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Estimating a Differentiated Products Model with a Discrete/Continuous Choice and Limited Data

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Abstract:

This paper specifies a vertically differentiated products model for a product with a discrete/continuous choice. The model is easily estimated with the relatively limited data used in classical demand equation estimation, supplemented by readily available market characteristics data. The model, with some modifications, is estimated with a new dataset (by state and region) for the U.S. Portland cement industry. Plausible patterns of own and cross price elasticities are obtained. The role of market characteristics is estimated generalizing the applicability of the results to other markets and periods.

1. Introduction

Estimation of differentiated products models, after beginning with the U.S. automobile industry has flowered into other consumer products (see Davis (2000) for a recent survey). This work has developed models for particularly visibly differentiated products, where the consumer makes a discrete choice, which are estimated with readily available specialized datasets of prices, sales and product characteristics. Even then, estimation remains demanding. Furthermore, for numerous products for which the framework is appealing, useful product characteristics data is not available. One of the implicit assumptions of the new empirical industrial organisation is that though industry specific models are developed, they are substantially generalisable. However, the focus on discrete choices and the specialized data and estimation requirements mean the generalization of structural differentiated products models has been limited to date.

This paper specifies a differentiated products model for a product that features both a discrete and continuous choice. The model is readily estimated with the type of data typically used in classical demand equation estimation. The discrete choice component is based on the vertical differentiation model used in Bresnahan (1987). The model is estimated with a new dataset on U.S. state and regional Portland cement consumption, construction, prices of building materials and other market characteristics. This dataset is typical of data available to researchers in a broad set of products. Furthermore the model is estimable using just instrumental variables estimation yet it yields estimates of structural own and cross price elasticities of demand. In addition, elasticities with respect to market characteristics are estimated. This paper demonstrates how structural models of demand for differentiated products can be extended to handle products with a discrete and continuous choice. This case is particularly important for modeling long run demand for products, for example, with switching costs. It demonstrates how these models can be applied to broader set of commodities, with more limited data, than the highly visible differentiated consumer products considered to date. Finally, a new set of estimates of demand elasticities for Portland cement is provided – an industry that has rarely been considered for demand estimation (see Gupta (1975) for the only direct example).

The paper proceeds as follows. In the next section, the cement industry and the data are introduced. At the end of this section, estimates of a set of linear demand equations are presented, demonstrating that further structure is required to explain demand - especially the cross-section variation. Then an estimable version of a generalized vertical differentiation model is derived - so to handle the mixed discrete/continuous choice and other features of the commodity and data. In section four, it is discussed how to handle complements, uncertain substitutes and other econometric issues. Then the estimated equations and elasticities are presented. Some suggestions for further work are presented in the conclusion.

2. The Demand for Portland cement: Background, Data and Preliminary Estimates

2.1 Background

Portland cement is the powder mixed with water, sand and aggregate that makes concrete. For a manufactured product, cement is essentially homogeneous, with no important differences across the types of Portland cement or across the products of different manufacturers (Prentice (1997)). Up until recently there has been no economic substitute for cement in making concrete in the United States.

Concrete is primarily used in construction and its importance differs significantly across different types of construction. The shares of cement in the cost of selected types of construction are presented in Table One. Streets and Highways feature the largest share, just over 2.5%.

Table One: Cost Share of Cement in Construction		
Category	Percentage	
All Construction	0.96	
New Construction	1.07	
One Family Housing	1.15	
Streets and Highways	2.51	
Farming	1.30	
Calculated by averaging requirements (including in concrete) in the Input-Output		
Statistics for 1972, 1977 and 1982, (Williams (1981, 1985) and Bureau of Economic		
Analysis (1991))		

Cement differs in importance, in part, because there are several substitutes for concrete such as asphalt, steel, wood and curtain wall products (U.S Department of Commerce (1987)). The substitutability of concrete varies considerably with the type of construction. While there are few substitutes for concrete for foundations and large dams, the use of concrete varies considerably across buildings and over time. For example, the Brutalist school, and other modernist architects made prominent and substantial use of concrete in a variety of buildings (Fleming et al (1980), Jencks (1985)). In addition, concrete houses were built in early 20th century U.S.A and Australia and are common in developing countries today. Although cement is substantially homogeneous, it is one of a set of differentiated products in the building materials industry, highlighted by the variety in the buildings around us.

2.2 Highlights of the Data

In this subsection, the highlights of the data are presented. The sample is composed of annual data on U.S. states and regions from 1956 to 1992. There are three sets of data. First, there is the cement and construction data. Second, there is the set of prices of building materials. Third, there is the set of market characteristics.

The characteristics of the cement and construction data are summarized in Table Two:

Table Two: Characteristics of the Cement and Construction Data			
Series	Aggregation	Availability	Sources
Cement	State	1956-1992 except	Bureau of Mines
		Hawaii, Alaska.	(1956-1992)
Construction	State	1967-1992, except	FW Dodge
		Hawaii, Alaska.	-
Construction	Regions	1956-1966	FW Dodge
NB: The components of the FW Dodge regions are listed in a Data Appendix available			
from the author.			

The construction data is the value of contracted new construction, supplied by F.W. Dodge Co., which is then deflated using a construction price index constructed in Prentice (1997). From 1956 – 1966, detailed construction data is provided for eight regions across the U.S. From 1967 on, state level data is provided. Hence, all other data was collected for 48 states and Washington DC for 26 years, and eight FW Dodge regions for 11 years, providing a total of 1362 observations. The coverage of construction is not

complete as Preston and Lipsey (1966) suggest rural building is particularly underrepresented. Comparison with the input-output data suggests over 90% of cement consumption is captured, but just 50-70% of new construction expenditure. However, the FW Dodge construction data is used commercially, as a base for official statistics and is the best available measure of detailed regional construction activity.

The quantities of construction and cement consumed are highly correlated over time. The ratio of national cement consumption to construction, presented in Graph One, demonstrates some cyclical variation without a trend. This is consistent with there being no evidence of any substantial technological change in cement use in construction over the period. This ratio is used repeatedly in the paper and is hereafter referred to as the cement-construction ratio.

There is, though, considerable regional variation as displayed in Map One. The plains states, and some energy producing states feature higher than average cementconstruction ratios. And the Atlantic states and Washington feature lower than average cement-construction ratios. These differences may reflect different mixes of construction or local prices but there is no immediate intuitive explanation for the regional variation.

In Table Three, the definitions and sources of the data on prices and market characteristics are summarized.

The second set of data is the prices of building materials. For cement and bricks, average prices are calculated at the source over states or a relatively small number of states. Where prices are calculated over groups of states, these prices are assigned to all states in the group. However, for the other substitutes, the Engineering News Record prices are collected across twenty cities.

Table Three: Data			
Variable	ariable Definition Source		
Prices of Building Materials			
Price-Cement	Average Annual Mill Price	Bureau of Mines Minerals	
		Yearbook (1956 – 1992)	
Price-Bricks	Average Annual Price of	Current Industrial Reports:	
	Building Bricks*	Clay Construction Products	
		(1957 – 1992)	
Price-Steel	Average – three types of	Engineering News Record	
	Building Steel*	(1956 – 1992)	
Price-Lumber	Average – four types of	Engineering News Record	
	Lumber*	(1956 – 1992)	
Price-Gravel	Average – two types of	Engineering News Record	
	Gravel*	(1956 – 1992)	
Price-Asphalt	Weighted average of three	Engineering News Record	
	types of Asphalt and Road	1956 – 1969, Dept. of	
	Oil.	Energy (1970 – 1992)	
	Market Characteristics		
Disaster Prone	Value of Losses for nine	Table 5.14, Petak and	
	types of natural disaster, by	Atkisson (1982)	
	state for 1970.		
Number of Hot Days		Statistical Abstract of the	
		United States (denoted SA)	
Average Temperature		SA	
Precipitation		SA	
Normal Minimum		SA	
Area		SA	
Annual Growth		SA, Bureau of the Census	
		(1975) (denoted HS)	
Share of Agriculture	Ratio Cash Receipts from	SA	
	Farm Marketing to State		
	Income		
Share of Industries	Employment Share of Eight	SA	
	Broad Industry Groups		
Catch-up Ratio	Ratio of Per Capita State	SA, (HS)	
	Income to Highest Income		
	State Income in 1950		
Income Growth	Growth 1950-1992	SA, (HS)	
Other Data			
Interest Rate	Real Moodys Aaa	Economic Report of the	
	Corporate Bond Rate	President. (various years)	

Where prices are not available for states, prices from neighboring (sometimes averages across various states) are used. All prices are deflated using the Implicit GDP Deflator, collected from the Economic Report of the President (various years).

The third set of data is the market characteristics data. There are three sets of characteristics: physical characteristics, economic structure and stage of development. First, the physical characteristics of the state, including its exposure to natural disasters, are likely to influence construction choices. For example, the demand will be lower in states which feature both high precipitation and extreme cold as this combination of weather makes concrete failure more likely (Lea (1970)). Demand will be higher in states which feature short lived strong winds, like tornadoes, as concrete tends to hold well with short-lived strong winds (Petak and Atkisson (1982)). Second, the economic structure, and short run demand fluctuations may affect the composition of construction in ways not captured by the different construction categories. Finally, the stage of economic growth may also affect the composition of construction. There is no formal theory of the effects of growth but Hayek (1939), as discussed in Montgomery (1995), suggests one pattern. Large, basic projects are then followed by smaller more specialized projects to obtain full value from the initial projects. Roads, dams, aqueducts are examples of basic projects featuring cement-intensive construction.

2.3 Estimating a Standard Demand Equation

In this section, instrumental variable estimates of standard demand equations are presented to demonstrate how they fail to adequately control for cross section variation and to suggest issues important for modeling and estimation in sections three - five. All regressions use the full sample of states and regions. The dependent variable is the cement-construction ratio. The explanatory variables are the prices of building materials, shares of construction types and time-varying market characteristics. As the prices of cement and gravel are locally determined, instruments are constructed.

The first regression (not reported here), with the prices of building materials as the sole explanatory variables, reveals this data features the heterogeneity associated with unit record data. Prices explain just 8.6% of the variation. Hence, fixed effects are introduced (using the within transformation to reduce multicollinearity).

In Table Four, extracts of the results of three regressions with fixed effects are presented. The first column contains the estimates of an equation with just the prices of building materials as explanatory variables. The second column contains the estimates of a second equation with the prices of building materials and the shares of different types of construction. The third column contains the estimates of a third equation with prices, construction shares and time varying market characteristics.

Table Four: Extracts from the Demand Equation Results			
Explanatory	Basic Specification	(1) & Construction	(2) & Time Varying
Variable	(1)	Shares	Market
		(2)	Characteristics
			(3)
Price – Cement	-0.3706 (-7.812)	-0.2286 (-5.756)	-0.2180 (-5.408)
Price – Brick	-0.03419 (-0.361)	-0.07376 (-0.947)	-0.0861 (-1.112)
Price – Asphalt	0.4138 (3.648)	0.5785 (5.114)	0.5938 (4.170)
Price – Steel	0.1765 (3.934)	0.1928 (5.173)	0.1863 (4.974)
Price – Lumber	0.0394 (1.588)	0.0624 (3.054)	0.0607 (2.947)
Price – Gravel	0.2379 (1.723)	0.1347 (1.200)	0.0532 (0.469)
$\overline{\mathbf{R}}^2$	60.04	75.71	77.26
Spearman Rank	23.158	19.121	10.3018
Correlation Test			
Condition Number	10.9593	1113.1	1817.55
F-Test Result	32.67 (vs. no fixed	38.19 (vs. column	9.73 (vs. column
	effects)	(1))	(2))

There are two things to note about these results. First, that the coefficients on prices have plausible signs and typically are statistically significant from zero. However, these demand equations fail to pick up the cross sectional variation identified in Map One. This is demonstrated by testing the relationship between the fixed effects and the dependant variable. First, the average cement-construction ratios over time for each state and region are calculated and ranked. Then the sizes of the fixed effects are ranked and the two sets of ranks used in a Spearman Rank Correlation Test. In all cases the null hypothesis of no correlation is rejected, suggesting the demand equation has failed to pick up the cross section variation.

To gain some information on what could be determining the cross sectional variation, the estimated fixed effects from the third specification are regressed on a set of non-time varying explanatory variables. Extracts of the results are presented in Table Five

Table Five: Extracts of the Results of the Regression of Fixed Effects		
on market characteristics.		
Variable	Coefficient (T-Statistic)	
Catch-up	0.0621 (5.078)	
Precipitation	-0.2292 (-11.339)	
Normal Minimum	0.3406 (5.080)	
Area	-0.0193 (-3.516)	
Flood	1.1686 (13.195)	
Storm Surge	0.7970 (13.825)	
Tornado	0.0759 (1.516)	
Hurricane	-0.6746 (-11.018)	
Strong Winds	11.2428 (4.048)	
Earthquake	-0.1442 (-4.037)	
Landslide	0.3654 (1.396)	
Expansive Soil	1.0310 (17.002)	
$\overline{\mathbf{R}}^2$	71.4	

A substantial proportion of the variation in the fixed effects is accounted for by the market characteristics. Not surprisingly, weather conditions and physical disaster losses influence the cement-construction ratios. Precipitation has a negative effect and strong winds a positive effect. The catch-up variable also positively affects cement intensity, supporting the hypothesis of Hayek (1939).

If the sole purpose of the analysis is to extract current elasticities for the cement industry then using fixed effects is fine. However, if we want to compare elasticities across countries or over longer periods of time, estimating the determinants of the cross section variation is important.

3. A Structural Model of the Demand for Portland cement

In this section, an estimable differentiated products demand model featuring a discrete and continuous choice is presented. In the first subsection, it is argued that cement consumption is based on a mixed discrete/continuous choice. Then the discrete choice component for each construction job is specified following the vertical differentiation model used by Bresnahan (1987). The quantities consumed for each job are then aggregated up to a state demand equation. This equation though features variables unobserved by the econometrician so the equation is manipulated until an estimable equation is obtained. Because meaningful product characteristics data does not exist, unlike Bresnahan (1987), the estimated equation is a structural equation. To explain the cross section variation, market characteristics are introduced as determinants of some coefficients.

An additional complication, specific to this industry, is to allow for a price inelastic component of the demand for cement because of building regulations or extreme physical conditions.

3.1 A Model with Discrete/Continuous Product Choice and Inelastic Demand

The demand for building materials has two features that require adapting the discrete choice framework that is the standard foundation for differentiated products models.

The first feature is that the demand for building materials is best characterized as featuring both a discrete choice and a continuous choice. For each component of a construction job a discrete choice of the building material to be used must be made. For example, the builder of a driveway could choose gravel, concrete, bricks or asphalt. This type of choice seems not dissimilar to a choice of model of automobile or other consumer product. However, a continuous component must be added as rather than buying one automobile, the quantity of the chosen material varies across consumers with the size of the construction job. It is assumed the continuous component is a linear function of the size of the construction job i.e. in effect there is constant returns to scale in construction. So the total quantity of the ith material consumed on a construction job, j, of type g is:

(1)
$$Q_{g,j}^i = \beta_g^i Q_{g,j}^{con}$$

where β_g^i is the per unit requirement coefficient for input i for construction type g. Different coefficients for different types of construction are assumed because Table One demonstrates different construction types feature different per unit consumption of cement. The mixed discrete/continuous specification, though required for building materials is also applicable to other commodities, particularly those featuring switching costs such as sunk costs before use. The second modification, though, is more specific to the construction industry. It is assumed there is a price inelastic component of demand. This is because building codes or industry practice requires the use of concrete for some jobs because only concrete provides, for example, sufficient strength or sealant powers. Hence there is a price inelastic component and a price elastic component of cement consumption. Then, the total cement consumed for a job j of type g is:

(2)
$$Q_{g,j}^{cem} = \begin{cases} \beta_{g,i}^{cem} k_g Q_{g,j}^{con} + \beta_{g,e}^{cem} (1 - k_g) Q_{g,j}^{con} \text{ if cement is chosen} \\ \beta_{g,i}^{cem} k_g Q_{g,j}^{con} \text{ otherwise} \end{cases}$$

where k_g is the share of construction type g that features a price inelastic component of cement and $\beta_{g,i}^{cem}$ and $\beta_{g,e}^{cem}$ are the input requirement coefficients for the inelastic and elastic components of cement consumption.

3.2 Modeling the Discrete Choice of a Building Material

The model of a discrete choice of a building material largely follows the adaptation of the vertical product differentiation model in Bresnahan (1987). This model has the disadvantage of a relatively restricted pattern of elasticities. While more general models exist (see Davis (2000) for a survey) for the (even multiple) discrete choice case, there is no existing model that handles the mixed discrete/continuous case required here and the Bresnahan (1987) model can be generalized to this case relatively easily.

Therefore, we start with a model of the decision maker and then aggregate up to a market demand equation. The decisionmaker, hereafter referred to as the client, has income, Y, to invest and chooses one of a set of construction projects composed of particular materials, or another investment, commonly referred to as the outside option. Because construction is substantially an investment good rather than a consumer durable

the utility from the outside option is modeled as dependent on the rate of return, r, rather than as a constant (as in Bresnahan (1987)):

(3)
$$U = Y(1+r)$$

The utility gained from investing in any one of the construction projects is specified as follows. Denote a as the taste for quality, x_i as the quality of the ith material and Pⁱ as its price.¹ The utility gained from the construction project is:

(4)
$$U(x, Y, a, i, P^{i}, \beta_{f}^{i}, Q_{f}^{con}) = ax_{i}Q_{g,j}^{con} + (Y - P^{i}\beta_{g}^{i}Q_{g,j}^{con})(1+r)$$

When the client chooses between the different materials, two factors are of concern: price and quality. Quality can be thought of as an index composed of different factors such as aesthetic appeal, strength, resistance to weather and insulation ability. It is assumed that, for broad classes of materials, clients, following the advice of their architects, agree on a ranking, in terms of quality, of the different materials that could be used. But, the importance of quality is assumed to differ across clients. In particular a is assumed to be distributed uniformly:

 $a \sim U[\underline{a}, a^{\max}]$ with density δ (Per capita).

Unlike Bresnahan (1987), the density of clients is expressed per capita rather than absolutely. This is to allow for variations in population across the states and over time.

The taste for quality plays an important role in determining the discrete choices of whether to invest in construction, and material to be used. For example, assume concrete is ranked as higher quality than asphalt but of lower quality than bricks. A client is

¹ The budget constraint does not enter the problem formally, but, as with Bresnahan (1987), is assumed to be satisfied. Introducing multipliers, etc., to control for the budget constraint would significantly complicate the specification without much gain.

indifferent between using concrete, c, or asphalt, b, if their taste for quality, a, equals a_{cb} such that:

(5)
$$a_{cb} = \frac{P^c \beta_g^c - P^b \beta_g^b}{x_c - x_b} (1 + r)$$

Denote, P^{sub} , x and β_g^{sub} as vectors of the prices, quality ratings and input requirements coefficients of the relevant materials. Then the indicator function, $I_j^c(P^{sub},x,\beta_g^{sub},a)$, for concrete, where the br subscript and superscripts refer to bricks, is defined as follows:

(6)
$$I_{j}^{c}(P^{sub}, x, \beta_{g}^{sub}, a) = \begin{cases} 1 \text{ if } \frac{P^{br}\beta_{g}^{br} - P^{c}\beta_{g}^{c}}{x_{br} - x_{c}}(1+r) > a > \frac{P^{c}\beta_{g}^{c} - P^{b}\beta_{g}^{b}}{x_{c} - x_{b}}(1+r) \\ 0 \text{ otherwise.} \end{cases}$$

The demand for cement for job j of construction type g, of size $Q_{g,j}^{con}$, is as below:

(7)
$$Q_j^{cem} = I_j^c \left(P^{sub}, x, \beta_g^{sub}, a \right) \beta_g^{cem} Q_{g,j}^{con}$$

where $\beta_g^{cem} = \beta_g^c \alpha_{cem}^c$ where α_{cem}^c is the quantity of cement required per unit of concrete.

3.3 A Model of State Cement Consumption

Because the data is aggregated at the state level the job specific model of the previous subsection will be aggregated up to the state level. Denote n_g as the number of jobs of construction type g. Each job requires different nonnegative quantities of each type of construction, $Q_{g,j}^{con}$.

The total quantity of cement consumed is then obtained by aggregating across all of the individual decisions represented in equation (7):

(8)
$$Q^{cem} = \sum_{g=1}^{G} \sum_{j=1}^{n_s} \Gamma_j^c (P^{sub}, x, \beta_g^{sub}, a) \beta_g^{cem} Q_{g,j}^{con} = \sum_{g=1}^{G} \beta_g^{cem} Q_g^{con-cem}$$

where $Q_g^{con-cem}$ is the quantity of construction of type g that uses cement. This does not yield an equation for estimation in terms of observable exogenous variables as $Q_g^{con-cem}$ is not observed. To get around this problem, first denote, \overline{Q}_g^{con} as the average quantity of construction of type g and $\varepsilon_{g,j}^{con}$ as the deviation from the mean for job j. Then, (8) can be rewritten as:

(9)
$$Q^{cem} = \sum_{g=1}^{G} \sum_{j=1}^{n_s} \mathbf{I}_j^c \left(P^{sub}, x, \beta_g^{sub} \right) \beta_g^{cem} \left(\overline{Q}_g^{con} + \varepsilon_{g,j}^{con} \right)$$

Because, for each construction type, the deviation averages out, (White (1984)), this leaves us with (10):

(10)
$$Q^{cem} = \sum_{g=1}^{G} \sum_{j=1}^{n_s} \mathbf{I}_j^c \left(P^{sub}, x, \beta_g^{sub} \right) \beta_g^{cem} \overline{Q}_g^{com}$$

Denote $n_{g,cm}$ as the number of projects that use cement. Equation (10) can then be rewritten as:

(11)
$$Q^{cem} = \sum_{g=1}^{G} n_{g,cm} \beta_g^{cem} \overline{Q}_g^{con}$$

The $n_{g,cm}$, being unobservable, will now be substituted out. First, multiply and divide (11) by n_g yielding:

(12)
$$Q^{cem} = \sum_{g=1}^{G} \left(\frac{n_{g,cm}}{n_g} \right) \beta_g^{cem} Q_g^{con}$$

Note, Q_g^{con} is observable. However $n_{g,cm}$ and n_g are not. Following Bresnahan (1987), $n_{g,cm}$ for each state s can be replaced as follows:

(13)
$$n_{g,cm} = \delta * pop_s * [a_{ac} - a_{bc}] = \delta * pop_s * (1 + r) * \left[\frac{P^a \beta_a - P^c \beta_c}{x_a - x_c} - \frac{P^c \beta_c - P^b \beta_b}{x_c - x_b} \right]$$

Rewriting equation (12) using (13) yields:

(14)
$$Q^{cem} = \sum_{g=1}^{G} \left(\frac{\delta}{n_g} \right) pop_s \beta_g^{cem} Q_g^{con} [a_{ac} - a_{bc}] (1+r)$$

This leaves n_g to deal with which is neither observable nor can be assumed to be constant. However, n_g can also be replaced. First note all clients with a* as defined below are indifferent between taking the construction project and just investing the funds in the outside option:

(15)
$$a^* = \frac{\left(P^b \beta_g^b (1 - k_g) + k_g P^{cem} \beta_g^{cem} \right)(1 + r)}{x_b (1 - k_g) + x_c k_g}$$

As long as P^b , the bottom ranked construction material, P^{cem} and r are negatively correlated – which is quite likely as the two materials are inputs to investment, a* will be roughly constant. Assuming a* is constant, n_g can be replaced as follows:

(16)
$$n_g = pop_s \delta(a_{\max} - a^*)$$

Substitution of (16) into (14) yields:

(17)
$$Q^{cem} = \sum_{g=1}^{G} \beta_g^{cem} Q_g^{con} [a_{ac} - a_{bc}] (1+r) (a_{max} - a^*)^{-1}$$

Finally, the price inelastic and price elastic components of cement consumption are added to yield a differentiated products demand equation for Portland cement:

$$(18) \ Q^{cem} = \sum_{g=1}^{G} k_g \beta_g^{cem} Q_g^{con} + \sum_{g=1}^{G} (1-k_g) \beta_g^{cem} Q_g^{con} (1+r) \left(\frac{\frac{\beta_g^a P^a}{x_{a,cem,g}} + \frac{\beta_g^b P^b}{x_{cem,b,g}} - \frac{\beta_g^{cem} x_{a,b,g} P^{cem}}{x_{a,cem,g} x_{cem,b,g}} \right) = \frac{\beta_g^{cem} x_{a,b,g} P^{cem}}{a_{max} - a^*}$$

where $x_{i,j}$ is the difference $(x_i - x_j)$ in the quality indices and state and time subscripts are suppressed. This equation is estimated with the same set of variables required for a classical demand equation – the prices of building materials and construction as a defacto income. The interactive terms arise from the mixed discrete/continuous choice that forms the foundation of the model.

Cross-section variation is accounted for by making the coefficients a function of the market characteristics. In the absence of rapid technological change in construction materials comparable to cement, the values of the quality index are unlikely to change. However, as argued earlier, the coefficients β_g^{cem} are likely to vary across construction types and with market characteristics. Hence, the β_g^{cem} can be re-expressed as follows:

(19)
$$\beta_g^{cem} = \beta_{0,g} + \beta_{h,g} Z$$

where Z is a set of market characteristics. Equations (18) and (19) are the theoretical basis for the estimation in Section 5.

4. Econometric Issues

Before beginning estimation, there are three major econometric issues that must be dealt with. First, a set of prices of building materials for each type of construction must be chosen – both substitutes and complements. Second, it is demonstrated that equations (18) and (19) reduce to a linear form that is relatively simple to estimate with instrumental variables. Finally, the treatment of market characteristics is discussed.

Strictly replicating Bresnahan (1987, 1981) requires beginning with a set of initial rankings so to select those to be compared with, in this case, cement. The market characteristics data available in this case does not permit this. Omitting a relevant material results in inconsistent estimates, while including an irrelevant variable only reduces efficiency. Hence we select a set of building materials that could be ranked around cement, and include these in the demand equation, interacting all of them with the relevant variables as if they were the relevant prices. The model is then supported if two of the coefficients are significantly positive.

The related problem is that certain materials may be complementary rather than substitutes for cement e.g. steel for reinforced concrete. Potential complements are treated the same way as potential substitutes. They are included in the demand equation, interacting with the relevant variables. Those inputs with coefficients significantly negative are considered complements. Similarly, if more than two inputs prove to be significant this suggests they are complementary to the substitutes. Ultimately, as in most estimation of this type, the industry knowledge and judgment of the researcher will have to be relied on to decide whether a negative coefficient is evidence of complementarity or misspecification.

The second issue is choice of estimation technique. The model specified in equations (18) and (19), though theoretically identified, is highly non-linear. However, the model reduces to a linear form that retains key features and, importantly, enables identification of the structural elasticities. The linear form estimated in section five is summarized below:

(20)
$$\frac{Q^{cem}}{Q^{con}(1+r)} = \sum_{g=1}^{G} \gamma_g \frac{s_g^{con}}{(1+r)} + \sum_{g=1}^{G} \sum_{i=1}^{I_s} \theta_{g,i} s_g^{con} P^i$$

(21)
$$\gamma_g = \gamma_{0,g} + \gamma_{char,g} Z_g$$

where s_g^{con} is the share of total construction of construction type g, γ and θ are the reduced form combinations of the structural coefficients in (18) and (19) and I_g and Z_g are

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the set of materials and market characteristics for construction type g. Though the individual parameters (β_g^{mat} , k_g , a^{max} , a^* , x_{ij}) cannot be identified from this specification, the structural elasticities can be estimated. The own price, $\varepsilon_{p,cem}^Q$, cross price, $\varepsilon_{p,mat}^Q$ and characteristics, ε_{char}^Q , elasticities are stated below:

$$\varepsilon_{p,cem}^{Q} = \sum_{g=1}^{G} \frac{\theta_{g,cem} Q_{g}^{con} P^{cem} (1+r)}{Q^{cem}}$$
$$\varepsilon_{p,mat}^{Q} = \sum_{g=1}^{G} \frac{\theta_{g,mat} Q_{g}^{con} P^{mat} (1+r)}{Q^{cem}}$$
$$\varepsilon_{char}^{Q} = \sum_{g=1}^{G} \frac{\gamma_{g,char} Q_{g}^{con} Z_{char}}{Q^{cem}}$$

The third issue is the treatment of the market characteristics variables. If the coefficients $\gamma_{g,char}$ are all equal to zero, then the sign of each reduced form coefficient is specified unambiguously by the model. The γ 's will all be positive, the θ 's on cement and any complements will be negative, and the θ 's on the other prices either positive or insignificantly different from zero. However the evidence from section three suggests several market characteristics are required to explain the cross section variation. The market characteristics variables used are Normal Minimum, Precipitation, Catch Up, Income Growth, Share of Agriculture and Losses due to Tornadoes and Expansive Soil. While economic theory does not provide signs for the $\gamma_{char,g}$ priors based on industry literature enable some of them to be evaluated – for the others, their role will be demonstrated by the results.

Finally, the treatment of endogenous explanatory variables and multicollinearity is discussed. As for the standard demand equations instruments were constructed for the prices of cement and gravel – the building materials for which the prices were most likely to be set locally. The other econometric problem is dealing with multicollinearity. The results from the standard demand equation estimation suggests, suitably transformed, the price data is not collinear but as construction and other forms of heterogeneity variables are introduced, the interaction terms are likely to cause problems. Hence the quantity of cement is divided by the quantity of construction and (1+r). Then all variables are centered on their means before estimation. Finally the data is scaled to similar levels. This should reduce problems from multicollinearity.

5. The Results

In this section, three sets of results are discussed. First, the demand equations (20) and (21) are estimated with the $\gamma_{char,g}$ set equal to zero. Second, a set of elasticities compiled from estimates of the demand equations (20) and (21) with the $\gamma_{char,g}$ allowed to differ from zero are discussed. Finally, the coefficients underlying these elasticities and a more general specification are discussed.

First, equations (20) and (21), without market characteristics, are estimated. Though potentially inconsistent, the theoretical model provides unambiguous predictions of the signs of the coefficients, which enables assessing, on their own terms, if the model is supported in the data. Two versions are presented – the first with one construction type, and the second with two types: Roads and Rest of Construction. The results are presented in Table Six:

Table Six: Standard Specifications			
Explanatory Variable	One type of Construction	Two types of Construction	
Constant*	-10.0717 (-4.7346)		
Share of Roads		1.7615 (9.4217)	
Rest of Construction		1.0273 (5.5364)	
Price-Cement	0.0002 (0.0447)	-0.0032 (-0.8235)	
Price-Brick	0.0266 (3.1914)	0.02714 (3.4059)	
Price-Asphalt	0.0011 (0.0713)	0.1202 (6.0855)	
Price-Steel	-0.0265 (5.0943)	-0.0348 (-6.8504)	
Price-Lumber	0.0093 (3.2624)	0.0012 (0.4444)	
Price-Gravel	0.1072 (7.6892)	0.0968 (7.7772)	
RSS	1501.66	1194.07	
Condition Number	533.064	103.999	
The constant term is actually the coefficient on $(1/1+r)$.			
T-statistics are in parentheses.			

The first set of estimates is unsatisfactory because the constant term is significantly less than zero and the price of cement is statistically insignificant. However, when two types of construction are introduced (only in the "intercept" terms), the coefficients on the types of construction are significantly positive. The coefficient on the price of cement is now negative though statistically insignificant. Additional support for the model is gained from there being at least two prices of substitutes that have significantly positive coefficients. The price of steel has a significantly negative coefficient suggesting it is a complement. These results are broadly supportive of the underlying model.

In the next set of regressions, market characteristics variables are included through equation (21). As is discussed below, results are obtained consistent with the model. Hence, as the elasticities are identified these are discussed first as presented in Table Seven:

Table Seven: Elasticities			
Prices		Characteristics	
Variable	Elasticity	Variable	Elasticity
Cement	-0.297	Normal Minimum	0.008
Brick	0.137	Precipitation	-0.005
Asphalt	0.168	Catch-Up	0.015
Steel	-0.122	Agriculture	0.009
Lumber	-0.006	Tornado	0.004
Gravel	0.100	Income Growth	0.001
		Expansive Soil	0.008

The own-price and cross price elasticities suggest relatively little substitutability across inputs, but the pattern across inputs is quite plausible. All cross-price elasticities are smaller than the own price elasticity. Asphalt features the largest cross price elasticity and steel is again found to be a complement. The coefficient on lumber is statistically insignificantly different from zero.

Next the characteristics elasticities are considered. First note that all coefficients on the characteristics are significantly different from zero, except economic growth. But cement demand is inelastic to small changes in these characteristics. This is not inconsistent with the map presented earlier as the usage ratios are similar in similar states but different across substantially different states. Catch-up is the largest, providing some support for the pattern of growth suggested in Hayek (1939).

Next the coefficients on the two regressions with market characteristics are presented in Table Eight. The first column (Regression One) allows for different market characteristics to interact with the different types of construction but not the coefficients on the prices. These results are quite successful and are used to calculate the elasticities in Table Seven. The second set (Regression Two) allows for different coefficients on the prices as well – but these results are less successful.

Table Eight: Coefficients			
Explanatory Variable	Regression One	Regression Two	
Rest	1.1128 (6.8541)	0.9852 (6.805)	
Roads	1.7640 (10.2046)	1.9949 (2.381)	
Roads – Normal Minimum	0.0012 (4.4865)	0.0013 (4.7159)	
Roads – Precipitation	-0.0005 (-2.5356)	-0.0009 (3.243)	
Rest – Catch-up	0.0001 (4.6817)	0.0001 (3.7287)	
Rest – Agriculture	0.0002 (8.6483)	0.0002 (6.5064)	
Rest – Tornado Damage	0.0004 (5.3423)	0.0004 (5.2137)	
Rest – Growth	0.0002 (0.6665)	-0.0001 (-0.3619)	
Rest – Expansive Soil	0.0008 (8.6416)	0.0008 (7.936)	
Cement	-0.0195 (-5.083)		
Brick	0.0321 (3.8457)		
Asphalt	0.1130 (5.9573)		
Steel	-0.0137 (-2.9321)		
Lumber	-0.0004 (-0.1691)		
Gravel	0.0410 (3.4361)		
Rest – Cement		0.00001 (0.0746)	
Rest – Brick		0.0003 (2.5801)	
Rest – Steel		-0.0001 (-1.8236)	
Rest – Lumber		-0.00002 (-0.5132)	
Roads – Cement		-0.0019 (-1.2965)	
Roads – Asphalt		0.0089 (4.8599)	
Roads – Gravel		0.0042 (3.1313)	
RSS	885.825	885.387	
Condition Number	164.801	579.045	
T statistics are in parentheses.			

First, consider regression one. The signs on the construction and heterogeneity variables are all plausible. Importantly, there are at least two significantly positive coefficients on the price variables. The signs on the coefficients on prices have been discussed further above. The effects of precipitation and tornadoes are consistent with that suggested in the engineering and disaster literature. It is nice to see roads with a larger coefficient.

Regression two appears less successful - probably because of multicollinearity. The residual sum of squares has barely fallen at all. The coefficients on the heterogeneity variables have not changed very much. For the group of prices based around roads, the results are broadly satisfactory as both asphalt and gravel are positive and significant from zero. Cement is negative though insignificant. However, the "rest of construction" coefficients are less satisfactory. Cement is just insignificant but positive. Furthermore, only brick is positive and significant, though steel remains negative and significant. These discouraging results could be due to two causes. First, the condition number has crept up in the second regression so multicollinearity may be starting to create problems. More seriously, we may be coming up to the limits of what we can get out of the price data.

To summarize, these results suggest we have successfully estimated a differentiated products model featuring a discrete/continuous choice and using largely classical demand data. The signs of the coefficients are consistent with the theory - and departures plausible. The pattern of elasticities is also plausible. Multicollinearity - in part from the functional form and in part from the limits of the data - appears to be a problem.

5. Conclusions

There are ongoing important advances in estimating differentiated products models. In general, though, these have focused on visibly differentiated consumer products best characteristed as requiring a discrete choice. The case of a mixed discrete/continuous case has not been considered. In this paper we develop a differentiated products model for a product featuring a discrete and continuous choice and that is readily estimable with the more limited data typically used in classical demand equation estimation. The model also handles complementary products and researcher uncertainty about which inputs are substitutes and complements. This model is estimated with a new dataset on U.S. state and regional Portland cement consumption, prices and market characteristics. A plausible set of own price and cross price elasticities are obtained. Finally, the model is easily applicable to a broad range of products - including those with switching costs.

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Chart One

Map One

