Time is money

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Abstract. Specialized topics on financial data analysis from a numerical and physical point of view are discussed when pertaining to the analysis of coherent and random sequences in financial fluctuations within (i) the extended detrended fluctuation analysis method, (ii) multi-affine analysis technique, (iii) mobile average intersection rules and distributions, (iv) sandpile avalanches models for crash prediction, (v) the (m, k)-Zipf method and (vi) the i-variability diagram technique for sorting out short range correlations. The most baffling result that needs further thought from mathematicians and physicists is recalled: the crossing of two mobile averages is an original method for measuring the "signal" roughness exponent, but why it is so is not understood up to now.

1 Introduction

It is fortunate to recall from the start that Louis Bachelier (1870-1946) was a mathematician at the University of Franche-Comté in Besançon. He was the first to develop a theory of Brownian motion, – in his Ph. D. thesis [1], in fact for the pricing of options in speculative markets. Later on he wrote down what is known as the Chapman–Kolmogorov equation. Alas, he motivated his approach on what is known nowadays as the *efficient market theory*, basically a Gaussian distribution for the price changes. This is known to be incorrect, at least for economic indices [2,3].

Recently the statistical physics community has been reattracted into investigating economic and financial problems. Two modern reasons can certainly be given: (i) economic systems, like stock markets produce quite complex signals due to a high number of parameters involved, and (ii) models developed so far in actual econometry do seem irrelevant for mimicking available signals, – at least on the level expected by usual physical models for natural phenomena. A list of recent progress is too long to be cited or discusssed here. Several books are already of interest. One aim should be first to review a few technical details in a global context. Even for a general audience, with mathematical orientation, it is in fact hopefully possible to

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give nonrigourous informations on how physicists pretend that an increase in revenue can be obtained if general rules are found from non linear dynamic-like analysis of financial time series. Some general views have been already presented in "Money Games Physicists Play" [4]. More specialized topics are discussed here as were already sketched in ref. [5].

There are six methods or so that we have been discussing and using in the Liège GRASP[1], when performing investigations in the context of sorting deterministic features from apparently stochastic noise contained in economic and financial data. The investigations pertain to considerations on different time correlation ranges. First it has been observed a long time ago that stock market fluctuations were not Brownian motion-like, but some long range correlation existed [3]. We have tested that idea on foreign exchange currency (FXC) rates [6]. Using the detrended fluctuation analysis method (DFA), it was shown that profit making in the FXC market can be made by bankers. This leads to the introduction of a turbulence-like picture in order to discuss the spareness and roughness of FXC rates. It can be shown that not all FXC rates belong to the same so-called universality class, but nobody knows at this time why, nor what universality classes exist.

Next, some medium range correlation can be discussed. First, a technique due to stock analysts, known as the mobile (or moving) average technique which allows for predicting gold or death crosses, whence suggesting buying or selling conditions will be discussed. It can be shown to be a rather delicate (a euphemism !) way of predicting what to do on a market. This will lead to a very interesting, and apparently unsolved problem for mathematicians and physicists. Moreover, the behavior of major stock market average indices will be recalled, and it will be observed that so-called crashes have well defined precursors. The crash of October 1987 could be seen as a phase transition [7]. The amplitude and the universality class can be discussed as well, thereby indicating how the financial crash of October 1997 could have been (and was) predicted. This will lead to indicate a microscopic model for such a set of crashes, model based on sandpile avalanches on fractal lattices. This will lead to emphasize a very interesting problem for the dynamics of numbers.

Moreover, a claim will be substantiated that the (m, k)-Zipf analysis and low order variability diagrams can be used for demonstrating short range correlation evidence in financial data.

2 Detrended Fluctuation Analysis Techniques

The Detrended Fluctuation Analysis technique consists in dividing a time series or random one-variable sequence $y(n)$ of length N into N/T nonoverlapping boxes, each containing T points [8]. Then, the local trend in each box

[1] GRASP = Group for Research in Applied Statistical Physics
http://www.supras.phys.ulg.ac.be/statphys/statphys.html

is *a priori* defined. A linear trend $z(n)$ like

$$z(n) = an + b, \tag{1}$$

or a cubic trend like

$$z(n) = cn^3 + dn^2 + en + f, \tag{2}$$

can be assumed [6,9]. In a box, the linear trend might be way-off from the overall intuitive trend, henceforth shorter scale fluctuations might be missed if the box size becomes quite large with respect to the apparent short time fluctuation scale of the signal. Thus the interest of using non linear trends. The parameters a to f are usually estimated through a linear least-square fit of the data points in that box. The process is repeated for all boxes. The detrended fluctuation function $F(T)$ is then calculated following

$$F^2(T) = \frac{1}{T} \sum_{n=kT+1}^{(k+1)T} |y(n) - z(n)|^2, \qquad k = 0, 1, 2, \cdots, (\frac{N}{T} - 1). \tag{3}$$

Usually only one type of trend is taken for the whole analysis, but mixed situations could be envisaged. Averaging $F(T)$ over all N/T box sizes centered on time T gives the fluctuations $\langle F(T) \rangle$ as a function of T. If the $y(n)$ data are random uncorrelated variables or short range correlated variables, the behavior is expected to be a power law

$$\langle F^2(T) \rangle^{1/2} \sim T^{\alpha} \tag{4}$$

with an exponent α =1/2 [8]. An exponent $\alpha \neq 1/2$ in a certain range of T values implies the existence of long-range correlations in that time interval as e.g. in the fractional Brownian motion [10]. Correlations and anticorrelations correspond to $\alpha > 1/2$ and $\alpha < 1/2$ respectively.[2]

If a signal has a fractal dimension D, its power spectrum is supposed to behave like

$$S(f) \sim f^{-\beta} \tag{5}$$

where

$$D = E + \frac{(3 - \beta)}{2} \tag{6}$$

[2] Notice that the procedure to estimate α in [11] includes an *a priori* integration of the tested signal, and these authors measure in fact an $\alpha' = \alpha + 1$.

or

$$\beta = 2H + 1, \tag{7}$$

in terms of the Hurst exponent H such that $D = E + 1 - H$ [10,12-14]; e.g. $\beta = 2$ for Brownian motion. Therefore, since $\alpha = H$

$$\beta = 2\alpha + 1. \tag{8}$$

In so doing one defines pink (or black) noise depending whether H is less (or greater) than 1/2. Black noise is related to long-memory effects, and pink noise to anti-persistence. processes are dominant over the external influences and perturbations [10].

Such power laws are the signature of a propagation of *information* across a hierachical system over very long times. Two cases are shown in Fig. 1. For time scales above 2 years, a crossover is however observed on studied financial data. This crossover suggests that correlated sequences have a characteristic duration of ca. 2 years along the whole financial evolution at least for the 16 years cases studied in ref. [6]. In order to probe the existence of *correlated and decorrelated sequences*, a so-called observation box of "length" 2 year was constructed and placed at the beginning of the data. The exponent α for the data contained in that box was calculated at each step. The box was then moved along the historical time axis by 20 points (4 weeks) toward the right along the financial sequence. Iterating this procedure for the sequence, a "local measurement" of the degree of "long-range correlations" is obtained, i.e. a local measure of the Hurst or α exponent. The results indicate that the α exponent value varies with the date. This is similar to what is also observed along DNA sequences where the α exponent drops below 1/2 in so-called non-coding regions.

The opposite has been observed for the breaking apart and disappearance of stratus clouds (over Oklahoma) [15]. The exponent α jumps from much below 1/2 to about 1/2 and drops back to a low value when the clouds scattered all over the area. By analogy with DNA and cloud behaviors, our findings suggest that financial markets loose the controlled structure (*following some loss of "information"*) at such a time. It should be noticed that in ref. [6] both sequences observed around 1983 and 1987 were not immediately seen from the rough data nor the value of α, and were missed by R/S and Fourier analysis.

Therefore, two of the main advantages of the DFA over other techniques like Fourier transform, or R/S methods [3] are that (i) local and large scale trends are avoided, and (ii) local correlations can be easily probed [6,9]. In economic data like stock exchange and currency fluctuations, long or short scale trends are *a posteriori* obvious and are of common evidence. The DFA method allows one to avoid such trend effects which can be considered as the envelope of the signal and could mask interesting details. Thus, we expect that DFA allows a better understanding of apparently complex financial signals.

In so doing, correlations can be sorted out and a strategy for profit making can be developed in terms of persistence and antipersistence signals [6].

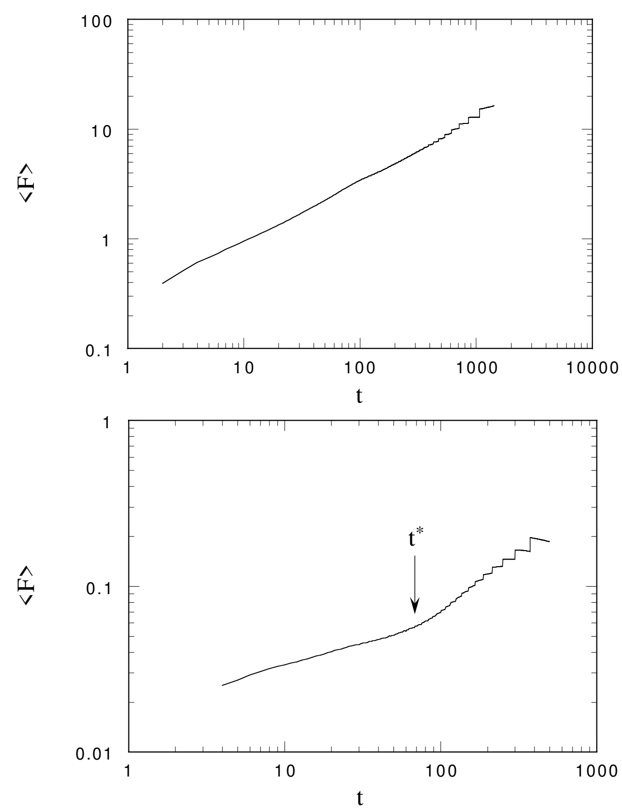

Fig. 1. Linearly Detrended Fluctuation Analysis function for two typical foreign currency exchange rates, i.e. JPY/USD and NLG/BEF between Jan. 01, 1980 and Dec. 31, 1995. The Brownian motion behavior corrresponds to a slope 1/2 on this log-log plot as indicated. The notation $\langle F \rangle$ is used for $\langle F^2(T) \rangle^{1/2}$ for conciseness in labeling the y-axis.

3 Multiaffine Analysis Techniques

A locally varying value of α suggests a multifractal process. A multi-affine analysis of several currency exchange rates has been performed in ref.[16–18], and also for Gold price, and Dow Jones Industrial Average (DJIA) in ref.[17]. In order to do so the roughness (Hurst) exponent H_1 and the intermittency exponent C_1 are calculated for the correlation function $c(\tau)$ supposed to behave like

$$c_1(\tau) = \langle |y(t) - y(t')| \rangle_\tau \sim \tau^{H_1}. \tag{9}$$

The technique consists in calculating the so-called 'qth order height-height correlation function" [19] $c_q(\tau)$ of the time-dependent signal $y(t)$

$$c_q(\tau) = \langle |y(t) - y(t')|^q \rangle_\tau \tag{10}$$

where only non-zero terms are considered in the average $\langle . \rangle_\tau$ taken over all couples (t, t') such that $\tau = |t - t'|$. The roughness exponent H_1 describes the excursion of the signal. For the random walk (Brownian motion), $H_1 = 1/2 = H$. In the case of white noise $H_1 = 0$ [10]. Notice also that $H_1 \sim H_2 = H$.

The generalized Hurst exponent H_q is defined through the relation

$$c_q(\tau) \sim \tau^{qH_q}. \tag{11}$$

The C_1 exponent [19–21] is a measure of the intermittency in the signal $y(t)$

$$C_1 = -\left.\frac{dH_q}{dq}\right|_{q=1}. \tag{12}$$

which can be numerically estimated by measuring H_q around $q = 1$.

It appears that in a (H_1, C_1) diagram (Fig. 2) the currency exchange rates are dispersed over a wide region around the Brownian motion case ($H_1 = 0.5, C_1 = 0$) and have a significantly non-zero thus intermittent component, i.e. ($C_1 \neq 0$) – the value of which might depend on the nature of the trading market, thereby indicating that economic policy seems to be probed through the analysis and its role should be taken into account in further microscopic modeling [17].

4 Moving Average Techniques

A stock market index has often been considered as cyclic, but so-called unpredictable events, like crashes are fascinating. It should be noted that they take place at the end of a period of euphory, when some anxiety builds in. Surely it is not obvious from the general trend nor from the apparently stochastic noise when a crash is forthcoming. Can we find some deterministic content beside the official trend from a basic noise characteristic, e.g. the fractal dimension has beentaken as a fundamental question.

Roughness or Hurst exponents are commonly measured in surface science [22] and also in time series analysis [23]. From a usual technique by analysts, known as the mobile (or moving) average technique, an interesting way can

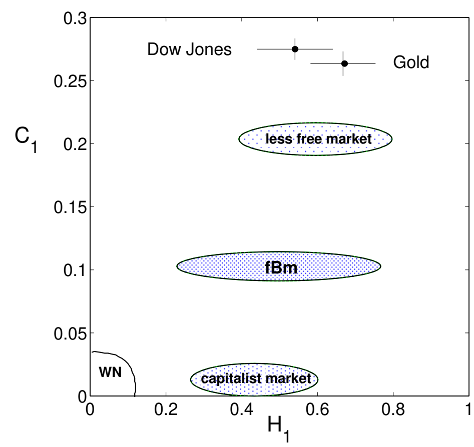

Fig. 2. Roughness(H_1), intermittency (C_1) parameter phase diagram of a few typical financial data and mathematical, i.e.fractional Brownian motion (fBm) and white noise (WN) signals.

be proposed for determining H and how to apply it right away to many cases with persistent or antipersistent correlations.

Consider a time series $y(n)$ given at discrete times n. At time n, the mobile average $\bar{y}$ is defined as

$$\bar{y}(n) = \frac{1}{N} \sum_{i=0}^{N-1} y(n-i), \tag{13}$$

i.e. the average of y for the last N data points. One can easily show that if y increases (resp. decreases) with time, $\bar{y} < y$ (resp. $\bar{y} > y$). Thus, the mobile average captures the trend of the signal over a time interval N. Such a procedure can be used in fact on any time series like in atmospheric or meteorological data, DNA, electronic noise, fracture, internet, traffic, and fractional Brownian motions.

Let two different mobile averages $\bar{y_1}$ and $\bar{y_2}$ be calculated respectively over e.g. T_1 and T_2 intervals such that $T_2 > T_1$. Since $T_1 \neq T_2$, the crossings of $\bar{y_1}$ and $\bar{y_2}$ coincide with drastic changes of the trend of $y(n)$. If $y(n)$ increases for a long period before decreasing rapidly, $\bar{y_1}$ will cross $\bar{y_2}$ from above. This event is called a "death cross" in empirical finance [24,25]. On the contrary, if $\bar{y_1}$ crosses $\bar{y_2}$ from below, the crossing point coincide with an upsurge of the signal $y(n)$. This event is called a “gold cross”. Financial analysts usually try to '“extrapolate” the evolution of y_1 and y_2 expecting “gold” or “death” crosses. Most computers on trading places are equiped for performing this kind of analysis and forecasting [25]. Even though mobile averages seem to be "arbitrary" measures, they present some very practical interest for physicists and raise new questions. It is paradoxical to have such a type of nalysis performed while on the other hand the "efficient market hypothesis" is the basis of most econometry theories.

It is well known [23] that the set of crossing points between a signal $y(n)$ and the $y = 0$-level is a Cantor set with a fractal dimension $1 - H$. The related physics pertain to so-called studies about first return time problems [26]. However, we have checked that the density ρ of crossing points between $\bar{y_1}$ and $\bar{y_2}$ curves is homogeneous along a signal and is thus *not* a Cantor set.

In so doing, the fractal dimension of the set of crossing points is one, i.e. the points are homogeneously distributed in time along $\bar{y_1}$ and $\bar{y_2}$. Due to the homogeneous distribution of crossing points, the forecasting of 'gold" and "death" crosses even for self-affine signals $y(n)$ seems unfounded.

However, it is of interest to observe how ρ behaves with respect to the choice in the relative difference $T_1 - T_2$. More precisely, consider the relative difference $0 < \Delta T < 1$ defined as $\Delta T = (T_2 - T_1)/T_2$. It has been found [27] that the density of crossing points $\rho(\Delta T)$ curve is fully symmetric, has a minimum and diverges for $\Delta T = 0$ and for $\Delta T = 1$, *with an exponent which is the Hurst exponent.* This remarkable and puzzling result does not seem to have been mentioned previously to ref. [27] due to the fact that some theoretical framework for the mobile average method is missing. The behavior of ρ is analogous to the age distribution of domains after coarsening in spin-like models [28] and to the density of electronic states on a fractal lattice in a tight binding approximation. This method of mobile averages can in fact serve to measure the Hurst exponent in a very fast and continuous way.

5 Sandpile Model for Rupture and Crashes

Another investigation of the relationship between the trend and local structure of a signal, like that of stock market measures like the DJIA and the Standard & Poor 500 (S&P500) has led us into examining regions where huge variations were taking place. These are usually associated to rupture phenomena'and 'crashes".

It has been proposed [29] that an economic index $y(t)$ increases as a complex power law, i.e.

$$y(t) = A + B\left(\frac{t_c - t}{t_c}\right)^{-m}\left[1 + C\sin\left(\omega\ln\left(\frac{t_c - t}{t_c}\right) + \phi\right)\right] \quad \text{for } t < t_c \tag{14}$$

where t_c is the crash-time or rupture point, A, B, m, C, ω and ϕ are free parameters. The law for $y(t)$ diverges (converges) at $t = t_c$ if the exponent m is positive (negative) while the period of the oscillations converges to the rupture point at $t = t_c$. The real part of the law is similar to that of critical points at so-called second order phase transitions [30] but generalizes the scaleless situation to cases in which a discrete scale invariance [31] is presupposed when a complex exponent $m + i\omega$ exists. This relationship was already proposed in order to fit experimental measurements of sound wave rate emissions prior to the rupture of heterogeneous composite stressed up to failure [32]. Such log-periodic corrections have been recently reported in biased diffusion on random lattices [33], and in our sandpile studies is found when the underlying base is quasi-fractal [34]. Thus, an avalanche sand pile model can be imagined for financial indices [35]

A logarithmic divergence, corresponding to the $m = 0$ limit, can be also proposed, [36] i.e. the divergence of the index y for t close to t_c should be such that

$$y(t) = A + B \ln\left(\frac{t_c - t}{t_c}\right)\left[1 + C \sin\left(\omega \ln\left(\frac{t_c - t}{t_c}\right) + \phi\right)\right] \quad \text{for } t < t_c \tag{15}$$

In August 1997, a series of investigations was performed in order to test the existence of crash precursors. Daily data of the DJIA and the S&P500 was used. A strong indication of a crash event or a rupture point in between the end of october 1997 to mid-november 1997 was numerically discovered [37], and later predicted to occur during the week of Oct. 27, 97, and it was observed to occur on Oct. 27, 97 [38]. This resulted from an analysis of the similarities between two long periods: 1980-87 and 1990-97. The number of open days per year on Wall Street is about 261 days, - the exact value depending on the number of holidays falling on week ends. For the first period, the analysis was performed on data ending two months before the so-called Black Monday, i.e. October 19, 1987. For the second period, the data was considered till August 20th, 1997. In fact, we have separated the search of the crash day into two problems, that of the divergence itself and that of the oscillation convergences on the other hand, i.e. (i) t_c^{div} for the power (or logarithmic) divergence and (ii) t_c^{osc} for the oscillation convergence.

Sometimes it might be natural to be contempting, and/or displeased by, the eye balling technique we are supposedly using [39–42]. We should totally disagree concerning this gross misunderstanding of our technique. Our statistical analysis takes into account the approximate location of the maxima, and in a recent paper it has been precisely shown one good way of taking the maximum location into account. It is true that in ref.[37] the arrows pointing at maxima and minima look rather thick, but this is for a display purpose. In fact the statistical data analysis takes into account the number of data points in the best possible interval, as it is standard in critical point (exponent) analysis [43,44]. In so doing the origin of the time interval is obtained indeed. This time origin the definition of the phase) should lead to some interesting questions in fact. It might be of interest to recall that the closing value of the DJIA was used. This is not necessarily the intraday maximum value in fact, nor the intraday average value. One might wonder if the former or the latter would give a better estimate of the upper bound of the predicted crash day. One might search whether rather than the closing value or the maximum the average DJIA, or average of range, or something else over a one day interval should be better used for better predictability, etc. It is known that there are larger fluctuations at the begining and at the end of a day. These are left for further investigations. The stability of the fit parameters can also be checked on these closing values with respect to random noise and through a Monte-Carlo data in order to take into account some sort of uncertainty in a bivariate data analysis (with error bars on the x and y axes). However due to the apparent precision of the technique at this time no robustness test has been performed as of now.

It should be pointed out that we do not expect any real divergence in the stock market indices; this is total non sense of course. However a divergence is predicted by us to occur at some upper bound of t_c. This is exactly the same as in phase transitions, where there is never any infinite divergence at the critical temperature. The divergence of the correlation length, specific heat, etc. is a virtual (mathematical) image of physical reality. There is no infinity (nor zero in fact) in physics due to finite size effects, inhomogeneities, noise, etc. Therefore to argue on the true existence of zeroes and infinity [33,40–42] is rather meaningless. We consider that to give an upper bound is certainly an as good predictive technique in data analysis and for modeling, as good as to give a determinis tic finite value at t_c.

Moreover a true drop certainly exists at a crash and is the signature of the crash, and the formula of ref. [29] would seem therefore appropriate. According to ref. [29,40], the drop goes to a finite value. Notice that there is some sense indeed to examine the size of jumps at crashes though. Such an attempt has been made in ref. [7].

index - (period)	$t_c^{div}(m=0)$	$t_c^{div}(m \neq 0)$	t_c^{osc}
DJIA (80-87)	87.85 ± 0.02	88.46 ± 0.04	87.91 ± 0.10
DJIA (90-97)	97.92 ± 0.02	98.68 ± 0.04	97.89 ± 0.06
S&P500 (80-87)	87.89 ± 0.03	88.78 ± 0.05	87.88 ± 0.07
S&P500 (90-97)	97.90 ± 0.02	98.67 ± 0.04	97.85 ± 0.08

Table 1: Fundamental parameters found for the DJIA and S&P500 indices during 1980-87 and 1990-97 periods. Time is expressed in years. The notations for t_c are such that e.g. 97.90 means the calendar date corresponding to the 90-th day as if there are 100-days in 1997. Two values of t_c^{div} correspond to a fit using a logarithmic divergence ($m = 0$) and a fit using a power law divergence ($m \neq 0$) respectively. The true date of the October 1987 crash in the above units gives $t_c = 87.79$ and for the October 1997 crash is $t_c = 97.81$, i.e. *quasi* the predicted dates.

6 (m, k)-Zipf Techniques

For testing and emphasizing short range correlations, the (m, k)-Zipf and the i-Variability Diagram (i-VD) techniques have been used. The Zipf analysis consists in counting the number of words of a certain type appearing in a text, calculating the frequency of occurence f_o of each word in a given text, and sorting out the words according to their frequency, i.e. a rank R is assigned to each word, with $R = 1$ for the most frequent one, and rank R_M for the word appearing the less. Moreover call f_M the frequency (occurrence) of the most often observed word.

For natural languages, one observes a power law

$$f_o \sim R^{-\zeta} \tag{16}$$

with an exponent ζ close to one for any language. This has been applied to various complex signals or "texts" [45–47], economy (size of sales and firms) data [48], financial data [5], meteorological [17,49], sociological [50] or even random walk [51] and percolation [52] after translating whatever signal into a text based on an alphabet of k characters. The appearance of this power law is due to the presence of a so-called hierarchical structure of long range correlations in words, sentences, paragraphs, and so on for the given set of characters in an alphabet used for writing a text [46]. A simple extension of the Zipf analysis is to consider m-words only, i.e. the words strictly made of m characters without considering the white spaces.

Let for the sake of argument, only a binary alphabet with u and d characters, and the translation of a signal into a text (Fig.3). Let the probability to find a u in the text be p. The deviation from $p = \frac{1}{2}$, i.e. $p = \frac{1}{2} + \epsilon$ where $0 \le \epsilon \le \frac{1}{2}$ is called the bias. The bias is in fact a local measure of the trend in a stock price or index value.

We have chosen to examine ($m = 6, k = 2$) cases. It may be remarked that this is useful for attempting to observe short range (weekly) fluctuations in (weather or financial) data for example. The aim of the study is to find the exponent ζ. By the way, it has been conjectured [53,54] that ζ is related to the Hurst exponent H, thus to the fractal dimension D [10,12,14] of the signal as

$$\zeta = |2H - 1|. \tag{17}$$

Therefore, for H different from 0.5, and thus ζ different from zero, the signal is not Brownian-like, whence some predictability can be expected because non trivial correlations exist between successive daily fluctuations.

One case can serve as an illustration herebelow. As *experimental data* among the many indices and stocks available on Internet, let us choose an insurance company Oxford Health Plan ($OXHP$), treated on the NASDAQ. From Aug. 8, 1991 till March 15, 1999, this consists in about 1900 data points [15]. The daily closing price signal is shown in Fig. 4. The fractal dimension D, or power spectrum characterized by β, the DFA exponent α and ζ can be examined as well. The latter from a Zipf analysis for the $OXHP$ closing price is given in Fig. 5, and values of exponents summarized in Table 2. The corresponding results for the Brownian motion are also given and serve as an estimate of the validity of the analysis.

Name of the signal	ζ	α	β
Brownian motion	0.08 ± 0.0007	0.50 ± 0.01	1.79 ± 0.20
OXHP: Closing price	0.27 ± 0.02	0.56 ± 0.03	1.75 ± 0.25

Table 2: The ζ, α and β values for OXHP closing price for the time spans from August 8, 1991 till March 15, 1999.

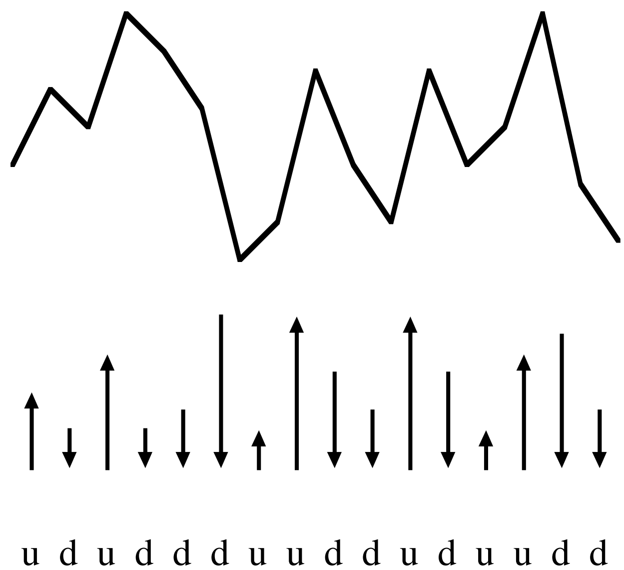

Fig. 3. Translation of part of a random walk sequence ("fluctuations") into a binary sequence made of two characters u and d.

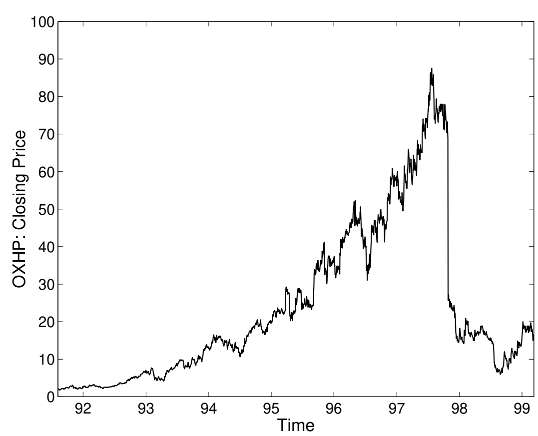

Fig. 4. Daily closing price of Oxford Health Plan ($OXHP$) stock, treated on the NASDAQ, from mid-91 till Jan. 99, i.e. about 1900 data points.

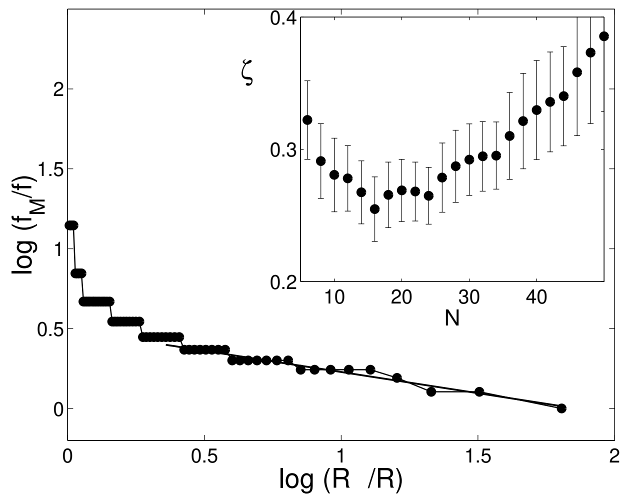

Fig. 5. (6,2)-Zipf analysis for the $OXHP$ stock closing price; the estimate of the ζ exponent is shown in the inset for R small and as a function of the number N of data points used to calculate the best slope from the main graph.

7 Basics of *i*-Variability Diagram Techniques

One disadvantage of the Zipf-method is that it is not possible to distinguish between persistent and antipersistent sequences. Only the departure from randomness is easily observed. Another way to sort out short range correlations is the i-Variability Diagram technique, used for example in heart beat [55] and meteorological [49] studies. Recall that the first return map (r_i, r_{i-1}) or the τ-return map $(r_i, r_{i-\tau})$ of a signal are often used for revealing a possible dominant correlation between the events of the data set. This leads to studies of *strange attractors* and the embedding dimension of a signal.

The return map of the *first derivative* of the signal, i.e. the so-called first order variability diagram (1-VD) [55] correlates every three consecutive points of the series,

$$\begin{aligned} s_{i+1} &= r_{i+1} - r_i \\ s_i &= r_i - r_{i-1} \end{aligned} \tag{18}$$

The curvature of the signal, thus relating every four consecutive events as

$$\begin{aligned} u_{i+1} &= r_{i+1} - 2r_i + r_{i-1} \\ u_i &= r_i - 2r_{i-1} + r_{i-2}. \end{aligned} \tag{19}$$

is called the second order variability diagram (2-VD).

The links between the 4-points e.g. defining the above relationship are seen through a phase space diagram for the curvature. A non-trivial shape of the point cloud and point distribution itself on such a diagram indicate an asymmetry between the different consecutive curvatures, and can be used for predictability.

It has been found [56] that for free market financial series (DJIA and Gold price) the local trend behaves like a $\sqrt{t}$. However the BGL/USD exchange rate variability seems to be different: the set of events leads to a line structure with slope $I = -1.21$ in the curvature return map. The differences can be conjectured to depend on economic policy grounds.

A combination of Zipf and i-VD has been recently attempted for the local curvature correlations in financial signals [56]. This method leads to suggest tests based on microscopic models.

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