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Abstract

A robust prediction from the tax evasion literature is that optimal auditing induces a regressive bias in effective tax rates compared to statutory rates. If correct, this will have important distributional consequences. Nevertheless, the regressive bias hypothesis has never been tested empirically. Using a unique data set, we provide evidence in favor of the regressive bias prediction but only when controlling for the tax agency's use of third-party information in predicting true incomes. In aggregate data, the regressive bias vanishes because of the systematic use of third-party information. These results are obtained both in simple reduced-form regressions and in a data-calibrated state-of-the-art model.

JEL Codes: D82, H26, K42

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1 Introduction

In this article, we provide the first empirical support of the regressive bias hypothesis established in the theoretical literature on tax evasion and optimal enforcement.[1]

The potential for tax evasion requires a distinction between the statutory tax system and the *effective* tax system. Tax evaders pay less taxes than they ought to, which implies a wedge between statutory and effective average tax rates. The regressive bias hypothesis predicts that this wedge is larger for high-income taxpayers than for low-income taxpayers. Thus, the distributional properties of the effective tax system may differ substantially from those intended by the tax code.[2]

The intuition behind this prediction is simple. The tax compliance game played by the tax agency and taxpayers is a screening problem in which high-income taxpayers can increase their expected payoff by imitating low-income taxpayers. If not all taxpayers can be audited, the tax agency should optimally prioritize tax returns reporting low income. Rather than eliminating tax evasion altogether, the goal becomes to discourage very low reports by high-income individuals. In equilibrium, this leads to the decreasing profile of effective average tax rates. Figure 1(a) illustrates that the wedge between the effective average tax rate, τ^{eff}, and the statutory tax rate, τ, is increasing in true income. As shown by Scotchmer (1992), the prediction of regressive bias is theoretically robust. Model variations in the literature consistently arrive at regressively biased effective average tax rates.

There is one important exception to the regressive bias result: Scotchmer (1987) shows that when the tax agency uses population observables such as gender, age, occupation, employer reported salaries to predict true incomes, there may be no bias or even progressive bias in the population as a whole. Specifically, she posits that a tax agency can use observables

[1] See Reinganum and Wilde (1986), Cremer, Marchand, and Pestieau (1990), Sanchez and Sobel (1993), Erard and Feinstein (1994) and others.

[2] Effective average tax rates are convenient in this context as they summarize in one statistic equity effects from both evasion/compliance, the increasing propensity to evade taxes as income increases, and the likelihood of detection and punishment.

to divide taxpayers into *audit groups* upon which the agency may condition its enforcement strategy. In this model, there is regressive bias within audit groups but likely progressive bias between groups. Consequently, the regressive bias hypothesis should be reinterpreted as a *within-audit-group* phenomenon. Figure 1(b) illustrates the aggregate relationship between effective average tax rates, τ^{eff}, and true income which is a composite of relationships within multiple audit groups, τ_i^{eff}. Whereas the regressive bias hypothesis remains valid within audit groups, *between audit groups*, effective tax rates may be *progressively biased*.

[Fig. 1 about here]

The mechanism driving the result is that some low-income taxpayers benefit from being high-income individuals within their audit group, while some high-income taxpayers instead are low-income taxpayers within their audit group. This reclassification changes the risk of being audited and, hence, the *ex ante* effective tax rate. In addition, the tax agency can more efficiently target high-income individuals by modifying the allocation of audit resources. If either third-party reported income or audits are more abundant among high-income taxpayers, progressive bias between groups may dominate in the aggregate.

Using micro-data on Danish taxpayers, we find evidence in simple reduced-form regressions that there is a regressive bias within audit groups. Between audit groups, tax rates are progressively biased whereas, in the aggregate, the two biases approximately cancel out. Thus, our findings support the regressive bias hypothesis at the theoretical level but not as an aggregate empirical outcome; specifically, our results correspond closely to the structure of effective tax rates conjectured in Scotchmer (1987).

However, our simple reduced-form analysis does not allow us to identify the effects from tax evasion and the audit regime on effective tax rates. By applying theoretical structure to the problem, we show that the empirical properties of effective tax rates are convinvingly replicated with an agent-based screening game between a tax agency and taxpayers. To do this, we combine insights from two main sources, Kleven, Knudsen, Kreiner, Pedersen, and Saez (forthcoming) and Erard and Feinstein (1994). In the former, the authors collect a uniquely detailed micro-data set, of which ours is a subset, of a random sample of Danish taxpayers containing pre- and post-audit incomes and taxes as well as incomes as reported by third parties, proxies for audit probabilities, etc. They show that third-party reported income is by far the best predictor of true income compared to other population variables. Since the Danish tax agency, SKAT, does in fact use these information reports extensively in its enforcement efforts, they are ideal for constructing audit groups.[3] Using these, we generalize Erard and Feinstein's within-audit-group model to describe tax evasion and optimal enforcement both within and between audit groups. We calculate an internally consistent set of model parameters directly from data and calibrate the tax agency's budget to match the simulated level of tax evasion to data. We evaluate the model numerically and find that applying structure to the data yields results in close correspondence with our minimal-assumptions reduced-form estimations. We conclude that (statically optimized) tax evasion and auditing is sufficient to generate the observed structure of effective average tax rates.

In both the reduced-form estimations and model simulation, the covariance structure of effective average tax rates is robust to changes in estimation method and parameter variations, respectively. In view of this, we predict that similar empirical relationships would be found in data from any tax agency that, as the Danish tax agency does, employs a strong signal in predicting true incomes.

Our results have important implications for policy. Due to the theoretical robustness of the regressive bias prediction, it has been argued (e.g. in Scotchmer 1992) that governments could increase the progressivity of the income tax code to counter the increasing tax rate bias. However, our

[3] Other recent papers demonstrate the importance of explicitly considering information reporting. Phillips (2010) demonstrates the predictive power of third-party reported information in US data and Pomeranz (2010) demonstrates the general importance of information as a deterrent of VAT evasion in a sample of Chilean firms.

results clearly show that population heterogeneity makes such a policy adjustment undesirable. Rather, allocating more resources to the tax agency or collecting more information ex ante is the recommended approach.

We now proceed to the main body of the paper. Section 2 outlines the Danish tax system and describes the main features of the data. Section 3 evaluates the correlation structure of effective average tax rates empirically. Section 4 presents our model. Section 5 describes the calibration of parameters, outlines the numerical strategy and establishes the correspondence of data and model-generated output. Section 6 concludes. The Appendix provides details of the numerical implementation and robustness checks.

2 Data

SKAT's tax collection efforts rely heavily on information reports by third parties. During some year t, incomes are earned and by the end of January in year $t+1$, SKAT receives information reports from employers, banks and other entities. By mid-March, SKAT sends out pre-populated tax returns based on third-party information and other information that they possess about the taxpayers, such as the taxpayers' residence and workplace for calculating commuting allowances. Subsequently, taxpayers have until May 1 to correct their tax return; in case of no corrections, pre-populated tax return counts as final.

After the deadline, SKAT's computerized system processes tax returns and attaches audit flags to returns that the system finds likely to contain errors. The system is entirely deterministic and does not as such assign a probability of audit. After the tax returns have been processed, tax examiners assess the flagged returns and decide whether or not to initiate an audit based on the severity of the different kinds of flags, local knowledge, and auditing resources. The process is depicted in Figure 2.

[Fig. 2 about here]

If an audit discovers underreporting the taxpayer may pay the taxes owed immediately or postpone the payment at an interest. If the tax examiner views the underreporting as deliberate, the tax agency may impose a fine according to a fining scheme depending on the assessed intentionality of the misreporting.

2.1 Experimental Design

The data originates from an experiment conducted by SKAT in the years 2006–2008, originally analyzed in Kleven et al. (forthcoming). The experiment involved a stratified random sample of 17,764 self-employed individuals and 25,020 employees and recipients of benefits in Denmark. In the present study, we narrow our focus to a subsample of non-treated employees and recipients of benefits and their incomes in the 2006 fiscal year. The sample is a stratified random sample of 10,470 selected Danish taxpayers.[4] For each taxpayer, SKAT conducted an unannounced audit after the deadline for changing the tax return (May 1, 2007). The tax audits were comprehensive in the sense that SKAT examined all items on the tax return, demanding documentation for all items on which SKAT did not possess information. SKAT made a significant effort to have tax examiners perform homogeneous audits by e.g. organizing training workshops and distributing detailed audit manuals. The audits took up 21% of the resources devoted to tax audits in 2007.

Of course, it is unlikely that tax examiners find all hidden income, such as that stemming from cash-only businesses and other black market activities. We focus our attention on the detectable part of tax evasion given the methods available to SKAT and thus denote our empirical counterpart of true income "detectable income". In what follows, we will write true income when in fact we mean detectable income.

For each taxpayer, we have income and tax records as reported by third parties, the final return as potentially changed by the taxpayer, and the post-audit return. In addition, the data contains information on the generated

[4] Note the randomness of our sample as opposed to tax compliance data obtained from the regular audits that is heavily biased by over-sampling taxpayers who are likely to have misreported their income in either direction. The sampling strategy involved a stratification on tax return complexity.

audit flags that would normally constitute a basis for selecting taxpayers for audits.

2.2 The Tax System and Tax Compliance in Denmark

The Danish income tax system (in 2006) operates with many different measures of income. Here, we will provide the headlines; see Table 1 for details. Labor market income, i.e. salary, fringe benefits and other earned income, are taxed proportionally by a labor market tax of 8%, while an earned income tax credit (EITC) of 2.5% is provided for labor market income up to 292,000 DKK.[5] Capital income is a net concept, and different tax rates apply depending on whether net capital income is positive or negative. For most taxpayers net capital income is negative due to interest payments on mortgages. Central government taxes (bottom, middle and top tax) are levied on the so-called personal income, which, in addition to positive net capital income, consists of labor market income plus social transfers, and pensions less labor market taxes and some pension contributions. Central government taxes constitute a progressive tax scheme with a personal allowance and three brackets. Local taxes (county and municipality) are levied on "taxable income" which is similar to the central government tax base except that it allows for negative net capital income deductions and other deductions such as transport allowances. In this way, Denmark has a version of the Nordic dual income tax[6]; negative capital income is taxed at a flat rate whereas positive capital income is taxed progressively just as regular income. Stock income (dividends and capital gains) is subject to a two-rate scheme with the high rate setting in at 44,300 DKK.

[Tab. 1 about here]

Table 2 presents some descriptive statistics of the sample on major income components. The table shows sample means with standard errors of means in parentheses – all numbers are calculated accounting for the stratification scheme. Column (1) presents pre-audit figures measured at the

[5] Approx. 53,000 USD (1 USD $\approx$ 5.5 DKK).

[6] For a discussion of the Nordic dual income tax., see e.g. Nielsen and Sørensen (1997).

deadline, May 1, and column (5) shows figures reported by third-parties. Self-reported figures (the difference between (1) and (5)) is shown in column (6). Negative figures mean that taxpayers on average adjust the number downwards to less than what third-parties have reported. Columns (2)–(4) describe how the figures in (1) were adjusted by the tax examiners during the audit. Columns (3) and (4) split the audit adjustments into positive (meaning underreporting) and negative (meaning overreporting) adjustments, while column (2) holds the average net adjustment, i.e. the sum of (3) and (4).

The top panel of Table 2 shows figures on net income and total taxes. The former is defined as the sum of personal income, capital income, stock income, self-employment income and foreign income less deductions. Pre-audit net income is on average a little less than 200,000 DKK with a significantly positive net adjustment from SKAT of almost 1,700 DKK. The positive net adjustment reflects an asymmetry in the reporting behavior with underreporting being more than ten times as high as the overreporting. Third-party reported net income is slightly higher than pre-audit net income mainly due to deductions not included in the third-party reports, implying a negative self-reported net income.

The bottom panel of Table 2 features a decomposition into main income components. The asymmetry in the over- and underreporting found for net income is noticeable for all components. Not surprisingly, the greatest relative amount of underreporting is found on items least subject to information reporting. Self-employment income tops the list with underreporting amounting to 18.6% of the mean pre-audit self-employment income level followed by stock income (6.8%), deductions (2.3%), and the rest being less than 2%.

[Tab. 2 about here]

3 Reduced-Form Analysis

In this section, we evaluate the correlation structure of effective tax rates in the subset of evaders in the population of taxpayers. Specifically, we look at a subset of 905 taxpayers for whom taxes liable were adjusted upwards following an audit.

We use SKAT's audit flag system as a proxy for the probability of an audit. Specifically, we calculate the approximate audit probability, f, by scaling the audit flags to indicate a probability of audit rather than a binary do- or do-not-audit value. During normal operations, the flag system always selects a much larger pool of taxpayers than SKAT's budget allows – as a consequence only a subset of the taxpayers selected by the flag system are actually audited. As we do not know the exact budget size, we scale the audit probabilities such that the expended budget correponds to auditing 3.45% of taxpayers; this value stems from the calibration of our model to match the level of evasion observed in the data, cf. Section 5.

In this manner we can calculate the effective tax rate for each individual as

$$\tau^{\text{eff}} = \frac{f\left(T + \theta\left(T - \tilde{T}\right)\right) + (1-f)\,\tilde{T}}{Y},$$

where T and $\tilde{T}$ are taxes on true and reported income, respectively, and Y is true income, and θ is the penalty rate on underreported taxes[7]. We denote $\tau - \tau^{\text{eff}}$ the effective tax rate bias where the nominal average tax rate, τ, is defined in the usual way, $\tau = T/Y$.

First, we will check whether the correlation between $\tau - \tau^{\text{eff}}$ and Y is positive *within* audit groups (i.e. for a fixed level of third-party reported income). This will serve as evidence of the regressive bias predicted by theory as the difference between the statutory tax rate and the effective tax rate would be higher for high-income taxpayers relative to low-income taxpayers within an audit group.

Conversely, effective tax rates may be progressively biased *between* audit

[7] We set $\theta = 1.06$ which is an approximate average value for the Danish tax system. The calculation of this parameter value is documented in Section 5.

groups, in which case this effect may dominate the within group variation such that pooled data displays progressive bias. If this is the case, $\tau - \tau^{\text{eff}}$ is decreasing in third-party reported income since the difference between the statutory tax rate and the effective tax rate will be higher for low audit groups than for high audit groups.

Table 3 shows the results of the simple reduced-form regressions. As expected, we find regressive bias within audit groups which is reflected by the significantly positive coefficient on the true income residual, cf. column (2), i.e. controlling for third-party reported income, the wedge between the statutory and the effective tax rates is increasing in income.

[Tab. 3 about here]

Between audit groups, tax rates are instead progressively biased reflected in a significantly negative coefficient on third-party reported income. This is also illustrated in Figure 3, which plots the average effective tax rate bias for 40th fractiles of the distribution of third-party reported income. In the aggregate, we find no significant relation between the effective tax rate bias and income, cf. column (1), i.e. neither regressive nor progressive bias.

[Fig. 3 about here]

Overall, Table 3 suggests a correlation structure of effective tax rates as depicted in Figure 1.[8] The data supports the theoretical prediction that effective tax rates are regressive within audit groups. Between audit groups, there is a progressive bias such that tax rate bias is virtually uncorrelated with total net income.

To put this into perspective, consider the median taxpayer among tax evaders. This individual earns approximately 280,000 DKK which was reported by third parties. In addition, he/she claims approximately 25,000 DKK in deductions not reported by a third party resulting in a net income of 255,000 DKK. Using Table 3 column (2) we can calculate the predicted

[8] Although there are some outliers in the data, they do not appear to be driving our results. Applying median regressions, which are less sensitive to outliers, does not qualitatively change our results.

tax rate bias for different compositions of net income, holding true income fixed. The predicted tax rate bias of the median taxpayer is approximately 2.2 percentage points. If all income was reported by third parties, the tax rate bias should be 0. If instead there were no income reports by third parties, the tax rate bias increases to 9.1 percentage points.

4 Theory: A Model of Income Tax Auditing Subject to Information Returns

To corroborate our interpretation of the observed correlation in the data, we now proceed to the model of optimal auditing and tax evasion on the population scale. In Section 5, we calibrate the model to data and show that its predictions are in close correspondence with the relationships established in the reduced-form analysis.

Several current theories are capable of analyzing behavior *within* an audit group. However, as we wish to analyze *aggregate* reporting behavior as well as the tax agency's overall response we need a model that can encompass several audit groups. To this end, we generalize the model in Erard and Feinstein (1994) to incorporate a population that is heterogeneous in third-party income reports.[9]

Erard and Feinstein (1994) introduce noise in taxpayer reports by incorporating the stylized fact that some taxpayers report their incomes honestly even when they have ample opportunity to evade taxes. This is also the case in our data and is demonstrated in Table 4. Column (1) and (2) separates taxpayers according to whether or not their entire income was reported to the tax agency by a third party providing both unweighted sample totals and population weighted sample shares (Panel A). The overall population weighted share of compliers, given by individuals not underreporting, amounts to 94%. To address taxpayers with ample opportunity to evade taxes, Panel B shows population weighted shares for the subcategories (1)

[9] We use a different specification for penalties in the case of detected evasion than Erard and Feinstein (1994). We model penalties as proportional to evaded taxes rather than evaded income as this is also the structure of the actual Danish penalty system.

and (2). Despite having the opportunity to evade taxes, a substantial share of over 80% of taxpayers with some income not reported by third parties choose to comply with the tax laws.

[Tab. 4 about here]

As argued in Erard and Feinstein (1994), including inherently honest taxpayers increases the realism and usefulness of the model: it eliminates several potential equilibria and leaves them with a unique mixed-strategy equilibrium prediction. Further, it eliminates the quaint feature of earlier models that the tax agency would know the true incomes of all taxpayers *before* the actual audit. Thus, for each tax return filed by a particular taxpayer, the agency decides whether or not to audit based on the expected reports of dishonest and honest taxpayers and the resulting probability that any particular tax return is fraudulent.

A tax agency employs information to predict taxpayers' true incomes. As shown by Kleven et al. (forthcoming), third-party reported income is by far the most powerful predictor available, making it an ideal candidate for defining audit groups. However, as this variable, like true income, is intuitively best understood as a continuous variable, we allow the tax agency to choose audit functions contingent on the third-party information of a particular taxpayer and interpret each *level* of third-party reported income as an audit group. Reflecting the very low evasion rates on third-party reported income, gleaned from Table 4,[10] we use the simplifying assumptions that these reports are always correct and are common knowledge to both taxpayer and tax agency. Overall, the probability that a particular taxpayer is audited depends both on the predetermined signal, i.e. third-party reported income, and the endogenously determined reported income.

The overall structure of the model is illustrated in Figure 4.

[Fig. 4 about here]

[10] For example, of those with their entire income reported by third parties, 98.4% do not underreport income.

The tax agency selects the audit regime subject to a budget constraint without being able to commit to an audit strategy. The audit schedule for a particular audit group (i.e. conditional on a particular third-party reported income level) is a function of taxpayers' reported *residual* incomes, i.e. income in excess of third-party reported income, reflecting our assumption that third-party reported income is common knowledge. Whereas the distribution of true incomes, conditional on information reports, is known, actual true incomes of individual taxpayers are private information. Taxpayers choose income reports subject to their expectations about the audit regime. Finally, the actual returns and the audit schedule are realized, audits are conducted, and tax revenue and *ex post* utilities, as measured by income net of taxes and any penalty payments, are realized.

4.1 Individual Reporting Behavior

Individual taxpayers have true taxable incomes y and report taxable incomes, $\tilde{y}$. Part of true income, z, is reported by third parties and is known to all parties. Therefore, $y = z + u$, where u is residual income which can be positive or negative as it includes both e.g. wages and deductions not reported by third parties. u is *ex ante* unknown and can only be ascertained by the tax agency by conducting a costly audit. We denote the reported residual x, such that $x = \tilde{y} - z$.

Two facts of the data defy a pure rational-choice model of evasion behavior, cf. Table 4: First, a proportion 28.4% of taxpayers have non-zero residual income, but do not evade taxes; second, a proportion 65.6% of taxpayers have zero residual income and do not claim unwarranted deductions.

Erard and Feinstein (1994) splits taxpayers into two broad groups, honest and dishonest taxpayers, and assume that these two types differ only in reporting behavior and not in the scope for evasion. This simple pair of assumptions cannot be directly reconciled with the two above facts as the ratio of compliant to noncompliant taxpayers is not constant due to a large mass of correct reports at $u = 0$. We prefer to remain agnostic as to whether the 65.6% are inherently honest or merely honest due to practical circumstance and keep these taxpayers in a separate group. In the subset of taxpayers for whom $u \neq 0$, we denote by Q the fraction of honest taxpayers. Thus, the conditional density of u given z of honest taxpayers is $Q \cdot f_{u|z}(u)$ while for dishonest taxpayers it is $(1-Q) \cdot f_{u|z}(u)$, where $f_{u|z}(u)$ is defined on the domain $[\underline{u}, \overline{u}]$. We denote by $F_{u|z}$ the conditional distribution function associated with $f_{u|z}$. Finally, we denote by $M(z)$ the mass point of compliant taxpayers reporting $x = 0$ given z.

We follow Erard and Feinstein (1994) in assuming that taxes are linear in income.[11] Whereas honest taxpayers always report $x = u$, we assume that dishonest taxpayers are risk neutral and maximize expected utility given by expected income net of taxes and penalties

$$(1-t)z + p(x|z)\left[(1-t)u - \theta t(u-x)\right] + (1 - p(x|z))\left[u - tx\right].$$

In optimum, the taxpayer's choice must satisfy the first order condition

$$u = x + \frac{p(x|z) - \frac{1}{1+\theta}}{p'(x|z)}. \tag{1}$$

It is clear from equation (1) that for $p(\cdot) = \frac{1}{1+\theta}$, $x = u$ and evasion is discouraged completely. However, $p \geq \frac{1}{1+\theta}$ is not compatible with equilibrium when the tax agency cannot commit to the audit regime: if evasion were completely discouraged, the tax agency would lower p for some x as a cost saving measure. Thus, in equilibrium $p(\cdot) \in \left[0, \frac{1}{1+\theta}\right)$. Furthermore, the incentive compatibility constraints on the tax agency's optimization problem implies that audit functions are decreasing on the domain of income reports (see Erard and Feinstein (1994) for a detailed demonstration of this point).

Given that $p'(x|z)$ is negative and $p(x|z) < \frac{1}{1+\theta}$, increasing the audit probability will, *ceteris paribus*, lower tax evasion as the risk of getting caught is higher. Lowering $p'(x|z)$ (increasing its absolute value) also reduces tax evasion by increasing risk of audit from taxes evaded on the

[11] Clearly, this an abstraction but not an extreme one. Although the income tax schedule has three brackets, the average tax rates are much smoother. It would also be possible to perform the analyses using a full, nonlinear specification of taxes. We do not expect that the conclusions of this paper would to be substantially affected by this change.

margin.[12]

4.2 Optimal Audit Response

The tax agency chooses a continuum of audit schedules $p(x|z)$ for all z. In this way, the informational aspect of using third-party reported incomes to predict true income is incorporated into the population-wide equilibrium.[13] The audit schedule is chosen to maximize expected revenue (taxes plus fines)

$$\int \left(\int_{\underline{x}}^{\bar{u}} \left(p(x|z) \left[t\mathrm{E}(y|x,z) + \theta t \left(\mathrm{E}(y|x,z) - \tilde{y} \right) \right] + (1 - p(x|z))\, t\tilde{y} \right) \mathrm{d}F_{x|z} \right) \mathrm{d}F_z$$

subject to the budget constraint

$$c \int \left(\int_{\underline{x}}^{\bar{u}} p(x|z)\, \mathrm{d}F_{x|z} \right) \mathrm{d}F_z \leq \int B(z)\, \mathrm{d}F_z \equiv \mathbf{B}$$

where $F_{x|z}$ is the induced conditional distribution function for reported residual income, x, given third-party reported income, z; F_z is the marginal distribution function for z; and $B(z)$ is the proportion of the overall audit budget, $\mathbf{B}$, allocated to income reports with third-party reported income z. For each (x, z), the tax agency must choose p to solve

$$\max_p \left(p \left[t\mathrm{E}(y|x,z) + \theta t \left(\mathrm{E}(y|x,z) - \tilde{y} \right) \right] + (1-p) t\tilde{y} \right) \mathrm{d}F_{x|z} \mathrm{d}F_z \\ -\lambda c \left[p\ \mathrm{d}F_{x|z} - B(z) \right] \mathrm{d}F_z$$

[12] Taxpayers' income returns must also satisfy the second order condition, $p''(x|z)(x-u) + 2p'(x|z) \leq 0$.

[13] In principle, the tax agency could also condition audit schedules on other population variables such as gender, age, occupation etc. However, as Kleven et al. (forthcoming) show, these variables are less powerful as predictors. Conditioning on whether the taxpayer was audited in previous years would complicate matters as it would introduce a dynamic aspect to reporting decisions. However, as observations on past audits are not employed in SKAT's actual audit scheme, this limitation is unlikely to affect the fit of our model. In addition, the statute of limitations for retrospective audits is limited to 14 months.

where λ is the Langrangian multiplier on the budget constraint. This implies a point-wise first order condition

$$tE(y|x,z) + \theta t E(y|x,z) - \theta t\tilde{y} - t\tilde{y} - \lambda c \gtreqless 0 \tag{2}$$

which is greater than, equal to, or less than zero as $p = \frac{1}{1+\theta}$, $p \in \left(0, \frac{1}{1+\theta}\right)$, or $p = 0$. We look for equilibria in which the tax agency chooses a mixed strategy, such that (2) holds with equality.[14]

As mentioned, our model is a generalization of the model in Erard and Feinstein (1994). Specifically, our model simplifies to theirs if *i)* z is identical for all individuals such that $F_{uz} = F_u$, *ii)* $\log(u) \sim \mathcal{N}(\mu, \sigma^2)$,[15] and *iii)* $B(z) \equiv \mathbf{B}$. In this case, the problem becomes that of a partial optimization for a fixed $B(z)$ within an audit group. In this simpler version of the model, it can be shown that the equilibrium audit and evasion functions have a number of useful properties. Due to the incentive constraints on reporting for high-income taxpayers, the audit function $p(x|z)$ is decreasing and continuous in reported income. The reporting function, $x(u|z)$ is strictly increasing in an upper region of the income domain and constant in a lower region as some taxpayers pool at the lowest possible report. As the audit and evasion functions are continuous and differentiable on the interior of the reporting domain, it is possible to solve for the equilibrium using methods of differential equations. In addition, as pooling occurs only at the lowest report, where the differential equation is undefined, sufficient conditions for equilibrium can be obtained by checking that the solution to the differential equation also satisfies the tax agency's first order condition for the lowest report, equivalent to (4) below. In the same way, we can leverage these properties to solve for the population-wide equilibrium as a range of within-audit-group equilibria coupled with the optimal budget distribution $B(z)$.[16]

[14] The second order condition is $\frac{\partial \mathrm{E}(y|x,z)}{\partial p(x|z)} \geq 0$. In our simulations the solutions always satisfy this criterion.

[15] Note that this implies that $M(z) = 0$, $\forall z$ in (3).

[16] Our analysis is complicated by the fact that the mass point of compliant taxpayers at $x = u = 0$ induces a singularity in the differential equation (6) given in the Appendix. We take a standard practical approach and approximate the numerical solution by "smoothing" the transition of $p(x|z)$ and $p'(x|z)$ at $x = 0$ by interpolating the indicator function

Thus, the unique equilibrium of the model is described by the collection of functions $u(x|z)$ and $p(x|z)$ and the budget distribution $B(z)$. Once $p(x|z)$ is determined, $u(x|z)$ is implicitly defined as the solution to the taxpayers' first order condition and the tax agency chooses $p(x|z)$ such that (2) holds with equality. The two equations are connected by the tax agency's conditional expectation of taxpayers' true income given the reported income and third-party reports, $\mathrm{E}(y|x,z)$, which is

$$\mathrm{E}(y|x,z) = z + \frac{Qf_{u|z}(x)\,x + (1-Q)\,f_{u|z}(u(x|z))\,\frac{\partial u(x|z)}{\partial x}u(x)}{Qf_{u|z}(x) + (1-Q)\,f_{u|z}(u(x|z))\,\frac{\partial u(x|z)}{\partial x} + \mathbf{1}(x=0)\,M(z)} \tag{3}$$

where $\mathbf{1}(\cdot)$ is the indicator function and the derivative $\frac{\partial u(x|z)}{\partial x}$ is derived from (1) by differentiating implicitly to get $\frac{\partial u}{\partial x} = 2 + \frac{p''(x)(x-u)}{p'(x)}$.[17]

We can then derive a second order differential equation, (6) in the Appendix, which determines the optimal equilibrium responses $p(x|z)$ and $x(u|z)$ in audit group z using the expressions for $\mathrm{E}(y|x,z)$, $u(x|z)$, $\frac{\partial u}{\partial x}$ and the tax agency's first order condition. However, as some taxpayers pool at the lowest report, to obtain sufficient conditions for equilibrium, we must check the tax agency's first order condition at $x = \underline{u}$ separately as

$$\mathrm{E}(u|x=\underline{u},z) = \frac{Qf_{u|z}(x)\,x + (1-Q)\int_{\underline{u}}^{u^{\mathrm{pool}}} u\cdot f_{u|z}(u)\,\mathrm{d}u}{Qf_{u|z}(x) + (1-Q)\int_{\underline{u}}^{u^{\mathrm{pool}}} f_{u|z}(u)\,\mathrm{d}u} = \frac{\lambda c}{t+\theta t} + \underline{u}. \tag{4}$$

where u^{pool} is the residual income at which taxpayers (in this audit group) begin to pool at the lowest possible report.

As mentioned above, the model contains Erard and Feinstein (1994) as a special case when attention is limited to a single audit group in which taxpayers are homogeneous in third-party income reports. To illustrate, Figure 5 depicts the within-group equilibrium for fixed $B(z)$ at 10%, $\log(u) \sim \mathcal{N}(3.42, 0.3^2)$ truncated on $[20, 44]$, $Q = 0.4$, and $t = 0.5$.

$1(x=0)$ by a piecewise cubic hermite interpolating polynomial. See the Appendix for details.

[17] Notice that $f_{x|z}(x(u)) = f_{u|z}(u(x))\left|\frac{\partial u(x,z)}{\partial x}\right| = f_{u|z}(u(x))\frac{\partial u(x,z)}{\partial x}$ since the SOC implies that $\frac{\partial u}{\partial x} \geq 0$ in interior optimum.

[Fig. 5 about here]

Figure 5(a) shows the audit schedule, $p(x|z)$: It starts in $\underline{u}$, is downward sloping, and terminates in $p(\bar{x}) = 0$. This form balances the need to audit in order to raise revenue with the cost of doing so. The negative slope reflects the need to discourage high-income taxpayers from reporting too low incomes.

Figure 5(b) shows the amount of evasion as a function of income. The linear increase in the first part of the graph reflects pooling of dishonest taxpayers: for a given audit schedule, there will be some income in $[\underline{u}, \bar{u}]$, u^{pool}, for which the most profitable report is $\underline{u}$ – consequently all taxpayers with residual incomes $u < u^{\text{pool}}$ also report $x = \underline{u}$. Therefore, there will be a point mass in the induced distribution of reports, $f_{x|z}(x)$. After this pooling point, evasion falls rapidly in income until evasion again becomes increasing in income as the probability of detection becomes sufficiently low.

Figure 5(c) shows the effect of the optimal audit schedule on the effective tax rate, calculated as

$$\tau^{\text{eff}} = \frac{p(x) \cdot (ty + \theta t(y - \tilde{y})) + (1 - p(x)) \cdot t\tilde{y}}{y}. \tag{5}$$

The declining profile of $p(x|z)$ together with the high propensity to evade taxes of high income taxpayers, result in a negative relationship between the effective tax rate and income. Therefore, high-income taxpayers pay significantly less than the statutory tax rate, which, in the case of Figure 5(c), is $t = 0.5$.

Figure 5(d) shows the induced distribution of incomes and reports. The top graph is the original income distribution, which in this case is lognormal. The lower graph shows the distribution of induced reports, i.e. the equilibrium response of all taxpayers to the audit schedule. The right part of the graph is just a scaling of the original income distribution by Q while the left part is a weighted average of reports by honest and dishonest taxpayers. The whole graph is somewhat lower than the original income distribution as there is a mass point of dishonest taxpayers reporting at $\underline{u}$, the mass point being equal to the area between the graphs.

5 Calibration and Simulation Results

Due to the considerable detail of our data, we can construct a set of parameters that are internally consistent, that is they all derive from the same data set. We estimate the income distribution and the parameters Q, θ, and t, for each accounting for the stratification scheme, and calibrate the model to the observed average level of tax evasion by varying the budget parameter, $\mathbf{B}$. As a normalization, we set the cost of an audit, c, to 1. Thus, by the tax agency's budget constraint, $\mathbf{B}$ can be interpreted as the percentage of the population of taxpayers that are examined by auditors.

5.1 Calibration

Income Distributions

We use the taxpayer data to construct the income distribution needed in the model. As income measure we choose net income defined as the sum of personal income, capital income, stock income, self-employment income and foreign income less deductions.

First, we allow the mass point of compliant taxpayers at $u = 0$ to vary in z, see Figure 6(a). This is important because richer taxpayers are much more likely to have non-zero residual income than poorer taxpayers. To fit the remaining simultaneous distribution of z and u, we exclude the former taxpayers and fit a mixed lognormal distribution.[18] Figure 6(b) depicts three conditional distributions of u given z in the lower, middle and upper part of the domain of z.

[Fig. 6 about here]

The exact characteristics of this distribution is documented in the Appendix. Briefly, the variance of $u|z$ is generally increasing in z; however,

[18] Our results do not appear to alter significantly if, instead, a kernel estimation is used. Kernel densities are inconvenient as they allow for "troughs" of zero density in the interior of the domain for $f_{u|z}$ which may cause our algorithm to fail.

the taxpayers with very low z seem to have relatively complicated income compositions resulting in high variance of $u|z$ in the first audit group.

Honesty

A key model parameter is the fraction of honest taxpayers, Q. In order to determine an appropriate value of this parameter, we must account for the fact that, in reality, some taxpayers seem to make reporting mistakes. For example, in the data some reports are adjusted downward by the auditor which means that, in the absense of an audit, the taxpayer would have payed more than intended by the statutory tax system.

We approach the problem in the following manner. First, we assume that no taxpayer will try to evade taxes on income that is reported by a third party (this assumption is bourne out in the data as shown in Table 4). Then we separate the taxpayers into two groups, one containing those whose true income is entirely reported by third parties so that $y = z + u$ and $u = 0$, and the other containing those with some residual income not subject to third-party reporting, $u \neq 0$. The second group is then separated according to whether or not the audit led to a change in their reported income, i.e. whether or not $x \neq u$. In other words, we are classifying taxpayers into groups of compliant ($u = x = 0$), inherently honest ($u \neq 0$, $x = u$) and dishonest taxpayers ($u \neq 0$, $x \neq u$).

Table 4 shows this decomposition. First, note that among taxpayers whose entire income is reported by third parties, the compliance rate is 97.9%. Among those taxpayers that have some of their income not reported by third parties, the compliance rate is 80.8%. We can define honest taxpayers in several ways. The simplest is to include only those reporting correctly. This definition fails to acknowledge the fact that some taxpayers make reporting errors. However, modelling reporting errors is beyond the scope of this paper. A revenue maximizing tax agency cares not whether revenue is collected from dishonest taxpayers who intentionally underreport or honest taxpayers who do so by mistake. We classify overreporting taxpayers as honest and underreporting taxpayers as dishonest. Thus, the number of honest taxpayers is the sum of those reporting correctly and those overreporting by mistake, which corresponds to $Q = 85.2\%$. The residual consists of both dishonest taxpayers and taxpayers underreporting by mistake whom we cannot distinguish.

Penalty

In the same way that we approximate the tax system by a proportional tax rate, we approximate θ by an average penalty rate. In Denmark, evasion penalties are generally calculated as a factor on taxes evaded; that factor, however, varies for the amount evaded and the intentionality of evasion as assesed by the auditor.

In the case of intentional tax evasion, the fine is calculated as one times evaded taxes under 30,000 DKK and two times the evaded taxes exceeding 30,000 DKK. In the case of gross negligence, the rates are instead 1 times evaded taxes not exceeding 30,000 and 0.5 times evaded taxes exceeding 30,000.

We use actual "compliance ratings" of Danish tax auditors of individual taxpayers to approximate θ. As part of SKAT's ongoing effort to monitor compliance, each taxpayer in our data has been assigned a compliance rating, varying from 0 (severe intentional evasion) to 6 (honest mistakes). These ratings are further sub-divided into two groups in which ratings below 3 signify intentional evasion and ratings above and including 3 signify mistakes (also including severe negligence). We take the simplest approach and assign the rates for intentional evasion to the first group and the rates for negligent underreporting to the second group and use the OLS slope coefficent between approximated penalties and underreported taxes.[19] The resulting penalty rate on underreported taxes is 1.06. We take the view that this value of θ is a lower bound on the appropriate value as it includes neither the cost of potential prison sentences for severe cases of evasion nor the psychological

[19] Of course, we calculate θ accounting for the stratification scheme. Assigning the penalty rates for severe negligence only to those for whom the compliance rating is 4 or 5 does not significantly alter the value of θ due to the small number of honest mistakes classified among underreporting taxpayers.

cost and effort of defending one's reported tax liability.

5.2 Simulation Strategy

An individual solution to Equation (6) in the Appendix, $\left(p, \frac{\partial p}{\partial x}\right)$, that corresponds to a particular z is found numerically using methods of Ordinary Differential Equations (ODE) as initial value problems. The solver is initialized using $p(\bar{x}) = 0$ and $p'(\bar{x}) = \left(\frac{1}{1+\theta}\right) / (\bar{u} - \bar{x})$, where $\bar{x} \equiv x(\bar{u})$. Thus, starting at the end-point of the equilibrium-path audit probabilities, a numerical solver finds values in steps until $\underline{u}$ is reached, ensuring that the taxpayers' as well as the tax agency's optimality conditions are met for reports $x \in (\underline{u}, \bar{x}]$. However, since a positive mass of taxpayers are pooling their reports at $x = \underline{u}$, the expectation $\mathrm{E}(u|x, z)$ is not differentiable in this point. Therefore, we check that the tax agency's FOC is met in the pooling point separately after finding some candidate solution, cf. (4).

The difficulty in identifying equilibria in this model stems from *a priori* indetermination of λ and $\bar{x}$: we must satisfy $\mathrm{E}(u|x = \underline{u}, z) - \underline{u} = \frac{\lambda c}{t+\theta t}$ which depends on both variables. Our solution method searches the space of possible $(\lambda, \bar{x})$ for candidate solutions, for each checking whether the tax agency's optimization constraints are satisfied on the entire domain of x, until satisfactory solutions are found. The optimal budget allocation, which in our simulations is always interior, equates marginal revenue with respect to the audit budget across levels of z.

While mathematically and intuitively z is naturally understood to be a continuous variable described by the simultaneous distribution of u and z, we approximate the optimal allocation of the total audit budget on the domain of z by constructing a representative grid of values by sub-dividing taxpayers into 40th fractiles. We provide detailed documentation of the numerical implementation in the Appendix.

We have estimated t, c, θ, Q and income distributions from data. Thus, the remaining free parameter is the budget value, $\mathbf{B}$. Since the mean level of evasion is inversely proportional to total tax revenue, it is monotonically declining in $\mathbf{B}$. To calibrate $\mathbf{B}$, we use the estimated income distribution to simulate a population of taxpayers: we vary **B** until the average level of evasion matches the level observed in the data. The unique resulting value is $\mathbf{B} = 0.0345$, corresponding to population-wide audit rate of 3.45%. This compares favorably to the reported audit rate in Kleven et al. (forthcoming) of 4.2% as this figure includes both wage earners and self-employed (for whom the average audit rate is high) and as SKAT auditors during normal operations do not check returns as thoroughly as in the experiment.[20]

5.3 Simulation Results

As mentioned, we calibrate the model to the level of evasion in the data. Figure 7 shows the average level of income evaded for 40th fractiles of the distribution of third-party reported income in data and simulations.

[Fig. 7 about here]

Although the level of evasion in data and simulations necessarily match, it is reassuring that the correlation between evasion and income in the simulations is not substantially different from the data.

The match between data and simulations may seem trivial as it is imposed by the calibration procedure. However, in the context of the economic literature on tax evasion, being able to match a model to moments of the data for reasonable parameter values is novel. For example, Alm, McClelland, and Schulze (1992) argue that observed evasion is too low to be explained by actual audit and penalty regimes. Our analysis lends support to the argument of Andreoni, Erard, and Feinstein (1998) and Slemrod (2007) that third-party reporting and tax-return-dependent audits can explain a substantial part of observed evasion. However, in accordance with Feld and Frey (2002), our analysis also requires us to take into account the substantial number of taxpayers that report honestly despite incentives to evade.

[20] This is not to imply that SKAT is cavalier in its audits, but rather that the audit flag system is intended to alert auditors to misreports in particular line items of taxpayers tax return, rather than to the return as a whole. Thus, it may make sense to audit a return only partly rather than investigate line items for which no signal of evasion has been received.

Budget Allocation

The optimal budget allocation across audit groups is depicted in Figure 8(a). Generally, the profile is increasing in z reflecting the fact that higher-income taxpayers find it relatively easier to evade taxes. The exception is the lowest audit group that is subject to a high audit intensity; perhaps because of a relatively high incidence of welfare fraud and black market income.

[Fig. 8 about here]

To compare, Figure 8(b) depicts the share of taxpayers with audit flags across audit groups. Interestingly, it exhibits the same qualitative features as the simulated optimal budget distribution, namely high audit intensities at the very bottom and top audit groups and an otherwise increasing profile in z.

Effective Tax Rate Bias

As in Section 2, we calculate the bias of effective average tax rates, $\tau - \tau^{\text{eff}}$, this time based on the simulated data. As Figure 9 shows, the simulated tax rate bias matches the data, both with respect to the order of magnitude and the progressivity between audit groups. Considering that the model is calibrated only to the mean level of *evasion*, the correspondence of effective tax rates in the data and the model is excellent.

[Fig. 9 about here]

In addition, the progressive bias between audit groups approximately cancels out the regressive bias within audit groups in the aggregate.[21] Thus, the model both quantitatively and qualitatively replicates the correlation structure of effective tax rates exhibited by the data.

[21] In fact, the model suggests a slight progressive bias in the aggregate whereas in the data the corresponding bias is not distinguishable from zero.

Robustness

The structure of tax rate bias within and between audit groups in the simulations is highly robust as we document in the Appendix. Although the magnitude of regressive and progressive bias is influenced by our key calculated parameters, t, θ, and Q, keeping fixed the income distribution and overall audit budget, the effects are relatively small and in no case do the the biases change signs (Table 6). Further, the biases respond symmetrically to changes in parameter values in the sense that a parameter variation that increases the regressive bias within audit groups also increases the progressive bias between audit groups leaving the aggregate tax bias relatively unaffected.

6 Concluding Remarks

This paper highlights the importance of information in tax enforcement. In doing so, we find evidence in favor of the regressive bias hypothesis and Scotchmer's (1987) conjecture that it is crucial to distinguish regressive bias *within* an audit group from *aggregate* or *between*-group variation. Using highly detailed data we find evidence suggesting that, whereas effective tax rates are regressively biased within audit groups as theory suggests, this relationship is largely negated by a progressive bias between audit groups induced by the distribution of audit resources and third-party information. As a result, no systematic bias can be detected in pooled data.

However, as emphasized by the literature, distortions may be substantial in settings in which third-party reporting is less comprehensive. Standard optimal auditing literature seems to suggest that regressive bias can be countered simply by adjusting marginal tax rates across the board. However, once we allow for population heterogeneity of behavior and income composition, this is no longer feasible. Our results suggest an obvious policy to ameliorate these distortions: increasing the share of income reported by third parties will reduce both the extent of evasion and the regressive bias in tax enforcement.

From a theoretical point of view, including third-party reported information and the likelihood of honest reporting conditional on the income composition is crucial in understanding tax evasion. We take a practical approach and do not as such try to explain the the large number of honest taxpayers in the data. Rather, we analyze whether the correlation structure in effective tax rates is consistent with that generated by an optimizing tax agency and expected utility maximizing tax evaders. We find that, for reasonable parameter values, our model can replicate the extent of observed evasion as well the subtle correlation structure of effective tax rates. In addition, our results indicate that the Danish tax agency employs a distribution of resources across audit groups that is surprisingly similar in key respects to the optimal distribution generated by the model. All in all, there seems to be a role for both standard economic theory and behavioral extensions in explaining tax evasion behavior. In particular, future behavioral research is needed to clarify whether the assumption of exogenous honesty is an appropriate simplification.

The correlation structure of effective tax rates seems robust: it is generated by our realistically complex model as well as in Scotchmer (1987). Furthermore, while variations in parameters change the level of average tax rate bias as well as the rate of progressivity between audit groups, in no variations is the correlation structure of effective tax rates qualitatively different from our baseline simulation. Thus, we are confident that similar empirical relationships would be found in data from any tax agency that, as SKAT does, employs a strong signal in predicting true incomes.

A natural objection to the model we employ is the lack of general equilibrium effects, for example feedback into labor market choices. As experience shows (e.g. Pencavel, 1979), adding such features to the model complicates the analysis substantially, which in our setting may be prohibitive. However, as argued in Chetty, Friedman, Olsen, and Pistaferri (2011) in the Danish context and also Rogerson and Wallenius (2009) and others, because short run responses to effective tax rates are likely to be small due to e.g. labor market rigidities, adjustments occur on the intensive margin rather than the extensive margin. In addition, dynamic aspects are likely to be negligible as due to the limited retrospectivity of SKAT's audit scheme and the restrictive statute of limitations on retroactive penalties for tax evasion. While beyond the scope of this article, extending the model in these directions are interesting questions for future research.

Appendix A

6.1 Numerical Implementation

The second order differential equation is obtained by combining (1), (2), (3) and the expression for $\frac{\partial u}{\partial x}$ to get

$$p''(x) = \left(\frac{\left[Q f_u(x) + 1(x=0) M\right] \frac{\lambda c}{\theta t + t}}{(1-Q) f_u(u(x)) \left[\frac{p(x) - \frac{1}{\theta+1}}{p'(x)} - \frac{\lambda c}{\theta t + t} \right]} - 2 \right) \times \\ p'(x)^2 \left(\frac{1}{1+\theta} - p(x) \right)^{-1} \tag{6}$$

suppressing z for convenience. Thus, sufficient conditions for equilibrium given $B(z)$ are the two equations (4) and (6).

We approximate the equilibrium by discretizing z into a 40 grid point vector, corresponding to mean values in 40th fractiles of the population distribution of z. Equilibrium functions for other values of z are approximated by interpolation and our simulation results are not sensitive to increasing the number of gridspoints. Within each fractile, we solve the 2nd order ordinary differential equation in (6) for many values of $\bar{x}$, where $\bar{x} \equiv x(\bar{u})$. The ODE algorithm is then initialized using $p(\bar{x}) = 0$ and $p'(\bar{x}) = \left(\frac{1}{1+\theta}\right) / (\bar{u} - \bar{x})$, cf. (1). For each value of $\bar{x}$ and z, we need a corresponding value of λ, the shadow value of increasing the budget size. However, λ and $\bar{x}$ are not separately identified. Therefore, we must take a heuristic approach, solving, for each $\bar{x}$, the ODE for many values of λ until one is found that satisfies the equilibrium conditions everywhere, particularly at $x = \bar{u}$. In practice, we do not merely guess repeatedly at λ, but employ a search algorithm to find the λ that satisfies (4); this provides a candidate λ corresponding to a particular $\bar{x}$ that satisfy the FOC everywhere with a small error tolerance.

Figures 10 and 11 illustrate an example of the set of solutions resulting from the algorithm.

[Figures 10 and 11 about here]

When this algorithm has executed for all grid points of z, we can determine the optimal budget allocation using the fact that, in an interior solution, λ must be equalized across different levels of z (the objective function is very steep for $B(z)$ close to zero and quite flat for large $B(z)$ so interior solutions for $B(z)$ has been the relevant case for the solutions we consider).

Equation (6) can be solved by standard numerical methods. We employ a Runge-Kutta-type algorithm developed in Shampine (2009) which outperforms standard ODE algorithms in Matlab in terms of errors. However, two main problems must be resolved. First, the discontinuity of $\mathrm{E}(u|x,z)$ at $x=0$ induces what is known as a "singularity" in the differential equation. We take a standard approach to this problem and approximate solutions for which $\bar{x}>0$ by substituting the logical function $1(x=0)$ with a piecewise cubic hermite interpolating polynomial. The resulting function displays a relatively smooth transition from 0 to 1 in a small band around $x=0$. This band can be made very small, thus minimizing approximation errors from this source. As it turns out, in our simulations, allowing $\bar{x}$ to exceed 0 is only relevant for the bottom and top fractiles where tax evasion is abundant. Second, the ODE algorithm may fail to converge if we allow the conditional density function to take values extremely close to 0 since the ratio $\frac{f_{u|z}(x)}{f_{u|z}(u(x))}$ may diverge toward infinity. Therefore, we truncate the domain of the conditional distributions where the densities are negligible. Specifically, we truncate the unrestricted conditional densities at the 0.5% and 99.5% fractiles. The resulting supports of the conditional distributions vary in z as illustrated in Figure 12.

[Figure 12 about here]

6.2 Robustness

Tables 5-6 present comparative statics from varying key calculated parameters, t, θ, and Q, while keeping fixed the income distribution and the overall audit budget.

Table 5 shows the mean level of evasion of tax evaders with the benchmark setting ($t = 0.46$, $\theta = 1.06$, and $Q = 0.85$) as index 100. Increases in the penalty for tax evasion, θ, and the fraction of honest taxpayers, Q, leads to lower mean evasion. The former effect is very intuitive and works directly through the first order condition of taxpayers. The latter may seem less obvious. Loosely speaking, a higher proportion of honest taxpayers results in less bunching at the lower end of the conditional income distributions, $u|z$, which makes it easier for the tax agency to detect evasion and less attractive to evade. Changes in the tax rate, t, has practically no effect on tax evasion. Because penalties are proportional to evaded *taxes*, t does not enter taxpayers' first order condition – there is no substitution effect as stressed by Yitzhaki (1974). Risk neutrality of taxpayers further implies no income effect in the tax evasion gamble. Hence, the only implication of a tax hike is an increase in revenue from taxes and penalties.

[Tab. 5 about here]

Although the three parameters, t, θ, and Q, affect tax evasion, the covariance structure of tax rate bias and income composition is much less affected. In Table 6, we present the tax rate bias within and between audit groups as measured by the OLS slope coefficients from a regression of the effective tax rate bias on u, z, and a constant. Qualitatively, our conclusions concerning the nature of the tax rate bias within and between audit groups are unaffected by parameter changes. The regressive bias within audit groups and the progressive bias between audit groups are present in all simulations. Further, the effect of changing the value of a parameter is symmetric throughout: a parameter change that increases (decreases) the regressive bias also increases (decreases) the progressive bias and vice versa.

[Tab. 6 about here]

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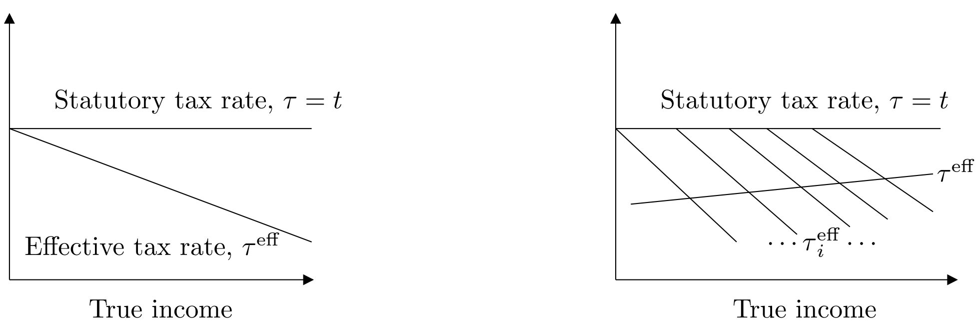
Figures

Figure 1. Correlation Structure of Effective Average Tax Rates.



(a) The Regressive Bias Result.

(b) Aggregation Across Audit Groups.

τ is the statutory average tax rate (here, constant at $\tau = t$), τ_i^{eff} is the effective average tax rate within audit group i, and τ^{eff} is the aggregate effective average tax rate.

Figure 2. Tax Collection in Denmark – The Timing of Events.

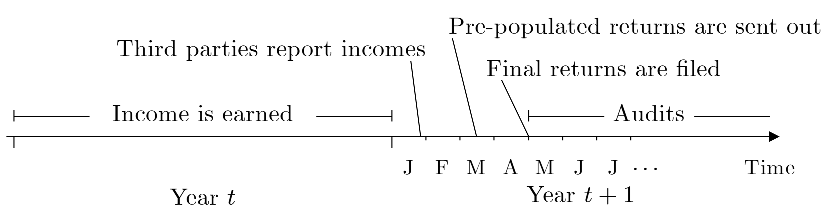
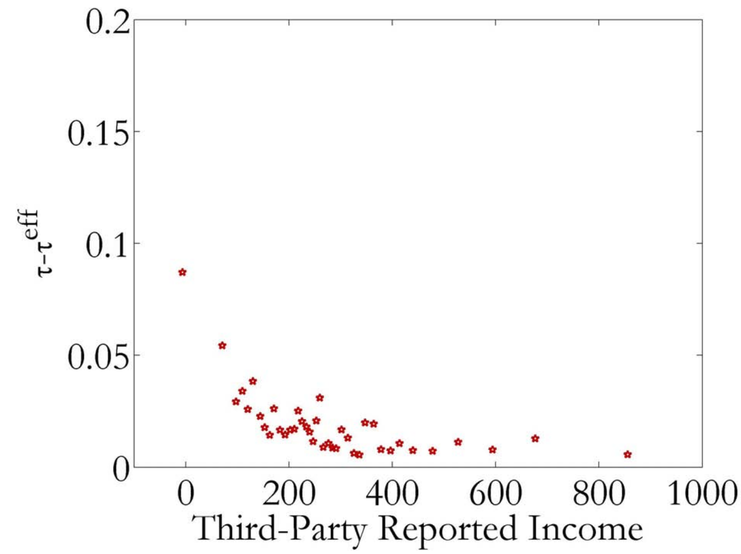


Figure 3. Progressive Bias in the Data for Tax Evaders: Between Audit Group Variation.



The observations for the between-groups analysis are calculated as the expectation of $\tau - \tau^{\text{eff}}$ and third-party reported income (in 1,000 DKK), respectively, for each audit group. Audit groups are approximated as 40th fractiles of third-party reported income.

Figure 4. Game Tree.

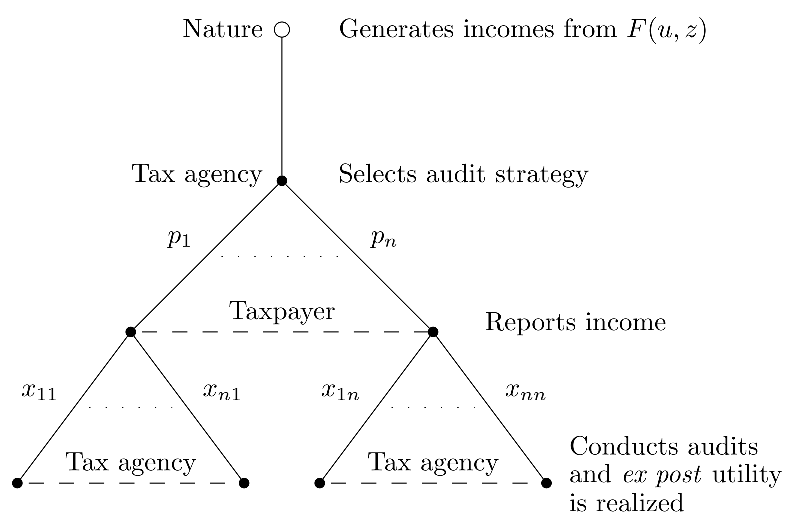
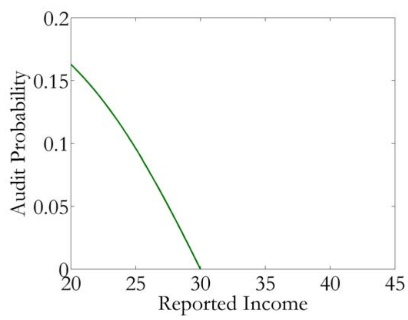
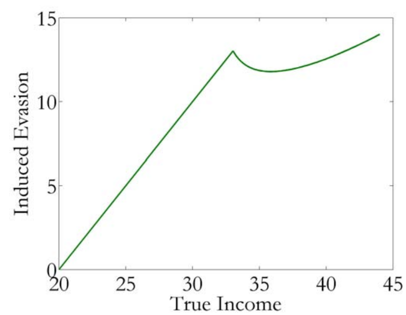


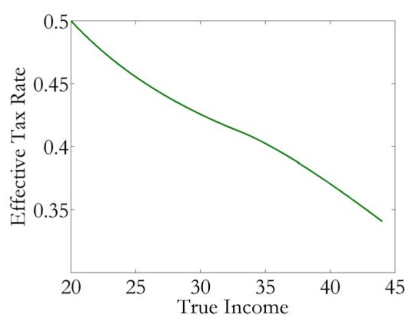
Figure 5. Equilibrium Responses and Tax Bias.



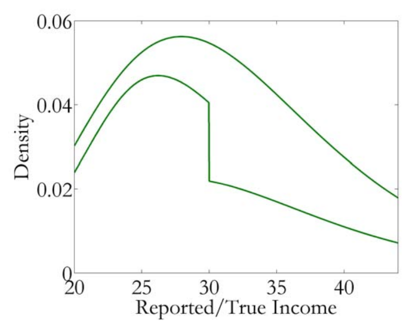
(a) The Optimal Audit Schedule.



(b) Evasion by True Income for Dishonest Taxpayers.

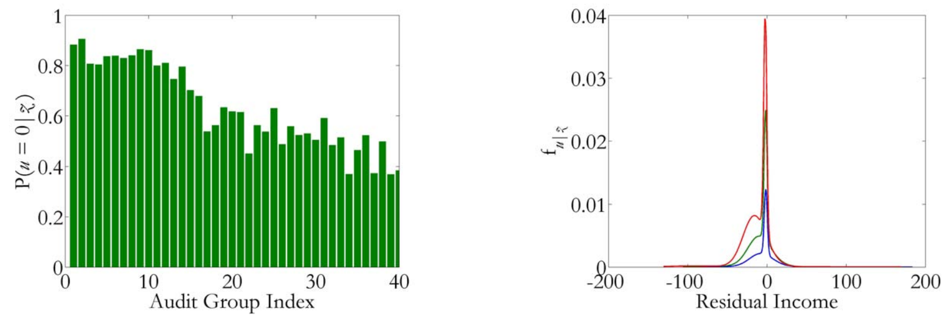


(c) Regressive Bias for Dishonest Taxpayers. In this case, the statutory marginal tax rate is set to $t = 0.5$.



(d) Induced Reporting Behaviour. The lower curve graphs the density of reports by dishonest taxpayers, excluding the mass point at $x = \underline{u}$, while the upper curve graphs the true income distribution.

Figure 6. Empirical Distributions.

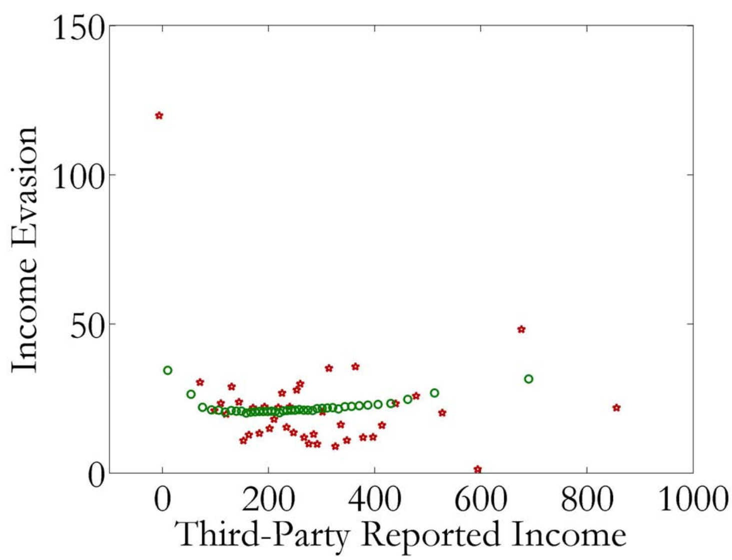


(a) The Proportion of Compliant Taxpayers at $u = 0$.

(b) Conditional Densities, $f_{u|z}$.

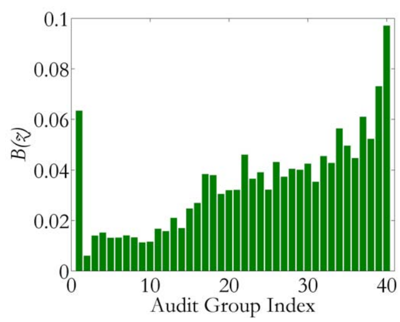
The numbers in (a) indicate shares relative to the total mass of taxpayers in the audit group.

Figure 7. Mean Level of Evasion Across Audit Groups for Evaders, Data and Simulation.

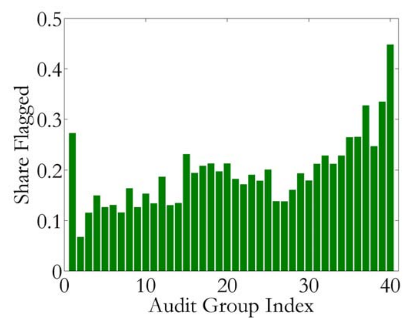


Red stars indicate data, green circles indicate simulated output. Third-party reported income is measured in 1,000 DKK. The left-most data point is extreme due to a single taxpayer with almost no third-party reported information that underreports a substantial amount.

Figure 8. Budget Allocation.



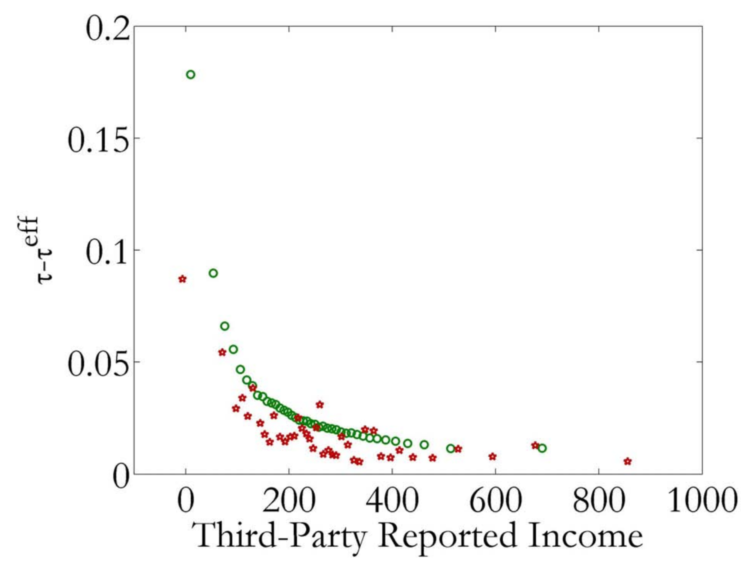
(a) The Optimal Budget Allocation Across Audit Groups.



(b) Share of Taxpayers with Flags Across Audit Groups in Data.

In (a), the budget is allocated such that 3.45 percent of all taxpayers are audited. The percentages shown denote the share of taxpayers within an audit group selected for audits.

Figure 9. Effective Tax Rate Bias Across Audit Groups, Data and Simulations.



Red stars indicate data, green circles indicate simulated output. Third-party reported income is measured in 1,000 DKK.

Figure 10. Examples of Optimal Audit Functions: $p(x|z)$.

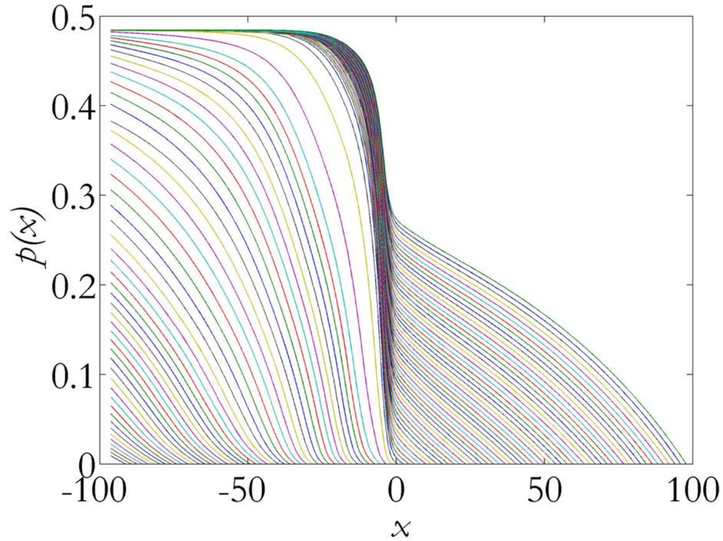
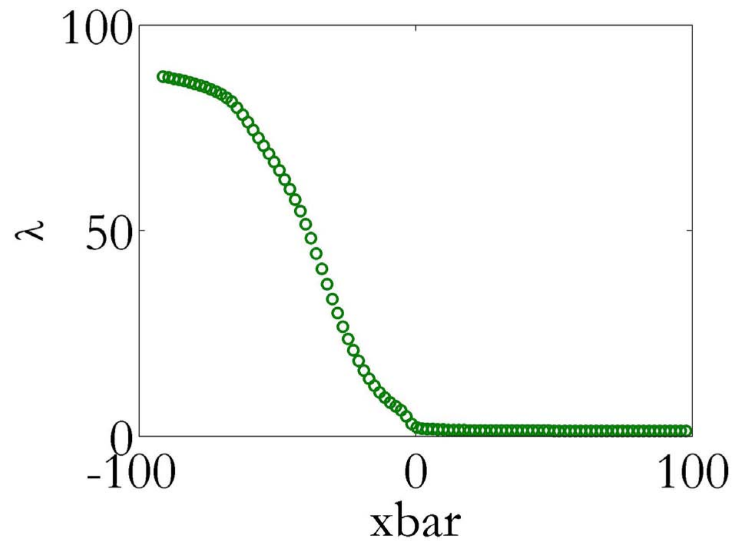
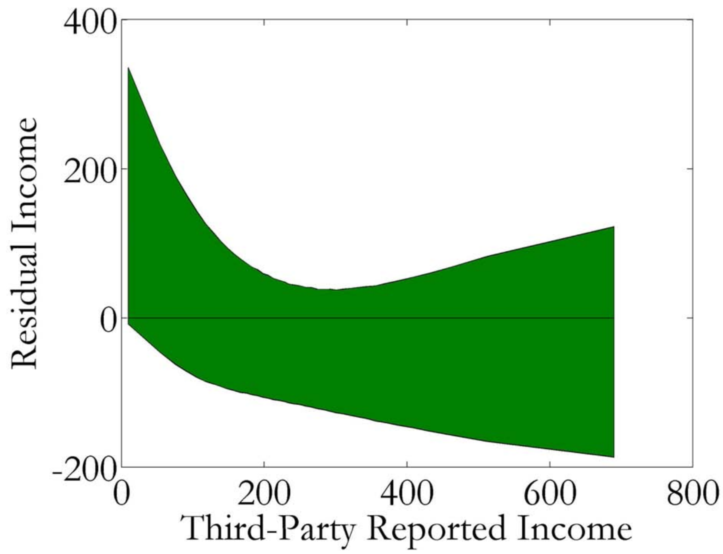


Figure 11. Shadow-Values of the Audit Budget, λ, in an Audit Group.



$\bar{x}$ (xbar) is defined as the lowest value of x that solves $p(x|\cdot) = 0$, i.e. the highest report of dishonest taxpayers.

Figure 12. The Support of u Across Audit Groups.



The conditional densities of $u|z$ are truncated at the 0.5 and 99.5 percent fractiles of the unrestricted conditional distributions. Residual income, u, and third-party reported income, z, are measured in 1,000 DKK.

Tables

Table 1. An Overview of the Danish Tax System, 2006.

Tax	Tax base	Bracket (DKK)*	Rate (pct.)
Labor market tax	Labor income	none	8.0
EITC	Labor income	up to 292,000	2.5
Bottom tax	Personal income + max(capital income, 0)	38,500–	5.5
Middle tax	— // —	265,500–	6.0
Top tax	— // —	318,700–	15.0†
Local taxes	Taxable income (= pers. income + cap. income – deductions)	38,500–	33.3‡
Stock income tax	Stock income	0-44,300; 44,300-	28.0; 43.0

*1 USD ≈ 5.5 DKK.
†The top tax rate may be lowered by the "tax ceiling" that limits the sum of state taxes (bottom, middle and top) and local taxes (excl. church taxes) to 59%. In the average municipality the tax ceiling lowers the top rate by 0.08 percentage points.
‡In the avg. municipality and county incl. optional church tax of on avg. 0.74.

Table 2. Tax Compliance in Denmark, Income Year 2006

	Reported Income	Audit Adjustment			Third-Party Reported Income	Self-Reported Income
		Net adj.	Under-reporting	Over-reporting		
	(1)	(2)	(3)	(4)	(5)	(6)
Net Income	193,277	1,664	1,825	-161	195,618	-2,341
	(1,906)	(480)	(479)	(22)	(1,844)	(584)
Total Tax	63,178	636	695	-59		
	(841)	(246)	(246)	(9)		
Income components						
Earnings	156,127	672	683	-11	155,987	140
	(2,275)	(203)	(203)	(6)	(2,217)	(559)
Personal Income	209,232	1,137	1,195	-58	209,726	-494
	(1,950)	(480)	(479)	(17)	(1,886)	(573)
Capital Income	-10,884	142	198	-56	-11,308	424
	(272)	(27)	(24)	(11)	(266)	(81)
Deductions	-9,264	143	213	-70	-5,605	-3,659
	(178)	(28)	(26)	(11)	(85)	(144)
Stock Income	3,612	239	262	-24	2,797	815
	(546)	(40)	(39)	(10)	(502)	(188)
Self-Employment	103	21	23	-2	8	95
	(60)	(8)	(8)	(1)	(4)	(60)
Foreign Income	479	-18	6	-25	0	479
	(92)	(19)	(4)	(19)	.	(92)

Notes: The sample contains 10,740 taxpayers denoted as employees or recipients of public transfers (unemployed, pensioners, etc.). Due to the stratification strategy employed by SKAT, the sample contains 74.6% "heavy" taxpayers (i.e. with high-complexity tax returns) and 25.4% "light" taxpayers, while the population has 32.6% heavy taxpayers and 67.4% light taxpayers.
Net income is defined as personal income + capital income – deductions + stock income + self-employment income + foreign income. In the Table, deductions are given as a negative amount. Reported income is the sum of third-party reported income and self-reported income. Standard errors of means in parentheses. All estimates are population weighted.
All amounts in DKK (1 USD ≈ 5.5 DKK).

Table 3. Effective Tax Rate Bias, Pooled, Within and Between Audit Groups. Dependent Variable: $\tau - \tau^{\text{eff}}$ (in percentage points).

	(1)	(2)
True Income	0.0057	
	(0.89)	
True Income Residual		0.0234***
		(5.28)
Third-Party Rep. Inc.		-0.0044*
		(-2.52)
Constant	0.7787	3.1476***
	(0.50)	(6.47)
R^2	0.0555	0.4381
Sample Size	905	905

Notes: t-statistics in parentheses calculated using robust, stratified standard errors. Incomes (true, residual and third-party reported) are measured in 1,000 DKKs.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Table 4. Calibration of Q, the Fraction of Honest Taxpayers

	(1) Entire income reported by third-parties		**(2)** Some income not reported by third-parties	
A.	Unweighted Sample Totals	Pop. Weighted Share	Unweighted Sample Totals	Pop. Weighted Share
# underreported	105	.010	796	.050
# correct	5210	.653	4148	.268
# overreported	27	.003	269	.016
Total reports	5342	.666	5213	.334
B.	Unweighted Sample Totals	Pop. Weighted Share of Sub-Sample	Unweighted Sample Totals	Pop. Weighted Share of Sub-Sample
Correct reports	5210	.979 (.0021)	4148	.808 (.0112)
Not underreporting	5237	.984 (.0019)	4417	.852 (.0102)
"Honest" taxpayers*	5264	.989 (.0015)	4686	.895 (.0089)

*Assuming that unintentional underreporting is as frequent as unintentional overreporting. I.e. # honest taxpayers (next to rightmost column) = $269 + 4148 + 269 = 4686$.
Notes: Standard errors in parentheses. All fractions and standard errors are calculated subject to the stratification scheme. Unweighted totals are simply counted in the sample.

Table 5. Comparative Statics – Mean Evasion

	$t = 0.41$			$t = 0.46$			$t = 0.51$		
	Q			Q			Q		
θ	.80	.85	.90	.80	.85	.90	.80	.85	.90
0.81	112.0	105.6	100.1	111.8	105.6	100.1	111.8	105.6	100.1
1.06	105.4	99.9	95.0	105.5	100	95.1	105.4	100.0	95.0
1.31	101.4	96.3	91.7	101.5	96.2	91.6	101.5	96.4	91.8

Note: Index 100 = benchmark.

Table 6. Comparative Statics – Tax Rate Bias Within and Between Audit Groups

Tax Rate Bias Within Audit Groups

	$t = 0.41$			$t = 0.46$			$t = 0.51$		
	Q			Q			Q		
θ	.80	.85	.90	.80	.85	.90	.80	.85	.90
0.81	0.083	0.081	0.081	0.093	0.091	0.090	0.103	0.101	0.100
1.06	0.080	0.079	0.078	0.090	0.089	0.088	0.100	0.099	0.098
1.31	0.078	0.077	0.077	0.088	0.087	0.086	0.097	0.096	0.095

Tax Rate Bias Between Audit Groups

	$t = 0.41$			$t = 0.46$			$t = 0.51$		
	Q			Q			Q		
θ	.80	.85	.90	.80	.85	.90	.80	.85	.90
0.81	-0.012	-0.010	-0.009	-0.013	-0.011	-0.010	-0.014	-0.013	-0.011
1.06	-0.010	-0.009	-0.008	-0.012	-0.010	-0.009	-0.013	-0.011	-0.010
1.31	-0.009	-0.008	-0.007	-0.010	-0.009	-0.008	-0.012	-0.010	-0.009

Note: Tax rate biases are given as the OLS slope coefficients from a regression of the effective tax rate bias on u (within), z (between) and a constant using approx. 100,000 observations of simulated data. All coefficients are significant on virtually any level.