Can social interaction contribute to explain business cycles?

Gomes, Orlando

Escola Superior de Comunicação Social - Instituto Politécnico de Lisboa

October 2006

Can Social Interaction Contribute to Explain Business Cycles?

Orlando Gomes*

Escola Superior de Comunicação Social [Instituto Politécnico de Lisboa] and
Unidade de Investigação em Desenvolvimento Empresarial [UNIDE/ISCTE].

- October, 2006 -

Abstract

Recent literature has been able to include into standard optimal growth models some hypotheses that allow for the generation of endogenous long run fluctuations. This paper contributes to this endogenous business cycles literature by considering social interactions. In the proposed model, individuals can choose, under a discrete choice rule, to which social group they prefer to belong to. This selection process is constrained essentially by the dimension of the group, which is the main determinant regarding the utility individuals withdraw from social interaction. The proposed setup implies the presence of cycles and chaotic motion describing the evolution of group dimension over time. Because being member of a group involves costs to households, the inclusion of these costs in a standard Ramsey growth model will imply that endogenous cycles might arise in the time trajectory of the growth rate of output.

Keywords: Social interaction, Business cycles, Growth models, Nonlinear dynamics and Chaos, Discrete choice.

JEL classification: E32, Z13, C61.

* Orlando Gomes; address: Escola Superior de Comunicação Social, Campus de Benfica do IPL, 1549-014 Lisbon, Portugal. Phone number: + 351 93 342 09 15; fax: + 351 217 162 540. E-mail: ogomes@escs.ipl.pt.
Acknowledgements: Financial support from the Fundação Ciência e Tecnologia, Lisbon, is grateful acknowledged, under the contract No POCTI/ECO/48628/2002, partially funded by the European Regional Development Fund (ERDF).

1. Introduction

The analysis of social interactions on economics takes us along several directions, since it can serve different purposes. One can define social interactions as Bisin, Horst and Ozgur (2006) do, that is, by considering them as "socioeconomic environments in which markets do not mediate all of agents' choices, which might be in part determined, for instance, by family, peer group, or ethnic group effects" (page 74). Such a general definition can be applied to almost any field on economics, because it deals essentially with the ability of agents to establish relationships and to withdraw some amount of utility of these relationships.

The literature on socio-economic relations is extensive and might be separated into two groups: first, comprehensive works that study the properties of models that allow for several types of interaction; in particular, these studies characterize static environments of interaction (essentially under a game theory approach) or address the stability features of dynamic frameworks. The distinction between complete and incomplete information is also relevant; furthermore, the issue of habit persistence is often discussed as well. These studies on the theory of social interactions include Glaeser and Scheinkman (1996), Blume and Durlauf (2001), Brock and Durlauf (2001*a*, 2001*b*) and Ioannides (2006).

On a second group of studies, one finds more subject oriented insights, that focus on a specific type of interaction. At this level, we may begin by mentioning Glaeser, Sacerdote and Scheinkman (1996), who concentrate on the analysis of crime (they find a strong covariance between crime rates and geography, pointing to a social element determining the conduct of the individuals in a given neighbourhood). Others search for the connection between social interaction and education results; it is the case of Kooreman and Soetvent (2002) and Entorf and Lauk (2006), who emphasize the idea that the behaviour towards school is highly influenced by the decisions of peers. We can identify as well important interaction analyses in subjects like income inequality [Durlauf (1996)], ethnic or ratial segregation [Schelling (1972), Benabou (1993)], participation in stock markets [Hong, Kubik and Stein (2004)], the analysis of unemployment [Krauth (2000), Topa (2001)], or the effects of advertising over consumers [Castaldi and Alkemade (2004)].

Macroeconomic issues do not escape the need to address the way individuals connect with each other outside the boundaries of strict market relations. Technological complementarities in the context of economic growth are addressed by Durlauf (1993) and Elison and Fudenberg (1993), specialization and international trade can also be subject to an interaction approach, as in Kelly (1997), and inflation as a result of social conflict is also analyzed [Crowe (2004)]. In this last case, the conclusion is that inflation tends to be higher in countries with higher inequality and with greater pro-rich bias.

In the present paper, we associate the notion of social interaction to a framework of optimal growth in order to address the issue of endogenous business cycles (EBC). The literature on EBC began in the early 1980s with the contributions of Stutzer (1980), Benhabib and Day (1981), Day (1982) and Grandmont (1985), among others, and it has gained a new impulse with the work on business cycles triggered by increasing returns to scale / production externalities developed by Christiano and Harrison (1996), Schmitt-Grohé (2000), Guo and Lansing (2002) and Coury and Wen (2005), among others. Other approaches can also be mentioned, as it is the case of Cellarier (2006) [Cycles through constant gain learning] and Gomes (2006*a*, 2006*b*) [Imperfect demand expectations and technological complementarities].

The setup to develop below refers to a scenario where two social groups exist. Individuals choose to be members of one or of the other group. The choice is determined by two factors: the costs associated to being a member of the group (these can be direct costs, like a submission fee, or indirect, like the ones needed to have the same appearance or other common links with the mainstream characteristics of the group), and the number of individuals that already constitute the group. We will consider that the utility of being part of a group rises when the group has an intermediate dimension, and declines otherwise (a network effect explains the need for a not too small group, while a conspicuous effect justifies why individuals prefer not to be a part of a group where everyone is welcomed).

The choice of an agent regarding the group to belong to is specified as arising from a discrete choice rule in the tradition of Manski and McFadden (1981), Anderson, de Palma and Thisse (1993), Brock and Hommes (1998) and Gomes (2005), among others. Under this rule, one is able to determine a one dimensional difference equation that describes the evolution of group dimension over time. This equation displays cyclical and chaotic dynamics for various values of parameters, and thus our social interaction setup gives rise to endogenous fluctuations regarding the dimension of each one of the two assumed groups.

To the social interaction setup, one can associate a standard Ramsey growth model with the usual consumption – capital accumulation trade-off. This growth model exhibits saddle-path stability (if this path is followed, endogenous variables, that is, consumption and the stock of capital, will converge to a fixed point steady state). If one considers the costs of group association in the decisions of households about the way they apply their income, the endogenous fluctuations interaction framework becomes essential in the growth setup and will determine the presence of endogenous business cycles in the long run.

With this analysis we do not intend to claim that social interaction is the single or the most important determinant of cycles (price stickiness, coordination failures, innovation shocks or changes in government policy, have indeed their fundamental role), but we can support the idea that they can have a word to say in what concerns the introduction of nonlinearities over the linear optimal growth model.

The remainder of the paper is organized as follows. Section 2 presents the social interaction setup. Section 3 derives dynamic results, namely the one that gives the presence of strange dynamics in the long run characterization of the interaction process. Section 4 combines the nonlinear interaction setup with a linear growth model, allowing for an economy's growth rate that is endogenously cyclical. Finally, section 5 concludes.

2. A Framework of Social Interaction

Consider an economy populated by a large but finite number of individual agents. Each individual may belong to one of two social groups (e.g., political parties or groups of football fans). Two characteristics determine the choice of an agent regarding the group she wishes to adhere. First, there is a cost or a fee that has to be paid to be accepted in the group. This cost may vary from one group to the other (costs will be addressed in section 4). Second, the utility of belonging to each one of the groups can be measured and, under the proposed setup, it can depend solely on the number of individuals that compose that group and not the other one. In this section we will focus on the measurement of utility.

When choosing a group in which individuals develop their social relations, they are constrained by two effects. One is a network effect. People tend to dislike being part of too small groups and therefore we consider that utility falls when the share of individuals in one group is too low, and that utility variations become positive as the referred share rises. After some point, a conspicuous effect arises and eventually tends to dominate. When a large fraction of the members of society choose to belong to one of the groups, the individual marginal utility will decline as new members enter this group. Therefore, the relation between the share of individuals that are members of one of the groups and the variation of utility for a representative agent in one of the groups will be given by an inverted U-shaped function.

Let Q_{1t} represent the utility of an agent from sharing the norms social group 1. Accordingly, Q_{2t} will be the utility withdrawn from a representative agent in group 2. Consider as well variable a_t as the share of individuals that in each moment t constitute the first of the two groups. Assuming $m^+>0$, $m^-<0$, $n^+>0$ and $n^-<0$ as utility variation boundary values, figure 1 is able to illustrate how such variation of utility reacts to the percentage of individuals that prefer to be associated to each one of the groups.

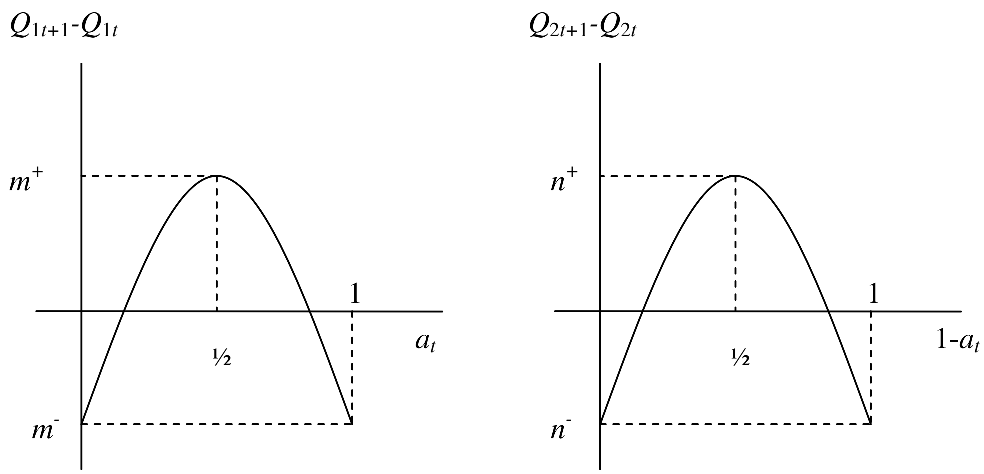

Figure 1 – Variation of individual utility as a function of group dimension.

Relatively to figure 1, some remarks are important. The figure on the left hand side represents the variation of utility from an individual in group 1 when the relative number of agents composing this group assumes any value between 0 and 1. When a small share of elements is associated to group 1, the utility of belonging to such group declines (Q_{1t+1}-Q_{1t}<0); this tendency is inverted when a larger amount of individuals will stay in the group (this is the mentioned network effect; it expresses the idea that man is a social being and therefore utility declines when the individual is almost in isolation). After a given point (that we assume a_t=½ in order to simplify computation and analysis but that does not need to be necessarily so), the growth of utility will decline with the rise in the share of individuals within the group, as the conspicuous effect sets in (individuals dislike to make part of a group that everyone shares), and, hence, as fraction a_t approaches 1, the change in utility becomes negative again (note a second simplification: the variation on utility when a_t=0 is the same as when a_t=1; the symmetry property intends solely to help in the tractability of the model).

The graphic on the right hand side offers a similar interpretation for the evolution of the utility withdrawn by an agent that shares group 2 characteristics. The consideration of different boundary values (n^+ and n^-) reflects the fact that variations of utility as a function of the number of agents in the group does not have to be necessarily the same across groups. In fact, it is this diversity that will allow to encounter rich and meaningful dynamic results.

We can easily translate the mechanics of social interaction utility in figure 1 into analytical expressions. According to the figure, the only force determining the utility of sharing a social group membership is the dimension of the share of elements that already compose the group. This idea is enough to produce the kind of dynamics we are searching for. Nevertheless, to capture a set of other eventual forces influencing such utility variation, we consider in equations (1) and (2) an additional term, associated to a parameter μ, which is a constant real number.

$$Q_{1t+1} - Q_{1t} = 4 \cdot (m^+ - m^-) \cdot a_t \cdot (1 - a_t) + m^- + \mu Q_{1t},\ Q_{10} \text{ given} \tag{1}$$

$$Q_{2t+1} - Q_{2t} = 4 \cdot (n^+ - n^-) \cdot a_t \cdot (1 - a_t) + n^- + \mu Q_{2t},\ Q_{20} \text{ given} \tag{2}$$

If μ=0, then equations (1) and (2) are the exact analytical translation of the functions in figure 1. In the eventuality of μ>0, one can state that in the absence of variation in the relative number of the members of the group, the utility of belonging to that group grows positively (as a function of other exogenous and non specified factors); when μ<0, the growth rate of the presented utility variable is negative, that is, the representative individual will be progressively less satisfied with its social choice (this, of course, for a fixed a_t share). Later on we will end up by considering μ=0 because this assumption is helpful for the analytical study of the model's properties. The assumption can be interpreted as the result of an offsetting device: exogenous factors that contribute to a positive growth of utility impose a variation in utility that is exactly symmetrical to the effect caused by factors that make utility to decline. For now one maintains μ in the analytical developments that follow.

The two previous equations are simple rules that describe the social preferences of individuals, mainly as a function of the number of members of each one of the existing groups. It is our intention to add a growth framework to the previous mechanism; the link between the two concerns the costs associated to social choices. However, one can study autonomously the dynamic behaviour underlying the proposed social interaction setup. This is the task undertaken in the section that follows. To proceed, one has to define a rule through which individuals may change group through time.

Considering a scenario of full rationality and absence of barriers to group mobility, agents would evaluate in every time moment the benefits of sharing their presence with each group [which are revealed by utility values, as measured through equations (1) and (2)] and the corresponding associated costs, and choose the option that would give the most favourable outcome. In practice, agents do not tend to react immediately and irreversibly to changes in net utility; if this is true for instance in what concerns commodity purchasing habits, this idea is reinforced in what concerns social relations: individuals do not quit immediately their group to adhere to the other just because in a specific time moment it is advantageous to do so. To capture this effect of sluggish and not totally rational switching behaviour, we adopt a standard discrete choice rule. The percentage of individual agents choosing to stay with one of the groups is a function of the corresponding utility levels through the following rule,

$$a_t(Q_{1t}, Q_{2t}) = \frac{\exp(bQ_{1t})}{\exp(bQ_{1t}) + \exp(bQ_{2t})} \tag{3}$$

Parameter $b \in [0,+\infty)$ is the intensity of choice. If $b=0$, then individuals will not change groups even though the utility of staying with a group might be systematically lower than the utility that the association to the other group allows to obtain. When $b \to +\infty$ the change is immediate, on the direction of the higher utility group. Hence, the higher the intensity of choice, the faster will be the decision of the individual in changing groups if this is the way to get the best utility result in each time moment. Note that with the specification in (3), a_t may be interpreted both as the share of agents associated to group 1, or the probability of a given agent being associated to group 1 in some time moment. This probability rises: (i) with the rise in the intensity of choice; (ii) with a higher utility level Q_{1t} (relatively to Q_{2t}).

3. The Dynamics of Interaction

Equations (1) and (2), with the rule for a_t in (3), can be used to obtain a one dimensional difference equation with a unique endogenous variable, which is a_t. This equation has, as one will perceive, relevant dynamic properties. The equation is derived in appendix. Its expression is

$$a_{t+1} = 1 \Big/ \left\{ 1 + \left(\frac{1-a_t}{a_t} \right)^{1+\mu} \cdot \exp\left[4b \cdot (n^+ - n^- - m^+ + m^-) \cdot a_t \cdot (1-a_t) + b \cdot (n^- - m^-) \right] \right\} \quad (4)$$

Equation (4) defines the evolution over time of the allocation of individuals to social groups. As remarked before we consider μ=0 to simplify the analytical treatment of the model; we also define constants $\theta \equiv m^- - n^-$ and $\rho \equiv n^+ - m^+$ to simplify notations.

The equation to study will then be

$$a_{t+1} = 1 \Big/ \left\{ 1 + \left(\frac{1-a_t}{a_t} \right) \cdot \exp\left[4b \cdot (\rho + \theta) \cdot a_t \cdot (1-a_t) - b \cdot \theta \right] \right\} \quad (5)$$

It is feasible to proceed with a local analysis of the dynamics of (5), however, as one will understand, the local stability analysis is not sufficient to withdraw all the relevant information about the equation's dynamic behaviour. Therefore, global dynamics will be addressed as well in what follows.

Proposition 1. If the constraints on parameters θ>0 and ρ<-$\theta \vee \rho$>0 are satisfied, then the existence of the steady state is guaranteed. The following two equilibria exist: $\bar{a} = \frac{1}{2} \cdot \left(1 \pm \sqrt{\frac{\rho}{\rho + \theta}} \right)$.

Proof: The steady state is defined by the condition $a_{t+1} = a_t \equiv \bar{a}$. Applying this condition to (5) one verifies that $\exp\left[4b \cdot (\rho+\theta) \cdot a_t \cdot (1-a_t) - b \cdot \theta\right] = 1$, that is, $4b \cdot (\rho+\theta) \cdot a_t \cdot (1-a_t) - b \cdot \theta = 0$. Solving this equation in order to $\bar{a}$, one gets the two solutions in the proposition. The constraints over parameters are essential to obtain admissible steady state values $0 < \bar{a} < 1$ ■

The analysis of local stability of the equilibrium points requires computing the derivative ${da_{t+1}}/{da_t}$ and evaluating it in the steady state points. Such computation leads to the statement of proposition 2.

Proposition 2. The condition required for stability is $b < \frac{2}{\theta} \cdot \sqrt{\frac{\rho + \theta}{\rho}}$. A bifurcation point is given by the combination of parameters $b = \frac{2}{\theta} \cdot \sqrt{\frac{\rho + \theta}{\rho}}$ and, from a local analysis point of view, $b > \frac{2}{\theta} \cdot \sqrt{\frac{\rho + \theta}{\rho}}$ implies instability (i.e., the steady state value of a_t is never accomplished; instead, the value of this share diverges to zero or to one).

Proof: The proof of this proposition requires essentially the computation of $\left. {da_{t+1}}/{da_t} \right|_{\bar{a}}$. Note that

$$ {da_{t+1}}/{da_t} = a_{t+1}^2 \cdot \exp[4b \cdot (\rho + \theta) \cdot a_t \cdot (1 - a_t) - b \cdot \theta] \cdot \left[\frac{1}{a_t^2} - \frac{(1 - a_t) \cdot (1 - 2a_t)}{a_t} \cdot 4b \cdot (\rho + \theta) \right] $$

Considering the derivative for the steady state values of a_t we verify that $\left. {da_{t+1}}/{da_t} \right|_{\bar{a}} = 1 \pm b\theta \cdot \sqrt{\frac{\rho}{\rho + \theta}}$. Stability requires $0 < \left. {da_{t+1}}/{da_t} \right|_{\bar{a}} < 1$; recalling that $\theta > 0$, the unit circle condition is never satisfied for one of the steady states (the one corresponding to the plus sign in the above expression); thus, we focus on the other steady state.

For the second steady state, one regards that the above derivative is always below unity but this does not imply that stability always holds: a bifurcation might exist if the condition $\left. {da_{t+1}}/{da_t} \right|_{\bar{a}} = -1$ is found to be true for some combination of parameter values. A flip bifurcation will then be observed under the condition $1 - b\theta \cdot \sqrt{\frac{\rho}{\rho + \theta}} = -1$ that, rearranging, is the bifurcation condition in the proposition. This boundary condition separates a region of stability (when the derivative remains inside the unit circle) from a region of instability ■

Given the nonlinear nature of difference equation (5), it is possible that the local analysis furnishes an incomplete picture concerning the equation's results. Looking at the global behaviour of (5) another reality will indeed arise: the bifurcation point will not only separate the long term results between stability and instability. The stable region is the one encountered in the previous discussion, but once the bifurcation point is passed, one finds cycles of various periodicities (and totally a-periodic cycles) before one reaches the area of instability.

Thus, from the perspective of the intensity of choice, one generically encounters the following dynamic property: low levels of the intensity of choice will correspond to fixed point stability and, as this parameter's value rises one passes through a bifurcation that leads to cyclical / chaotic behaviour until the instability result is finally achieved for high values of b. Only through numerical examples one can illustrate this result. Just consider admissible values for θ and ρ, e.g., θ=2 and ρ=1.

Figure 2 presents, for these values, our dynamic result. In this case, the bifurcation point is $b=\sqrt{3}$. To the left of this value, fixed point stability holds; to the right, a bifurcation process implies multiple qualitative results regarding the dynamics of a_t for different values of b. After a given value of b, instability sets in (in this case, a_t diverges to one).[1]

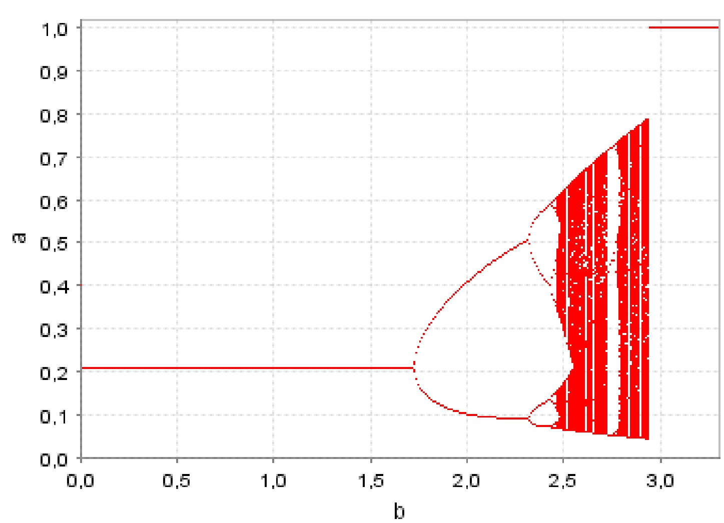

Figure 2 – Bifurcation diagram (b,a_t), with θ=2 and ρ=1.

[1] Figure 2 and all the following are drawn using IDMC software (interactive Dynamical Model Calculator). This is a free software program available at www.dss.uniud.it/nonlinear, and copyright of Marji Lines and Alfredo Medio.

Among the several possibilities in terms of qualitative results, the absence of any identifiable periodicity in the evolution of a_t is the one that is more appealing from a mathematical point of view. The existence of chaos can be measured through the computation of Lyapunov characteristic exponents (LCEs). Our one dimensional system allows for the computation of one LCE; its sign will indicate the kind of dynamics associated to the equation. If LCE>0, then one concludes that there is sensitive dependence on initial conditions, that is, orbits that start nearby tend to rapidly diverge (the LCE is in fact a measure of exponential divergence of nearby orbits). When LCE≤0, cycles of any finite periodicity may exist. Figure 2 allows to perceive which periodicity is present in each case. Table 1 presents the value of the LCE for different intensities of choice and the periodicity associated to the long run time path of the share of individuals that compose the first social group (the values of parameters θ and ρ are the same as before).

Value of *b*	**LCE**	**Dynamics**
1.5	-0.314	Fixed point
1.7	-0.044	Fixed point
1.9	-0.510	Period 2 cycle
2.1	-0.740	Period 2 cycle
2.3	-0.054	Period 2 cycle
2.5	0.186	Chaos
2.7	0.386	Chaos
2.9	-0.121	Period 4 cycle
3.1	--	Instability
3.3	--	Instability

Table 1 – LCE and dynamic characterization of a_t trajectory for different values of *b*.

To reinforce the previous results, one selects a value of b for which chaotic motion is found (b=2.7) to draw two additional diagrams. The first (figure 3) is just a long term time series of a_t; one observes that no regularity is observable in the way the variable evolves over time. The second (figure 4) gives an additional insight about the dynamic behaviour, since it represents a phase diagram and illustrates the fact that independently of the number of time moments one considers, the variable's value will never converge or diverge completely relatively to the steady state value (that in this case is $\bar{a} = 0.211$).

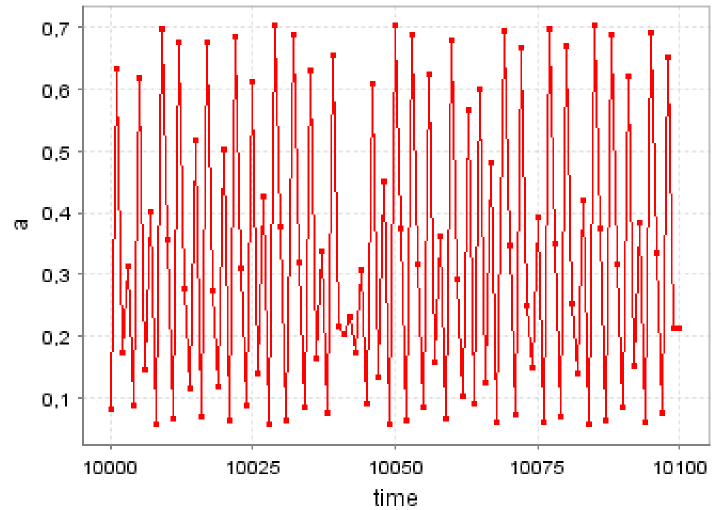

Figure 3 – Long run time series of a_t (b=2.7, θ=2, ρ=1).

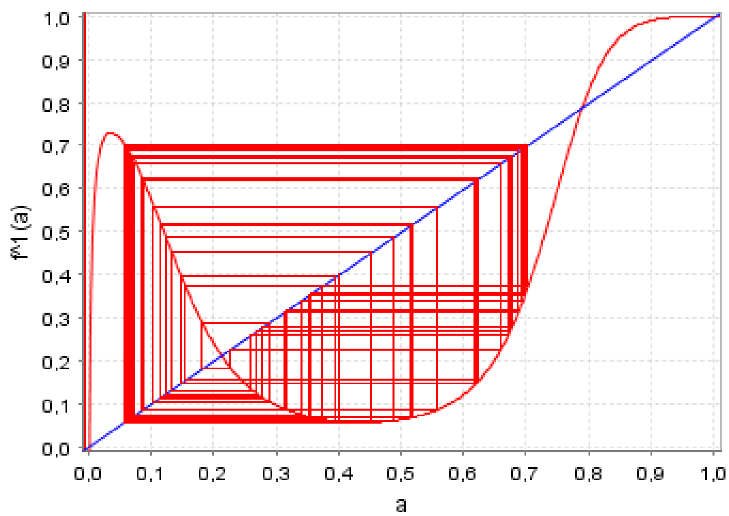

Figure 4 – Convergence dynamics (b=2.7, θ=2, ρ=1).

Having understood the dynamics of social group changing, we should discuss the meaning of the results. Our main initial hypothesis is that two forces determine the utility that agents withdraw from sharing social values. The two conflicting forces are a network effect and a conspicuous effect. It is this conflict, together with a boundedly rational mechanism of choice, that gives place to a nonlinear equation describing the evolution of a_t, and this equation may produce several varieties of long term cycles. Most importantly, our framework allows for a group dimension dynamics that does not end in the moment the steady state is accomplished: the dimension of the two groups may vary forever, under completely predictable low periodicity cycles or through a completely a-periodic and unpredictable time path. As one will regard along the next section, the constant migration of individuals between groups can be used to address the important issue of business cycles.

4. The Growth – Cycles Model

Consider a conventional endogenous growth model. This comprises two pieces: an intertemporal consumption utility maximization problem and the corresponding capital accumulation constraint. The constraint takes the form $k_{t+1} - k_t = s_t \cdot f(k_t) - \delta k_t$, k_0 given. Variable k_t defines the per capita stock of physical capital, s_t is the savings rate ($0 \leq s_t \leq 1$) and δ corresponds to a positive capital depreciation rate. The endogenous growth nature of the model requires $f(k_t)$ to exhibit constant returns to scale; hence, we just assume that $f' = A > 0$ constant.

The intertemporal control problem is assumed as an infinite horizon problem, for which one must define a discount factor $0 < \beta < 1$, $Max \sum_{t=0}^{+\infty} \beta^t \cdot U(c_t)$; this maximization problem is constrained by the capital accumulation equation. To simplify the analysis one considers a simple functional form for the utility function, in particular, $U(c_t) = \ln c_t$ [this obeys to the standard utility function requirements, i.e., marginal utility of consumption is positive but diminishing: $U' > 0$, $U'' < 0$; c_t is per capita consumption].

The only difference between the conventional growth setup and the framework here proposed consists in the relation between the savings rate and per capita consumption. Besides consumption and savings, we consider a third use for the income of households: the share of expenditure directed to pay the group's fees. This share of income is defined by $q_t = q_1 \cdot a_t + q_2 \cdot (1 - a_t)$. We consider constant expenditure shares for each group costs, however, the total share is a variable given that the payment by the representative agent will be a weighted average of each cost share given the probability of association to one or to the other group.

Let $y_t = f(k_t)$ be simultaneously output and the households' income (the government is absent from the analysis). Then, this income is divided in three shares: savings (s_t), expenditures needed to be integrated in a social group (q_t), and consumption ($1-s_t-q_t$). Per capita consumption is, thus, given by $c_t = (1-s_t-q_t)\cdot f(k_t)$.

To solve the growth model, we write the corresponding Hamiltonian function (p_t is a shadow-price of k_t),

$$\aleph(s_t,q_t,k_t) = \ln\left[(1-s_t-q_t)\cdot f(k_t)\right] + p_t\cdot\left[s_t\cdot f(k_t)-\delta k_t\right] \tag{6}$$

Computing first order conditions,

$$\aleph_s = 0 \Rightarrow \beta p_{t+1} = \frac{1}{(1-s_t-q_t)\cdot f(k_t)} \tag{7}$$

$$\beta p_{t+1} - p_t = -\frac{1}{k_t} - (As_t-\delta)\cdot\beta\cdot p_{t+1} \tag{8}$$

$$\lim_{t\to+\infty} p_t\beta^t k_t = 0 \text{ (transversality condition)} \tag{9}$$

From (7) and (8), it is possible to derive a difference equation concerning the movement of s_t,

$$s_{t+1} = 1-\left[(1-q_{t+1})\cdot A+1-\delta\right]\cdot\frac{\beta\cdot(1-s_t)}{As_t+1-\delta} \tag{10}$$

The steady state value of the savings rate is

$$\bar{s} = \beta\cdot\left[1-(q_1-q_2)\cdot\bar{a}-q_2\right]-\frac{(1-\delta)\cdot(1-\beta)}{A} \tag{11}$$

Note that one must assure that the values of parameters are such that $0<\bar{s}<1$. If such condition is verified, two important points must be highlighted: first, for the values of parameters b, θ and ρ that allow for endogenous fluctuations in $\bar{a}$, one has endogenous cycles characterizing the evolution over time of the long term savings rate. Second, equation (10) is unstable $\left[\frac{\partial s_{t+1}}{\partial s_t} = \frac{A+1-\delta}{A\bar{s}+1-\delta} > 1\right]$. This implies that the equilibrium savings rate is not accomplished if one starts from a point s_0 different from $\bar{s}$ just by allowing the economy to evolve; nevertheless, the savings rate is a control variable and therefore the representative agent, knowing in anticipation the steady state value in (11), can follow this stable path.

Allowing for the social group dynamics characterized in section (3) and the long term savings rate in (11), the remaining economic aggregates will evolve accordingly in the long term. The growth rate of per capita physical capital is

$$\gamma_k = A\beta \cdot \left[1-(q_1-q_2)\cdot \bar{a} - q_2\right] - (1-\delta)\cdot(1-\beta) - \delta \tag{12}$$

This is also the rate at which per capita consumption grows. The consumption – capital ratio is given by

$$\omega = \frac{\bar{c}}{\bar{k}} = \left[1-\bar{s}-(q_1-q_2)\cdot \bar{a} - q_2\right]\cdot A \tag{13}$$

When the evolution of the composition of social groups is subject to cycles, the same happens with the savings rate, the growth rate of capital and with the consumption – capital ratio, unless the cost of being part of a group is exactly the same for both groups (q_1=q_2). The illustration of the results is given in figures 5 to 7. For the same parameter values as in figures 3 and 4 plus the vector [A δ β q_1 q_2]=[2 0.02 0.7 0.03 0.05] one observes the presence of endogenous cycles in the savings rate, in the growth rate of the main economic aggregates and in the consumption – capital ratio.

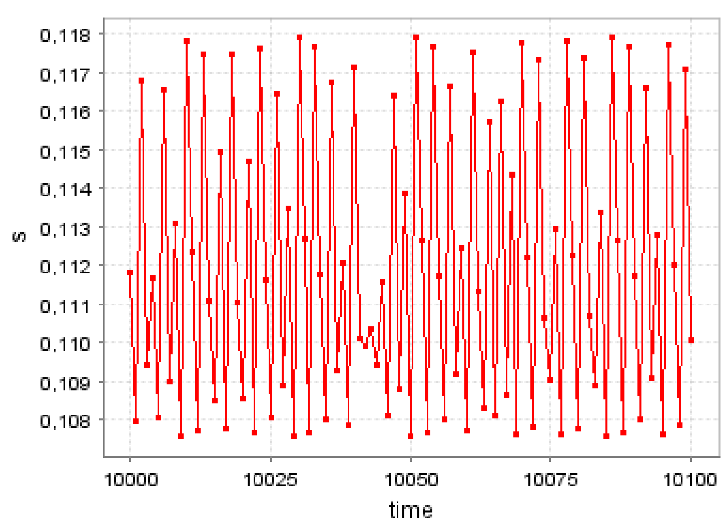

Figure 5 – Long term time trajectory of the savings rate

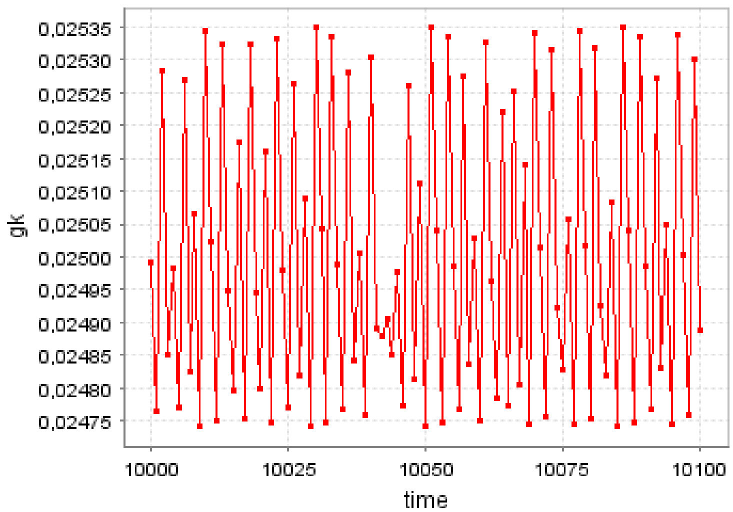

Figure 6 – Long term time trajectory of the growth rate

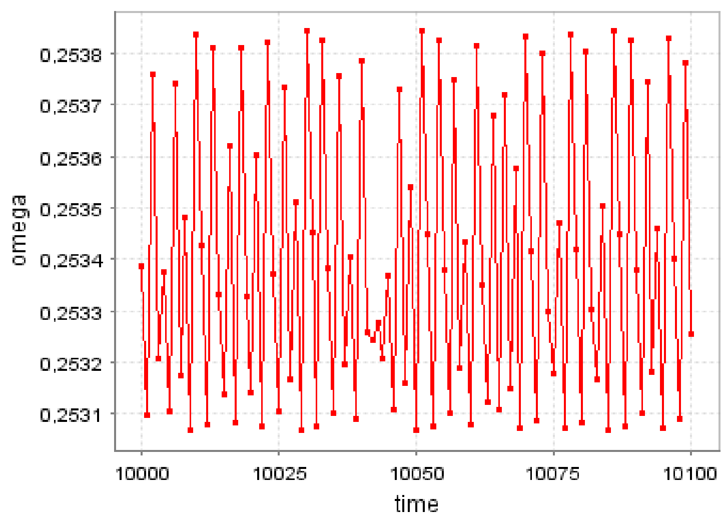

Figure 7 – Long term time trajectory of the consumption – capital ratio

5. Final Remarks

Traditionally, growth analysis focus on the trade-off between consumption and capital accumulation. Households income has in this case only two destinations in each time moment: consumption and savings. In the previous analysis we have introduced a third element into household decisions; part of their income is destined to build social relations, that is, they directly invest in a fee or any other kind of expenditures that give them the right to belong to a certain social group. We have considered that two social groups exist and that the main criterion in selecting one of the groups is, besides the referred fee, the dimension of the group. Small groups imply decreasing utility, as well as too large groups. Individuals prefer to be members of groups with an intermediate number of elements because in this way they can benefit from a network effect that loses relevance when the number of elements becomes too high, situation where a conspicuous effect dominates.

The setup allows to establish a rule of time evolution for the share of members of each group that is nonlinear, implying a large variety of possible long term results. Depending on the values of three parameters (including an intensity of choice attached to a discrete choice rule), fixed point stability, cycles of various periodicities and even chaotic motion are admissible results.

The costs of staying with a group are the element that links the social interaction setup with the growth framework; this link implies that the savings rate may become nonlinear as well; consequently, the economy's growth rate will exhibit a long term path also subject to fluctuations. The main idea enclosed in the proposed model is that even if one considers a standard 'linear' consumption – capital model, the assumption of complex social relations may be passed on to real economic activity, allowing to add a new element to the explanation of business cycles.

Appendix – derivation of equation (4)

Expression (3) can be rewritten in the following form:

$$\ln a_t = bQ_{1t} - \ln\left[\exp(bQ_{1t}) + \exp(bQ_{2t})\right] \tag{A1}$$

Similarly, for 1-a_t,

$$\ln(1-a_t) = bQ_{2t} - \ln\left[\exp(bQ_{1t}) + \exp(bQ_{2t})\right] \tag{A2}$$

Combining (A1) and (A2), we obtain equation (A3).

$$Q_{2t} - Q_{1t} = \frac{1}{b}\cdot\ln\frac{1-a_t}{a_t} \tag{A3}$$

which can be presented one period ahead,

$$Q_{2t+1} - Q_{1t+1} = \frac{1}{b} \cdot \ln \frac{1-a_{t+1}}{a_{t+1}} \tag{A4}$$

Subtracting (A3) to (A4),

$$(Q_{2t+1} - Q_{2t}) - (Q_{1t+1} - Q_{1t}) = \frac{1}{b} \cdot \left[\ln \frac{1-a_{t+1}}{a_{t+1}} - \ln \frac{1-a_t}{a_t} \right] \tag{A5}$$

Now, one replaces (1) and (2) in (A5) to obtain

$$\begin{aligned} &4b \cdot (n^+ - n^- - m^+ + m^-) \cdot a_t \cdot (1-a_t) + b \cdot (n^- - m^-) + b\mu \cdot (Q_{2t} - Q_{1t}) \\ &= \ln \frac{1-a_{t+1}}{a_{t+1}} - \ln \frac{1-a_t}{a_t} \end{aligned} \tag{A6}$$

Replacing (A3) in (A6) and rearranging, the intended difference equation, (4), is finally obtained.

References

Anderson, S.; A. de Palma and J. Thisse (1993). *Discrete Choice Theory of Product Differentiation*. Cambridge, Mass.: MIT Press.

Benabou, R. (1993). "Workings of a City: Location, Education and Production." *Quarterly Journal of Economics*, vol. 108, pp. 619-652.

Benhabib, J. and R. H. Day (1981). "Rational Choice and Erratic Behaviour." *Review of Economic Studies*, vol. 48, pp. 459-471.

Bisin, A.; U. Horst and O. Ozgur (2006). "Rational Expectations Equilibria of Economies with Local Interactions." *Journal of Economic Theory*, vol. 127, pp. 74-116.

Blume, L. and S. Durlauf (2001). "The Interactions-based Approach to Socioeconomic Behaviour." in S. Durlauf, P. Young (eds), *Social Dynamics*, Cambridge, MA: Brookings Institution Press and MIT Press.

Brock, W. A. and S. Durlauf (2001*a*). "Discrete Choice with Social Interactions." *Review of Economic Studies*, vol. 68, pp. 235-260.

Brock, W. A. and S. Durlauf (2001*b*). "Interactions-based Models." in J. Heckman, E. Leamer (eds.), *Handbook of Econometrics*, vol. V, Amsterdam: North-Holland.

Brock, W. A. and C. H. Hommes (1998). "Heterogeneous Beliefs and Routes to Chaos in a Simple Asset Pricing Model." *Journal of Economic Dynamics and Control*, vol. 22, pp. 1235-1274.

Castaldi, C. and F. Alkemade (2004). "An Agent-Based Model of Directed Advertising on a Social Network." *Computing in Economics and Finance*, nº 221.

Cellarier, L. (2006). "Constant Gain Learning and Business Cycles." *Journal of Macroeconomics*, vol. 28, pp. 51-85.

Christiano, L. and S. Harrison (1999). "Chaos, Sunspots and Automatic Stabilizers." *Journal of Monetary Economics*, vol. 44, pp. 3-31.

Coury, T. and Y. Wen (2005). "Global Indeterminacy and Chaos in Standard RBC Models." *University of Oxford and Cornell University working paper*.

Crowe, C. (2004). "Inflation, Inequality and Social Conflict." *CEP discussion paper* nº 0657.

Day, R. H. (1982). "Irregular Growth Cycles." *American Economic Review*, vol. 72, pp.406-414.

Durlauf, S. (1993). "Nonergodic Economic Growth." *Review of Economic Studies*, vol. 60, pp. 349-366.

Durlauf, S. (1996). "Neighborhood Feedbacks, Endogenous Stratification and Income Inequality." in W. Barnett, G. Gandolfo, C. Hillinger (eds.), *Dynamic Desiquilibrium Modelling: Proceedings of the 9th InternationalSymposium on Economic Theory and Econometrics*, New York: Cambridge University Press.

Ellison, G. and D. Fudenberg (1993). "Rules of Thumb for Social Learning." *Journal of Political Economy*, vol. 101, pp. 612-644.

Entorf, H. and M. Lauk (2006). "Peer Effects, Social Multipliers and Migrants at School: an International Comparison." *IZA discussion paper* nº 2182.

Glaeser, E.; B. Sacerdote and J. Scheinkman (1996). "Crime and Social Interactions." *Quarterly Journal of Economics*, vol. 111, pp. 507-548.

Glaeser, E. and J. Scheinkman (1999). "Measuring Social Interactions." in S. Durlauf, P. Young (eds), *Social Dynamics*, Cambridge, MA: Brookings Institution Press and MIT Press.

Gomes, O. (2005). "Volatility, Heterogeneous Agents and Chaos." *The Electronic Journal of Evolutionary Modelling and Economic Dynamics*, nº 1047, pp. 1-32, http://www.e-jemed.org/1047/index.php

Gomes, O. (2006*a*). "Local Bifurcations and Global Dynamics in a Solow-type Endogenous Business Cycles Model.", *Annals of Economics and Finance*, vol. 7, pp. 91-127.

Gomes, O. (2006*b*) "Too Much of a Good Thing: Endogenous Business Cycles generated by Bounded Technological Progress." *ESCS-UNIDE Working paper*.

Grandmont, J. M. (1985). "On Endogenous Competitive Business Cycles." *Econometrica*, vol. 53, pp. 995-1045.

Guo, J. T. and K. J. Lansing (2002). "Fiscal Policy, Increasing Returns and Endogenous Fluctuations." *Macroeconomic Dynamics*, vol. 6, pp. 633-664.

Hong, H.; J. D. Kubik and J. C. Stein (2004). "Social Interaction and Stock Market Participation." *Journal of Finance*, vol. 59, pp. 137-163.

Ioannides, Y. (2006). "Topologies of Social Interactions." *Economic Theory*, vol. 28, pp. 559-584.

Kelly, M. (1997). "The Dynamics of Smithian Growth." *Quarterly Journal of Economics*, vol. 112, pp. 939-964.

Kooreman, P. and A. Soetevent (2002). "A Discrete Choice Model with Social Interactions: an Analysis of School Teen Behavior." University of Groningen *CCSO working paper* nº 200214.

Krauth, B. V. (2000). "Social Interactions, Thresholds and Unemplyment in Neighborhoods." *SFU working paper* 00-12.

Manski, C. and D. McFadden (1981). *Structural Analysis of Discrete Data with Econometric Applications*. Cambridge, Mass.: MIT Press.

Schelling, T. (1972). "A Process of Residential Segregation: Neighborhood Tipping." in A. Pascal (ed.), *Racial Discrimination and Economic Life*, Lexington, MA: Lexington Books.

Schmitt-Grohé, S. (2000). "Endogenous Business Cycles and the Dynamics of Output, Hours, and Consumption." *American Economic Review*, vol. 90, pp. 1136-1159.

Stutzer, M. J. (1980). "Chaotic Dynamics and Bifurcations in a Macro-Model." *Journal of Economic Dynamics and Control*, vol. 2, pp. 353-376.

Topa, G. (2001). "Social Interactions, Local Spillover and Unemployment." *Review of Economic Studies*, vol. 68, pp. 261-295.