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Monetary Policy and Economic Growth: Combining Short and Long Run Macro Analysis

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Abstract

The new Keynesian monetary policy model studies the response of the inflation – output gap trade-off to policy decisions taken by the Central Bank, concerning the nominal interest rate time trajectory. Under an optimal setup, this model displays a saddle-path stable equilibrium and, if the stable trajectory is followed, the steady state is characterized by an inflation rate that coincides with the selected inflation target. A high inflation target has positive effects over the rise of effective output relatively to its potential level (the monetary policy problem captures this effect), but it has a perverse impact over investment decisions (the referred problem does not capture this effect, taking it as granted). This second relation can be understood by associating to the first macro model a second setup, which takes consumption and investment decisions, i.e., by considering a long term growth setup. The link between the two is present on the impact of inflation over investment decisions. With this integrated framework one is able to simultaneously study short and long-run macroeconomic phenomena and to jointly analyze the behaviour of nominal and real aggregates. The most important results consist on the determination of an optimal inflation target and on the consideration of short term supply shocks as having a long-run impact producing business cycles.

Keywords: Monetary policy, Economic growth, Inflation targeting, Output gap.

JEL classification: C61, E52, O41

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1. Introduction

One of the most striking advances in macroeconomic theory along the past few years is the change of paradigm in the analysis of monetary policy. The new Keynesian model developed by Clarida, Gali and Gertler (1999), Svensson (1999), Woodford (1999, 2003), Gali (2002) and Svensson and Woodford (2003), among many others, became a central tool for the understanding of how short-run economic conditions are determined by the intervention of the monetary authority.

By controlling the nominal interest rate, the Central Bank has an important word to say about the trade-off between inflation and real stabilization of the economic system. It is known since the work of Kydland and Prescott (1977) that the dynamic inconsistency problem implies that no long-run trade-off exists and, thus, increasing the money supply (through a lower reference interest rate) in order to push output above its potential level has as only effect in the long-run rising inflation. As a consequence, it is today widely accepted that commitment to the policy goal of maintaining a low and stable rate of inflation should be the main, if not the only, concern of monetary authorities.

The new monetary policy paradigm has clearly Keynesian features: nominal aggregates (prices and wages) produce relevant effects in real economic activity (output and employment). In particular, it is important to understand that prices and wages are not adjusted continuously; they remain fixed for a more or less long period of time, that is, nominal variables tend to be sticky or sluggish to adjust, and when they are reconsidered they are set on the basis of expectations about future conditions of demand and supply, i.e., in a forward-looking way. This is why expectations play a fundamental role in the new monetary policy setup.

This monetary policy framework has received several modifications and improvements in its structure. The original framework considers a quadratic objective function and a linear Phillips curve. Various authors, like Cukierman (2000), Ruge-Murcia (2002, 2004), Nobay and Peel (2003), Dolado, Pedrero and Ruge-Murcia (2004) and Surico (2004), claim that a symmetric objective function does not represent properly the true policy problem, while other authors point batteries to the shape of the Phillips curve; Clark, Laxton and Rose (1996), Debelle and Laxton (1997), Schalling (1999), Tambakis (1999) and Akerlof, Dickens and Perry (2001), among others, present evidence and argue against a linear relation between the inflation rate and the output gap, in the short-run. Also, the forward-looking expectations hypothesis has been

relaxed, as it is the case in Jensen (2005). Despite this extensive literature that modifies the original setup, it is with this that we will work in order to present a unified macroeconomic framework.

The Keynesian character of monetary policy analysis collides with the long run view proposed by growth models since Solow (1956) to the endogenous growth approach of Lucas (1988) – Romer (1990). Growth models describe long term trends of growth in frictionless economies; the most widely discussed growth analytical structures resort to general equilibrium setups, where the absolute level of prices is irrelevant for the allocation of resources. Such allocation will be dependent only on relative price changes.

It is the goal of this paper to unify the two interpretations of the macroeconomic reality, since they can be thought in a complementary way: the first, the monetary policy approach, focus on the short-run and studies the impact of money and interest rates over the relation between prices and output; the second, has its main attention concentrated in the long run outcome of decisions through time regarding consumption, savings and investment.

The necessary link to unify the two theoretical benchmark models (the new Keynesian monetary model and a conventional growth model of the neoclassical or endogenous growth type) resides in the observation that general price level instability can cause severe distortions in real economic decisions, namely the ones concerning investment, and thus it has fundamental consequences over the long run growth capacity of the economy.

In the framework that will be proposed along the next sections, inflation is seen as a source of inefficiency relating investment decisions [the same applies for deflation, since the impact of this is taken simply as symmetric of the one created by inflation; see Gali, Gertler, Rotemberg, Uhlig and Woodford (2004) about the also perverse effects of deflation]. Generated income can be used as consumption and investment; the second component will correspond to a potential level of investment that will be fully concretized only in the circumstance of zero inflation. The more inflation departs from zero, the lower will be the share of investment that is effectively undertaken by private agents.

The introduction of the previously explained link in a standard growth model allows for a joint discussion of nominal and real macroeconomic events. We associate the monetary policy model to, both, neoclassical and endogenous growth frameworks to derive some interesting results:

(i) Considering disturbance terms in the monetary policy problem, as it is usual, these will end up to be present in the long run steady state results on growth. Particularly, output and consumption will be subject to supply side shocks, which allows for discussing business cycles under the growth framework.

This can be a possible answer to conciliate the two mainstream views on the cycles literature. On one side, we have the Real Business Cycles (RBC) theory, developed by Kydland and Prescott (1982), Long and Plosser (1983), King, Plosser and Rebelo (1988), and Christiano and Eichenbaum (1992), and that continues to be discussed and upgraded e.g. by King and Rebelo (1999), Jones, Manuelli and Siu (2000) and Rebelo (2005); on the other side, we encounter the Keynesian perspective, of Phelps (1970) and Lucas (1972), that relies on the analysis of market imperfections, nominal sluggish adjustment, incomplete contracts and strategic interaction of boundedly rational agents.

With our model, we take some of the RBC framework (the Walrasian growth model) and add some of the Keynesian perspective (supply shocks that are introduced in the growth analysis because price stickiness implies a short-run Phillips curve relation). Outside the analysis we leave problems concerning the functioning of markets and the central piece of the RBC discussion: the labour-leisure trade-off. Therefore, business cycles arise in the simple one equation consumption –capital accumulation benchmark growth problem, when this is adjusted in order to include the penalty of an unstable price level over investment aggregate levels.

(ii) It is possible to present a rational explanation to why the inflation target should not be set to zero, even though zero inflation is the one that allows for a full use of investment resources. Fundamentally, two contradictory effects are present. Inflation reduces effective investment, but it also has the short-run ability of helping to stimulate output above its potential level;

(iii) In the dynamic analysis to undertake, saddle-path stability will be the most common obtained result, what implies that stable trajectories among variables can be derived. These trajectories allow to establish important relations between the growth model variables (output, consumption and physical capital stock) and the variables in the monetary policy problem (inflation and output gap).

The remainder of the paper is organized as follows. Sections 2 and 3 present, respectively, the monetary policy framework and the growth setup with a real impact of inflation. Section 4 studies the integrated model under a neoclassical perspective. Section 5 considers a production function with constant returns and, thus, gives an

endogenous growth interpretation of the proposed macroeconomic model. Section 6 is destined to a few final remarks.

2. The Monetary Policy Problem

Short-run macroeconomic analysis can be conducted through the consideration of the Central Bank monetary policy problem. We describe this problem as an optimal control setup in which the monetary authority commits, on an initial moment $t=0$, with a time path for the nominal interest rate, i_t , in order to maximize the value of function V_0 ,

$$V_0 = -\frac{1}{2} \cdot E_0 \left\{ \sum_{t=0}^{+\infty} \beta^t \cdot [a \cdot (x_t - x^*)^2 + (\pi_t - \pi^*)^2] \right\} \quad (1)$$

Parameter $\beta < 1$ is an intertemporal discount factor and $a \geq 0$ refers to the weight put in a real stabilization goal relatively to the price stability objective. The state variables of the problem are the output gap, x_t , and the inflation rate, π_t . This second variable is simply the percentage change of the price level between two consecutive time periods, while $x_t = \ln \tilde{y}_t - \ln y_t$. Variable \tilde{y}_t is the effective level of output and y_t represents the potential level of output, the level of output that would be observable if hypothetically wages and prices were completely flexible. Thus, the output gap is defined as the difference between the logs of effectively observed real level of income and the level of income of a frictionless Walrasian economy. We consider a constant labour force, so that every real variable, like output, is presented as a per capita variable. Parameters x^* and π^* correspond to policy choices in the sense that they represent the output gap target and the inflation rate target selected by the Central Bank in order to achieve some meaningful economic goals.

Two rules concerning the evolution of state variables constrain the monetary policy objective. On the demand side, a dynamic IS equation relates the output gap to the real expected interest rate,

$$x_t = -\phi \cdot (i_t - E_t \pi_{t+1}) + E_t x_{t+1} + g_t, \quad x_0 \text{ given.} \quad (2)$$

Parameter $\varphi > 0$ is an interest rate elasticity, $E_t x_{t+1}$ and $E_t \pi_{t+1}$ represent private sector expectations regarding next period output gap and inflation rate, and g_t corresponds to a demand stochastic component. Variable g_t is defined as an autoregressive Markov process, $g_t = \mu g_{t-1} + \hat{g}_t, 0 \leq \mu \leq 1, \hat{g}_t \sim iid(0, \sigma_g^2)$.

On the supply side, the aggregate supply equation is assumed as a new Keynesian Phillips curve. This relates present inflation to the output gap and to the next period inflation expectations,

$$\pi_t = \lambda x_t + \beta \cdot E_t \pi_{t+1} + u_t, \pi_0 \text{ given.} \quad (3)$$

Parameter $\lambda \in (0, 1)$ defines the degree of price flexibility / stickiness, that is, it is an inflation–output elasticity. The higher the value of this parameter the lower will be the degree of price stickiness or rigidity. Variable u_t relates to a supply stochastic component, that is, it reflects possible cost push shocks. As in the demand case, an autoregressive process is assumed: $u_t = \rho u_{t-1} + \hat{u}_t, 0 \leq \rho \leq 1, \hat{u}_t \sim iid(0, \sigma_u^2)$.

The main properties of optimal control problem (1) subject to (2) and (3) are known from the literature. The computation of first order conditions allows for obtaining the second equation in system (4),

$$\begin{cases} \pi_{t+1} = \frac{1}{\beta} \cdot \pi_t - \frac{\lambda}{\beta} \cdot x_t - \frac{1}{\beta} \cdot u_t \\ x_{t+1} = \left(1 + \frac{\lambda^2}{\alpha\beta}\right) \cdot x_t - \frac{\lambda}{\alpha\beta} \cdot (\pi_t - \beta\pi^* - u_t) \end{cases} \quad (4)$$

The first equation in (4) is just the Phillips curve (3) rewritten (hereafter we neglect the expectations operators by considering perfect foresight); the second relates the next period output gap to the contemporaneous values of the output gap and inflation rate, when the time path of the interest rate variable is chosen in order to optimize the monetary authority behaviour. An important evidence concerning system (4) is that the time movement of the output gap and of the inflation rate are not in any way determined by demand shocks. The only source of stochasticity is the one associated with the supply side variable u_t . Since later we will associate the monetary policy problem with a growth framework, we will claim that this disturbance term will

affect output, capital and consumption time trajectories and, therefore, supply side shocks will be the main (in the case, the only) determinant of business cycles.

System (4) is linear with respect to the endogenous variables, and thus local and global dynamic properties coincide. These are synthesized through proposition 1.

Proposition 1. *System (4), which characterizes the short-run relation between the economy's real and nominal aggregates under an optimal monetary policy, has a unique steady state point, and the underlying dynamics are characterized by saddle-path stability. The steady state is $(\bar{\pi}, \bar{x}) = \left(\pi^*, \frac{1-\beta}{\lambda} \cdot \pi^* - \frac{1}{\lambda} \cdot \bar{u} \right)$ and the stable trajectory takes the form $x_t - \bar{x} = \frac{1-\beta\varepsilon_1}{\lambda} \cdot (\pi_t - \bar{\pi})$, where ε_1 is a positive constant value lower than 1.*

Proof: We define the steady state as a collection of trajectories $(\bar{\pi}, \bar{x}, \bar{u})$, such that $\pi_t = \pi_{t+1}$, $x_t = x_{t+1}$ and $u_t = u_{t+1}$. With respect to the disturbance variable, this is not time invariant in the steady state; according to the definition of the dynamics of u_t , we observe that $\bar{u} = \frac{\hat{u}_t}{1-\rho}$, what means that after the convergence process to the steady state is fulfilled, u_t becomes a stochastic stationary process with a standard deviation of σ_u and a zero mean. The other conditions that define the steady state allow for computing the values of $\bar{\pi}$ and \bar{x} by solving (4) under such conditions. Therefore, the long term equilibrium point is a unique fixed point.

To investigate what kind of stability is associated to (4), we need to present the system in matrix form,

$$\begin{bmatrix} \pi_{t+1} - \bar{\pi} \\ x_{t+1} - \bar{x} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta} & -\frac{\lambda}{\beta} \\ -\frac{\lambda}{\alpha\beta} & 1 + \frac{\lambda^2}{\alpha\beta} \end{bmatrix} \cdot \begin{bmatrix} \pi_t - \bar{\pi} \\ x_t - \bar{x} \end{bmatrix}$$

The Jacobian matrix in the above expression, which we designate by J , obeys to the following conditions,

$$1 - \text{Det}(J) = -\frac{1-\beta}{\beta} < 0;$$

$$1 - \text{Tr}(J) + \text{Det}(J) = -\frac{\lambda^2}{\alpha\beta} < 0;$$

$$1 + \text{Tr}(J) + \text{Det}(J) = 2 \cdot \frac{1+\beta}{\beta} + \frac{\lambda^2}{\alpha\beta} > 0.$$

These conditions imply that the system is saddle-path stable, that is, one of the eigenvalues of J is located inside the unit circle, while the other is a value outside the unit circle. More specifically, these eigenvalues are $0 < \varepsilon_1 < 1$ and $\varepsilon_2 > 1$,

$$\varepsilon_1, \varepsilon_2 = \frac{a \cdot (1+\beta) + \lambda^2}{2a\beta} \mp \sqrt{\left[\frac{a \cdot (1+\beta) + \lambda^2}{2a\beta} \right]^2 - \frac{1}{\beta}}$$

From the Jacobian matrix, one withdraws an eigenvector associated to ε_1 , which is, $p = \left[1 \quad \frac{1-\beta\varepsilon_1}{\lambda} \right]$. The second element of p is the slope of the stable trajectory; this trajectory passes through the steady state point and, thus, the stable trajectory can be written as in the proposition ■

In what concerns the steady state result, note that the optimization process implies that the long term inflation rate will correspond exactly to the selected target. The time trajectory of the nominal interest rate is designed in $t=0$ by the Central Bank in order to accomplish a steady state interest rate that guarantees an inflation rate that remains on its target. Relatively to the steady state output gap, this will not assume a constant value because it will be dependent on the supply disturbance. Therefore, even though monetary policy can give place to nominal long run stability, fluctuations will be observable in real aggregates.

The concern of the monetary authorities will not focus solely on the long-run outcome, but also on the stability properties that allow for achieving the long-run result. In particular, the Central Bank should choose an initial interest rate value, i_0 , that automatically puts the system over the only path that guarantees stability: the saddle-path. Once over the saddle-path, both state variables will converge to the unique steady state. Given the constraints that the parameters values obey to, one observes that

convergence to the steady state following the stable trajectory implies that the output gap rises with an increase in the value of the inflation rate and that the output gap falls for $\pi_0 > \bar{\pi}$, that is, when the inflation rate diminishes as it adjusts to the target value.

Replacing the stable trajectory in the Phillips curve, we may concentrate the dynamic analysis of the monetary policy problem in a single equation regarding the evolution of inflation through time,

$$\pi_{t+1} = (1 - \varepsilon_1) \cdot \pi^* + \varepsilon_1 \pi_t - \frac{1}{\beta} \cdot (u_t - \bar{u}) \quad (5)$$

The trajectory of x_t can then be withdrawn from the stable path relation. Figures 1 and 2 display the time paths of π_t and x_t , respectively, for some reasonable values of parameters. The most striking feature in these figures is that the inflation rate tends to a constant long term value (the impact of the disturbance term disappears), while for the output gap the volatility component is forever present.¹

*** Figures 1 and 2 here ***

3. The Growth Model, Inflation and Investment Decisions

The above monetary policy problem reflects the short-run dynamics of an economy. Nominal and real variables interact through aggregate demand and aggregate supply equations, and monetary authorities commit with an interest rate trajectory that is optimal, given the goals of maintaining the output gap and the inflation rate close to the chosen targets. Independently of the weight put on the output concern, the equilibrium level of inflation corresponds to the target.

Nevertheless, the model is silent about the long-run consequences of monetary policy, that is, consequences over the growth trend. If the monetary problem was the only important piece of economic reality, one would be compelled to ask why bother with a low inflation target if the economy's main concern is associated to increased output and, in the long run, as regarded, this is as much higher as the higher is the inflation target rate.

¹ Figures 1, 2 and all the following, are drawn using iDMC (interactive Dynamical Model Calculator). This is a free software program available at www.dss.uniud.it/nonlinear, and copyright of Marji Lines and Alfredo Medio.

In other words, the policy setup does not give a single clue about why should the Central Bank be concerned with maintaining price stability. Every economist recognizes that this doubt has a straightforward answer: instability in the evolution of prices can be a serious impediment for investment decisions. The uncertainty caused by changes in the monetary value of prices (hiding shifts in relative prices and making it hard to resort to credit) is the most serious threat for an environment that intends to stimulate investment. This argument is the one used in this paper to put together the standard new Keynesian monetary policy problem, in the form just described, and the Ramsey growth paradigm, which precisely deals with investment decisions and explains long-term trends of growth.

We interpret the Ramsey growth model as a mechanism that describes the evolution of the potential levels of per capita consumption, per capita physical capital and per capita output. In reality, these aggregates are only benchmark levels from which we withdraw effectively observable aggregate values; for instance, the effective level of output is $\tilde{y}_t = y_t \cdot e^{x_t}$, where y_t is determined on the Ramsey model and x_t on the monetary problem.

Consider y_t , c_t , k_t and j_t the potential per capita levels of output, consumption, physical capital and investment. The last aggregate, investment, is the potential level of investment in the absence of any aggregate price change. Inflation (or deflation) will mean that some investment projects will simply be overlooked and not undertaken, given the uncertainty that becomes attached to them. For extremely high price variations the level of investment will fall asymptotically to zero. To reflect this effect over investment, we define j_t' as the potential level of investment adjusted by inflation. Analytically, the following relation is established: $j_t' = j_t \cdot e^{-\theta\pi_t^2}$, where $\theta > 0$ reflects the impact of price changes over investment decisions (the higher the value of θ , the faster investment falls to zero as the inflation rises). Figure 3 depicts graphically this relation,

*** Figure 3 ***

Besides this relation between inflation and investment decisions, the Ramsey model will have its usual structure. We are referring to an intertemporal utility maximization framework, where utility is withdrawn only from consumption; the infinitely lived representative consumer solves the discounted problem (6).

$$\text{Max}_c \sum_{t=0}^{+\infty} \beta^t \cdot U(c_t) \quad (6)$$

Per capita consumption is the control variable of the problem and the instantaneous utility function should obey to $U' > 0$, $U'' < 0$ and $U' \rightarrow +\infty$ as $c_t \rightarrow 0$. To simplify the analysis, we just consider $U(c_t) = \ln(c_t)$. Problem (6) is constrained by a capital accumulation equation which is derived from a demand equation and from the definition of capital accumulation. Abstracting from inflation effects, the relation between output and demand comes $y_t = c_t + j_t$. Capital accumulation, in turn, is defined as the potentially undertaken investment, when inflation is considered, less a depreciation term, that is, $k_{t+1} - k_t = j_t - \delta k_t$, with $\delta > 0$ a depreciation rate and k_0 given.

Combining the two previous expressions, one arrives to the capital accumulation constraint under inflation effects over investment,

$$k_{t+1} - k_t = [f(k_t) - c_t] \cdot e^{-\theta\pi_t} - \delta k_t \quad (7)$$

In (7), we consider the production function $y_t = f(k_t)$, where $f(k_t)$ should have the standard properties on growth analysis, that is, positive and non increasing returns must be guaranteed. Later, we will distinguish neoclassical growth (decreasing marginal returns) from endogenous growth (constant marginal returns) in order to inquire about how monetary policy has different implications given different notions of long-run growth. For simplicity, we take a Cobb-Douglas production function, that is, $y_t = A k_t^\alpha$, with $A > 0$ a technological parameter and $\alpha \leq 1$ a positive output-capital elasticity.

As in the monetary policy setup, the dynamics of the Ramsey model are widely known, and therefore we do not spend time in deriving the consumption difference equation. The computation of first order conditions implies the following rule, which relatively to the standard form is just augmented by the presence of the inflation term,

$$c_{t+1} = \beta \cdot \frac{c_t}{e^{\theta\pi_t}} \cdot \left[\alpha A k_{t+1}^{-(1-\alpha)} + (1-\delta) \cdot e^{\theta\pi_{t+1}} \right] \quad (8)$$

where k_{t+1} is obtainable through (7) and π_{t+1} comes from (5).

As stated, two different growth interpretations can be studied just by assuming, alternatively, $\alpha < 1$ and $\alpha = 1$. This is done below. In synthesis, one has taken two widely

used macro setups: the first is focused on the role of monetary authorities and on the short-run relation between nominal settings and the stabilization of real output levels. The second concentrates on the private sector choices regarding consumption, savings and investment. We have established a link between the two analytical structures by assuming that inflation may injure the full extent in which investment resources can be efficiently allocated.

Therefore, the two problems can be analyzed separately but the results of both interfere with the outcome of the other: inflation will be exogenous for the Ramsey model but it will have an important influence over consumption, capital and output trajectories. The output result of the Ramsey model, in turn, should be used together with the output gap in the policy model to get to the time series that are the truly relevant from the representative agent utility point of view: the effective levels of output and consumption.

4. Decreasing Returns and the Optimal Inflation Target Rate

Consider, first, the neoclassical case ($\alpha < 1$). The Ramsey model result is given by proposition 2.

Proposition 2. *The neoclassical Ramsey model with investment decisions determined by aggregate price changes has a unique steady state,*

$$(\bar{k}, \bar{c}) = \left(\left[\frac{\alpha A \cdot e^{-\theta \cdot (\pi^*)^2}}{1/\beta - (1-\delta)} \right]^{1/(1-\alpha)}, A\bar{k}^\alpha - \delta\bar{k} \cdot e^{\theta \cdot (\pi^*)^2} \right). \text{ System (7)-(8) is saddle-path stable}$$

and the stable trajectory corresponds to $c_t - \bar{c} = \frac{1-\beta\zeta_1}{\beta} \cdot e^{\theta \cdot (\pi^*)^2} \cdot (k_t - \bar{k})$, with $\zeta_1 < 1$ a positive constant.

Proof: The balanced growth steady state corresponds to the trajectories (\bar{k}, \bar{c}) which obey to $k_t = k_{t+1}$ and $c_t = c_{t+1}$. Solving system (7)-(8) under these constraints, we obtain a unique result that is precisely the one in the proposition (note that the inflation steady state rate is withdrawn from the monetary policy problem).

To inquire about the nature of the model's dynamics, we write the Jacobian matrix associated to our system,

$$J_R = \begin{bmatrix} 1/\beta & -e^{-\theta(\pi^*)^2} \\ -(1-\alpha) \cdot [1/\beta - (1-\delta)] \cdot \frac{\bar{c}}{k} & 1 + \beta \cdot (1-\alpha) \cdot [1/\beta - (1-\delta)] \cdot \frac{\bar{c}}{k} \cdot e^{-\theta(\pi^*)^2} \end{bmatrix}$$

The following conditions hold,

$$1 - \text{Det}(J_R) = -\frac{1-\beta}{\beta} < 0;$$

$$1 - \text{Tr}(J_R) + \text{Det}(J_R) = -\beta \cdot (1-\alpha) \cdot [1/\beta - (1-\delta)] \cdot \frac{\bar{c}}{k} \cdot e^{-\theta(\pi^*)^2} < 0;$$

$$1 + \text{Tr}(J_R) + \text{Det}(J_R) = 2 \cdot \frac{1+\beta}{\beta} + \beta \cdot (1-\alpha) \cdot [1/\beta - (1-\delta)] \cdot \frac{\bar{c}}{k} \cdot e^{-\theta(\pi^*)^2} > 0.$$

These conditions support a saddle-path stable dynamic result. The two eigenvalues of J_R are $\zeta_1 \in (0,1)$ and $\zeta_2 > 1$, such that,

$$\zeta_1, \zeta_2 = \frac{\text{Tr}(J_R)}{2} \mp \sqrt{\left(\frac{\text{Tr}(J_R)}{2}\right)^2 - \frac{1}{\beta}}$$

An eigenvector associated to ζ_1 is $q = \left[1 \quad \frac{1-\beta\zeta_1}{\beta} \cdot e^{\theta(\pi^*)^2} \right]$; the second element of

q is the slope of the stable trajectory as represented in the proposition ■

An important steady state result respects to the influence of the inflation target over long term potential consumption and investment. According to the expressions in proposition 2, the higher is the inflation target, the lower will be the amount of accumulated capital in the long term, which comes as no surprise given the assumption about the relation inflation-investment; and the higher is that target, the lower is also the level of per capita long-run consumption. Given the straightforward relation between the capital stock and income present in the production function, the potential steady state output / income level is also negatively related to the inflation target.

Therefore, from the point of view of the potential levels of macroeconomic real variables, there is all the advantage in having a Central Bank that sets low inflation targets. The selection of the inflation target implies a conflict: the loss in investment is the effect that is under discussion when taking the potential levels of macro variables;

nevertheless, we are interested in effective levels of variables, and these receive the stimulus of an output gap effect that is as much stronger as the higher is the chosen π^* (recall the steady state of x_t). We will return to this discussion below. For now, one can withdraw some additional information from the result in proposition 2.

Note, in the first place, that the steady state results relate to variables in levels and that these results correspond to constant quantities. This means that the introduction of inflation does not change in any way the neoclassical nature of the growth model: all per capita aggregates are constant values in the long-run, unless some change occurs in a parameter value, e.g., the technology index or the discount factor. Because these are potential levels, they are not subject as well to supply shocks that we have identified as important in the short-run analysis.

Additionally, we have stated that endogenous variables converge to the steady state only in the circumstance where the representative agent selects c_0 in order to locate variables, from the beginning, over the stable arm. Supposing that this occurs, we obtain a stable trajectory that is qualitatively similar to the one in the original Ramsey model: an increasing capital stock is accompanied by an increasing level of consumption, given the positive slope of the stable trajectory. Once again, the only additional note goes to the role of the inflation target: a higher inflation target means a steeper relation between k_t and c_t in the convergence towards the steady state, and this is synonymous of a faster convergence to the long-run position.

As in the monetary policy model, the analysis of the Ramsey problem can be reduced to a single equation, if one replaces the stable trajectory in equation (7). The resulting dynamic relation is:

$$k_{t+1} - k_t = (Ak_t^\alpha - \bar{c}) \cdot e^{-\theta\pi_t^2} - \frac{1 - \beta\zeta_1}{\beta} \cdot (k_t - \bar{k}) - \delta k_t \quad (9)$$

Because (9) is derived from the stable arm, it is an equation with a stable equilibrium.

Short and long-run macro analysis can now be reduced to the examination of system (5)-(9). The respective steady state is a stable node, i.e., independently of (π_0, k_0) the steady state is always accomplished. The dynamics of the other two variables, x_t and c_t , may, then, be analyzed given the saddle-path relations.

In synthesis, both considered frameworks are saddle-path stable and assuming that the control variables (interest rate and consumption) can be manipulated (the first by

public authorities and the second by the private sector) in order to follow such stable arms, we obtain a fully stable system in which all relevant variables converge to the steady state. None of these variables is, however, the fundamental ones concerning the welfare of economic agents; these are the effective levels of output and consumption, to which we now turn.

Recall that $\tilde{y}_t = y_t \cdot e^{x_t}$. The definition of output in terms of demand implies that $\tilde{y}_t = c_t \cdot e^{x_t} + j_t \cdot e^{x_t}$; hence, we define $\tilde{c}_t = c_t \cdot e^{x_t}$ as the effective level of consumption and $\tilde{j}_t = j_t \cdot e^{x_t - \theta \pi_t^2}$ as the effective level of investment. This last one allows for a clear discussion of the benefits and costs of inflation targeting. Recover the relation between the inflation rate and the output gap in proposition 1. This relation implies that the exponent expression in the above condition is equivalent to $\frac{1 - \beta \varepsilon_1}{\lambda} \cdot \pi_t - \frac{(1 - \varepsilon_1) \cdot \beta}{\lambda} \cdot \pi^* - \frac{1}{\lambda} \cdot \bar{u} - \theta \pi_t^2$. Maintaining inflation above zero has a cost, which corresponds to the last term in the expression, but it has also a benefit, because with the stimulation of a positive output gap investment rises. Proposition 3 is a central result,

Proposition 3. *The inflation rate that maximizes effective investment is*

$$\pi_t^m = \frac{1 - \beta \varepsilon_1}{2\theta \lambda}.$$

Proof: Consider $g(\pi_t) = \frac{1 - \beta \varepsilon_1}{\lambda} \cdot \pi_t - \frac{(1 - \varepsilon_1) \cdot \beta}{\lambda} \cdot \pi^* - \frac{1}{\lambda} \cdot \bar{u} - \theta \pi_t^2$. Function g has an inverted U-shaped form, and hence a unique maximum exists. This maximum is the solution of $\partial g / \partial \pi_t = 0$. The solution is the inflation rate displayed in the proposition ■

According to proposition 3, the economy has an advantage in maintaining a positive rate of inflation that, however, must not be superior to π_t^m . The inflation rate π_t^m is the one to which the perverse effects of inflation over investment are better fought by the positive impact of pushing output above its potential level.

Economic agents cannot choose a single π_t^m over all time moments, since the evolution of inflation depends on a state constraint. Nevertheless, the Central Bank controls the inflation target. This is chosen in order to bound the price variation and, in

our monetary policy framework, it becomes the inflation level in the steady state. Therefore, one can investigate which is the inflation target that should be selected in order to maximize the long term level of per capita investment.

Proposition 4. *The inflation target concerning monetary policy decisions that maximizes the long term investment level is $\pi^* = \frac{1-\beta}{2\theta\lambda}$.*

Proof: Function $g(\pi^*) = \frac{1-\beta}{\lambda} \cdot \pi^* - \frac{1}{\lambda} \cdot \bar{u} - \theta(\pi^*)^2$ is such that $\bar{j} = j \cdot e^{g(\pi^*)}$.

Thus, the effective level of investment (relatively to its potential level) will be as higher in the steady state as the higher is the value of g . Once again $g(\pi^*)$ is an inverted U-shaped function, and its maximum corresponds to $\partial g / \partial \pi^* = 0 \Rightarrow \pi^* = \frac{1-\beta}{2\theta\lambda}$ ■

Proposition 4 gives an important policy indication. It says that, in order to maximize effective investment, the Central Bank should select a positive inflation target, which depends negatively on three parameters: the discount factor (hence, the higher the discount rate of future decisions the higher is also the required target rate), the parameter that translates the effect of inflation over investment losses, and the price flexibility parameter (stronger sluggishness of prices requires a higher inflation target).

The steady state analysis can be extended to variables output and consumption. Relatively to these, we can compare effective levels of the variables with the ones of a perfectly competitive economy without any kind of inefficiency or nominal effect. The original Ramsey model implies the following steady state values,

$$\bar{y}_c = A^{1/(1-\alpha)} \cdot \left[\frac{\alpha}{1/\beta - (1-\delta)} \right]^{\alpha/(1-\alpha)} \quad \text{and} \quad \bar{c}_c = \left[\frac{\alpha A}{1/\beta - (1-\delta)} \right]^{1/(1-\alpha)} \cdot \left[\frac{1/\beta - (1-\delta)}{\alpha} - \delta \right].$$

With the imposed conditions, effective output and effective consumption correspond, in

the steady state, to the following quantities, $\bar{y} = \bar{y}_c \cdot e^{(1-\beta)\pi^* / \lambda - \bar{u} / \lambda - \alpha\theta(\pi^*)^2 / (1-\alpha)}$; and

$$\bar{c} = \bar{c}_c \cdot e^{(1-\beta)\pi^* / \lambda - \bar{u} / \lambda - \alpha\theta(\pi^*)^2 / (1-\alpha)}.$$

Proposition 5. *The inflation rate target that maximizes the effective level of output, relatively to the competitive economy case, is $\pi^* = \frac{(1-\alpha) \cdot (1-\beta)}{2\alpha\theta\lambda}$.*

Proof: We want to maximize the value of function $h(\pi^*) = \frac{1-\beta}{\lambda} \cdot \pi^* - \frac{1}{\lambda} \cdot \bar{u} - \frac{\alpha\theta(\pi^*)^2}{1-\alpha}$, which implies solving $\partial h / \partial \pi^* = 0$ in order to obtain the result in the proposition. Once again, this is a maximum of the function because this corresponds to an inverted quadratic function ■

Comparing with the investment analysis, we have introduced a new factor that determines the optimal inflation target: the output – capital elasticity; the higher the value of this elasticity, the lower should be the target rate.

The important aspect to emphasize in the steady state analysis of effective levels of the variables is that two conflicting forces collide: the positive effect of the rising prices over the generation of income and the negative influence of inflation over investment decisions. There is an interval in which the first effect overcomes the second, namely, when

$$\pi^* \in \left[\frac{\frac{1-\beta}{\lambda} - \sqrt{\left(\frac{1-\beta}{\lambda}\right)^2 - \frac{4\alpha\theta\bar{u}}{(1-\alpha)\cdot\lambda}}}{\frac{2\alpha\theta}{1-\alpha}}, \frac{\frac{1-\beta}{\lambda} + \sqrt{\left(\frac{1-\beta}{\lambda}\right)^2 - \frac{4\alpha\theta\bar{u}}{(1-\alpha)\cdot\lambda}}}{\frac{2\alpha\theta}{1-\alpha}} \right].$$

The optimum π^* in proposition 5 is inside this interval. When the inflation target is outside this set, then the negative effect of price instability implies lower levels of output and consumption than in the frictionless case. In this way, it is not optimal to choose a zero target rate for inflation because this decision eliminates investment losses, but it eliminates also the benefits from the stimulus of output production over its potential level.

Our main argument, with important policy implications, is this: there is a role for monetary policy, in the sense it helps to produce a long term result concerning real economic growth that is preferable to the one that would be found on an economy that is capable of keeping zero inflation even without any public intervention. Our result is welfare enhancing relatively to the optimal result in the benchmark frictionless growth model.

The steady state output and consumption values have an additional important information to give, namely that our neoclassical growth model with monetary policy decisions will exhibit business cycles as a result of the supply side stochastic component

that is present in the Phillips curve. Figures 4 and 5 represent the time series of effective levels of output and consumption for reasonable values of parameters.

*** Figures 4 and 5 ***

5. Constant Returns and a Short-run / Long-run Stable Trajectory

The remarks in the previous section apply to an economy that does not grow in the long term. A parallel set of conclusions are now discussed under an endogenous growth setup ($\alpha=1$). In this case, capital and consumption dynamics reduce to

$$k_{t+1} = (Ak_t - c_t) \cdot e^{-\theta\pi_t^2} + (1 - \delta) \cdot k_t \quad (10)$$

$$c_{t+1} = \beta \cdot \frac{c_t}{e^{\theta\pi_t^2}} \cdot \left[A + (1 - \delta) \cdot e^{\theta\pi_{t+1}^2} \right] \quad (11)$$

For a constant steady state inflation rate, capital and consumption will grow at a same positive rate in the long-run; thus, we define the variable consumption – capital ratio, $\psi_t = c_t/k_t$, which is a constant value in the steady state.

The dynamics of ψ_t are given by,

$$\psi_{t+1} = \frac{\beta \cdot \left[A + (1 - \delta) \cdot e^{\theta\pi_{t+1}^2} \right]}{(A - \psi_t) + (1 - \delta) \cdot e^{\theta\pi_t^2}} \cdot \psi_t \quad (12)$$

and the corresponding steady state value is $\bar{\psi} = (1 - \beta) \cdot \left[A + (1 - \delta) \cdot e^{\theta \cdot (\pi^*)^2} \right]$. The higher the inflation rate, in the steady state, the more physical goods will be allocated to consumption rather than capital accumulation.

Note that (12) is an unstable difference equation and, thus, it is unlikely that the steady state is reached, unless the initial level of consumption is chosen in order to fulfil such goal. We can form a system of two equations with two endogenous variables putting together (5) and (12). This system will be, in the steady state vicinity, the following,

$$\begin{bmatrix} \psi_{t+1} - \bar{\psi} \\ \pi_{t+1} - \bar{\pi} \end{bmatrix} = \begin{bmatrix} 1/\beta & -2\theta \cdot (1-\beta)^2 \cdot (1-\delta) \cdot \pi^* \cdot e^{\theta(\pi^*)^2} \\ 0 & \varepsilon_1 \end{bmatrix} \cdot \begin{bmatrix} \psi_t - \bar{\psi} \\ \pi_t - \bar{\pi} \end{bmatrix} \quad (13)$$

Proposition 6. *System (5)-(12) is saddle-path stable and the stable trajectory is*

$$\psi_t - \bar{\psi} = \frac{1 - \beta\varepsilon_1}{2\theta\beta \cdot (1-\beta)^2 \cdot (1-\delta) \cdot \pi^* \cdot e^{\theta(\pi^*)^2}} \cdot (\pi_t - \pi^*).$$

Proof: The Jacobian matrix of (13) has two associated eigenvalues: ε_1 , which is inside the unit circle, and $1/\beta$, which is a value above 1. Therefore, saddle-path stability holds. An eigenvector of the eigenvalue ε_1 is

$$s = \begin{bmatrix} 1 \\ \frac{1 - \beta\varepsilon_1}{2\theta\beta \cdot (1-\beta)^2 \cdot (1-\delta) \cdot \pi^* \cdot e^{\theta(\pi^*)^2}} \end{bmatrix}. \text{ The second element of the vector is the}$$

slope of the stable arm in the proposition ■

If the level of consumption is initially chosen in order to follow the stable trajectory, the dynamics of convergence towards the steady state are such that a reduction of the inflation rate succeeds at the same time as the ratio ψ_t falls, that is the consumption level declines relatively to the accumulated stock of physical capital. Also, because there is a saddle-path positive relation between π_t and x_t , if it is sustainable to increase the output gap, consumption will rise relatively to capital accumulation.

Because in the present case all relevant variables grow at a same steady state rate, the impact of the monetary policy problem over the growth model must be analyzed in terms of growth rates. Note that, in what concerns potential levels of output, capital and consumption, the steady state growth rate result is:

$$\gamma_y = \gamma_k = \gamma_c = \frac{\beta}{e^{\theta(\pi^*)^2}} \cdot \left[A + (1-\delta) \cdot e^{\theta(\pi^*)^2} \right] - 1 \quad (14)$$

From (14), one understands that the potential growth of the real variables declines with higher long term inflation. In what concerns the growth of effective levels, we should note that $\gamma_{\bar{y}} \approx \gamma_y + \gamma_{e^x}$ and $\gamma_{\bar{c}} \approx \gamma_c + \gamma_{e^x}$, and therefore the growth rates we seek are

$$\gamma_{\bar{y}} = \gamma_{\bar{c}} = \frac{\beta}{e^{\theta(\pi^*)^2}} \cdot \left[A + (1 - \delta) \cdot e^{\theta(\pi^*)^2} \right] + e^{(\bar{u}_{t+1} - \bar{u}_t) / \lambda} - 2 \quad (15)$$

Analyzing (15), we conclude that effective output (and consumption) will grow at a rate around the potential output (consumption) growth rate, but that is not constant over time given the supply shock term. Figure 6 compares the two growth rates.

*** Figure 6 ***

In the case of endogenous growth one observes that selecting an inflation target above zero brings no gain of higher growth. The best policy in this case consists in aiming at a zero inflation rate. Nevertheless, the absence of growth effects does not mean that level effects are absent. As in the neoclassical case, although they will grow the same (on average), the level of effective output will be always higher than the level of potential output if the target inflation is set on an optimal value as in proposition 5.

6. Final Remarks

The new Keynesian monetary policy model takes as given an important strong assumption: the Central Bank should be concerned primarily with price stability, because inflation is a serious threat for the correct allocation of resources, what certainly harms the process of growth in the long run. In a strictly monetary analysis, the previous assumption is implicit and the focus is only on the short-run relation between the inflation rate and the output gap; no consideration is made about the trend of long run growth and how this is constrained by a high or low degree of price stability.

By turning the previous hypothesis explicit through the inclusion of a Ramsey growth setup into the monetary policy paradigm, one has combined short and long term macroeconomic analysis, what allowed finding some interesting dynamic relations.

Under a neoclassical growth model, the simultaneous monetary policy analysis has taken us to steady state results where business cycles are present and where it is clear the tension between two competing effects (the welfare enhancing effect of lower interest rates over effective output, although with a cost regarding price stability; and the harmful impact of price instability over investment). As a result, we found that the inflation target (which, in our framework, is always accomplished under the optimality

scenario) should be above zero, even if this means a sustained loss in the potential level of investment (when this loss is offset by the gain in terms of output gap).

The endogenous growth case revealed a stable trajectory relation where consumption falls relatively to accumulated capital, in the circumstance where the convergence to the steady state occurs for a decreasing inflation rate. In this model, we have also verified that effective output and consumption steady state growth rates are not constant when the supply side disturbance of the monetary model is present. Endogenous growth with business cycles is, in this way, supported by adding, to the positive and constant rate of potential growth, the growth rate of a non constant output gap term determined in the policy framework.

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Figures

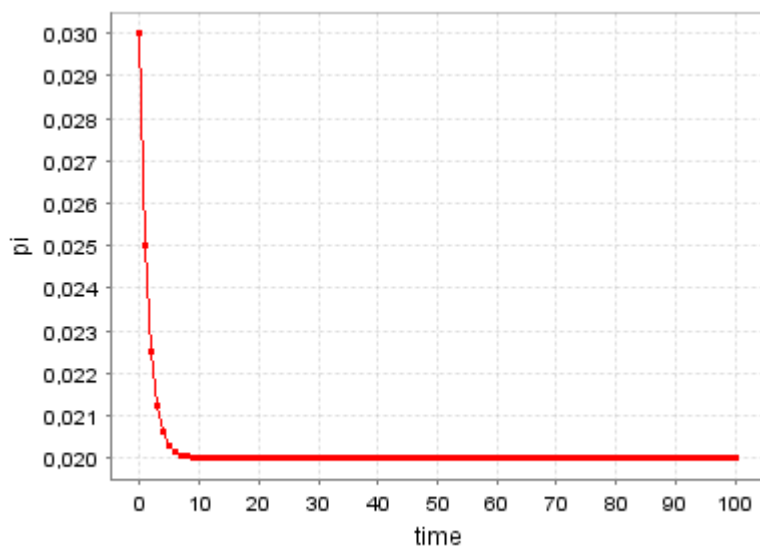


Figure 1. Inflation time path in the optimal monetary policy model [$\pi_0=0.03$; $x_0=0.01$; $\varepsilon_1=0.5$; $\beta=0.96$; $\pi^*=0.02$; $\lambda=0.75$; a disturbance term is included ($\rho=0.75$ and $\sigma_u^2=0.002$)].

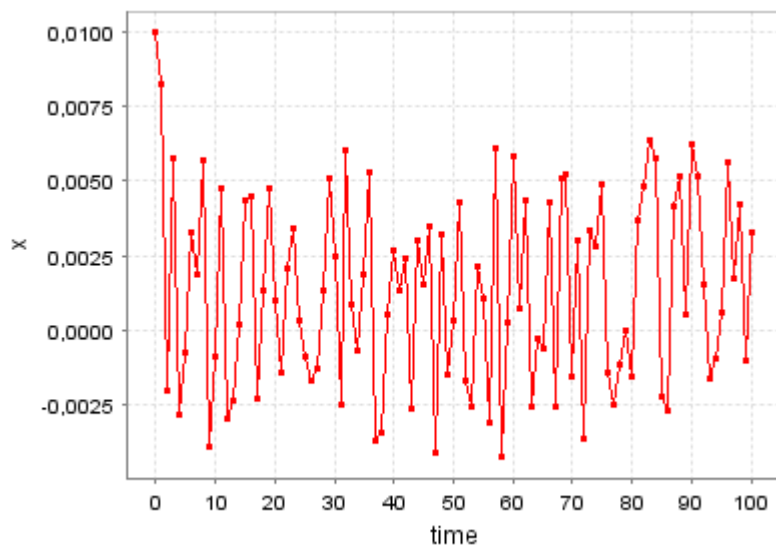


Figure 2. Output gap time path in the optimal monetary policy model [$\pi_0=0.03$; $x_0=0.01$; $\varepsilon_1=0.5$; $\beta=0.96$; $\pi^*=0.02$; $\lambda=0.75$; a disturbance term is included ($\rho=0.75$ and $\sigma_u^2=0.002$)].

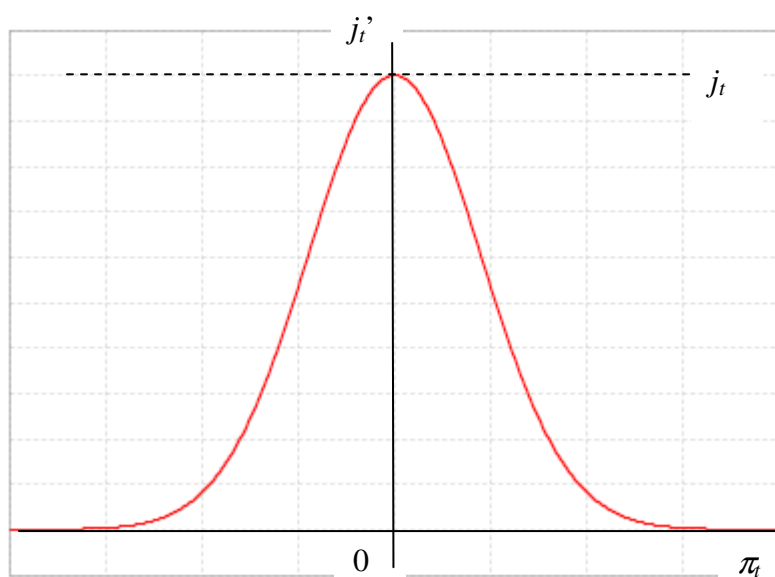


Figure 3. Investment levels and inflation.

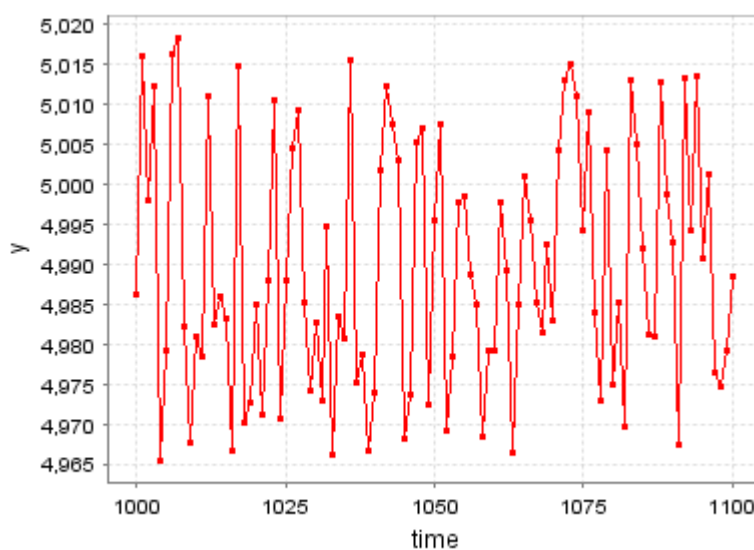


Figure 4. Long term effective output time path [$\bar{y}_c = 5$; $\alpha=0.75$; $\beta=0.96$; $\pi^*=0.02$; $\lambda=0.75$; $\theta=2.25$; a disturbance term is included ($\rho=0.75$ and $\sigma_u^2=0.002$)].

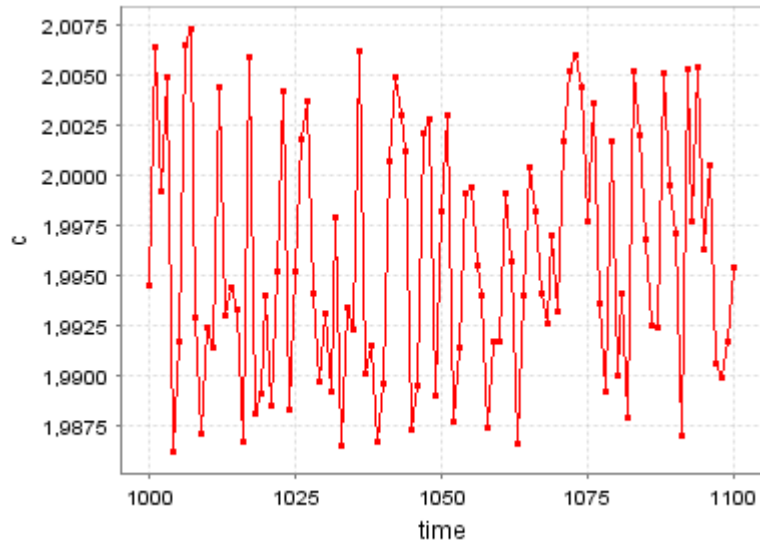


Figure 5. Long term effective consumption time path [$\bar{c}_c = 2$; $\alpha=0.75$; $\beta=0.96$; $\pi^*=0.02$; $\lambda=0.75$; $\theta=2.25$; a disturbance term is included ($\rho=0.75$ and $\sigma_u^2=0.002$)].

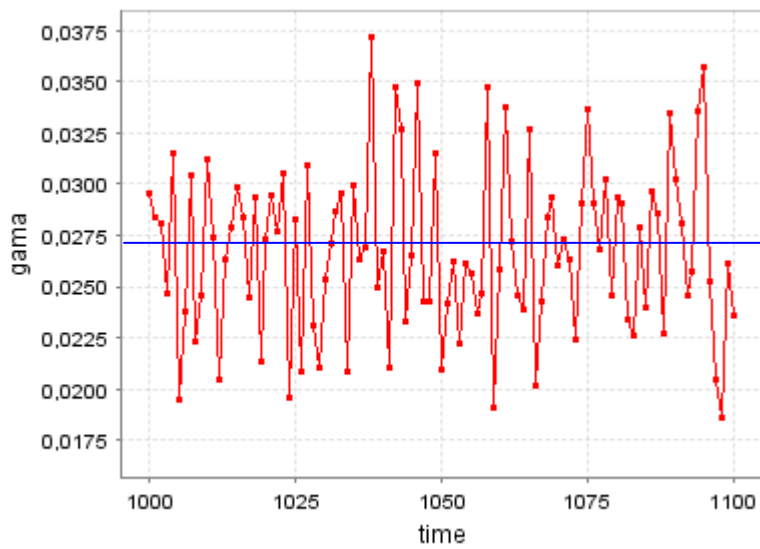


Figure 6. Growth rate of per capita output [potential (the one that displays a constant growth rate) and effective (the one with cycles)] in the endogenous growth model [$A=0.12$; $\beta=0.96$; $\pi^*=0.02$; $\lambda=0.75$; $\theta=2.25$; $\delta=0.05$; a disturbance term is included ($\rho=0.75$ and $\sigma_u^2=0.002$)].