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# The Dynamics of Television Advertising with Boundedly Rational Consumers

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## Abstract

The paper adapts a static model of television advertising into a dynamic scenario. In its original form, the model consists on a profit maximization problem of a television network working in a competitive environment. The network sells commercial time to advertisers and tries to minimize the effects of viewers' aversion to ads. Viewers are assumed heterogeneous with regard to the preferences over the types of products companies sell through ad time. Into this framework we introduce an intertemporal rule reflecting the possible preference changes of consumers (these are boundedly rational and their utility for different types of products varies over time). The introduction of the intertemporal rule originates interesting dynamic results, namely in what concerns the evolution over time of crucial variables like the total time of broadcasting that networks allocate to advertising or the amount of revenues that satisfies the profit maximization condition. As in the original model, attention will be given to the possibility, that cable television allows, of ad addressability.

Keywords: Television advertising, Networks' profit maximization, Heterogeneous viewers, Ad addressability, Bounded rationality, Nonlinear dynamics.

JEL classification: L82, M37, C61

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## 1. Introduction

The relevance of advertising revenues in the television network business justifies the need for a theoretical framework aimed at explaining the connection between networks' contents and the relationship that media establish between consumers with distinct preferences and firms who wish to advertise the products they sell.

Various models explore such connections. Some of the most prominent include the path breaking work by Steiner (1952), in his specific case regarding radio broadcasting, and recent contributions like Goettler (1999), McDowell and Sunderland (2000), Esteban, Gil and Hernandez (2001), Kieshnick, McCullough and Wildman (2001), Byzalov and Shachar (2004), Gabszewicz, Laussel and Sonnac (2004), Peitz and Valletti (2004) and Anderson and Gabszewicz (2005), among others. This theoretical work follows on the footsteps of important empirical contributions, which address some illustrative case studies: Smith (1999) analyzes advertising profitability for other media besides television and radio broadcasting (in particular, he addresses the market for non daily newspapers); Sonnac (2000) looks as well to advertising in the press, studying in detail the attitude of consumers towards ads; Fare, Grosskopf, Seldon and Tremblay (2004), and Reid, King, Martin and Soh (2005) search for evidence on the effectiveness of media advertising and the degree in which television is in this respect a substitute relatively to other media.

The present paper explores further and illustrates numerically and graphically one of the existing advertising models, namely the Kim and Wildman (2006) model, henceforth KW. In its basic structure, this theoretical framework explores the profit maximization behaviour of a representative television network. Assuming that costs are constant (i.e., in this case, independent from advertising time, which is the endogenous variable of the model), the profit maximization becomes a revenue maximization setup, where revenues are essentially determined by the decisions of the network concerning the time of broadcasting that is destined to be sold to advertisers.

Two conflicting forces are taken in consideration in this problem: the revenues generated by selling advertising time and the loss of viewers that occurs when ad time is raised, given that viewers are considered as disliking commercials. The loss of viewers is interpreted in two ways: first, when ad time is augmented, individuals simply change network, and this has a direct impact on the amount of ad time networks can sell to advertisers; second, if one considers a subscription television service (cable TV), as the network rises ad time it will lose paying clients.

The main feature of the KW model is the fact that it considers heterogeneous television viewers. These are not necessarily heterogeneous in what concerns their tastes about TV programs, but they distinguish themselves in what concerns the type of commodities they generally acquire. Therefore, one will encounter as well various types of firms that advertise their goods trying to target the viewers that reveal a preference for such goods. The heterogeneity assumption has important consequences over the solution of the maximization problem, namely regarding the optimal profit level and the time of broadcasting time that, in the optimum, is allocated to commercials.

This theoretical framework gains a new appeal when we acknowledge the possibilities that the digital cable television supply. In particular, new technologies are now achieving two relevant results: they can help in collecting information about consumers' tastes and preferences and they are beginning to turn it possible to target advertising, that is, the same advertising time can be used to send different commercial information to different types of consumers. The combination of these two factors implies the possibility of selling the same advertising time to different types of producers, what has unambiguous advantages to the network's profit objective. The possibility of ad addressability has also effects over the total commercial time in a competitive market environment, but these effects are not clear a priori; they will depend on the specific assumptions and structure of the model.

The KW model furnishes a benchmark tool to the study of the relation between television networks programming policies and the role of advertising and it can be extended in multiple directions. In this paper, one of such directions is explored. We introduce a dynamic setup by considering that consumer preferences over different types of goods change intertemporally, given three types of effects: first, we consider a constant rate of utility appreciation / depreciation. Individuals do not have to maintain forever the same degree of utility from a type of good. They can progressively like it more or like it less. This rate translates such an effect. Second, there is an imitation or diffusion effect that leads each individual to prefer a kind of good that others also prefer. Third, the imitation effect can be overcome by a conspicuous consumption effect [see Benhabib and Bisin (2002)]: when individuals realize that many other individuals also consume that good, their preference for it may drop down as the consumption no longer represents outstanding status.

The previous three effects are associated to a bounded rationality mechanism that works through a discrete choice rule [see McFadden (1973), Manski and McFadden

(1982) and Anderson, de Palma and Thisse (1991)]. This choice rule has proved important in explaining other economic phenomena, like the working of financial markets [Brock and Hommes (1998), Gaunersdorfer (2000)] or how heterogeneous inflation expectations contribute to endogenous business cycles [Branch and McGough (2006), Gomes (2006), Gomes, Mendes, Mendes and Sousa Ramos (2006)].

The dynamic framework regarding consumers / viewers' preferences with the previous properties reveals a very rich set of long run equilibrium properties: a fixed point stable steady state is accomplishable for some sets of parameter values, but for others, also reasonable, sets of parameter values cycles of various periodicities and even chaotic motion is encountered. Thus, one may have a long term time series for viewers preferences about commodities that, although being determined under a purely deterministic setup, exhibits fluctuations, and therefore it becomes difficult or even impossible to predict how viewers will behave in the long run with respect to the products they wish to acquire.

Because consumer preferences are a central piece in the representative network revenue function (networks sell ad time to advertisers under the assumption that these only pay advertising directed to the viewers potentially interested in acquiring their goods), the possible periodic or a-periodic long term results will also characterize the evolution over time of the share of programming time dedicated to advertising and the revenues of the network.

The main novelty over the KW model that this new assumption about dynamic preferences under bounded rationality allows is that there is the possibility of becoming unfeasible to determine exactly how much the network will spend in advertising time and how much will be the optimal profit in the long run.

The model will be explored mainly through numerical examples and graphical illustrations because general results about global dynamics are not determinable. Nevertheless, the sensitivity analysis will be able to reveal some meaningful regularities.

The remainder of the paper is organized as follows. Section 2 reviews the basic structure of the KW model, section 3 presents the intertemporal mechanics of preferences over a bounded rationality scenario; dynamic properties of the model are discussed in sections 4 and 5; section 4 treats the conventional case of ad non addressability; section 5 explores the targeted commercials case. Section 6 briefly discusses another insight: the possibility of dynamic preferences not only over products but also over programs. Finally, section 7 concludes.

## 2. The Kim-Wildman Model

As stated in the introduction, the KW model furnishes the basic structure for our analysis of television advertising in the presence of intertemporal changes in consumers' preferences. We briefly review the structure of the model.

The baseline idea is that a finite number of networks (i.e., programming service providers) compete for revenues, which come from two sources: advertisers and final consumers (viewers). Networks obey to some symmetry conditions: (i) production costs are fixed and equal to all networks (every network maximizes profits by maximizing revenues); (ii) viewers can change from one network to another, but it is supposed that they do not abandon the market, that is, when one network loses viewers, these will continue to watch television distributed equally among all the other service providers; (iii) networks are Cournot competitors in the sense that their business decisions are taken on the assumption of no change in the behaviour of competitors.

The last of the previous assumptions allows us to consider a representative network, relatively to which one is able to analyze optimizing behaviour (the other networks will operate in a same way).

Consumers or viewers are separated in two groups. This separation has nothing to do with tastes regarding programs that are broadcasted by networks [another simplification of the model is that viewers are indifferent regarding networks' contents, as long as the time of effective (without ads) program service provided by each one of them is the same]. The distinction is associated with the final consumption role of viewers and the way they are affected by advertising. Following the original formulation, we distinguish between right-handed viewers (RH) and left-handed viewers (LH). They consume different bundles of goods: goods RH and LH, respectively. There is also a finite number of firms in the market that sell and advertise the two sets of goods; thus, firms are also of two types.

Let  $\alpha_t$  be the share of viewers that in a given moment  $t$  prefer to acquire goods RH (accordingly,  $1-\alpha_t$  is the share of LH type viewers). Differently from the KW model, one is assuming an intertemporal setup (that will be discussed in the next section), and therefore subscript  $t$  allows us to locate ourselves in time.<sup>1</sup>

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<sup>1</sup> We should be careful in our analysis with the use of the term 'time'. The problem is intertemporal, and therefore there is a sequence of time periods from  $t=0$  until a non specified moment in the future. Each period contains a unit of time that can be divided between the effective broadcasting of television

Advertisers RH and LH will pay to the representative network to broadcast their commercials; they pay  $r_R(a_{Rt})$  and  $r_L(a_{Lt})$ , respectively. In these revenue functions,  $a_{Rt}$  and  $a_{Lt}$  are the amounts of programming time the network allocates in each time period to commercials of each type. Following KW, the revenue functions will display positive and diminishing returns to ad time, given the intuitive idea that repeating the same message over and over again has a marginal decreasing effect over consumers and, thus, firms will not be willing to pay the same for each additional unit of time on the air.

Because our model becomes hard to analyze once we introduce the intertemporal component, we will work with specific functional forms. In the present case, the following functions are considered:  $r_R(a_{Rt}) = \rho_R \cdot a_{Rt}^\theta$  and  $r_L(a_{Lt}) = \rho_L \cdot a_{Lt}^\theta$ ,  $\rho_R, \rho_L > 0$ ,  $0 < \theta < 1$ . These functions clearly obey the specified conditions  $r' > 0$  and  $r'' < 0$ .

Having understood how advertisers contribute to the network's revenues, we must look now to the viewers' side. If we have considered a free television service acting in a monopolistic market and if there were no legal constraints to how much advertising a TV station could transmit, the network's revenue would be maximized with a level of commercials that were certainly above a social optimum value. But our model is different from this extreme scenario in several ways. We are working with a competitive market and we make the reasonable assumption that viewers dislike commercials. Furthermore, we take the additional assumption that viewers pay to watch television (a subscription cable TV service is considered). Under this scenario, two effects arise: (i) the license fee of the network decreases when time dedicated to commercials rise (i.e., the amount a viewer is willing to pay to access a channel falls with advertising time); (ii) as individuals can switch from one network to the others, the network will lose viewers for other networks if it rises commercials time.

These two effects can be translated analytically. The first will be given by a function  $f(a_t)$ , with  $a_t$  the total time of advertising in each period that a network transmits. The function relates to the per subscriber fee collected by the network; obviously, this fee falls with a rise in  $a_t$ , so that  $f' < 0$ . An admissible functional form for this function is  $f(a_t) = \varphi / a_t$ , with  $\varphi > 0$ . The second effect states that there is a function  $s(a_t)$  that defines the network's share of all viewers, in such a way that  $s' < 0$ . We consider  $s(a_t) = \sigma \cdot a_t^{-\omega}$ , with  $\sigma > 0$  and  $0 < \omega < 1$ .

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programs and advertising. Therefore, the term 'advertising time' will be generally used to designate the share of each period time dedicated to commercials broadcasting.

As stated previously, the optimization problem of the network is, in fact, a problem of revenue maximization. In each moment of time, the representative network solves problem (1).<sup>2</sup>

$$\text{Max } R(a_{Rt}, a_{Lt}) = V \cdot s(a_t) \cdot [\alpha_t \cdot r_R(a_{Rt}) + (1 - \alpha_t) \cdot r_L(a_{Lt})] + V \cdot f(a_t) \quad (1)$$

In (1), parameter  $V$  defines the total number of television viewers.

The way in which we will solve problem (1) depends on two features. The first feature is related to the adopted notion of dynamics. We will introduce a temporal dimension by considering that  $\alpha_t$  varies in time. This is, in the original model, a constant, and in our formulation a state variable (the network is unable to control its evolution). Therefore, the control variables for the network continue to be the same as in the KW model: the ad time the service provider can sell to each firm. In this sense, from the network point of view, the problem continues to be static: problem (1) can be solved in exactly the same way in all time periods from today to infinity. The second main feature is associated with a central piece of the argument of KW, that is, the possible addressability of advertising time.

In its conventional form, television can only provide untargeted (non addressable) advertising. Untargeted commercials are delivered to the whole audience, independently of the type of consumers. In its traditional form, television is unable to separate audiences and to furnish to each audience the advertisements relating to the goods they frequently purchase. In terms of the network business, the main consequence of the non addressability property of television is that the same units of commercial time cannot be occupied by more than one advertiser. The translation of this idea to our model is straightforward: the total ad time is just the sum of the time sold to each type of advertiser, that is,  $a_t = a_{Rt} + a_{Lt}$ ; in this case, we define a new variable  $u_t$  that is the share of ad time associated to RH advertisers, so that  $a_{Rt} = u_t \cdot a_t$  and  $a_{Lt} = (1 - u_t) \cdot a_t$ . Problem (1) can be restated for the particular case of non addressability as follows,

$$\text{Max } R^U(a_t, u_t) = V \cdot s(a_t) \cdot [\alpha_t \cdot r_R(u_t \cdot a_t) + (1 - \alpha_t) \cdot r_L((1 - u_t) \cdot a_t)] + V \cdot f(a_t) \quad (2)$$

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<sup>2</sup> For a similar presentation of revenues maximization see the original KW model.



The advantage of analyzing (2), over (1), is that only two control variables, relatively to which we should maximize  $R^U$ , are present: the total time of advertising and the share of advertising time associated to one of the goods' types.

New technologies applied to cable television are beginning to make advertising addressable. If, by any means, networks and advertisers are able to compile information about the preferences of consumers and if television addressability technology is available, it will be possible to sell the same advertising time two times, that is, to the two types of goods providers. The simultaneous delivery of distinct commercials to different viewers implies relevant changes in the model; now, advertising time becomes non rival, that is, the same advertising time can be simultaneously sold to both types of producers / advertisers. If we consider the limit case where all commercial time can be targeted, then  $a_{Rt}=a_t$  and  $a_{Lt}=a_t$ , turning the maximization problem into the one in expression (3),

$$\text{Max } R^T(a_t) = V \cdot s(a_t) \cdot [\alpha_t \cdot r_R(a_t) + (1 - \alpha_t) \cdot r_L(a_t)] + V \cdot f(a_t) \quad (3)$$

As in the benchmark KW model, one of the purposes of the analysis consists in comparing untargeted and targeted ads scenarios. It is relevant to understand how total time of advertising varies between the two cases and how the targeting possibility affects optimal revenues. After characterizing consumer preferences in the next section, we will derive and compare such results.

### 3. The Dynamics of Consumer Preferences

Although we have included time subscripts into variables on last section's model, this was presented essentially as a static one period problem. In this section, we turn the model dynamic by deriving a difference equation that describes the motion in time of variable  $\alpha_t$ . Recall that  $\alpha_t$  is the share of viewers / consumers that prefer to acquire RH products and therefore react essentially to advertisement of this type of goods. Now we allow preferences to change. They will not change as a result of advertising (we assume that advertising has the ability to stimulate the viewer to buy a larger quantity of the product that the viewer already prefers, but it has no capacity to change preferences); instead, preferences will change given a set of three effects and a bounded rationality mechanism.

Let  $Q_{Rt}$  and  $Q_{Lt}$  be the utility withdrawn by one consumer from potentially purchasing RH and LH goods, respectively. These utility values will change given three effects:

i) A depreciation / appreciation of utility effect: households can gain or lose interest in acquiring a good independently of any other factor besides their own subjective perception of the good's quality;

ii) A network / imitation effect: this occurs when individuals withdraw higher utility when the share of consumers interested in the goods in question rise. This effect generally occurs for a relatively low level of the referred share;

iii) A conspicuous consumption effect: when the share of individuals consuming the goods is high, as it rises even further individuals may want to consume less of the goods because it is no longer a symbol of status to acquire them; consumers will outstand if they become interested on the other, less preferred, good.

To better understand the second and third effects, figure 1 presents two diagrams where the imitation and conspicuous properties are depicted. As we have stated, for a given individual the variation on utility is determined by the share of consumers / viewers that already show preference for the good: first, the imitation process leads the individual consumer to 'follow the mob' and to withdraw higher utility just because other individuals are beginning to prefer that good or goods. After some point (that here we consider  $\alpha_i=0.5$  to simplify computation and analysis, but that does not need to be exactly this value), the conspicuous effect dominates: each consumer will be less interested in the good, i.e., he will attribute less utility to it, because the act of consumption will no longer allow him to outstand relatively to other consumers. In figure 1, the left diagram relates to utility changes regarding goods of type RH; the right diagram is concerned with goods of type LH.

Figure 1

Note in figure 1 that if the share of individuals preferring one or the other type of goods is very low or very high, then utility varies negatively, and it will vary positively if the number of consumers preferring each type of good is an intermediate value.

The functions in figure 1 can be translated analytically. Equations (4) and (5) present the utility variation functions, including the three referred effects. These equations are the ones presented in the figure, plus a last term in the right hand side of each equation that corresponds to the utility appreciation / depreciation effect that is

independent from share  $\alpha$ , it only depends on the own dynamics regarding each consumer formation of preferences.

$$Q_{R_{t+1}} - Q_{R_t} = 8m\alpha_t \cdot (1 - \alpha_t) - m + \mu Q_{R_t}, \quad Q_{R_0} \text{ given}, \quad m > 0 \quad (4)$$

$$Q_{L_{t+1}} - Q_{L_t} = 8n\alpha_t \cdot (1 - \alpha_t) - n + \mu Q_{L_t}, \quad Q_{L_0} \text{ given}, \quad n > 0 \quad (5)$$

If  $\mu=0$ , in (4) and (5), these equations are just the ones presented in figure 1, which are able to address the issues regarding the way individual utility is dependent on the share of individuals with exactly the same preferences. Considering parameter  $\mu$  a positive or negative real number, we are stating that in consumer preferences there is a natural tendency for preferring more ( $\mu>0$ ) or less ( $\mu<0$ ) of each type of good over time.

To complete the dynamic preferences framework, we have to select a preference change rule. Equations (4) and (5) give the evolution of utility values. If one just considers a full rationality setup, each consumer would evaluate and compare utility results in every moment of time and choose RH when  $Q_{R_t} > Q_{L_t}$  and LH when  $Q_{L_t} > Q_{R_t}$ . Instead of full rationality, we assume that individuals are boundedly rational, that is, the share of consumers choosing one of the types of goods is given by the discrete choice rule

$$\alpha_t = \frac{\exp(bQ_{R_t})}{\exp(bQ_{R_t}) + \exp(bQ_{L_t})} \quad (6)$$

Regard that  $\alpha_t$  can be, in this case, simultaneously interpreted as the share of individuals preferring RH goods or the probability of each individual choosing RH goods instead of LH goods. Parameter  $b$  assumes a special relevance in (6). It is known as the intensity of choice and it is a non negative value. When  $b$  is close to zero, such implies a low degree of rationality, in the sense that consumers will not tend to change preferences even though the non selected set of goods gives systematically better utility results over time. For high values of the intensity of choice, the probability of changing preferences as the utility results vary becomes higher. This can be interpreted as an increase on the degree of rationality (such that when  $b \rightarrow \infty$ , we would be on the case of complete rationality), but it is also sometimes interpreted, specially in the finance

literature, as a case where individuals just adopt a herd behaviour, without weighting all the benefits and costs of changing preferences between types of goods [see Hirshleifer (2001)].

From equations (4), (5) and (6), it is possible to withdraw a difference equation that characterizes the evolution of share  $\alpha$  over time (eliminating from the analysis, in this way, the utility variables). One reaches equation (7) (an appendix at the end of the paper derives this equation),

$$\alpha_{t+1} = 1 / \left\{ 1 + \left( \frac{1 - \alpha_t}{\alpha_t} \right)^{1+\mu} \cdot \exp(b \cdot (n - m) \cdot [8\alpha_t \cdot (1 - \alpha_t) - 1]) \right\} \quad (7)$$

Equation (7) is a rule of preferences switching derived from a set of reasonable assumptions about human behaviour. The introduction of such a rule into the advertising model of section 2 allows for adding a dynamic component to the model with various implications that are important to discuss. In dynamic models we may distinguish between transitional dynamics, the period that goes from the initial state to a stationary long run state, and the steady state, where some regularities are observable and that can be interpreted as a state that does not suffer any qualitative relevant changes unless some external disturbance occurs.

In the following sections, we will be mainly concerned with the long term state of the model, after the transition phase is fulfilled, that is, we will analyze the long run scenario that remains unchanged unless some shock over parameter values succeeds. Nevertheless, we must remark that it is not the same thing to analyze a static setup as the original KW model and the long run steady state of a dynamic model. This last state is reached only because there is a past history that gave rise to it. Past periods are not forgotten; by the contrary, they are the ones that allowed for the existence of the achieved state. This remark is important when addressing the model's results in the next sections.

#### 4. Long Run Results with Untargeted Commercials

In this and in the following sections we study the KW model as described in section 2, given preference dynamics as characterized in section 3. We begin by considering the case in which ad addressability is not feasible, that is, the case of untargeted commercials. This is the case translated into expression (2). Recall that the

television network controls both the total time of advertising that is sold in each period and the share of commercials time that is attributed to each type of advertisers. Hence, revenues maximization implies, in this case, solving  $\partial R^U / \partial u_t = 0$  and  $\partial R^U / \partial a_t = 0$ .

Making the computation for the specific functional forms that were assumed, one gets

$$u_t = 1 / \left[ 1 + \left( \frac{\rho_L \cdot (1 - \alpha_t)}{\rho_R \cdot \alpha_t} \right)^{1/(1-\theta)} \right] \quad (8)$$

$$a_t = \left\{ \frac{(\theta - \omega) \cdot \sigma}{\varphi} \cdot \left[ \rho_R \cdot \alpha_t \cdot u_t^\theta + \rho_L \cdot (1 - \alpha_t) \cdot (1 - u_t)^\theta \right] \right\}^{-1/(1+\theta-\omega)} \quad (9)$$

Expression (8) reveals the optimal share of ad time allocated to advertisers RH. We immediately observe the intuitive result that the higher the number of viewers preferring goods RH, the larger will be the relative amount of ads of this type of goods that are broadcasted by the network, in order for this to maximize profits.

Relatively to the total time of advertising per period, result (9) indicates that such time is dependent on shares  $u_t$  and  $1 - u_t$  and that we must assume the condition  $\theta > \omega$  in order to assure a positive value for the total advertising time, that is, positive returns on advertising time sold have to be more pronounced than negative returns on the loss of viewers to other channels as a result of increasing advertising time.

If there were no changes in preferences across periods, it would be straightforward to interpret (8) and (9). With the setup introduced in section 3 we lose in simplicity but gain in richness of results. To obtain meaningful conclusions it is necessary to consider specific numerical examples. We take, for now, the following benchmark vector of parameter values:  $[V \ \sigma \ \omega \ \theta \ \rho_R \ \rho_L \ \varphi \ m \ n \ b \ \mu] = [1,000; 1; 0.1; 0.25; 0.75; 0.5; 1; 3; 1; 2.75; 0.5]$ . Some of these values will be changed or allowed to vary, along the analysis. The initial value of  $\alpha_t$  must also be defined ( $0 < \alpha_0 < 1$ ).

Only the last four parameters have impact over the model's dynamics, since these are the ones on equation (7), that defines the evolution of preferences. The others will just allow to determine the amount of total advertising time, the share of time allocated to commercials of each type and the total level of revenue of the network in each period. Thus, we begin by describing the motion underlying the preference dynamics equation.

To characterize the dynamics of (7), we present two bifurcation diagrams.<sup>3</sup> Figures 2 and 3 take the above vector of parameter values and make parameters  $b$  and  $\mu$  to vary, respectively.

Figures 2 and 3
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The bifurcation diagrams, which are drawn for the 1,000 observations after the first 1,000 transients, clearly indicate that a fixed point steady state result is not the only possible long term outcome. Cycles of various orders and total a-periodicity (chaotic motion) are found as well for several values of parameters. When the intensity of choice is varied (maintaining all the other values as given in the benchmark vector), we observe that a fixed point result exists for  $b$  lower than 2.1 (roughly), and that a bifurcation follows implying a period two cycle, which is broken for  $b$  around 2.5, giving place to a period 4 cycle. Cycles of other periodicities are observed as well, and for values of  $b$  between 2.7 and 3.1 and between 3.6 and 4.4 we regard that we have mostly situations of full a-periodicity, in the sense that our endogenous variable can possess any value of the ones indicated in the figure.

Relatively to figure 3, a same kind of interpretation is possible. When varying the appreciation / depreciation of utility parameter multiple possibilities about steady state dynamics arise. In particular, we should note that in this specific case chaotic motion is found for both the possibility of utility appreciation and utility depreciation. Note, for instance, that after a given value of utility appreciation, the degree of predictability of the system rises as a-periodicity gives place to cycles of progressively lower order until reaching a fixed point result.

The presence of cycles indicates that a fully deterministic rule of preferences dynamics, such as (7), does not produce a unique invariant long run result: the preferences share will alternate between two or more steady state outcomes (in cases of two period cycles or other periodic cycles), or it can follow a time series that does not display any kind of identifiable regularity, that is, it can follow a chaotic pattern.

Figures 4 and 5 present, for the chosen vector of values of parameters, a phase diagram and a time series. With the time series it becomes clear that the selected example implies the presence of endogenous fluctuations (this is the long run behaviour

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<sup>3</sup> The program used to draw these and all the following figures is iDMC (interactive Dynamical Model Calculator). This is a free software program available at [www.dss.uniud.it/nonlinear](http://www.dss.uniud.it/nonlinear), and copyright of Marji Lines and Alfredo Medio.

of the preferences share, drawn after withdrawing the first 1,000 observations). The phase diagram may be used to understand that in this case the equilibrium is not stable or unstable; the time series will just gravitate forever around the steady state value [To confirm this just draw a 45 degrees line, select an initial value for the endogenous variable and draw successive vertical lines from the 45 degree line to the function and horizontal lines from the presented function to the 45 degrees line. You will see that the steady state (the points in the intersection between the line and the function) is never accomplished, but a divergence process away from the steady state is not observed as well].

Figures 4 and 5

A rigorous confirmation of the presence of chaotic motion in the case we are considering requires the computation of the Lyapunov characteristic exponent (LCE) associated to equation (7).<sup>4</sup> A positive value of the LCE for a one equation system means that nearby orbits diverge exponentially or, in other words, there is sensitive dependence on the initial condition (SDIC). This coincides with the most widely accepted definition of chaotic motion: chaos relates to the fact of two trajectories departing from two points close to each other following completely different orbits. In the present case, for the selected set of parameter values, we confirm the presence of chaotic motion:  $LCE=0.426$ .

As it is clear from expressions (8) and (9), advertising optimal results are heavily dependent on preferences. If these do not obey to a fixed point result in every circumstance, then cycles and chaos will be also reflected on the long term values of variables  $u_t$ ,  $a_t$  and  $R_t^U$ . Sticking with our benchmark case, figures 6, 7 and 8 present the time trajectories for these variables. These will be compared with the results for the targeted commercials case of the next section.

Figures 6, 7, 8

Note that each one of the figures 6 to 8 is drawn for 100 observations after the first 1,000. The presence of chaotic motion governing preferences makes the long term of each one of the advertising model variables to be subject to endogenous fluctuations.

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<sup>4</sup> This computation is done using iDMC. See Medio and Lines (2001) about the meaning and computation of LCEs.

For the concrete example, the share of advertising time sold to producers of type RH fluctuates around 0.75 and 1 (approximately); the total ad time varies between 6.75 and a value a little above 8 (we have not defined a unit for the ad time variable, but this can be thought as daily advertising broadcasting hours); and the total revenue will fluctuate between 950 and 1150 monetary units.

### 5. Long Run Results with Targeted Commercials

The possibility of ad addressability opens new opportunities for networks and for advertisers. From the point of view of the network, it seems obvious that the possibility of selling the same advertising time to two types of firms (or more) will have a positive impact over profits. A not so obvious result relates to the time allocated to advertising: will this grow or decline? In the present model, absolute answers cannot be presented, but we can study the subject under our benchmark case and related scenarios. In what follows, we confirm the positive impact of targeted commercials over revenues and inquire about the time spent with advertising.

We solve maximization problem (3), that is, we derive an optimal level of advertising time from the condition  $\partial R^T / \partial a_t = 0$ . The outcome is

$$a_t = \left\{ \frac{(\theta - \omega) \cdot \sigma}{\varphi} \cdot [\rho_R \cdot \alpha_t + \rho_L \cdot (1 - \alpha_t)] \right\}^{-1/(1+\theta-\omega)} \quad (10)$$

Expression (10) is similar to (9), but now variable  $u_t$  disappears as a result of the addressability assumption. Once again it is fundamental that condition  $\theta > \omega$  holds.

Using the same benchmark example with chaotic preferences as in the previous section, we characterize the targeted ads case dynamics with the presentation of figures 9 and 10, that reveal how the total ad time and the network's profits evolve in the long run. These time series should be compared with the ones in figures 7 and 8. To better make this comparison, we display two additional figures (figures 11 and 12).

Figures 9, 10, 11, 12.

From figures 11 and 12 we observe that, for the case in appreciation, advertising time falls when advertising is addressable and we confirm the rise in revenue that is



intuitively true. The blue dashed lines in these two figures show the case in which the targeted and untargeted ad cases would furnish the same results. Although chaotic motion exists, we regard that ad time effectively observed points are always below the dashed line, meaning that to maximize profits under ad addressability it is not necessary to broadcast so much commercials as in the conventional non addressability case. Furthermore, chaos does not distort in any way the revenues result; we see that the corresponding set of points (the long run attractor) is always above the dashed line, and therefore ad addressability means an optimal result where revenues are higher with lower advertising time. If one could generalize this result, one might say that ad addressability does not only benefit the network but also the viewer, who can assess more broadcasting time without advertising.

To attempt to generalize the previous results we present table 1, where we compare ad time and profits between addressability and non addressability cases, for various sets of parameter values.

Values of parameters	Type of dynamics	$a_t^U$	$a_t^T$	$R_t^U$	$R_t^T$
Benchmark	Chaos	6.75-8.10	6.7-7.3	950-1,140	1,055-1,145
$b=2$	Fixed point	7.21	6.88	1,063	1,113
$b=3.5$	Period 6	6.68; 6.69; 6.85; 6.92; 9.13; 9.25	6.68; 6.69; 6.74; 6.77; 7.86; 7.94	829; 840; 1,108;1,119; 1,145;1,146	965; 976; 1,132;1,137; 1,146;1,147
$\mu=0.75$	Period 2	6.91; 7.45	6.77; 6.98	1,029; 1,110	1,097; 1,133
$\mu=-0.25$	Chaos	6.71-9.9	6.69-8.9	775-1,143	859-1,145
$m=0.1$	Fixed point	9.8	9.3	786	826
$m=3.3$	Period 3	6.7; 6.86; 8.71	6.69; 6.75; 7.6	880; 1,118; 1,145	1,009;1,137; 1,146
$n=0.5$	Period 3	6.69; 6.87; 9.15	6.69; 6.75; 7.87	838; 1,116; 1,145	974; 1,136; 1,146
$n=2$	Fixed point	7.07	6.83	1,085	1,123
$\varphi=0.5$	Chaos	3.68-4.42	3.66-3.98	867-1,040	963-1,045
$\varphi=2$	Chaos	12.3-14.8	12.2-13.3	1,038-1,246	1,153-1,252
$V=500$	Chaos	6.73-8.08	6.7-7.27	475-570	527-572
$V=2000$	Chaos	6.73-8.08	6.7-7.27	1,898-2,277	2,108-2,287

$\sigma=0.5$	Chaos	12.3-14.76	12.25-13.29	519-623	577-626
$\sigma=2$	Chaos	3.69-4.42	3.67-3.98	1,733-2,080	1,925-2,090
$\omega=0.05$	Chaos	4.89-5.83	4.87-5.27	1,029-1,226	1,138-1,232
$\omega=0.2$	Chaos	22.99-28.08	22.86-25.02	748-913	839-918
$\theta=0.2$	Chaos	10.61-12.57	10.56-11.51	875-1,037	956-1,041
$\theta=0.5$	Chaos	2.38-3.05	2.37-2.53	1,148-1,468	1,382-1,478
$\rho_R=0.5$	Chaos	9.58-10.82	9.51-9.51	708-800	806-806
$\rho_R=0.9$	Chaos	5.75-7.03	5.72-6.39	1,090-1,334	1,198-1,340
$\rho_L=0.25$	Chaos	6.74-8.56	6.72-7.99	896-1,138	960-1,141
$\rho_L=0.9$	Chaos	6.73-7.30	6.38-6.67	1,051-1,139	1,149-1,202

**Table 1 – Sensitivity analysis.**

In table 1, we have advertising time results and optimal revenues for several combinations of parameters. The first case considers our benchmark vector of parameter values that was characterized graphically. All the other 22 cases are studied for a change in one of the parameter values, maintaining all the rest in its original values. Recall that in the benchmark case we have chaotic dynamics, and only four parameters can change qualitatively this result, which are the parameters in equation (7), an equation that defines the movement over time of consumers' preferences.

Therefore, a first straightforward conclusion from the table is that any change in parameters, except in  $b$ ,  $\mu$ ,  $m$  or  $n$ , will not modify the qualitative dynamic nature of our problem; this is why the last 14 cases in the table continue to respect to a situation of chaotic motion. For changes in the values of the referred four parameters, we can abandon chaos and have various types of periodic motion and fixed point stability. For the advanced examples, notice that three cases of fixed point are presented, along with one case of period 2 cycles, two cases of period 3 cycles and one case of cycles of period 6.

Although studying a set of possible cases does not allow for universal conclusions, an important regularity is found for every studied example. In all cases, from fixed point stability, to cycles or chaos, one finds the same result as in the benchmark case: ad addressability implies higher long term profits and less advertising time. As in figures 11 and 12, in every case there is a positive relation between ad time under the two assumptions (the addressability and non addressability assumptions) and between

optimal revenues under the two assumptions. This relation is always found for a higher ad time under non addressability and for a higher level of profits under addressability (these higher values appear in the columns in grey).

Hence, all the proposed examples give rise to a same set of conclusions regarding the basic features of the long term behaviour of the variables of the advertising model, despite the different qualitative behaviour implied by the evolution of preferences.

Note, in the table, that in the case of cycles with identifiable order, each value in the column relating non addressability corresponds to the value in the same position in the column of addressability (both for ad time and revenues); in the case of chaotic motion, we present an interval in which the fluctuation is bounded, and there is a positive relation between the two sets of addressability – non addressability values.

Our main conclusion is that although the model is not robust to changes in parameter values regarding the qualitative long term dynamic behaviour, it seems robust to changes in the values of parameters in what concerns the main implications of the possibility of targeting commercials.

## 6. A Further Insight

In the KW model, besides product preference heterogeneity, program preference heterogeneity is also discussed. Consider that networks specialize in one of two types of programs (type 1 and type 2). Let  $\beta_{Rt}$  and  $1-\beta_{Rt}$  be the shares of RH viewers who prefer type 1 and type 2 programs, respectively, and let  $\beta_{Lt}$  and  $1-\beta_{Lt}$  be the shares of LH viewers who prefer type 1 and type 2 programs, respectively.

In this section we adopt this environment to understand how consumer / viewer heterogeneity are combined in determining long term endogenous changes on audience shares. Thus, there is an important change on focus. We are no longer worried with an individual network profits but with the relation between audiences of different types of networks (between networks of the same type, the same competitive scenario of previous sections continues to hold).

If  $\gamma_t = \alpha_t \cdot \beta_{Rt} + (1 - \alpha_t) \cdot \beta_{Lt}$  is the share of audience for programs of type 1 and if  $1 - \gamma_t = \alpha_t \cdot (1 - \beta_{Rt}) + (1 - \alpha_t) \cdot (1 - \beta_{Lt})$  is the share of audience for programs of type 2, we can study the long term behaviour of the audiences ratio,  $\gamma_t / (1 - \gamma_t)$ . To accomplish this, we assume that preferences for programs are determined by the same factors that determine the preferences for products: imitation – I will like what others

like; conspicuous behaviour – if all the others prefer program 1, I will try program 2; and cumulative utility / disutility – utility withdrawn from one program does not have to remain still over time.

Therefore, in a parallel way to (7), one may consider (11) and (12),

$$\beta_{Rt+1} = 1 / \left\{ 1 + \left( \frac{1 - \beta_{Rt}}{\beta_{Rt}} \right)^{1 + \mu_R} \cdot \exp(b_R \cdot (n_R - m_R) \cdot [8\beta_{Rt} \cdot (1 - \beta_{Rt}) - 1]) \right\}, \quad (11)$$

$\beta_{R0}$  given

$$\beta_{Lt+1} = 1 / \left\{ 1 + \left( \frac{1 - \beta_{Lt}}{\beta_{Lt}} \right)^{1 + \mu_L} \cdot \exp(b_L \cdot (n_L - m_L) \cdot [8\beta_{Lt} \cdot (1 - \beta_{Lt}) - 1]) \right\}, \quad (12)$$

$\beta_{L0}$  given

Parameters  $\mu_R$  and  $\mu_L$  can possess positive or negative values. All other parameters in (11) and (12) are positive quantities.

Combining (7), (11) and (12), multiple possibilities regarding dynamic outcomes arise. To illustrate some of these possibilities we consider the same set of parameter values as in previous sections plus the following vectors:  $[b_R \ m_R \ n_R \ \mu_R] = [5.5 \ 2 \ 1 \ -0.25]$  and  $[b_L \ m_L \ n_L \ \mu_L] = [3.3 \ 1 \ 3 \ 0.1]$ . For these values, one may present the evolution over time of the audiences' ratio (figure 13). Afterwards, a sensitivity analysis indicates how audiences are shaped by changes in parameter values. The values were selected in order to get chaotic motion for both program preference shares.

Figure 13

In figure 13, one can observe a long term time path for the audiences' ratio with no identifiable periodicity. The chaotic motion, triggered by the bounded rationality mechanism involving preferences over consumption and television programs, implies that the overall preference for programs of type 1 and 2 changes dramatically from one period to the next.

In table 2 a more detailed analysis of long term possibilities regarding program shares is provided by comparing several numerical examples.

Values of parameters	Dynamics of $\alpha_t$	Dynamics of $\beta_{Rt}$	Dynamics of $\beta_{Lt}$	Audience's ratio
Benchmark	Chaos	Chaos	Chaos	0.12-97.4
$b=2; b_R=1;$ $b_L=2$	Fixed point	Fixed point	Period 2	2.47; 2.85
$b=3.5; b_R=10;$ $b_L=5$	Period 6	Fixed point	Fixed point	0.001; 0.0013; 0.03; 0.05; 1; 1.2
$\mu=0.75; \mu_R=0.25;$ $\mu_L=-0.5$	Period 2	Chaos	Fixed point	1.54-294.5
$\mu=-0.25; \mu_R=0;$ $\mu_L=0.5$	Chaos	Period 2	Period 3	0.1-72.8
$m=0.1; m_R=1;$ $m_L=2$	Fixed point	Fixed point	Period 2	0.1; 0.32
$m=3.3; m_R=3.3;$ $m_L=3.3$	Period 3	Fixed point	Fixed point	0; 0.03; 0.58
$n=0.5; n_R=1;$ $n_L=0$	Period 3	Chaos	Period 2	0.18-214
$n=2; n_R=2;$ $n_L=2$	Fixed point	Fixed point	Period 2	0.88; 0.92

**Table 2 – Heterogeneity in product and program preferences: dynamic results.**

Table 2 clarifies the kind of dynamics that audiences may be subject to. We have considered various examples that modify the values of the benchmark case. The dynamics of each one of the three preference equations is independent from the others and depends only on the parameters that are present in each one of the equations. The ratio of audiences is dependent on the three preference variables and the higher degree periodicity tends to dominate, that is, if chaos exists for one of the equations (7), (11) or (12), the audience's ratio will also display chaotic motion; if two of the equations are characterized by a fixed point equilibrium and for the other a period 2 cycles is found, then the ratio will be given by a period 2 cycle as well.

Regard that when chaotic motion is present, the ratio alternates between values lower and higher than 1, indicating that the most preferred type of programs will vary over time. When low periodicity cycles dominate, this can imply that one of the two

types of program is always preferred but with different intensities, or that periods of preference for one or the other type of program alternate over time.

## 7. Concluding Remarks

As any other firm, television networks must weight revenues and costs in order to maximize profits. Concerning revenues, advertising has a central role for networks. In the KW model, advertising influences revenues through three channels: advertisers pay to announce their products to potentially interested buyers, viewers tend to abandon the network if it broadcasts too much commercials and viewers tend to switch channels when comparing networks' time dedicated to advertising. These features allow for a straightforward static analysis of the optimal level of profits and of the advertising time that networks must sell in order to maximize profits.

The analysis becomes more interesting when two scenarios are compared: the conventional case, under which the same television transmission time cannot be sold simultaneously to different advertisers, and the case where advertising becomes addressable, as a result of exploring the new possibilities of cable television broadcasting. The analysis is also more insightful because heterogeneous consumers are considered: even if television viewers are homogeneous in terms of programming preferences, they differ in terms of preferences over types of goods sellers advertise on television.

This paper has extended the KW analysis into a dynamic environment. In particular, one has derived a dynamic rule concerning the eventual change of product preferences by television viewers. If viewers are exposed to a set of effects that change the utility they withdraw from different types of products and if a bounded rationality mechanism allows for switching preferences, then it is possible to make an intertemporal analysis of the impact of advertising on networks' profits.

The proposed framework allowed for a large set of different qualitative long term results that went from fixed point stability, to low periodicity deterministic fluctuations and to chaotic motion. The main implication of this preferences' structure is that long term outcomes for the advertising time on television programming, for the share of time attributed to one or to the other type of advertiser and for optimal profits are not necessarily predictable. For combinations of parameter values implying chaotic preference switching, the network is unable to know from the beginning how much profits and how much ad time it has to broadcast in the long term to guarantee an

optimal behaviour. It just knows that profits and ad time will vary within a specified interval.

Despite the lack of predictability and the possibility of multiple long term results, some regularities are derived: first, the expected result that profits rise in the eventuality of targeted commercials is confirmed; second, all the proposed examples indicate that under ad addressability the share of programming time dedicated to commercials declines relatively to the non addressability case; third, if one incorporates dynamic rules for changes in program preferences as well as product preferences, we conclude that the type of dynamics matters: under cycles or chaos, the most desired type of program may vary over time in the long run.

### Appendix – Derivation of Equation (7)

In this appendix, we derive equation (7) from (4), (5) and (6).

Similarly to (6), one may present the share of consumers selecting goods' type LH as

$$1 - \alpha_t = \frac{\exp(bQ_{L_t})}{\exp(bQ_{R_t}) + \exp(bQ_{L_t})} \quad (\text{A1})$$

Taking logs, equations (6) and (A1) are equivalent to (A2) and (A3) respectively,

$$\ln \alpha_t = bQ_{R_t} - \ln[\exp(bQ_{R_t}) + \exp(bQ_{L_t})] \quad (\text{A2})$$

$$\ln(1 - \alpha_t) = bQ_{L_t} - \ln[\exp(bQ_{R_t}) + \exp(bQ_{L_t})] \quad (\text{A3})$$

From (A2) and (A3) one withdraws the following relation,

$$Q_{L_t} - Q_{R_t} = \frac{1}{b} \cdot \ln \frac{1 - \alpha_t}{\alpha_t} \quad (\text{A4})$$

which can be presented as well for period  $t+1$ ,

$$Q_{L_{t+1}} - Q_{R_{t+1}} = \frac{1}{b} \cdot \ln \frac{1 - \alpha_{t+1}}{\alpha_{t+1}} \quad (\text{A5})$$

Now, one can subtract each member of (A4) to each member of (A5) and obtain

$$Q_{L_{t+1}} - Q_{R_{t+1}} - (Q_{L_t} - Q_{R_t}) = \frac{1}{b} \cdot \left[ \ln \frac{1 - \alpha_{t+1}}{\alpha_{t+1}} - \ln \frac{1 - \alpha_t}{\alpha_t} \right] \quad (\text{A6})$$

Making use of (4) and (5), we rewrite (A6),

$$b \cdot (n - m) \cdot [8\alpha_t \cdot (1 - \alpha_t) - 1] + b\mu \cdot (Q_{L_t} - Q_{R_t}) = \ln \frac{(1 - \alpha_{t+1}) \cdot \alpha_t}{\alpha_{t+1} \cdot (1 - \alpha_t)} \quad (\text{A7})$$

In (A7), the term  $Q_{L_t} - Q_{R_t}$  can be replaced by the equivalent expression in (A4), such that

$$b \cdot (n - m) \cdot [8\alpha_t \cdot (1 - \alpha_t) - 1] + (1 + \mu) \cdot \ln \frac{1 - \alpha_t}{\alpha_t} = \ln \frac{1 - \alpha_{t+1}}{\alpha_{t+1}} \quad (\text{A8})$$

Applying exponentials to (A8) in order to eliminate the logs from the relation, and solving in order to  $\alpha_{t+1}$ , we then promptly reach equation (7).

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Figures

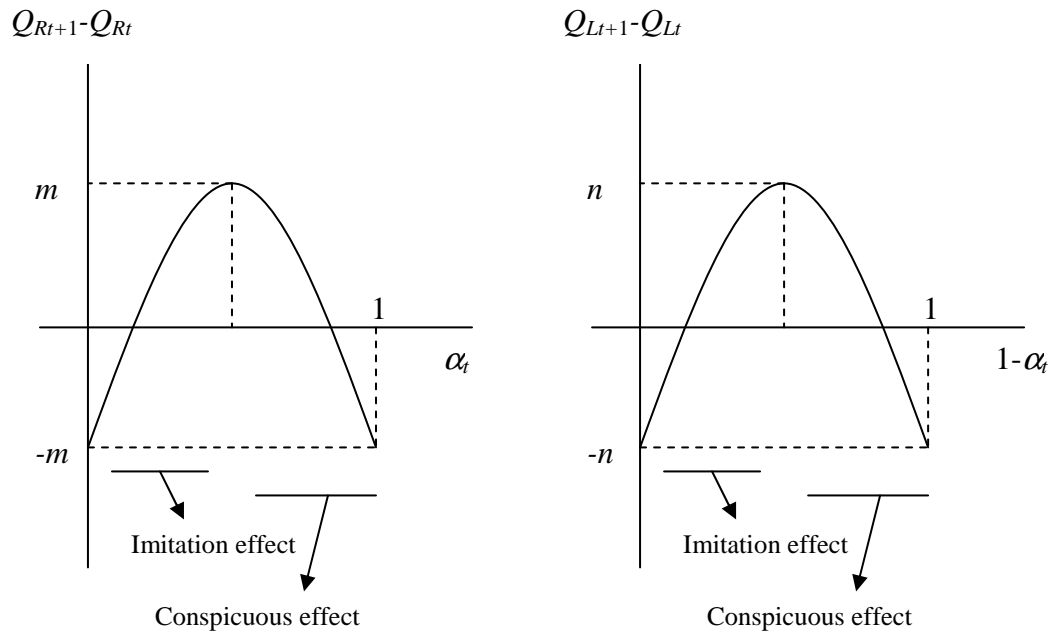


Figure 1 – Graphical illustration of imitation and conspicuous effects over utility changes.

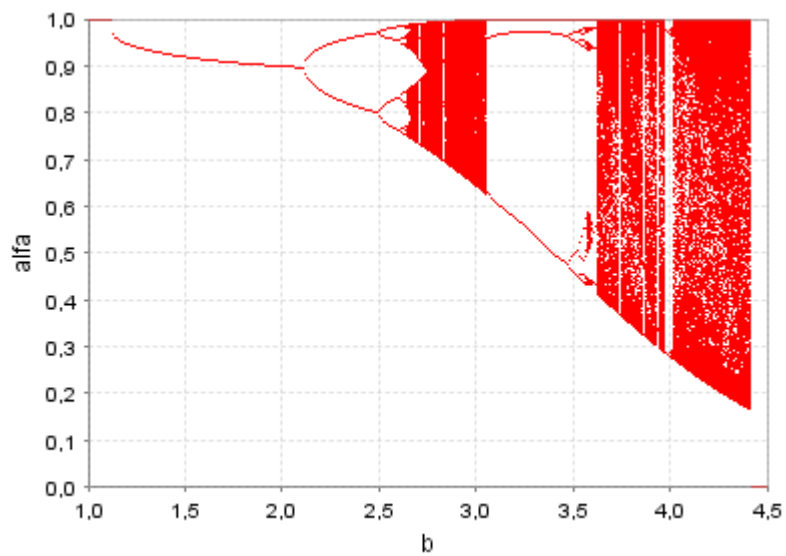


Figure 2 – Bifurcation diagram  $(b, \alpha)$ .

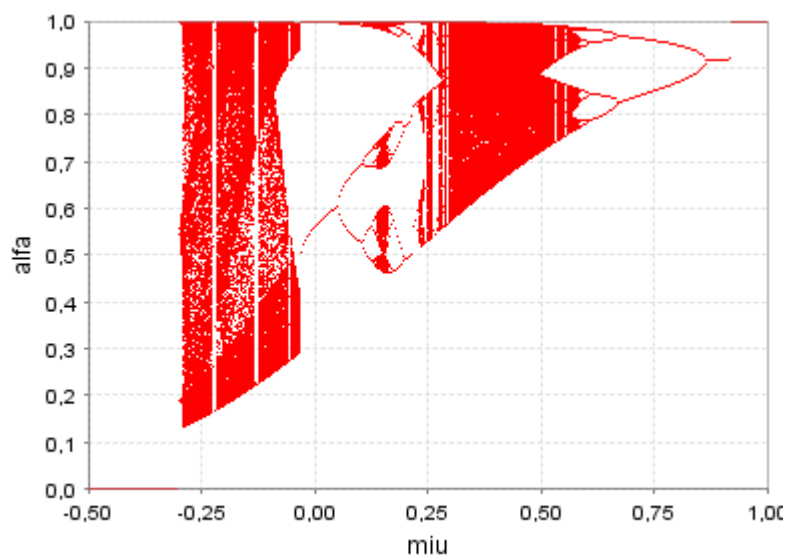


Figure 3 – Bifurcation diagram  $(\mu, \alpha)$ .

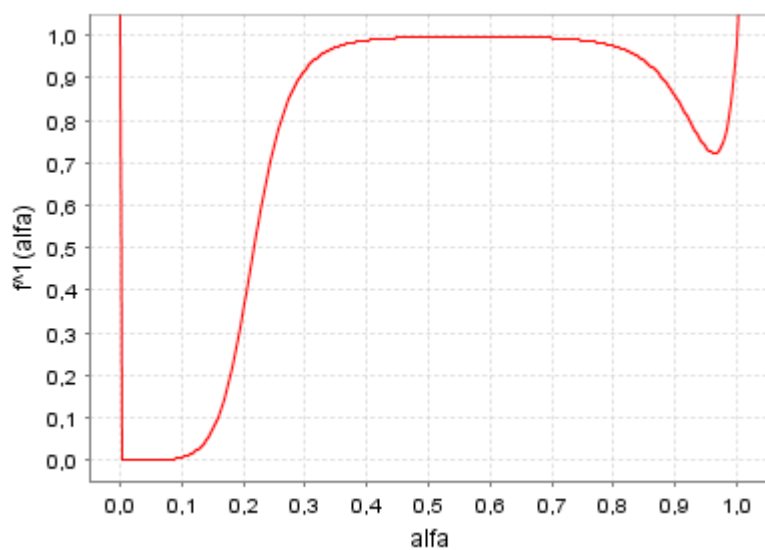


Figure 4 – Phase diagram  $(\alpha, \alpha_{t+1})$ .

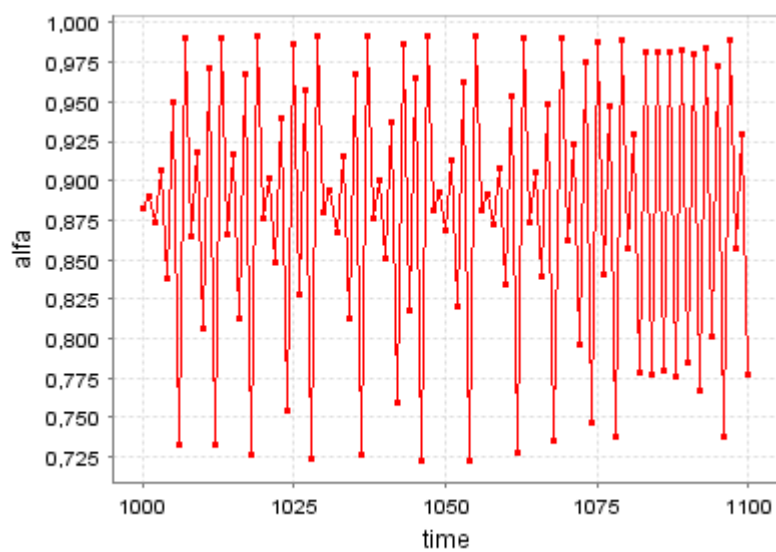


Figure 5 – Time series  $(\alpha)$ .

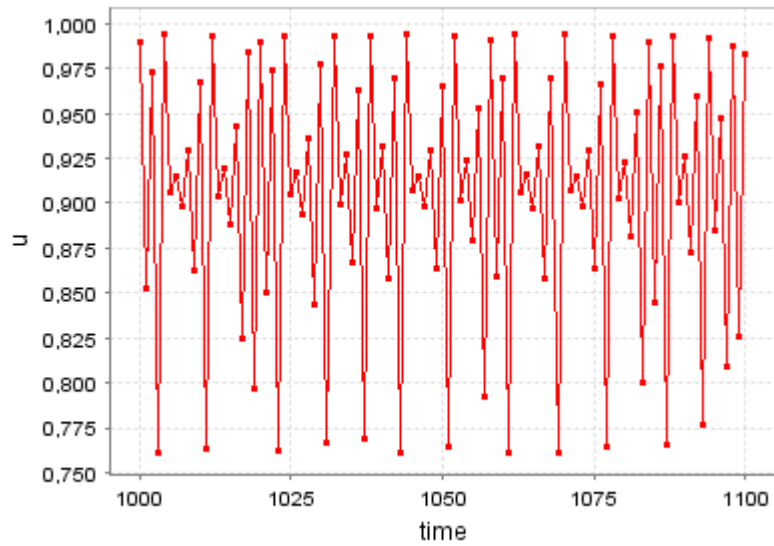


Figure 6 – Time series ( $u_i$ ; untargeted commercials case).

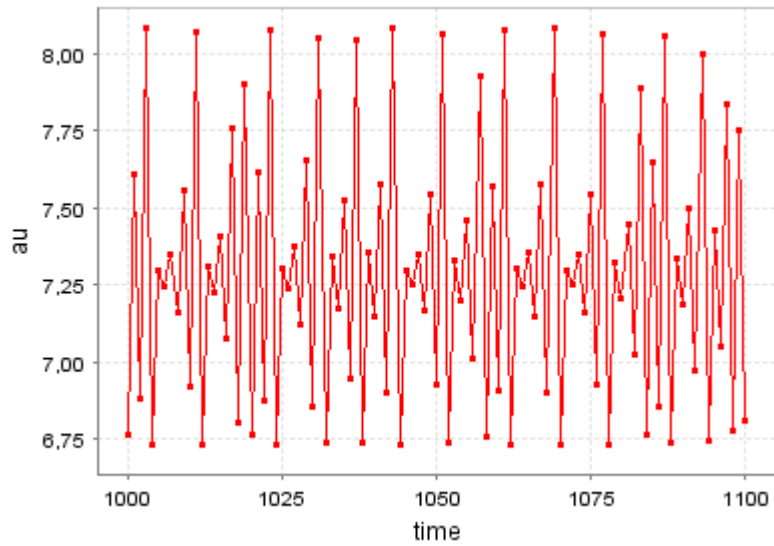


Figure 7 – Time series ( $a_i^U$ ; untargeted commercials case).

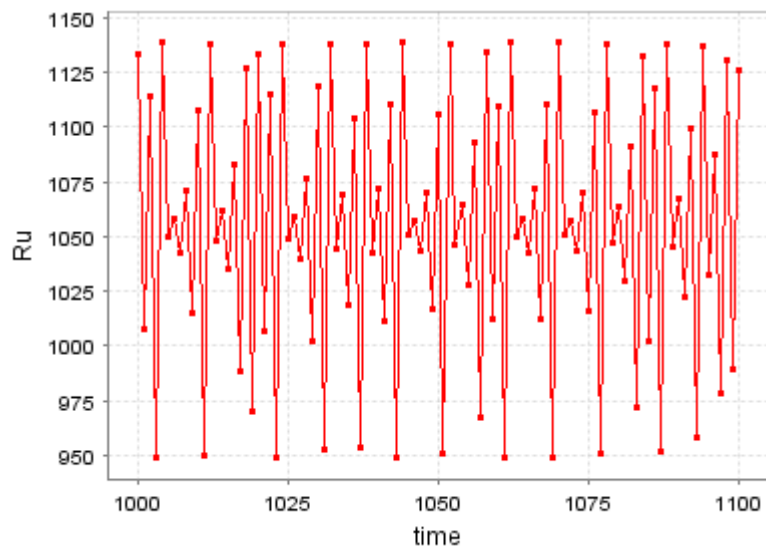


Figure 8 – Time series ( $R_i^U$ ; untargeted commercials case).

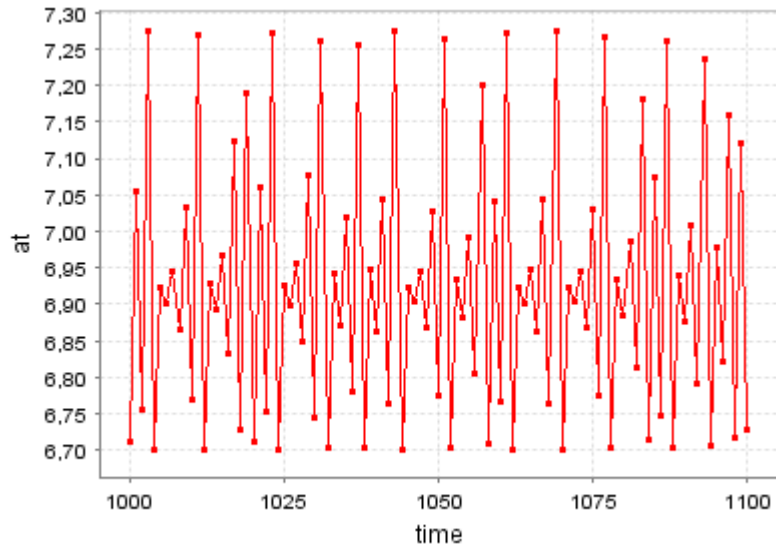


Figure 9 – Time series ( $a_t^T$ ; targeted commercials case).

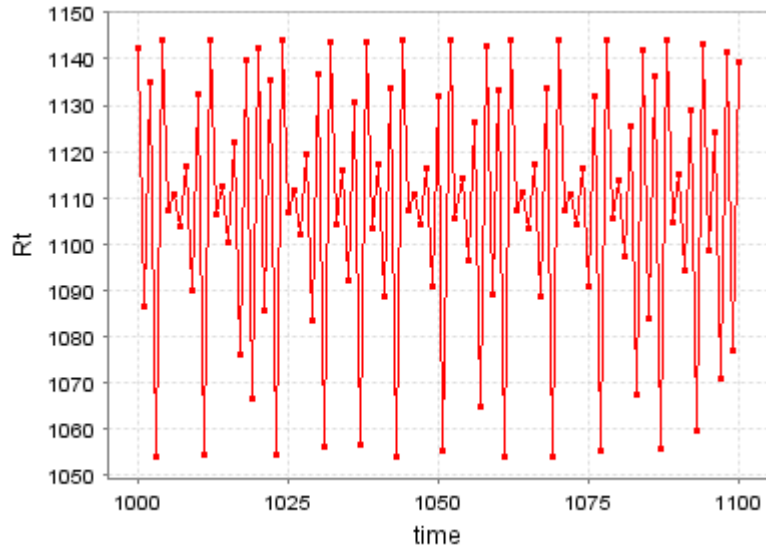


Figure 10 – Time series ( $R_t^T$ ; targeted commercials case).

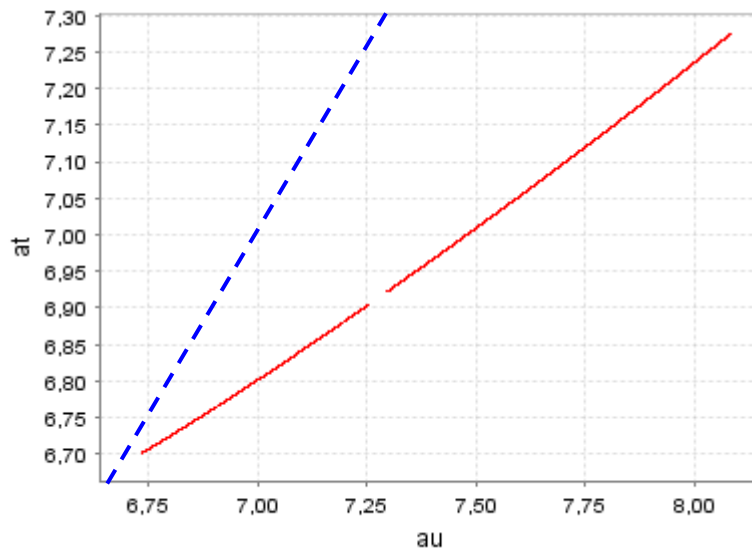


Figure 11 – Long term relation between  $a_t^U$  and  $a_t^T$ .

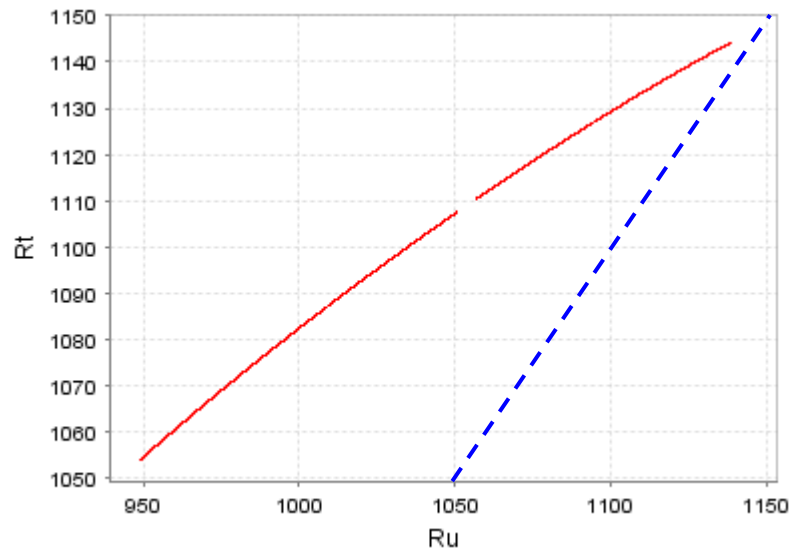


Figure 12 – Long term relation between  $R_t^U$  and  $R_t^T$ .

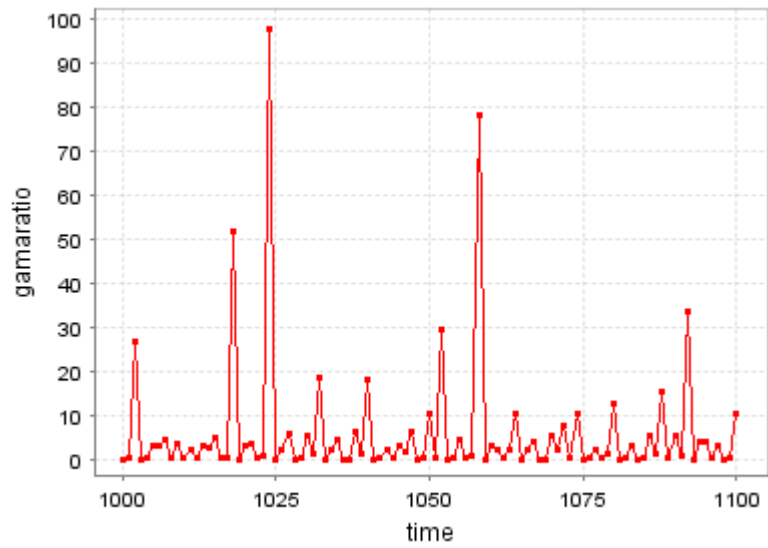


Figure 13 – Time series of the audiences' ratio.