Nonlinear inflation expectations and endogenous fluctuations

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August 2006

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- August, 2006 -

Abstract

The standard new Keynesian monetary policy problem is, in its original presentation, a linear model. As a result, only three possibilities are admissible in terms of long term dynamics: the equilibrium may be a stable node, an unstable node or a saddle point. Fixed point stability (a stable node) is generally guaranteed only under an active monetary policy rule. The benchmark model also considers extremely simple assumptions about expectations (perfect foresight is frequently assumed). In this paper, one inquires how a change in the way inflation expectations are modelled implies a change in monetary policy results when an active Taylor rule is taken. By assuming that inflation expectations are constrained by the evolution of the output gap, we radically modify the implications of policy intervention: endogenous cycles, of various periodicities, and chaotic motion will be observable for reasonable parameter values.

Keywords: Monetary policy, Taylor rule, Inflation expectations, Endogenous business cycles, Nonlinear dynamics and chaos.

JEL classification: E52, E32, C61.

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Acknowledgements: Financial support from the Fundação Ciência e Tecnologia, Lisbon, is grateful acknowledged, under the contract No POCTI/ECO/48628/2002, partially funded by the European Regional Development Fund (ERDF).

1. Introduction

The success of monetary policy intervention in controlling inflation in most of the developed world along the past few decades is the result, among other factors, of the change in the theoretical paradigm followed in macroeconomic science. Since the famous analysis about the inconsistency problem [Kydland and Prescott (1977), Barro and Gordon (1983)], it is widely accepted that the main goal of monetary policy should consist in fighting price instability, rather than worrying about real stabilization. This idea became clearer with the development of the model that has gained the central position in the explanation of central banks behaviour: the new Keynesian monetary policy problem [see, among many others, Goodfriend and King (1997), Clarida, Gali and Gertler (1999) and Woodford (2003)].

The benchmark new Keynesian model has been built over the staggered pricesetting analysis of Calvo (1983), which has allowed recovering the Phillips curve relation. It is admissible to establish a relation between the contemporaneous values of the inflation rate and of the output gap, through a parameter that reflects the degree of price stickiness; when this relation is augmented by a term that relates the present period's inflation with expectations about future inflation, we can establish the central piece of the new monetary policy paradigm, which is the 'new Keynesian Phillips curve' [this denomination was initially proposed by Roberts (1995)].

Alongside with the aggregate supply relation that the Phillips curve defines, another state constraint is essential to describe the short run environment in which monetary authorities are compelled to take decisions; this is an IS equation that describes how the real economic activity responds to changes in the real interest rate.

With the knowledge of the previous two state equations, the central bank has a problem to solve, which is to maintain price stability and, if possible, to guarantee some positive difference between effective output and its potential level (if this does not hurt the inflation objective). The most immediate solution for this problem would be to consider an optimal control setup, under which the central bank minimizes the distance between the observed inflation rate and output gap relatively to the target values it defines. The constraints of this intertemporal problem are the Phillips curve and the IS equations. The control variable is the nominal interest rate, i.e., the monetary authority chooses the time path of the interest rate that optimizes its utility function in time.

If one considers the benchmark version of the optimizing model, a problem arises: the optimal interest rate path does not correspond to a stable path, and therefore the intended long term optimal values of inflation and output are not accomplished. In this sense, the stability of the equilibrium becomes a central issue in the way monetary policy is conducted. If optimal policy is not stable, it is necessary to find a less than optimal result that guarantees stability. This is generally assured by assuming an ad-hoc interest rate rule instead of following the optimal path.

The influential work of Taylor (1993) and the huge amount of literature that it has originated seems to give a satisfactory answer to the stability concern [see, among many others, McCallum and Nelson (1999), Benhabib, Schmitt-Grohé and Uribe (2001), Svensson and Woodford (2003), Benigno and Woodford (2005)]. It has become widely accepted that an active Taylor rule (i.e., a monetary policy rule under which in response to an increase in inflation the central bank raises the nominal interest rate by more than the increase in inflation), has stabilizing effects. Intuitively, this appears correct: inflationary pressures are fought by a monetary policy that triggers an increase in the real interest rate, which should have the effect of slowing down aggregate demand and, therefore, sustain the rise in the general price level.

The described monetary policy problem is essentially linear. Replacing in the IS curve the nominal interest rate by a rule in which this rate is dependent on inflation (and also on the output gap), the reduced form of the problem will be a system of two difference equations where, under perfect foresight, the output gap and the inflation rate depend linearly on previous period values of these two variables (and, also linearly, on eventual stochastic shocks on demand and supply). When changing the linear form of the model the stability result can give place to endogenous cycles, which essentially mean that a public policy oriented to attain price stability may not achieve a full stability result, but it can produce fluctuations, that will be more or less predictable depending on the periodicity of those fluctuations.

Concerning the introduction of nonlinearities, authors follow essentially two paths:

(i) When assuming the optimal problem, the original framework considers a quadratic objective function. Various authors, like Cukierman (2000), Ruge-Murcia (2002, 2004), Nobay and Peel (2003), Dolado, Pedrero and Ruge-Murcia (2004) and Surico (2004), claim that a symmetric objective function does not represent properly the true policy problem (authorities do not perceive as equally important positive and negative deviations from the target values of inflation and output gap). Thus, in this way nonlinearities and the possibility of long term endogenous fluctuations arise in a way that is consistent with empirical evidence.

(ii) Also consistent with empirical evidence is the fact that the Phillips curve can hardly be modelled through a linear relation. Clark, Laxton and Rose (1996), Debelle and Laxton (1997), Schalling (1999), Tambakis (1999) and Akerlof, Dickens and Perry (2001), among others, present evidence and argue against a linear relation between the inflation rate and the output gap, in the short-run. Gomes, Mendes, Mendes and Sousa Ramos (2006*a*) prove that for a specific functional form of a non linear Phillips curve, endogenous cycles are found, and this corresponds mainly to cases in which no identifiable periodicity is encountered (i.e., when assuming a non linear Phillips curve chaotic motion can be generated for values of parameters that do not depart significantly from empirical data).

In this paper, we consider the non optimal monetary policy model (i.e., we assume a Taylor rule) and linear Phillips and IS equations that are linear in the relation between contemporaneous values. The nonlinearity is introduced by departing from the perfect foresight assumption regarding inflation expectations. This is also a subject debated in the literature, for instance by Jensen (2005), who considers that policy affects expectations about future policy. In Branch and McGough (2006), Gomes (2006) and Gomes, Mendes, Mendes and Sousa Ramos (2006*b*), inflation expectations are modified by considering heterogeneous agents, who predict future inflation in different ways; under bounded rationality (i.e., under a discrete choice mechanism for the switching between expectation rules) chaotic motion is also identified in this case.

In the present case, we depart from perfect foresight by assuming that agents will form expectations about inflation having in consideration the output gap. The rule is as follows: when the output gap is equal to its target value, as defined by the central bank and perceived by private agents, the perfect foresight will hold; if the output gap rises above that benchmark value, then the expected inflation will also rise above the perfect foresight value; if the output gap falls below the target, agents will predict an inflation value below the perfect foresight value; finally, for strong recessions (output gap clearly negative), agents expect inflation to rise faster (that is, strong recessions will be a symptom of an economy where institutions do not work, and therefore the control of price stability does not function properly).

This simple assumption over the original monetary policy problem imposes relevant changes on the dynamic behaviour of variables, namely chaos and cycles of various periodicities are obtained. Therefore, one concludes that monetary policy (under an active interest rate rule) does not yield necessarily a fixed point result, but cycles of several periodicities are observable, when considering parameter values that intuitively are reasonable.

The remainder of the paper is organized as follows. Section 2 discusses the intuition behind the inflation expectations rule; section 3 presents the analytical structure of the model; section 4 characterizes global dynamics; in section 5, growth issues are addressed; and, finally, section 6 concludes.

2. Inflation Expectations

The simplest approach to modelling expectations consists in assuming perfect foresight. Under perfect foresight, agents have a complete knowledge about the economy. They know how every other agent will act and how monetary authorities will conduct their policy. In turn, authorities should also understand without doubts the decisions that the private economy take in every moment of time, being as well able to predict and anticipate the decisions of all economic agents. This implies a world where agents' choices become the best response to the choices of third parties.

This approach to expectations is too narrow, implying full information and full efficiency in the use of information. This is why macroeconomics has become increasingly concerned with alternative methods of modelling expectations [learning mechanisms have been adopted to more realistically describe how agents predict future outcomes; see Evans and Honkapohja (2001)]. In the present analysis we ignore any kind of learning mechanism and stick with the perfect foresight assumption that gives place to the fixed point outcome in the presence of an active monetary policy rule; over perfect foresight, one takes an additional assumption that reflects how the private economy responds, in terms of the way it perceives price evolution, to fluctuations in output.

We assume that output gap expectations are solely the outcome of a perfect foresight evaluation: $E_t x_{t+1} = x_{t+1}$. The output gap variable is defined as the difference between effective output and potential output (in logs), that is, $x_t = \ln y_t - \ln \hat{y}_t$. Relatively to the inflation expectations, the perfect foresight prediction is adjusted by a term that translates the way individuals think the difference between effective and potential output will affect the rise in prices. Thus, we consider $E_t \pi_{t+1} = \pi_{t+1} \cdot \xi(x_t)$. Function $\xi(x_t)$ must be such that when the output gap is equal to some predefined value (that here we consider to be the target value of the central bank for this variable:

x^*), the value of this function is 1, that is, perfect foresight holds. When $x_t > x^*$, the output gap has assumed a value above 'normal', and thus agents will suspect that only a rise in prices will be able to maintain such abnormally high output gap, and hence they will expect prices to rise above the perfect foresight value. If $x_t < x^*$, private agents will perceive a slowdown of the economic activity, and therefore they introduce a penalty term in their predictions, which means that the expected inflation value will remain below the benchmark value.

Finally, when the output gap becomes extremely low relatively to the corresponding target value, this will be understood as a serious problem of economic malfunctioning, probably associated to an inability of the institutions to fulfil their regulatory role, and therefore very low levels (in principle, negative) of the output gap will be understood as eventually producing a faster rise in prices because the monetary authority becomes unable of controlling the production of money and the interest rates.

Figure 1 presents the shape of function $\xi(x_t)$, when this obeys to the characteristics described above. Parameter $\sigma>0$ is defined in order to present the location of the point in which the function reaches a minimum and therefore the expected inflation is the lowest relatively to the perfect foresight value. Note, as stated, that three areas are identifiable: high inflation is expected in periods of expansion or strong recession; moderate recession implies low expected inflation.

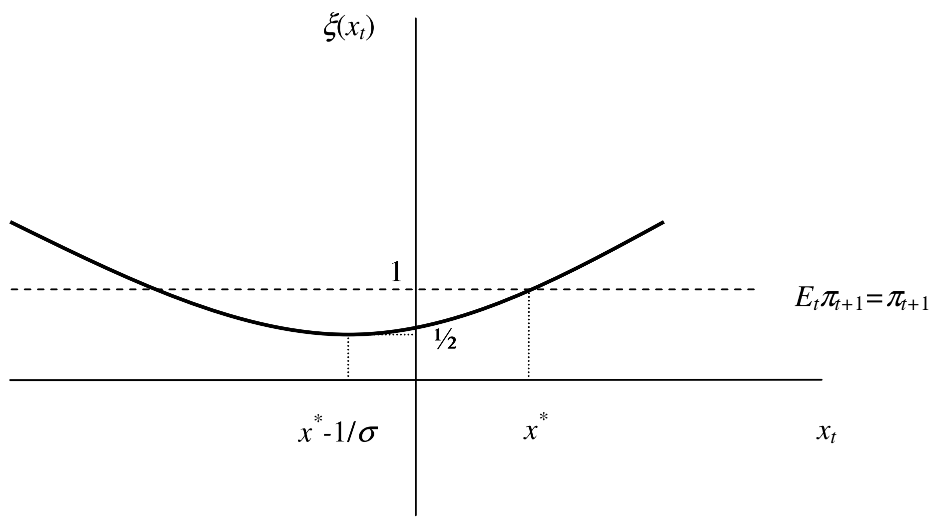

Figure 1 - Function $\xi(x_t)$.

The function in figure 1 can be translated analytically as follows: $\xi(x_t) = 1 + \sigma \cdot (x_t - x^*) + \frac{\sigma^2}{2} \cdot (x_t - x^*)^2$.

Synthesizing, over the original new Keynesian monetary model one introduces only one modification: we bend the line $E_t\pi_{t+1}=\pi_{t+1}$ in order to illustrate how individuals and firms react (in terms of price evolution predictions) to the output gap departures from a reference value.

Next section incorporates this assumption in the monetary policy framework.

3. The Monetary Policy Model

In what follows we describe the main features of the conventional new Keynesian monetary model. The state constraints are, on the demand side, a dynamic IS equation, and, as an aggregate supply relation, a new Keynesian Phillips curve. The first relates the output gap to the expected real interest rate,

$$x_t = -\varphi \cdot (i_t - E_t\pi_{t+1}) + E_t x_{t+1} + g_t, \ x_0 \text{ given.} \qquad (1)$$

Parameter $\varphi>0$ is the output gap - interest rate elasticity and variable i_t defines the nominal interest rate. Variable g_t corresponds to a demand stochastic component and it is defined through an autoregressive Markov process, $g_t = \mu g_{t-1} + \hat{g}_t, 0 \le \mu \le 1, \hat{g}_t \sim iid(0, \sigma_g^2)$. Subsequently, we ignore the stochastic component of the equation in order to highlight the presence of endogenous fluctuations.

On the supply side, the Phillips curve relates contemporaneous inflation to the output gap and to the next period inflation expectations,

$$\pi_t = \lambda x_t + \beta \cdot E_t\pi_{t+1} + u_t, \ \pi_0 \text{ given.} \qquad (2)$$

Parameter $\lambda \in (0,1)$ defines the degree of price flexibility / stickiness, that is, it is an inflation–output elasticity. The higher the value of this parameter the lower will be the degree of price stickiness or rigidity. Parameter $\beta<1$ is an intertemporal discount factor, and variable u_t translates a supply stochastic component, that reflects possible cost push shocks. As in the demand case, an autoregressive process is assumed:

$u_t = \rho u_{t-1} + \hat{u}_t, 0 \leq \rho \leq 1, \hat{u}_t \sim iid(0, \sigma_u^2)$; also as in the demand case, this term is ignored under the discussion of endogenous fluctuations.

To complete the model, one takes a conventional Taylor rule, which is given by the following expression [a similar Taylor rule can be found in Clarida, Gali and Gertler (1999)],

$$i_i = i^* + \gamma_\pi \cdot (E_t \pi_{t+1} - \pi^*) + \gamma_x \cdot x_t \tag{3}$$

In expression (3), i^* defines the equilibrium nominal interest rate, π^* is the inflation target that the central bank sets (low, but positive in order to guarantee relative price variations without the need of nominal decreasing of prices and wages), and γ_π and γ_x are the policy parameters that reflect how the central bank reacts in terms of interest rate changes, when economic conditions provoke changes in inflation and effective output.

As stated in the introduction, an active interest rate rule is, normally, stabilizing, meaning that stability is attained when there is an interest rate response to inflation changes that are stronger than a one-to-one change; this implies imposing the constraint $\gamma_\pi > 1$.

Replacing the Taylor rule (3) in the IS expression (1), and assuming perfect foresight for the output gap, we get the following relation between output gap and inflation rate, regardless from the expectations about inflation,

$$x_{t+1} = \varphi \cdot (i^* - \gamma_\pi \pi^*) + [1 + \varphi \gamma_x - (\gamma_\pi - 1) \cdot \lambda / \beta] \cdot x_t + [(\gamma_\pi - 1)/\beta] \cdot \pi_t \tag{4}$$

The Phillips curve can be rewritten, having in consideration the way we have defined inflation expectations in the previous section,

$$\pi_{t+1} = (1/\beta) \cdot [\pi_t / \xi(x_t)] - (\lambda / \beta) \cdot [x_t / \xi(x_t)] \tag{5}$$

The system one wants to analyze is the difference equations system (4)-(5). This is the conventional problem for σ=0, and it departs from this case as we rise the value of the parameter (the higher the value of σ, the more the inflation expectations rule 'bends' relatively to the perfect foresight case). Except in the known particular case, the analysis of the steady state and of local dynamics becomes difficult. Solving for the steady state one would obtain multiple equilibria (a third order polynomial would be obtained and thus three equilibrium points would arise); nevertheless, the combinations of parameters that define the steady state points are cumbersome and it becomes difficult to extract some meaningful information from them. Without the steady state values, local dynamic analysis is not feasible as well. The next section concentrates on a global analysis of the underlying dynamics, which is essentially a numerical and graphical analysis.

4. Global Dynamics

System (4)-(5) involves a linear and a non linear equation. As we will understand below, the presence of this nonlinear equation opens the possibility for the finding of strange dynamics defining the long term behaviour of endogenous variables. Otherwise stated, the following parameter values are considered: β=0.96; γ_x=0.5; γ_π=2.2; σ=25; φ=0.01; λ=0.75; π^*=0.02; x^*=0.03; i^*=0.01. Note that for reasonable initial values of variables inflation and output gap, we find no limit for the basin of attraction, and therefore any value economically meaningful can be considered for the matter at hand.

We begin by presenting some bifurcation diagrams.[1] Figures 2 and 3 display the long term possible outcomes of the output gap and the inflation rate for different values of the parameter that defines the nature of the monetary policy (γ_π<1 respects to a passive monetary policy and γ_π>1 to an active policy). The most striking and important evidence in this figures is that instability prevails for a passive interest rate rule, that is, when the central bank responds to the rise of inflation with a less than one-to-one variation in the nominal interest rate. Instability is characterized in this case by a divergence of the output gap to infinity and of the inflation rate to zero. When the policy parameter assumes a value higher than one, the modified nonlinear expectations model implies the presence of cycles of multiple orders until an extremely high value of the parameter is attained.

Basically, we note that some regions in figures 2 and 3 define cases in which low periodicity cycles exist, while in other areas of the graphics it is evident the presence of chaos: the variable can assume practically any value on a given interval.

We will highlight further the presence of endogenous fluctuations in the figures that follow; nevertheless just by looking to the bifurcation diagrams (that are drawn for

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[1] The various figures presented in this section are drawn using IDMC software (interactive Dynamical Model Calculator). This is a free software program available at www.dss.uniud.it/nonlinear, and copyright of Marji Lines and Alfredo Medio.

the 1,000 observations after the first 1,000 transient ones) it is evident that a-periodicity arises.

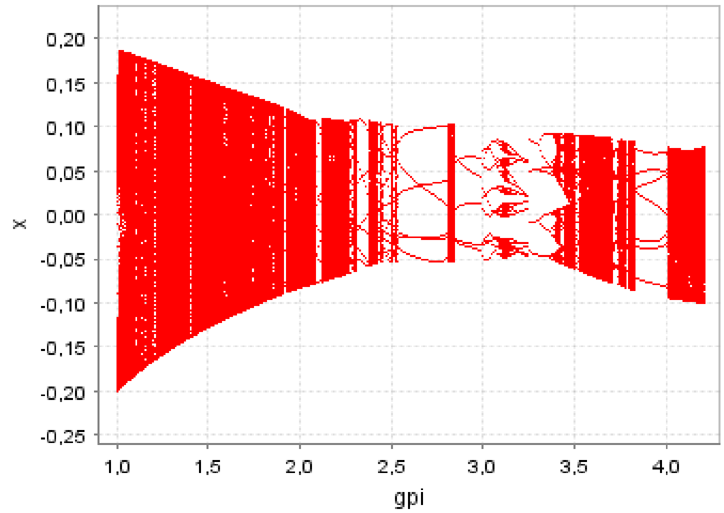

Figure 2 – Bifurcation Diagram (γ_π,x_t).

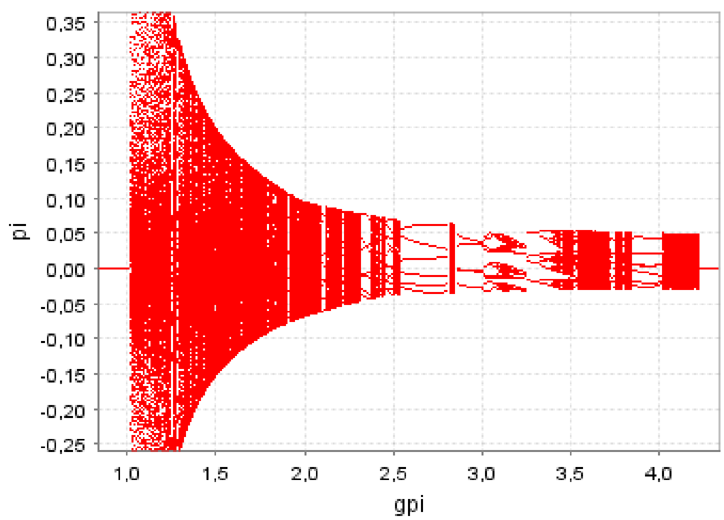

Figure 3 – Bifurcation Diagram (γ_π,π_t).

To explore further the dynamics of the modified expectations monetary policy problem, other bifurcation diagrams are drawn (figures 4 to 7). Both for the parameter attached to the inflation expectation rule and for the price stickiness parameter it is clear the presence of cycles and chaotic motion. Note, more precisely, that the degree of chaoticity is higher for a low value of σ, and that for λ chaos is present for almost every possible value of this price stickiness parameter.

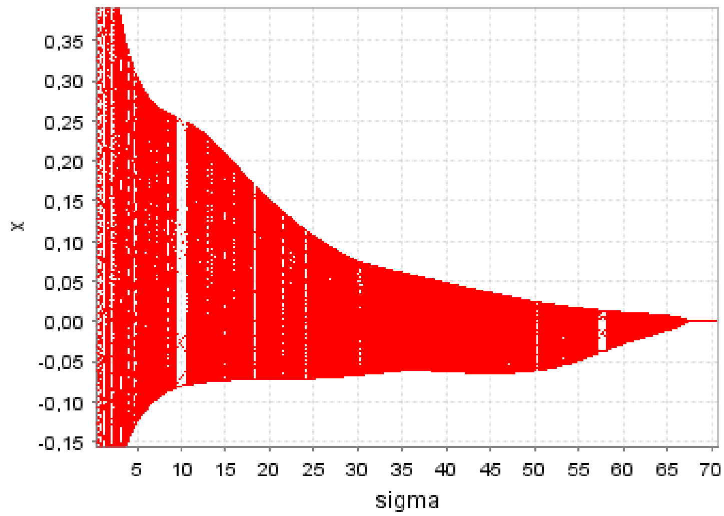

Figure 4 – Bifurcation Diagram (σ,x_t).

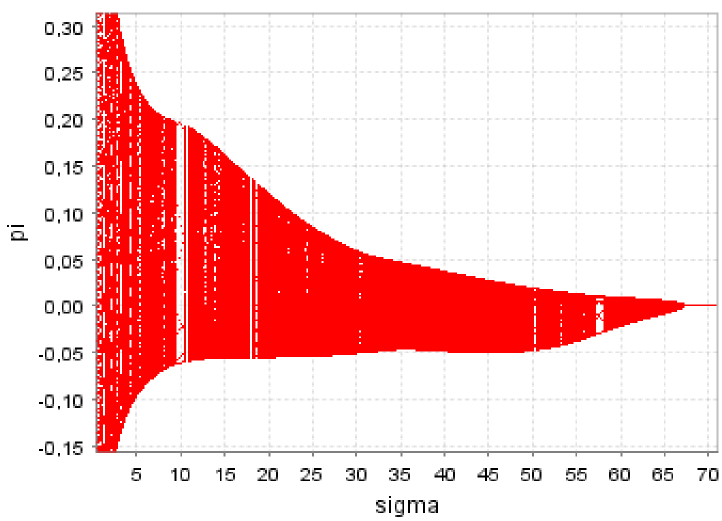

Figure 5 – Bifurcation Diagram (σ,π_t).

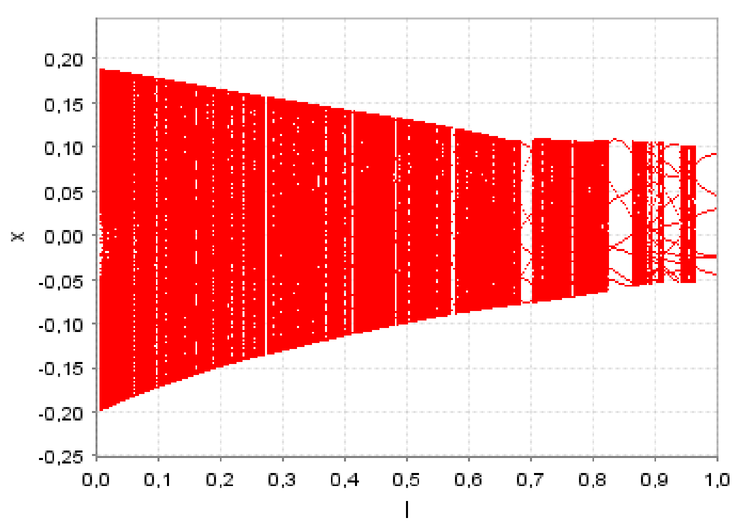

Figure 6 – Bifurcation Diagram (λ,x_t).

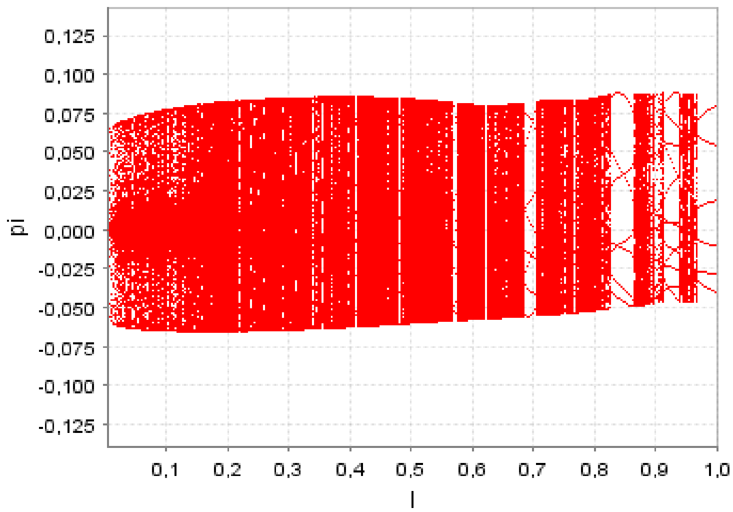

Figure 7 – Bifurcation Diagram (λ, π_t).

One can explore as well the presence of cycles of different orders in the space of parameters. With figures 8 to 10, we are able to observe that all sorts of periodicities are obtainable for different values of parameters. Regions in white contain the possibility of chaotic motion. These figures reveal that the dynamic system is deeply sensitive to small changes in almost all parameter values. Recall that instability is ruled out when an active interest rate rule is assumed, but for high values of γ_π (above 4 – 4.5) cyclical motion arises (note that the fixed point stability result that characterizes the linear model only arises for extremely high levels of σ).

In what concerns the relation between the interest elasticity and the price stickiness parameter, figure 9 reveals that although we have chosen to work with a low value of the interest elasticity, the same kind of dynamic behaviour is observable for higher values of this parameter, while changes in λ tend to modify the periodicity of cycles but they continue to be present. Finally, the relation between the discount factor and the target value for the output gap is also capable of generating cycles of multiple orders. The figure is presented for a discount factor higher than 0.5, because for lower values it begins to appear a wide region of instability; relatively to the output gap target, this apparently reveals a symmetric behaviour for values above and below 0.04 (more or less).

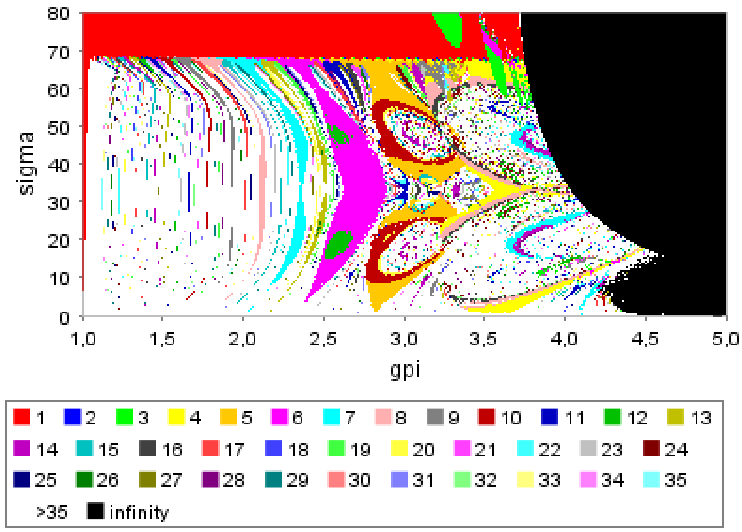

Figure 8 – Cycles in the space of parameters (γ_π, σ).

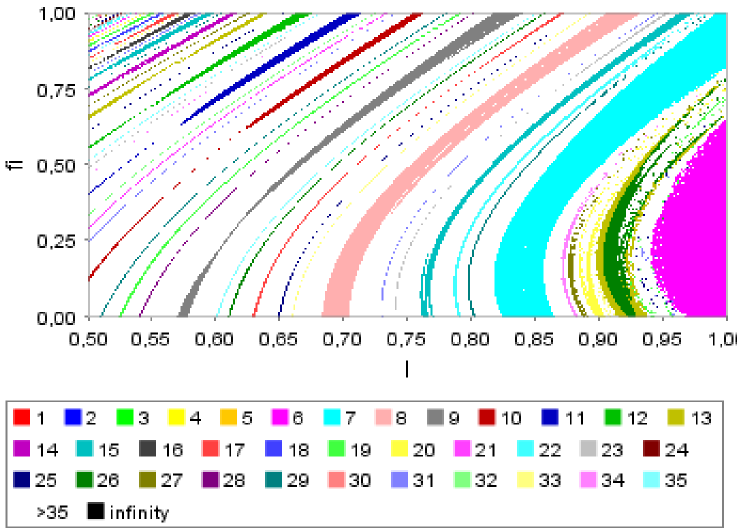

Figure 9 – Cycles in the space of parameters (λ, φ).

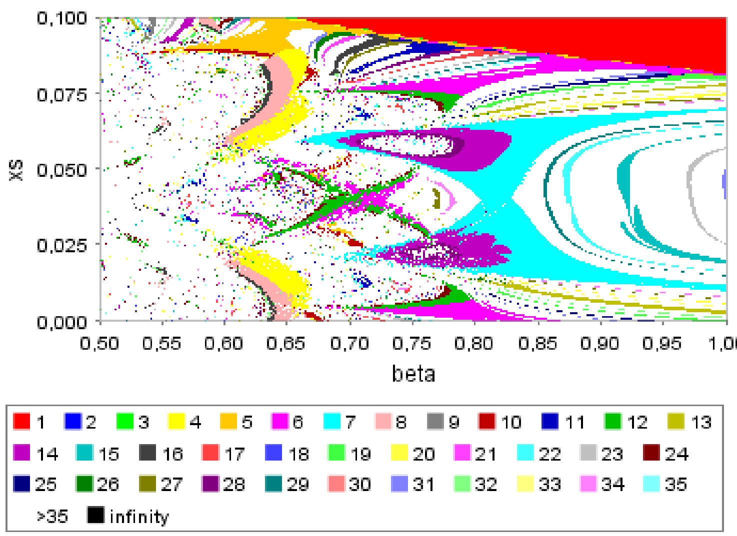

Figure 10 – Cycles in the space of parameters (β, x^*).

We now take the set of parameter values defined in the beginning of this section to present some attractors, i.e., the long term relation between our two endogenous variables. Figure 11 considers precisely the initial set of values. To understand how the dynamics can be modified, we vary some of the parameter values to present the graphics in figures 12 to 15. All the attractors are drawn with 100,000 iterations after excluding the first 1,000 transients.

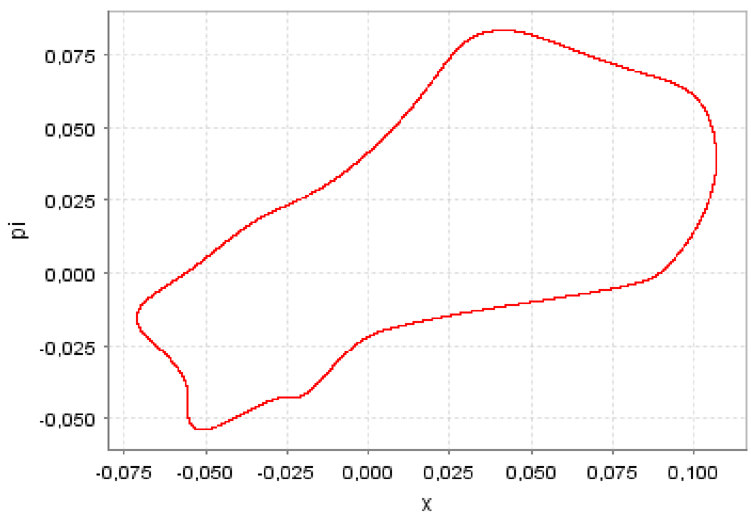

Figure 11 – Attractor (x_t,π_t).

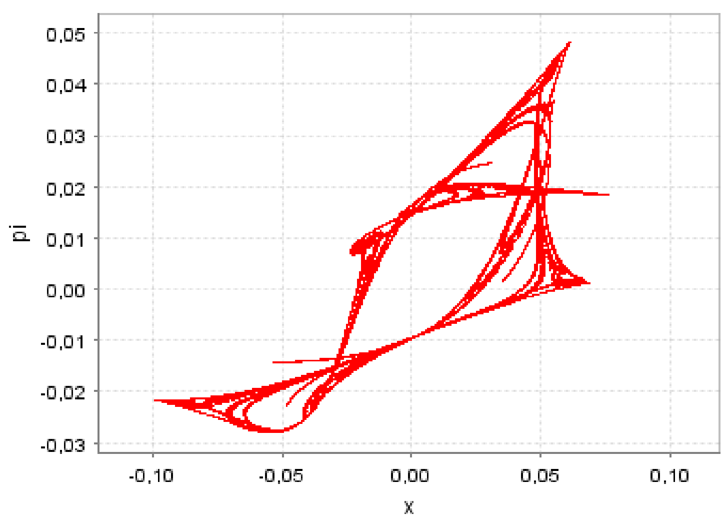

Figure 12 – Attractor (x_t,π_t), γ_π=4.2.

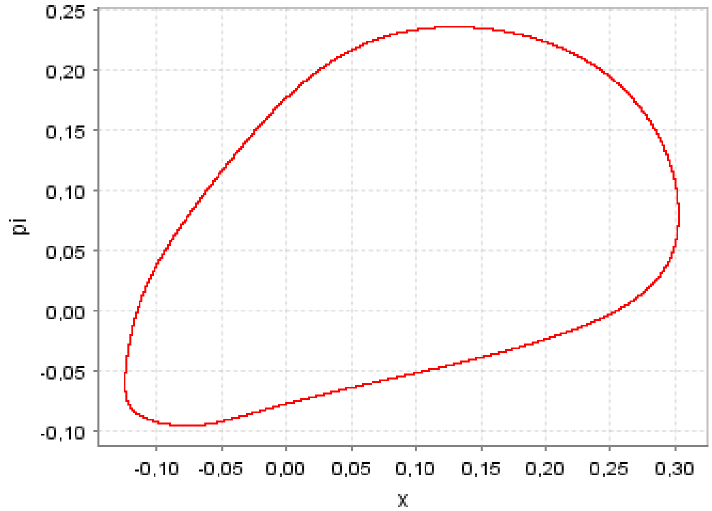

Figure 13 – Attractor (x_t,π_t), σ=5.

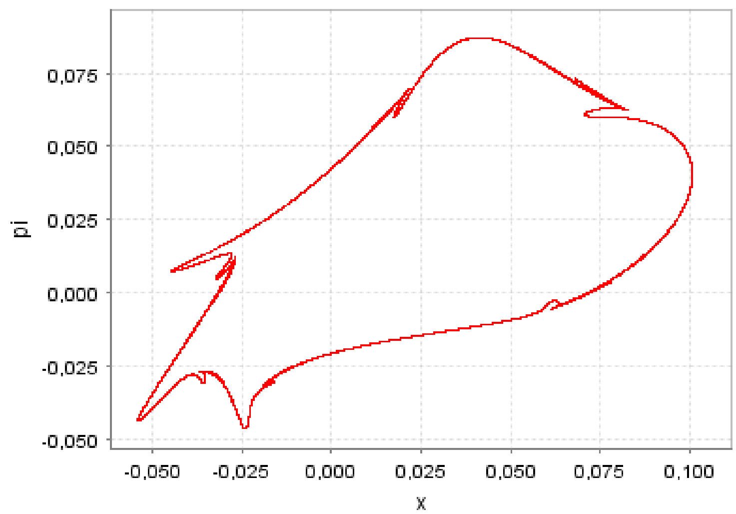

Figure 14 – Attractor (x_t,π_t), λ=0.96.

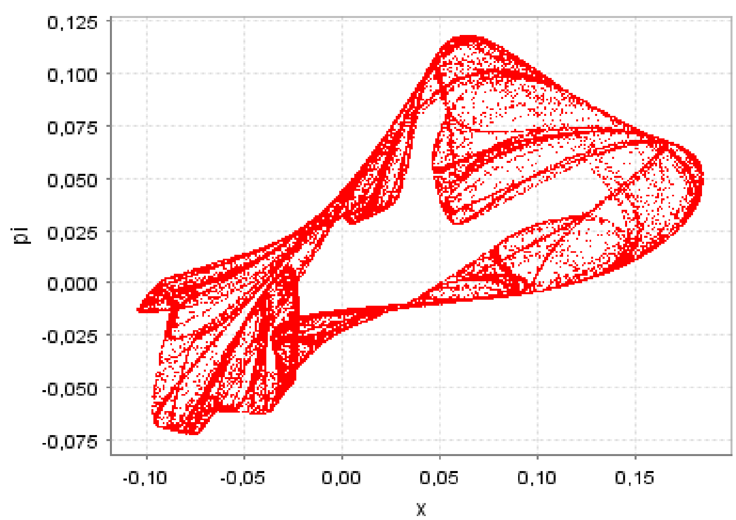

Figure 15 – Attractor (x_t,π_t), β=0.7.

Finally, two pairs of long term time series are presented, in order to illustrate the existence of endogenous cycles. Note that both variables can assume positive and negative values, that is, periods of inflation and deflation are observed, as well as periods when the effective output is above or below the potential level (figures 16 to 19).

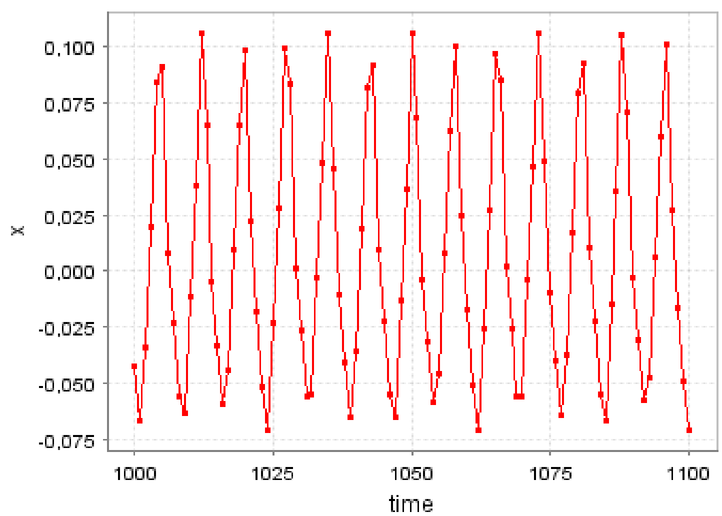

Figure 16 – Time series x_t.

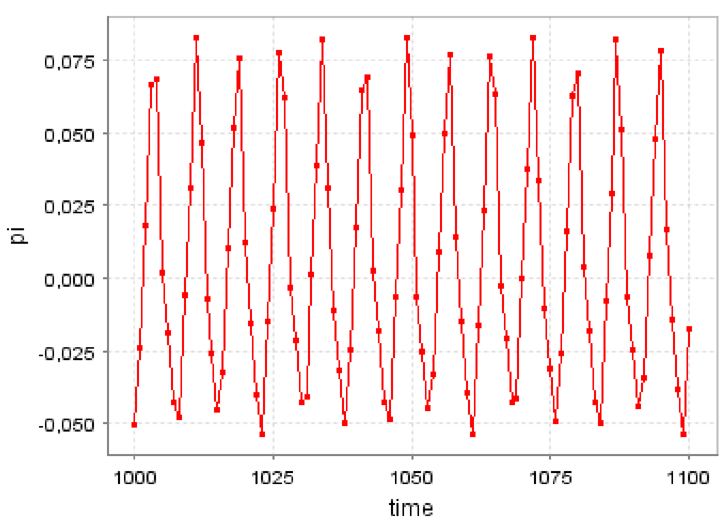

Figure 17 –Time series π_t.

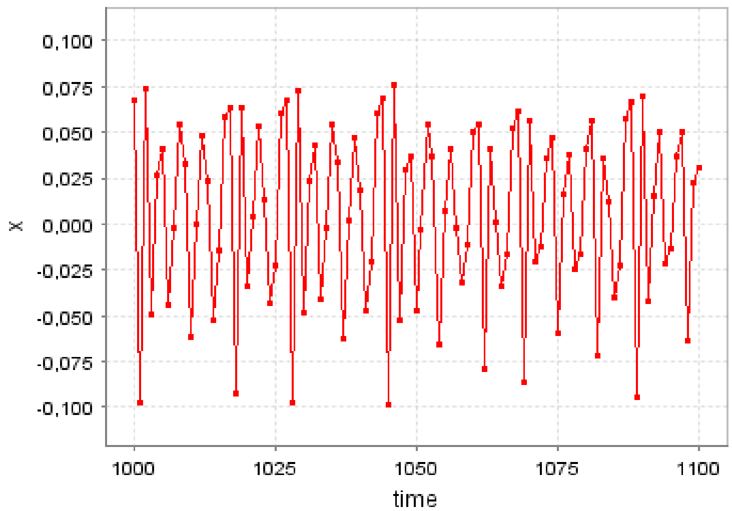

Figure 18 – Time series x_t, γ_π=4.2.

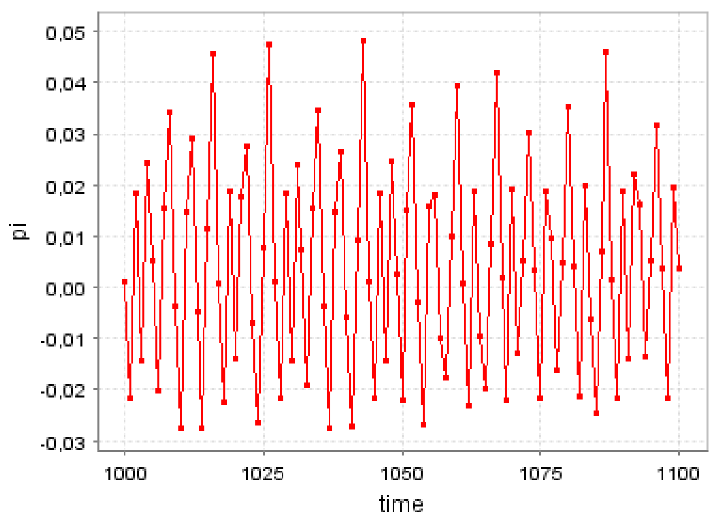

Figure 19 – Time series π_t, γ_π=4.2.

5. Growth Implications

Monetary policy analysis is undertaken through two state equations that define short-run economic conditions. These can be integrated with a long term growth analysis. Growth models are generally developed under a competitive framework and they are specially designed to analyze the trend of growth, i.e., they are built in order to characterize potential output motion. Consider a capital accumulation equation

$$k_{t+1} = Ak_t^{\alpha} - c_t + (1-\delta)\cdot k_t, \quad k_0 \text{ given} \tag{6}$$

In this equation, k_t and c_t represent, respectively, per capita physical capital and consumption. Parameter A>0 is a technological index, 0<α<1 and δ defines a positive depreciation rate. From growth literature it is well known that, given a representative agent that maximizes an intertemporal flow of consumption utility functions, the growth problem is reduced to a two equations system describing the motion in time of the consumption and the capital variable. Then, the long term behaviour of output can be withdrawn from the production function, once we know how the rule of capital accumulation and the optimization behaviour of the representative consumer imply a given path for the capital stock.

Therefore, we can use the growth problem to get to the potential level of output, $\hat{y}_t = Ak_t^{\alpha}$. In the real world, we are not concerned with how much it is possible to produce, but how much it is effectively produced. Given the proposed notion of output gap, effective output comes: $y_t = \hat{y}_t \cdot e^{x_t}$. In terms of growth rates, $\frac{y_{t+1}}{y_t} - 1 = \frac{\hat{y}_{t+1} \cdot e^{x_{t+1}}}{\hat{y}_t \cdot e^{x_t}} - 1$.

If the competitive growth model is stable, and neoclassical features define it (i.e., output does not grow in the steady state due to endogenous forces) this means that in the long run we find a fixed point stable result for the potential output, and thus $\hat{y}_{t+1} = \hat{y}_t$; the growth rate of effective output becomes, then, $\frac{y_{t+1}}{y_t} - 1 = \frac{e^{x_{t+1}}}{e^{x_t}} - 1$, that is, the growth rate of effective output depends solely on the growth rate of the output gap. If, instead of neoclassical growth, we take the assumption that the growth model is endogenous (a positive constant growth rate defines the steady state), then potential output grows at a given rate γ, meaning that $\hat{y}_{t+1} = (1+\gamma) \cdot \hat{y}_t$. Also in the case of endogenous growth, one can present effective output growth as a function of the output gap, as follows: $\frac{y_{t+1}}{y_t} - 1 = \frac{(1+\gamma) \cdot e^{x_{t+1}}}{e^{x_t}} - 1$.

The previous reasoning intends to conciliate growth analysis, that under market clearing conditions clearly aims at explaining growth tendencies, with the short run analysis provided by the monetary policy problem: because nominal and real economic conditions are jointly determined in the short term, and since expectations are not necessarily the simple result of a perfect foresight evaluation, then fluctuations can be explained in this policy framework and later added to the growth setup. In this way, we strongly emphasize the idea that business cycles are a short run phenomenon that influences the shape of effective growth in time.

To finish, we present a simple graphical example, taking the benchmark numerical values of the previous section. For those values, one has concluded that endogenous irregular cycles were present. Now, consider that the potential growth rate (derived from a growth / capital accumulation setup) is, e.g., 3% (γ=0.03). Using the definition of effective output derived above, and taking the time series of the output gap in figure 16, we display in figure 20 the long term time series of the effective output variable: the variable gravitates around the potential value, but since the output gap is not constant, then effective output is subject to endogenous fluctuations.

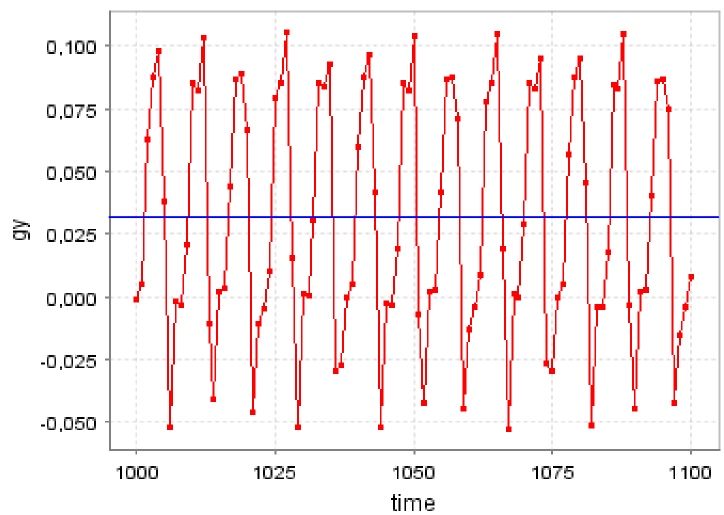

Figure 20 – Time series of the growth rate of the effective output (γ=0.03).

6. Conclusions

The new Keynesian monetary policy model has two fundamental features: it establishes aggregate demand and aggregate supply relations that are dynamic and subject to the influence of expectations about next period values for real and nominal variables (these relations are derived from well structured micro foundations); and it introduces the relevant role of authorities in choosing the path of the nominal interest rate that best serves the purpose of guaranteeing price stability. It is important to keep in mind that price stability is not necessarily guaranteed by solving an intertemporal optimal control problem, because this can guarantee a steady state that is close to the target defined by the central bank, but that can eventually never be reached given the stability properties of the underlying difference equations system.

Therefore, the model under consideration constitutes not only a good description of private economic behaviour in the short run, but it is also a relevant tool for policy analysis and intervention. The model can be presented in multiple forms, and slightly modified in many ways. Recent literature has proved that slight changes in the benchmark presentation can lead to significant changes in the underlying dynamics, what modifies as well the policy implications one is able to withdraw. In the present paper we have tried to include an additional change relatively to the original model – the idea was essentially to assume that agents do not forecast inflation in a perfect way; even if they possess all the necessary information to decide, they will adjust expectations about inflation to the moment of the business cycle we are in: periods of expansion are understood as periods where inflation will rise faster, while moderate periods of recession imply a feeling that inflation will fall.

This change in the model's structure introduces significant changes into the dynamics. The model gains a non linear character, and as a result we find cycles and chaos for different values of parameters, that replace the unique fixed point result that the original model is able to reproduce. The implications are many: first, monetary policy, that is, the choice of a nominal interest rate rule, no longer gives a long term absolutely predictable outcome; second, price stability will depend on the degree in which private agents are influenced by output gap changes when formulating expectations; third, it is the short run relation between nominal and real variables that induces cycles and not the process of capital accumulation, from which one can only withdraw a constant trend of growth.

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