is from INDSTAT3, a database maintained by UNIDO, see UNIDO (1996-). Total manufacturing in the North is just the sum of each of its members' manufacturing output. North's exports to the rest of the world are the sum of each of its members' exports to the rest of the world, minus each members' exports to other countries in the North. Finally, total trade flows from South to North are the sum of what each country in the North gets from each country in the South. The absorbtion figure in say the North is $\text{Abs}_{No} = \text{Mfg}_{No} + \text{Imp}_{No,So} - \text{Exp}_{Wd,No}$, where Mfg_{No} is total manufacturing output in the North, $\text{Imp}_{No,So}$ are total trade flows from South to North, and $\text{Exp}_{Wd,No}$ are total exports to the world from the North. The fraction of the manufacturing goods available in the North that was imported from the South is then $\text{Imp}_{No,So}/\text{Abs}_{No}$. In my model, this corresponds to $\psi_{No,So}(s_T)^{\eta-1}$. A similar procedure is applied to find the import share in the South coming from the North.

The fixed costs (in units of labor) f_k I target the average establishment size in manufacturing for both the North and South. To construct these statistics, I take data on the number of manufacturing establishments as well as the number of paid employees in manufacturing, both from INDSTAT3. The total number of establishments in the North is the sum of establishments of its members; the same summation is applied to the number of employees. Average establishment size is then the number of employees divided by the number of establishments. In the model, I count both managers and workers as employees; where the amount of workers includes workers used to cover fixed costs as well

Table 1: Calibrated Parameters

Parameter		Target	Data	Model
$\delta_{No}(s_T) = 0.45$	Exp. share on tradables in No	same as left	0.45	0.45
$\delta_{So}(s_T) = 0.55$	Exp. share on tradables in So	same as left	0.55	0.55
$\eta = 5.88$	Elasticity of substitution	Fraction of employees that are workers in US	0.90	0.91
$A_{So} = 0.135$	TFP term	Real relative income (South/North)	0.116	0.114
$\tau_{So,No} = 2.67$	Iceberg cost No to So	North's Share in South's manufacturing	0.132	0.130
$\tau_{No,So} = 1.23$	Iceberg cost So to No	South's Share in North's manufacturing	0.027	0.027
$f_{No} = 0.59$	Fixed cost in No	Average establishment size in North	20	20
$f_{So} = 17.9$	Fixed cost in So	Average establishment size in South	124	124

Notes: (1) Fixed costs are in units of labor. (2) All parameters except for the expenditure shares are jointly calibrated by solving for the entire equilibrium.

as workers used in actual production.

The results of the calibration are given in Table 1. The implications for relative prices are as follows: the price of nontradables relative to tradables in the North is 0.92 (in the data it is 0.62), and the price of nontradables relative to tradables in the South is 0.69 (in the data it is 0.39). The model over-predicts both of these, but the question the paper addresses is the relative magnitude of these ratios; that is, the interest lies in explaining the ratio $0.62/0.39 = 1.59$. To this end, the model generates a ratio of relative prices equal to $.92/.69 = 1.34$. On a log scale, this means that the model explains about 63% of the observed difference in the price of nontradables relative to tradables. Moreover, across countries, the model predicts that the price of tradables differs by a factor of about 1.3, while the price of nontradables differs by a factor of 1.72. This is consistent with the fact that both tradables and nontradables are more expensive in rich countries, and the fact that the cross-country difference is larger in nontradables; Simonovska (2008) discusses the case for cross-country differences in prices of tradables.

Some interesting statistics for the calibrated (baseline model) are described in Table 2. First, the fraction of managers that are in the nontradable sector, relative to the tradable sector, is of greater magnitude in the South than in the North. This is partly what is generating the relative price differences. As the fixed cost increases, domestic competition in tradables falls relative to nontradables which deflates the relative price. Second, the presence of nontrivial import costs in the South blocks competition from abroad which in turn decreases competition in the South's tradable sector and also shows up in relative prices. Finally, when productivity is measured as output per-worker, the model predicts that there are larger cross-country productivity differences in nontradables than in tradables, going against the Balassa-Samuelson hypothesis.

Another interesting result of the baseline calibration is that trade costs are asymmetric, and the South faces a larger import cost than does the North. This is counter to existing studies in the gravity literature; see, for example, Waugh (2009). His study reveals that in order to reconcile ICP prices, poor countries must face larger export costs, which are also modeled as iceberg costs. Since my model includes an extensive margin, the variation in export costs are getting picked up by fixed costs, and therefore predicting an opposite conclusion with respect to trade (iceberg) costs on the intensive margin, while still being consistent with ICP measures of prices. On the other hand, my calibration only has two countries. With more countries, it is possible for a given country to have both larger average export costs, and larger average import costs. In a two-country setting this is simply not possible.

4.2 Discussion of Establishment Size Data

Figure 7 reveals that, on average, manufacturing plants are larger in poor countries. This may seem awkward as Bhattacharya (2009) argues that average establishment size co-varies positively with income per-worker. However, this can be consistent since I am only looking at establishment size in the manufacturing sector, as opposed to Bhattacharya (2009) who looks at all establishments. To see why, in comparing the US and Mexico, Buera, Kaboski and Shin (2010) show that the average size of manufacturing establishments is larger in Mexico than in the US, while the opposite is true for services. Since services employ a relatively larger fraction of labor in developed countries than they do in developing countries, it is not at all surprising that the data display these features.

One criticism of this fact is based on the presence of an unmeasured informal sector. That is, informal establishments tend to be more common in developing countries, and are typically very small. Ignoring these would lead to an over-estimate of mean size in poor countries. While this point is certainly true, my defense is that the informal sector is not likely to matter so much in the manufacturing sector. That is, informal establishments are often those small family run businesses which operate from the owner's home, or as a vendor, with no clear distinction between personal and business assets, and these operations are typically service oriented: a tailor, food stand, or a repair-man to name a few. It seems much less conceivable for an informal establishment to engage in hard manufacturing activity, and be involved in exporting, especially to a degree which would cause interpretation problems in the data. So the correct interpretation of my model includes the informal sector as part of nontradables, which I have not targeted directly.

Some harder evidence comes from Alfaro, Charlton and Kanczuk (2008). To offset potential bias arising from the absence of small firms in poor countries, they restrict attention to establishments with 20 or more employees in all countries, and still find that mean size is larger across poor countries than across rich countries. However, they do show that the variance in mean size is much larger in poor countries, which makes the finding of Bhattacharya (2009) not so surprising, as he only samples 13 countries.

5 Counterfactual Experiments

In the following counterfactual experiments, I shut down cross-country differences in one parameter at a time, holding all other parameters fixed at their calibrated values, and observe the implications for relevant variables and compare them to their corresponding baseline values. For each exercise I report the fraction of managers in each sector for each country, the cross-country difference in output per-worker for each sector, and the ratio of relative prices. Results are reported in Table 2.

Figure 7: Average size of manufacturing establishments across countries

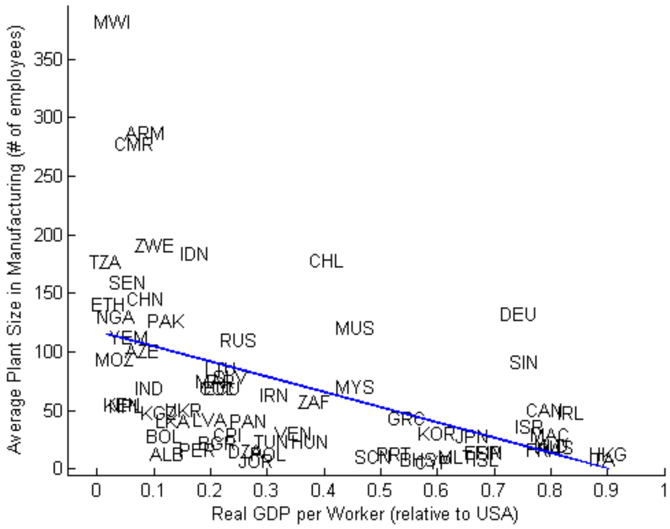


Table 2: Counterfactual Exercises

		Managers Nontradables	Managers Tradables	Ratio of Rel Price
Baseline	North	6.8	2.2	1.34
	South	5.4	0.5	
		(0.18)	(0.16)	
CF1:	North	6.35	2.48	1.43
$\delta_k(s_N) = 0.5$	South	5.87	0.41	
		(0.18)	(0.16)	
CF2:	North	6.8	2.2	1.37
$A_{So} := A_{No} = 1$	South	5.4	0.5	
		(1.07)	(0.90)	
CF3:	North	6.8	2.2	0.91
$f_{So} := f_{No}$	South	6.0	2.8	
		(0.24)	(0.23)	
CF4:	North	6.8	2.2	1.34
$f_{So} := f_{No}$(no re-sorting)	South	5.4	0.5	
		(0.18)	(0.19)	
CF5:	North	6.8	2.2	1.36

Table 2: (continued)

		Managers Nontradables	Managers Tradables	Ratio of Rel Price
$\tau_{No,So} := \tau_{So,No} = 2.67$	South	5.4	0.5	
		(0.13)	(0.12)	
CF6:	North	6.8	2.2	1.16
$\tau_{So,No} := \tau_{No,So} = 1.23$	South	5.4	0.5	
		(0.13)	(0.14)	
CF7:	North	6.8	2.2	0.95
$\tau_{k,l} := 1$ (free trade)	South	5.4	0.5	
		(0.15)	(0.14)	

Notes: 1) Both populations are normalized to 100 so the reported values from managers by sectors are simply the fraction of the entire workforce that manage in those sectors. 2) The ratio of relative price is simply the relative price of nontradables in the North, divided by the relative price of nontradables in the South. 3) The number in parentheses is the ratio of average productivity, which is computed as output per-worker in the South divided by that in the North, for a given sector. Output per- worker is defined to be the value of output in a sector, divided by the measure of workers which includes managers and workers (both productive and those used to cover fixed costs)

5.1 Expenditure Shares

In this exercise (CF1 in Table 2) I shut down differences in expenditure shares across countries. To do this I set $\delta_k(s_T) = 0.5$. For what follows, it does not matter if I had set both to 0.45 (North) or 0.55 (South), so I report the results only for 0.5. The result is that the ratio of relative prices becomes even larger. Here is the reason. In the North, I shifted expenditures from nontradables to tradables. This leads to managers leaving the nontradables sector and entering tradables, and in turn decreasing the measure of competition in nontradables. In the South the opposite occurs since expenditures are shifted from tradables to nontradables. These changes in competition across sectors increases the relative price in the North and decreases it in the South.

5.2 TFP

In this exercise (CF2 in Table 2) I shut down differences in TFP. I also interpret this as changing relative endowments since the calibrated TFP terms include unmeasured stocks of capital. To do this I set $A_{So} = A_{No} = 1$. Only the ratio matters so the choice of the value is without any loss of generality. First, thresholds are unaffected in both countries. The reason is that the productivity term has only a wage effect. Making the South more productive on average makes them more competitive on the international market. More Southern agents would enter tradables at the current prices, and marginal workers would want to become managers. This increases demand for labor and decreases the supply. The shift in the two curves increases the wage in the South and quantity of labor does not change. This has little effect on relative prices for the following reason. When TFP is increased, the measure of competing varieties produced by managers in the South increases by the same proportion in both the tradable and nontradable sector. However, since some goods are imported in the tradable sector, the total measure of competition increases by a slightly larger proportion in nontradables than in tradables, since the amount of competing varieties coming from abroad is unchanged (trade costs are the same). But this difference is negligible and hence the ratio of relative prices is only slightly higher.

5.3 Fixed Costs

For this exercise (CF3 in Table 2) I set $f_S = f_N = 0.59$, the value in the North. The first thing to notice is that thresholds in the South are different from the baseline. However, thresholds in the North are unchanged compared to the baseline. The lower fixed cost in the South allows more agents in the South to enter tradables which drives up competition in that sector. This pushes down the price index for nontradables relative to tradables as the increase in competition in tradables is larger than the increase in competition in nontradables. In the end, the price of nontradables relative to tradables is actually larger in the South than it is in the North. So essentially, differences in fixed costs are necessary just to produce a larger price of nontradables relative to tradables in developed versus developing countries. So in order to reconcile the large difference quantitatively, the differences in fixed costs must be huge. The reason that the relative prices actually become higher in the South is because the South has a larger expenditure share in tradables. In fact, the only reason thresholds look different across countries in this case is because expenditure shares are different across countries.

In terms of the wage effect, the nominal wage in the South is essentially unchanged from the baseline. Note that the amount of managers in the South has increased from the baseline, and in a larger proportion in tradable sector as opposed to the nontradable sector. All else equal this would require an increase in the wage to clear the labor market, but there is also less demand for labor since fixed costs (in units of labor) are smaller. So this is evidence of a simultaneous wage effect.

To separate the wage effect from the effect coming from changes in thresholds, I compute what the equilibrium prices would look like, holding thresholds at their baseline values, while imposing the same fixed costs (CF4 in Table 2). Basically, I am not allowing agents to pick new occupations in response to the change in fixed costs. Therefore, any differences from baseline values are due entirely to the wage effect. The results are essentially indistinguishable from the baseline in terms of relative prices. However, the nominal wage in the South does fall below its baseline value implying that there is a wage effect, but it is not showing up in relative prices since it is neutral across sectors.

5.4 Trade Costs

There is a lot of action coming from trade costs since these determine the amount of competition coming from abroad. A high import cost reduces competition from abroad. I will perform several counterfactuals to analyze: first, how the mere presence of trade costs matters, and second, how asymmetry matters.

5.4.1 Symmetric Trade Costs

There are two cases to think about for the case of symmetric trade costs. First, suppose both countries have the larger import cost ($\tau_{No,So} = 2.67$), and second suppose both countries have the smaller import cost ($\tau_{So,No} = 1.23$). There is a considerable difference in the two exercises. I want to point out again that changes in trade costs do not affect thresholds for the same reason that changes in the TFP term did not. However, competition in tradables will be affected to the extent that competition from abroad depends crucially on the trade costs.

In the former, when both countries face the larger import cost, (CF5 in Table 2) barriers to trade are high. But the South is still facing the same import barrier so competition coming from abroad is unaffected. Also, fixed costs are untouched so domestic competition in unchanged. In the North however, there is now a larger import cost so some competition from abroad is blocked. This leads to slightly less competition in the tradable sector in the North and in turn a slightly higher relative price. This is negligible however since the measure of competition coming from abroad was very small to begin with as the South has very few varieties in tradables being produced due to high fixed costs.

For the latter case where both countries face the same lower import cost (CF6 in Table 2) there is considerable action. The North faces the same import cost as in the baseline so not much action takes place there. However, import costs are reduced in the South. Since the South produces few tradable varieties of its own, a large fraction of its competition in tradables is coming from the North. Once the barrier is reduced, the measure of competing varieties coming from abroad increases dramatically. However, changing trade costs does not affect thresholds so competition in nontradables is unchanged. So the relative price in the South increases, and hence the ratio of relative prices (North/South) declines by over 30%. So asymmetry in trade costs matters to the extent that the South faces a larger import cost than the North.

5.4.2 Free Trade

Moving to free trade means setting $\tau_{No,So} = \tau_{So,No} = 1$. This means that all varieties produced in the tradable sector in each country will reach the other country (CF7 in Table 2). This matters more for the South since it consumes more foreign produced varieties than the North does.[2] The measure of competing varieties in tradables more than doubles in the South, and increases in the North but less than doubles. The amount of domestic competition in nontradables is unchanged in both countries so the ratio of relative prices falls substantially. In fact the ratio goes below 1. The reason for going below 1 is because of differential expenditure shares as described above. So the presence of

[2] The reason is because the South produces fewer tradable varieties, which is a result of a larger fixed cost.

trade costs matters a lot to the extent that these barriers block competition from abroad. This argument implies that even if trade costs were symmetric, relative prices would look different due simply to different import-penetration ratios stemming from different fixed costs.

6 Discussion

Before concluding, this section briefly discusses how the main findings would change in a more general environment, and then ties the main findings into the broader PPP puzzle.

6.1 Generalizing Preferences

I appealed to log preferences over the two composite goods which implied a fixed expenditure share. This specification allowed for tractability and clear insight into the sorting mechanism since expenditure per-competing variety across sector was a major determinant for occupation decisions. An obvious concern is then how robust this specification is as opposed to an non-unit elasticity of substitution across composite goods.

Suppose that there is a constant elasticity of substitution between the two composite goods, say, ϵ. Then the household's first order condition would imply that, after taking logs,

$$\log\left(\frac{C_k(s_T)P_k(s_T)}{C_k(s_N)P_k(s_N)}\right) = \epsilon \log\left(\frac{\delta_k(s_T)}{\delta_k(s_N)}\right) + (\epsilon - 1)\log\left(\frac{P_k(s_N)}{P_k(s_T)}\right),$$

where the left-hand side is the ratio of country k's expenditure on tradables relative to nontradables. When $\epsilon = 1$, expenditure shares are independent of prices. Expenditure shares were a major component of the sorting outcome, as occupation decisions depends on expenditure per-competing variety. Sorting in turn determines prices. However, if $\epsilon \neq 1$ then there is a feed back effect in which prices determine expenditure shares. This is precisely why the log-utility specification was tractable. However, there does not seem to be very strong empirical support for this channel.

Recall that the price of nontradables relative to tradables varies positively with income; see Figure 1, and the ratio of expenditures on tradables relative to nontradables varies negatively with income; see the left panel of Figure 2. It is possible that correlation between relative prices and the ratio of expenditure shares is driven by a non-unit elasticity of substitution, which is consistent with the right panel of Figure 2, which shows a positive correlation between the ratio of expenditure shares (tradable to nontradable), and relative prices (nontradable to tradable).

To clarify, let the ratio of expenditure shares (tradables to nontradables) be x_k and the relative price (nontradables to tradables) be ρ_k, and define $\alpha_k = \delta_k(s_T)/\delta_k(s_N)$. Then

$$\frac{x_k}{x_l} = \left(\frac{\alpha_k}{\alpha_l}\right)^{\epsilon} \left(\frac{\rho_k}{\rho_l}\right)^{\epsilon - 1}.$$

So to be able to match expenditure shares, if $\epsilon > 1$, then less variation in relative prices would be required. On the other hand, if we abstracted from differences in δ's across countries, then more variation in relative prices would be required to reconcile expenditure shares since the data suggest that α is larger in poor countries. Given the small correlation among expenditure shares and relative prices, although it seems consistent $\epsilon > 1$, my stance is that this effect would essentially be offset by putting less pressure on the δ's. Of course this is really a quantitative question, but one limitation of the model is that too much tractability is lost in allowing for $\epsilon > 1$. However, in my opinion, the overall quantitative conclusion would not dramatically change.

6.2 Implications for the PPP puzzle

The question I have addressed is intimately linked to the purchasing-power-parity (PPP) puzzle, one the the six major puzzles in International Macroeconomics as identified by Obstfeld and Rogoff (2000). The PPP puzzle states that there is a weak relationship between exchange rates and national price levels across countries, i.e., PPP does not hold, and more importantly, deviations from PPP are systematic with respect to income per-worker. The question addressed in this paper is related to the PPP puzzle due to the fact that exchange rates are determined by prices of tradable goods, while purchasing power is determined by a basket of goods which includes both tradable and nontradable goods. In particular, if the predictions of PPP held, then any correlation between income and the domestic price of nontradables, would have to be equal to the correlation between income and the domestic price of tradables, given that exchange rates perfectly reflect prices of tradable goods.

Though this paper can shed some light on this matter, it is far from a definitive answer. First, there is still a large fraction of the difference in relative prices that the model can not account for. Second, I have been mostly silent regarding the source of fixed costs. Typical examples of fixed costs include a firm having to build its own power plant, or railways in order to get its product to the market. But these are difficult to measure in practice as the costs of such investments are often incurred at the firm level, as opposed to the establishment level. In any case, it is very conceivable that such costs are larger in countries with poor infrastructure, i.e., developing countries.

7 Conclusion

This paper identifies two sources for the positive correlation between income and the price of nontradables relative to tradables; trade costs which affect the intensive margin of competition, and fixed costs which affect the extensive margin. The model features a sorting mechanism, governed by fixed costs, which determines the levels of domestic competition across sectors. Trade costs then determine the level of foreign competition in the tradable goods sector. Differences in competition are then able to explain a large portion of the difference in relative prices.

In the presence of sorting, cross-country differences in TFP generate a wage effect which is neutral across sectors, and therefore, are not capable of reconciling the pattern of relative prices across countries. Due to the way TFP is modeled, this same result also shows that differences in relative endowments, which also generate a wage effect, contribute quantitatively little to the reconciliation of differences in relative prices.

At the sectoral level, there is a one-to-one link between competition and marginal productivity, providing support for the Balassa-Samuelson hypothesis. However, output per-worker is only partly determined by marginal productivity in the presence of fixed costs. If productivity is measured the latter way, then there does not appear to be support for the Balassa-Samuelson hypothesis. It is for this reason I explain the mechanisms through measures of competition.

The main drivers of differences in relative prices are i) substantially larger fixed costs in the less developed country, which blocks domestic competition in tradables since only a very small measure of tradables will be produced domestically, and ii) the presence of trade costs which affect developing countries more due to larger import-dependence stemming from larger fixed costs. In addition to this, developing countries face larger import costs which further reduces the amount of competition in the tradable sector. The combination of larger fixed costs and larger import costs in developing countries reduces the amount of competition in the tradable sector and explains over 60% of the observed difference in relative prices.

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A Price and Expenditure Share Data

The countries that comprise the North in my calibration are: Hong Kong, Italy, Ireland, Australia, Netherlands, Macao, Canada. France, Israel, Singapore, Germany, Finland, Iceland, Spain, Japan, Malta, South Korea, Cyprus, Bahamas, Greece, Portugal, Slovenia, Malaysia, Mauritius, and Chile. The countries that comprise the South are: Morocco, Latvia, Peru, Indonesia, Ukraine, Sri Lanka, Albania, Pakistan, Bolivia, Kyrgyzstan, Zimbabwe, India, Armenia, China, Azerbaijan, Cameroon, Yemen, Senegal, Nepal, Kenya, Nigeria, Mozambique, Malawi, Ethiopia, and Tanzania. The main limitation is the availability of production data for the manufacturing sector in the year 1996 in the INDSTAT3 data set.

Data on relative prices are constructed as follows. Using the 1996 Benchmark study from Penn World Tables, I look at disaggregate expenditure data country by country. There are 27 categories I consider and separate into tradables and nontradables. For each country k, and each category i, there is expenditure data in both domestic currency and international currency. I use this to infer real prices and then aggregate across categories to construct price indices for tradables and nontradables. For example, to construct the price index for tradables, I take nominal expenditure data $p_k^i c_k^i$ and international expenditures $\tilde{p}_k^i c_k^i$ and then define $P_k^T = \left(\sum_{i \in T} \frac{p_k^i c_k^i}{\tilde{p}_k^i c_k^i} \right)$ where the set T is the categories of tradable goods. The construction of P_k^N is similar. The relative price of nontradables to tradables in each country is simply P_k^N / P_k^T. To map this into my model, I simply compute averages of relative prices across both groups (North and South). This gives a values of 0.62 and 0.38 for North and South respectively.

B Derivations

Define the following variables:

$$\begin{aligned}
\Psi_{ll} &= \left[\int_{\zeta_l} z^{\eta-1} dG\right]^{\frac{1}{\eta-1}}, \\
\Psi_{kl} &= \frac{A_l}{A_k}\frac{w_k}{w_l}\frac{1}{\tau_{kl}}\Psi_{ll}, \\
\Phi_l &= \left[\sum_{l=1}^{K}\Psi_{kl}^{\eta-1}\right]^{\frac{1}{\eta-1}}, \\
\psi_{kl} &= \Psi_{kl}/\Phi_k, \quad \text{and} \\
\Omega_k &= \left[\sum_{l=1}^{K}\delta_l Y_l\left(\frac{A_l}{w_l}\frac{w_k}{A_k}\tau_{lk}\Phi_l\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}
\end{aligned}$$

First I derive payoff/profit functions for managers. The rent earned by manager z in sector s_i is

$$\pi_k(s_i, z) = \sum_{l=1}^{K} p_{lk}(s_i, z)c_{lk}(s_i, z) - w_k l_k(s_i, z).$$

Suppressing the arguments for sector and ability, and substituting for the available technology this becomes

$$\pi_k(s_i, z) = \sum_{l=1}^{K} p_{lk}c_{lk} - w_k\frac{q_k}{zA_k} - w_k f_k.$$

Now apply the market clearing condition $q_k = \sum_l \tau_{lk}c_{lk}$ and rearrange to obtain

$$\pi_k(s_i, z) = \sum_{l=1}^{K} c_{lk}\left[p_{lk} - \frac{w_k\tau_{lk}}{zA_k}\right] - w_k f_k.$$

Denoting the sectoral price indices across countries by P_k and using the demand for variety equation $c_{lk}(s_i, z) = \delta_l(s_i)Y_l P_l^{\eta-1}p_{lk}^{-\eta}$ we obtain

$$\pi_k(s_i, z) = \sum_{l=1}^{K} \delta_l(s_i)Y_l P_l^{\eta-1}p_{lk}^{-\eta}\left[p_{lk} - \frac{w_k\tau_{lk}}{zA_k}\right] - w_k f_k.$$

Now use the fact that prices of individual varieties are set according to $p_{lk}(s_i, z) = \frac{\eta}{\eta-1}\frac{w_k\tau_{lk}}{zA_k}$, along with the price index equation to obtain

$$\begin{aligned}
\pi_k(s_i, z) &= \sum_{l=1}^{K}\delta_l Y_l\left(\frac{\eta}{\eta-1}\right)^{\eta-1}\left(\frac{w_k\tau_{lk}}{A_k}\right)^{\eta-1}\Phi_l^{1-\eta} \\
&\quad \times\left(\frac{\eta}{\eta-1}\right)^{-\eta}\left(\frac{w_k\tau_{lk}}{zA_k}\right)^{-\eta}\left[\frac{\eta}{\eta-1}\frac{w_k\tau_{lk}}{zA_k} - \frac{w_k\tau_{lk}}{zA_k}\right] - w_k f_k.
\end{aligned}$$

Now simplify and obtain

$$\begin{aligned}\pi_k(s_i,z) &= \frac{1}{\eta}\sum_{l=1}^{K}\delta_l Y_l\left(\frac{w_l}{A_l}\right)^{\eta-1}\left(\frac{w_k}{A_k}\tau_{lk}\right)^{1-\eta}\Phi_l^{1-\eta}z^{\eta-1}-w_kf_k \\ &= \frac{1}{\eta}\sum_{l=1}^{K}\delta_l Y_l\left(\frac{A_l}{w_l}\frac{w_k}{A_k}\tau_{lk}\Phi_l\right)^{1-\eta}z^{\eta-1}-w_kf_k\end{aligned}$$

Now I compute aggregate profit for sector s_i.

$$\begin{aligned}\Pi_k(s_i) &= \int\limits_{z\in\zeta_k(s_i)}\pi_k(s_i,z)dG \\ &= \frac{1}{\eta}\Omega_k(s_i)^{1-\eta}\int\limits_{z\in\zeta_k(s_i)}z^{\eta-1}dG-w_kf_k\mu_k(s_i) \\ &= \frac{1}{\eta}\Omega_k(s_i)^{1-\eta}\Psi_{kk}-w_kf_k\mu_k(s_i),\end{aligned}$$

where $\mu_k(s_i)$ is the Lebesgue measure of agents managing in sector s_i. Then

$$\begin{aligned}\Pi_k(s_i) &= \frac{1}{\eta}\sum_{l=1}^{K}\delta_l(s_i)Y_l\left(\frac{A_k}{A_l}\frac{w_l}{w_k}\frac{1}{\tau_{lk}}\frac{\Psi_{kk}(s_i)}{\Phi_l}(s_i)\right)^{\eta-1}-w_kf_k\mu_k(s_i) \\ &= \frac{1}{\eta}\sum_{l=1}^{K}\delta_l(s_i)Y_l\left(\frac{\Psi_{lk}(s_i)}{\Phi_l(s_i)}\right)^{\eta-1}-w_kf_k\mu_k(s_i) \\ &= \frac{1}{\eta}\sum_{l=1}^{K}\delta_l(s_i)Y_l\psi_{lk}^{\eta-1}-w_kf_k\mu_k(s_i),\end{aligned}$$

I am now ready to derive aggregate income. Recall that $G(\underline{z}_k)$ is the mass of the population that are workers. Then aggregate income is the sum of all payments to labor, plus, all rents generated paid to managers across both sectors. That is,

$$Y_k = w_kG(\underline{z}_k)+\sum_{i\in\{N,T\}}\Pi_k(s_i).$$

Using the formula just derived for aggregate profits this becomes

$$\begin{aligned}Y_k &= w_kG(\underline{z}_k)+\sum_{i\in\{N,T\}}\frac{1}{\eta}\sum_{l=1}^{K}\delta_l(s_i)Y_l\psi_{lk}(s_i)^{\eta-1}-w_kf_k(s_i)\mu_k(s_i) \\ &= \frac{1}{\eta}\sum_{l=1}^{K}\sum_{i\in\{N,T\}}\psi_{lk}(s_i)^{\eta-1}\delta_l(s_i)Y_l+w_k\left[G(\underline{z}_k)-f_k(s_T)(G(1)-G(\bar{z}_k))\right],\end{aligned}$$

where $G(1)-G(\bar{z}_k)$ is the measure of managers who pay the fixed cost (in tradables) and I have use the fact that $f_k(s_N)=0$. Now use the fact that $\psi_{lk}(s_N)=0$ for $l\neq k$ and rearrange to get

$$\begin{aligned}Y_k\left[1-\frac{\sum_{i\in\{N,T\}}\psi_{kk}(s_i)^{\eta-1}\delta_k(s_i)}{\eta}\right] &= \frac{1}{\eta}\sum_{\substack{l=1\\l\neq k}}^{K}\psi_{lk}(s_T)^{\eta-1}\delta_l(s_T)Y_l \\ &\quad+\left[G(\underline{z}_k)-f_k(s_T)(G(1)-G(\bar{z}_k))\right]w_k.\end{aligned}$$

After simplifying we obtain

$$
\begin{aligned}
Y_k &= \sum_{\substack{l=1\\ l\neq k}}^{K} \underbrace{\frac{\psi_{lk}(s_T)^{\eta-1}\delta_l(s_i)}{\eta - \sum_{i\in\{N,T\}} \psi_{kk}(s_i)^{\eta-1}\delta_k(s_i)}}_{B_{lk}} Y_l \\
&\quad + \underbrace{\frac{\eta\left[G(\underline{z}_k) - f_k(s_T)(G(1) - G(\bar{z}_k))\right]}{\eta - \sum_{i\in\{N,T\}} \psi_{kk}(s_i)^{\eta-1}\delta_k(s_i)}}_{d_k} w_k. \qquad (7)
\end{aligned}
$$

This leaves K linear equations in K unknowns $\{Y_k\}_{k=1}^K$ to determine incomes as a function of wages and the thresholds. In particular, income in country k is an affine transformation of the incomes in all other countries. In autarky, $B_{lk} = 0$ for $l \neq k$.

Proof of Proposition 2.3: Equation (7) may be stated more compactly as

$$Y = B^\top Y + \mathrm{diag}(d \otimes w^\top),$$

where

$$B = \begin{bmatrix} 0 & B_{12} & \cdots & B_{1K} \\ B_{21} & 0 & & B_{2K} \\ \vdots & & \ddots & \\ B_{K1} & B_{K2} & & 0 \end{bmatrix}, \quad d = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_K \end{bmatrix}, \quad w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_K \end{bmatrix} \text{ and } \quad Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_K \end{bmatrix}.$$

Clearly this has a unique solution $Y = (I - B^\top)^{-1}\mathrm{diag}(d \otimes w^\top)$, where I is the identity matrix, since $\tau_{kl} < \infty \Rightarrow B_{kl} > 0 \Rightarrow I - B^\top$ has full rank. ■

In section 2.3 I showed that each country assigns a positive measure of agents to each occupation. This, together with the love-for-variety in preferences will imply that at least some of every tradable variety with a finite price will be consumed. Moreover, $Y \geq 0$ as long as $w \geq 0$.

Lastly, I derive the trade flows from country k to country l. The value of a given variety z that l purchases from k is $p_{lk}(s_T, z)c_{lk}(s_T, z)$. Therefore, the value of the aggregate flow of tradables from k to l is $\int_{z\in\zeta_k(s_T)} p_{lk}(s_T, z)c_{lk}(s_T, z)dG$. Using the demand for variety equation – $c_{lk} = \delta_l Y_l P_l^{\eta-1} p_{lk}^{-\eta}$ – we obtain

$$\int_{z\in\zeta_k(s_T)} p_{lk}(s_T, z)c_{lk}(s_T, z)dG = \delta_l(s_T)Y_l P_l(s_T)^{\eta-1} \int_{z\in\zeta_k(s_T)} p_{lk}(s_T, z)^{1-\eta}dG$$

Using the pricing rule $p_{lk}(s_T, z) = \frac{\eta}{\eta-1} \frac{w_k \tau_{lk}}{z A_k}$, and the price index formula, this becomes

$$
\begin{aligned}
\int\limits_{z \in \zeta_k(s_T)} p_{lk}(s_T, z) c_{lk}(s_T, z) dG &= \delta_l(s_T) Y_l \left(\frac{\eta}{\eta-1}\right)^{\eta-1} \left(\frac{w_l}{A_l}\right)^{\eta-1} \Phi_l(s_T)^{1-\eta} \\
&\quad \times \int\limits_{z \in \zeta_k(s_T)} \left(\frac{\eta}{\eta-1}\right)^{1-\eta} \left(\frac{w_k \tau_{lk}}{A_k}\right)^{1-\eta} z^{\eta-1} dG \\
&= \delta_l(s_T) Y_l \Phi_l(s_T)^{1-\eta} \left(\frac{A_l}{A_k} \frac{w_k}{w_l} \frac{1}{\tau_{lk}} \Psi_{kk}(s_T)\right)^{\eta-1} \\
&= \delta_l(s_T) Y_l \Phi_l(s_T)^{1-\eta} \Psi_{lk}(s_T)^{\eta-1} \\
&= \delta_l(s_T) Y_l \psi_{lk}(s_T)^{\eta-1}.
\end{aligned}
$$

C Existence of Equilibrium

C.1 Existence of Equilibrium in Stage 2

To this end, suppose that thresholds are predetermined such that $G(\underline{z}_k) \geq f_k(s_T)(G(1) - G(\bar{z}_k))$ for all k. This basically ensures that each country has allocated enough workers to cover fixed costs and hence produce positive quantities. I restrict attention to this case since if it were not true for some k, then in stage 1 everyone in k would choose to be a worker which would violate Lemma 2.5. Now, define an excess demand system as follows:

$$
Z_k(w) = \frac{1}{w_k} \left(\sum_{l=1}^{K} \delta_l(s_T) Y_l \psi_{lk}(s_T)^{\eta-1} - \delta_k(s_T) Y_k \right). \tag{8}
$$

I will show that there is a solution to $Z(w) = 0$, and hence, from equation (4), this will imply balanced trade and therefore be an equilibrium of the subgame.

Proof of Proposition 2.4: I will show that there is a wage vector $w \gg 0$ that satisfies $Z(w) = 0$. To do this I verify the following conditions: 1) $Z(w)$ is continuous on $\mathbb{R}^K_{++}$, 2) $w \cdot Z(w) = 0$ for all $w \gg 0$, and 3) if w^n is a sequence of wage vectors in $\mathbb{R}^K_{++}$ converging to $\bar{w} \neq 0$, and $\bar{w}_k = 0$ for some k, then for some country l with $\bar{w}_l = 0, Z_l(w^n)$ is unbounded above. Existence then follows by Jehle and Reny (2000, Theorem 5.3 p. 192).

1) The terms Y_k and $\psi_{kl}(s_i)$ for each i and all k, l are continuous in w by inspection, and hence so in Z.

2) Writing the inner product as a summation it follows that

$$\begin{aligned}\sum_{k=1}^{K} w_k Z_k(w) &= \sum_{k=1}^{K}\sum_{l=1}^{K} \delta_l(s_T) Y_l \psi_{lk}(s_T)^{\eta-1} - \sum_{k=1}^{K} \delta_k(s_T) Y_k \\ &= \sum_{l=1}^{K} \delta_l(s_T) Y_l \sum_{k=1}^{K} \psi_{lk}(s_T)^{\eta-1} - \sum_{k=1}^{K} \delta_k(s_T) Y_k,\end{aligned}$$

and the result follows since $\sum_{k=1}^{K} \psi_{lk}(s_T)^{\eta-1} = 1$ whenever $w \gg 0$.

3) Let w^n be a sequence of wage vectors in $\mathbb{R}^K_{++}$ converging to $\bar{w} \neq 0$, and $\bar{w}_j = 0$ for some j. Without loss of generality, suppose $\bar{w}_k > 0$ for all $k \neq j$. Then $\phi_{lj}^{\eta-1} = 0$ for all l, and this in turn implies that $Y_j = 0$ using equation (7). Since the wage vector $\bar{w}$ is bounded, and so are each limiting $\bar{\psi}_{kl}$ and strictly positive for $k, l \neq j$. Then since $\bar{w}_k > 0$ for $k \neq j$ the limiting aggregate incomes $\bar{Y}_k$ are also bounded and strictly positive. Therefore,

$$Zj(w^n) = \underbrace{\frac{1}{w_j^n}}_{\substack{\downarrow \\ \infty}} \underbrace{\left(\sum_{l=1}^{K} \delta_l(s_T) Y_l^n \psi_{lj}^n(s_T)^{\eta-1} - \delta_j(s_T) Y_j^n\right)}_{\substack{\downarrow \\ \text{positive and finite}}},$$

which evidently is unbounded. ■

C.2 Existence of Equilibrium in Stage 1

I prove existence in the occupation selection game for a closed economy. The result carries through to the open economy due to the fact that there are no empty sectors – Lemma (2.5), and that there is an equilibrium wage in the subgame – Proposition 2.4.

Before proving the claim in Proposition 2.9, I will first introduce some more careful notation, provide an equivalent definition of equilibrium given this new notation, and then establish a sequence of lemmas which will be used to deliver the result.

Let $(Z, \mathscr{Z}, \mu)$ be a measure space where μ is the unique measure induced by the distribution G described in the paper. For what follows, I generalize the environment and allow for an arbitrary number of sectors I. Let Δ_S be the simplex over S, where $S = \{s_o, s_1, \ldots, s_I\}$ is the set of sectors available for occupation selection. Δ_S can be interpreted as the constant correspondence from Z to the simplex over S, which defines the set of strategies available to agent z. Let $\sigma : Z \to \Delta_S$ be an OSF, where $\sigma_i(z)$ is the probability that agent z chooses an occupation in sector s_i for $i = 0, \ldots, I$. Denote by $\sigma_{-0} = \big(\sigma_1, \ldots \sigma_I\big)$, the vector that excludes the first coordinate of σ. Furthermore, let Σ be the space of all such OSFs that are measurable. Let $\xi_M(\sigma) = \int_Z \sigma(z)\, d\mu$, and $\xi_C(\sigma) = \left(\int_Z \sigma_{-0}(z)\, z^{\eta-1} d\mu\right)^{\frac{1}{\eta-1}}$. Now define the following sets

$$\begin{aligned}\Theta_M &= \{\xi_M(\sigma) : \sigma \in \Sigma\} \subset \mathbb{R}^{I+1}, \quad \text{and} \\ \Theta_C &= \{\xi_C(\sigma) : \sigma \in \Sigma\} \subset \mathbb{R}^{I},\end{aligned}$$

where $\theta_{M,i} \in \Theta_{M,i}$ is the measure of agents whose occupation is in sector $s_i, i = 0, \ldots, I$, and $\theta_{C,i} \in \Theta_{C,i}$ is the measure of competition in sector $s_i, i = 1, \ldots, I$. I will also denote $\xi(\sigma) = \big(\xi_M(\sigma), \xi_C(\sigma)\big)$ as well as $\Theta = \Theta_M \times \Theta_C$. Now let $W : \Theta_M \to \mathbb{R}$ be aggregate income function which is defined by $W(\theta_M) = \frac{\eta}{\eta-1}\left[\theta_{M,0} - \sum_{i=1}^{I} f(s_i)\,\theta_{M,i}\right]$. Now let $u : \Theta \times \Delta_S \times Z \to \mathbb{R} \cup \{-\infty, \infty\}$ be the payoff function which is given by

$$u(\theta, x, z) = x_0 + \frac{W(\theta_M)}{\eta}\left(\sum_{i=1}^{I} x_i\,\delta(s_i)\,\theta_{C,i}^{1-\eta}\right) z^{\eta-1} - \sum_{i=1}^{I} x_i\,f(s_i).$$

Now, denote the best response correspondence $R : \Theta \times Z \twoheadrightarrow \Delta_S$ by $R(\theta, z) = \underset{x \in \Delta_S}{\operatorname{argmax}}\{u(\theta, x, z)\}$. Finally, define a mapping $\Lambda : \Theta \twoheadrightarrow \Theta$ by

$$\begin{aligned} \Lambda_{M,i}(\theta) &= \int_Z R_i(\theta, z)\,d\mu,\ i = 0, \ldots, I \\ \Lambda_{C,i}(\theta) &= \left(\int_Z R_i(\theta, z)\,z^{\eta-1} d\mu\right)^{\frac{1}{\eta-1}},\ i = 1, \ldots, I \end{aligned}$$

where R_i is the i'th coordinate of the correspondence R, with $i = 0, \ldots, I$.[3] With this machinery, I will restate the definition of a Nash equilibrium in the occupation selection game that is equivalent to Definition 3.

Definition 4: *An OSF σ is a Nash equilibrium of the occupation selection game if for each z, $u(\xi(\sigma), \sigma(z), z) \geq u(\xi(\sigma), x, z)$ for all $x \in \Delta_S$.*

I will prove the existence of a Nash equilibrium of the occupation selection game by proving the existence of a fixed point of Λ.

Clearly u has a discontinuity with respect to θ when $\theta_{C,i} = 0$ for some $i = 1, \ldots, I$. However, for any z, $u(\cdot, \cdot, z)$ is continuous over $\{\theta \in \Theta : \theta_C \gg 0\} \times \Delta_S$. By Berge's Maximum Theorem, for each z, $R(\theta, z)$ is upper hemicontinuous on all θ such that $\theta_C \gg 0$.

In what follows, I will define an extension $\overline{\Lambda}$ to the correspondence Λ so that $\overline{\Lambda}$ is upper hemicontinuous on all of Θ. In particular, I will make it upper hemicontinuous at all θ such that $\theta_{C,i} = 0$ for some i. Then I will show that there is no fixed point of $\overline{\Lambda}$ at such a point.

The discontinuity of u is what causes the problem in Λ. To understand why, consider a sequence θ^n with $\theta^n_{C,i} > 0$ for all i and all n, that converges to a point θ with $\theta_{C,i} = 0$ for some i, and $W(\theta) = 0$. It is possible, for a carefully chosen sequence, that some agent z strictly prefers managing in sector i for all θ^n, but at the limit strictly prefers working under the convention that $0 \cdot \infty = 0 < 1$. This causes the best response correspondence to fail to be upper hemicontinuous at points with $\theta_{C,i} = 0$ for some i, and in turn the failure of Λ. So In order to find an upper hemicontinuous extension $\overline{\Lambda}$, I will modify the best response correspondence at these problem points to make it upper hemicontinuous.

[3] The notion of integration in which I appeal to for integrating correspondences are Aumann integrals which are defined as follows. Let $X \subseteq \mathbb{R}^N$ and $Y \subseteq \mathbb{R}^M$ and let $F : X \twoheadrightarrow Y$ be a correspondence. The Aumann integral $\int F = \left\{\int f : f \text{ is a selction of } F\right\}$. If F admits no integrable selector, then $\int F = \varnothing$.

Before doing so, let me first modify the aggregate income function. Define $W(\theta_M)^+ = \max\{W(\theta_M), 0\}$. Now redefine the Bernoulli payoffs to be

$$v_i(\theta, z) = \begin{cases} 1 & i = 0 \\ \frac{W(\theta_M)^+}{\eta}\delta(s_i)\theta_{C,i}^{1-\eta}z^{\eta-1} - f(s_i) & i = 1, \ldots, I, \end{cases}$$

and the payoff function by $v(\theta, x, z) = x_0 + \sum_{i=1}^{I} x_i\, v_i(\theta, z)$ for $x \in \Delta_S$. Notice that this does not affect any fixed point properties of Λ since it does not affect the best response correspondence R; that is $R(\theta, z) = \operatorname{argmax}\{v(\theta, x, z) : x \in \Delta_S\} = \operatorname{argmax}\{u(\theta, x, z) : x \in \Delta_S\}$ for all θ and all z. To understand why, note that there can be no fixed point when $W(\theta_M) \leq 0$, which occurs at any θ such that $\theta_{M,0} \leq \sum_{i=1}^{I} f(s_i)\theta_{M,i}$. To see this, suppose that $W(\theta_M) \leq 0$. Then for each z, $u_i(\theta, z) \leq 0$ for $i = 1, \ldots, I$. Note that this is true even if $\theta_{C,i} = \infty$, using the convention that $0 \cdot \infty = 0$. But $w = 1$ so an agent will always prefer to be a worker. Hence, everyone will become workers, which of course would mean that, for each z, $R_0(\theta, z) = \{1\}$ and $R_i(\theta, z) = \{0\}$ for $i = 1, \ldots, I$. But then $\Lambda_{M,0}(\theta) = \{1\}$, and $\Lambda_{M,i}(\theta) = \{0\}$ for $i = 1, \ldots, I$, in which case $\lambda_{M,0}(\theta) > \sum_{i=1}^{I} f(s_i)\lambda_{M,i}(\theta)$, for all $\lambda(\theta) \in \Lambda(\theta)$. This is clearly not a fixed point since I started with θ such that $\theta_{M,0} \leq \sum_{i=1}^{I} f(s_i)\theta_{M,i}$. Therefore, there is no loss in generality in modifying the aggregate income function to be $W(\theta)^+ = \max\{W(\theta), 0\}$. Of course, when $W(\theta) > 0$, the payoff function for the modified problem will coincide with the original problem, as will the best response function.

Now I will define the extension $\overline{\Lambda}$ by means of a modified best response correspondence $\overline{R} : \Theta \times Z \twoheadrightarrow \Delta_S$. Suppose $\theta \in \Theta$ such that $\theta_C \gg 0$. In this case define $\overline{R}(\theta, z) = R(\theta, z)$ for all z, as everything is well behaved over this region. The changes will have to be made at points θ such that $\theta_{C,i} = 0$ for some i.

case 1) Suppose $\theta \in \Theta$ such that $\theta_C = 0$; that is, $\theta_{C,i} = 0$ for each $i = 1, \ldots, I$. In this case define $\overline{R}_0(\theta, z) = \{0\}$ for all z, and $\overline{R}_{-0}(\theta, z) = \mathrm{Si}\big(\{s_i\}_{i=1}^{I}\big)$ for all z. Here, Si denotes the simplex. This modification allows for any mixed strategy that puts zero probability on choosing to be a worker. The notation R_{-0} means $R_1, \ldots, R_I$. To see that $\overline{R}(\cdot, z)$ is upper hemicontinuous in this case, consider a point θ such that $\theta_C = 0$. First note that by the definition of Θ, we also have $\theta_{M,i} = 0$ for each $i = 1, \ldots, I$, and $\theta_{M,0} > 0$. Therefore, $W(\theta_M)^+ > 0$. Since $\overline{R}$ is compact valued, I will use the sequential definition. So let θ^n be any sequence in Θ converging to θ. Appealing to continuity of W, there exists a number N such that $v_i(\theta^n, z) > 1$ for all z.[4] Now let $r(\theta^n, z) \in \overline{R}(\theta,^n, z)$ for all n. Then for $n > N$, $r_0(\theta^n, z) = 0$ for almost all z. Since there are only a finite number of sectors, there must be one sector s_i such that for some subsequence θ^{n_k}, $r_i(\theta^{n_k}, z) > 0$ for all $n_k > N$ and almost all z. Now, the sequence $r_i(\theta^{n_k}, z)$ is bounded, therefore, we may extract a monotone subsequence from it which converges. But this limit is clearly in $\overline{R}(\theta, z)$ by definition, since any mixed strategy which puts zero probability on working is allowed, in turn establishing upper hemicontinuity.

case 2) Suppose $\theta \in \Theta$ such that $\theta_{C,j} = 0$ for some $j \in \{1, \ldots, I\}$, but not all; that is, $\theta_{C,j} = 0$ for some j, and $\theta_{C,i} > 0$ for some $i \neq j$. Let $\mathcal{J}$ denote the set $\{j : \theta_{C,j} = 0\}$. In this case define $\overline{R}_i(\theta, z) = 0$ for $i \notin \mathcal{J}$ and all z, and $\overline{R}_{\mathcal{J}}(\theta, z) = \mathrm{Si}\big(\{s_0\} \cup \{s_j\}_{j\in\mathcal{J}}\big)$ for all z. This modification allows for any mixed strategy that includes choosing to be a worker, and choosing to manage in sectors with $\theta_{C,j} = 0$. $R_{\mathcal{J}}$ refers to $\{R_j : j \in \mathcal{J}\}$. To see that $\overline{R}(\cdot, z)$ is upper hemicontinuous in this case, there are two subcases to consider, $W(\theta_M)^+ > 0$ and $W(\theta_M)^+ = 0$. Begin with the former. Let θ^n be a sequence converging to some point θ with $\theta_{C,j} = 0$ for some but not all j. Then, since $W(\theta_M)^+ > 0$, there is a number N such that when $n > N$, $v_j(\theta^n, z) > v_i(\theta^n, z)$ for each $j \in \mathcal{J}$ and each $i \notin \mathcal{J}$, and almost all z. Using the same argument as in case 1, note that

[4] I say almost all z, but the only exception would be z=0. But this one agent has measure zero.

$R_{-\mathcal{J}}(\theta^n, z) = \{0\}$ for all $n > N$, and in the limit any mixed strategy that puts zero probability on managing in sectors s_i where $i \notin \mathcal{J}$ is allowed.[5] So finding a convergent subsequence whose limit is in $\overline{R}(\theta, z)$ is easy by definition by using the same argument as in case 1. In the latter case where $W(\theta_M)^+ = 0$, the argument is the same, except, putting positive probability on working is allowed. The technical details are the same as the other cases and therefore omitted, and hence upper hemicontinuity is established.

Now that I have a modified best response correspondence $\overline{R}$ which is upper hemicontinuous, define $\overline{\Lambda} : \Theta \twoheadrightarrow \Theta$ by $\overline{\Lambda}_M(\theta) = \int_Z \overline{R}(\theta, z)\, d\mu$, and $\overline{\Lambda}_C(\theta) = \int_Z \overline{R}_{-0}(\theta, z)\, z^{\eta-1}\, d\mu$.

Now I will argue that there is no fixed point θ of the map $\overline{\Lambda}$ such that $\theta_{C,i} = 0$ for some i. Consider again the possible cases.

case 1) Suppose $\theta_C = 0$. By the definition of Θ, $\theta_{M,i} = 0$ for each $i = 1, \ldots, I$, and $\theta_{M,0} > 0$. This implies $W(\theta_M)^+ > 0$. But by definition, $\overline{R}_0(\theta, z) = \{0\}$ for all z. Hence, $\overline{\Lambda}_{M,0}(\theta) = 0$, and cannot be a fixed point.

case 2) Suppose $\theta_{C,j} = 0$ for some $j \in \{1, \ldots, I\}$, but not all; that is, $\theta_{C,j} = 0$ for some j, and $\theta_{C,i} > 0$ for some $i \neq j$. Let $\mathcal{J}$ denote the set $\{j : \theta_{C,j} = 0\}$. Then $\theta_{C,i} > 0$ for each $i \notin \mathcal{J}$. By definition, $\overline{R}_i(\theta, z) = \{0\}$ for each $i \notin \mathcal{J} \cup \{0\}$ and all z, and therefore $\overline{\Lambda}_{C,i}(\theta) = 0$ for each $i \notin \mathcal{J}$ and cannot be a fixed point.

Lemma C.1: *The set Θ is nonempty, compact, and convex.*

Proof: Δ_S is obviously integrably bounded when viewed as a constant correspondence mapping Z to $\mathbb{R}^{I+1}$ given by $\Delta_S(z) = \{x \in \mathbb{R}^{I+1} : x_i \geq 0, \sum_{i=x}^{I} x_i = 1\}$. Moreover, it is clearly nonempty- and compact-valued. Since it is a constant correspondence, its graph belongs to the product σ-algebra $\mathscr{Z} \times \mathscr{B}$, where $\mathscr{B}$ is the Borel σ-algebra on $\mathbb{R}^{I+1}$. Therefore, Θ is nonempty Klein and Thompson (1984, Corollary 17.1.4 p. 186). Since in addition to being integrably bounded, Δ_S is also closed-valued, it follows that Θ is compact Klein and Thompson (1984, Proposition 18.3.2 p. 206). Lastly, since the measure space $(Z, \mathscr{Z}, \mu)$ is nonatomic, Θ is convex Klein and Thompson (1984, Theorem 17.1.6 p. 187). ■

Lemma C.2: *For each $\theta \in \Theta$, $\overline{R}(\theta, \cdot)$ has a measurable graph; that is for each $\theta \in \Theta$, the set $\{(z, \overline{R}(\theta, z)) : z \in Z\}$ belongs to the product σ-algebra $\mathscr{Z} \times \mathscr{B}$.*

Proof: Consider first those points θ for which $\theta_C \gg 0$. At such points, $v(\theta, \cdot, \cdot)$ is measurable on the graph of Δ_S since it is continuous. Then $\overline{R}(\theta, z)$ is measurable in z, and is nonempty- and compact-valued for all such θ Aliprantis and Border (2007, Theorem 18.19 p. 605). Since $\overline{R}$ is compact-valued, it is identically equal to its closure correspondence, hence $\overline{R}(\theta, \cdot)$ has a measurable graph Aliprantis and Border (2007, Theorem 18.6 p. 596). Next consider points for which $\theta_{C,i} = 0$ for some i. Then by construction, $\overline{R}(\theta, z)$ is a constant correspondence which is isomorphic to a simplex in $\mathbb{R}^N$ for $N < I$. This correspondence clearly has a measurable graph at such points. ■

Lemma C.3: *The correspondence $\overline{\Lambda}$ is nonempty- and convex-valued and upper hemicontinuous.*

[5] The notation $R_{-\mathcal{J}}$ refers to the coordinates i of the best response such that $i \notin \mathcal{J}$.

Proof: Let $\theta \in \Theta$. From Lemma C.2, $\overline{R}(\theta, \cdot)$ has a measurable graph. It is obviously integrably bounded as well so $\overline{\Lambda}$ has nonempty values Klein and Thompson (1984, Theorem 17.1.4 p. 186). Since the measure space is nonatomic, $\overline{\Lambda}$ has convex values at θ Klein and Thompson (1984, Theorem 17.1.6 p. 187). Finally, since $\overline{R}(\theta, z)$ is upper hemicontinuous in θ for all z and is integrably bounded and similarly $z^{\eta-1}\overline{R}(\theta, z)$ is also upper hemicontinuous in θ for all z and is also integrably bounded by assumption 1, then $\overline{\Lambda}_M(\theta) = \int_Z \overline{R}(\theta, z)\, d\mu$ and $\overline{\Lambda}_C(\theta) = \int_Z z^{\eta-1}\overline{R}(\theta, z)\, d\mu$ are both upper hemicontinuous by Aumann (1976) at θ. Since θ was arbitrary, $\overline{\Lambda}$ is upper hemicontinuous. ■

I am now ready to prove existence.

*P*roof of Proposition 2.9: By Lemmas C.1 and C.3 we may appeal to Kakutani's fixed point theorem to conclude that the map $\overline{\Lambda} : \Theta \twoheadrightarrow \Theta$ has a fixed point, call it θ^*. The two correspondences $\overline{\Lambda}$ and Λ differ only over the set $\{\theta : \theta_{C,i} = 0 \text{ for some } i = 1, \dots, I\}$. But I already argued that there is no fixed point of $\overline{\Lambda}$ that belongs to this set. Hence the fixed point must be such that $\theta^*_C \gg 0$, in which case, it is also a fixed point of Λ. But by the definition of Θ, there is an OSF $\sigma^* \in \Sigma$ such that $\theta^*_M = \int_Z \sigma^*\, d\mu$ and $\theta^*_C = \left(\int_Z \sigma^*_{-0}\, z^{\eta-1}\, d\mu\right)^{\frac{1}{\eta-1}}$, with $\sigma^*(z) \in R(\theta^*, z)$ for all z. But this implies that σ^* constitutes a Nash equilibrium. ■