Economic Growth and The Quality of Human Capital

Laabas, Belkacem and Weshah, Razzak

Arab Planning Institute

8 February 2011

Economic Growth and The Quality of Human Capital

B. Laabas
W A Razzak†

Arab Planning Institute
Kuwait
February 2011

Abstract

We calibrate an endogenous growth model to study the effect of the quality of human capital on productivity growth in a sample of thirty developed and developing countries for the period 1980 to 2007. We measure quality of human capital by relative cognitive skills. These are country scores in mathematics and science reported in Trends in International Mathematics and Science (TIMMS). The correlation between the relative quality of human capital and productivity growth is evident in the data for the developed countries. And, cross-country differences in the quality of human capital for a number of developed countries are highly positively associated with cross-country differences in productivity growth. The picture is significantly different for the developing countries in our sample.

JEL Classification O40, E10, I20, J24,
Keywords, quality of human capital, economic growth,

† We thank R. Thamri and M. El Bentour for excellent research assistance, and the participants of the first Lisbon workshop on the economics and econometrics of education 7-8 January 2011. Contact: razzakw@gmail.com and weshah@api.org.kw

1. Introduction

Hanushek and Kimko (2000) and Hanushek and Woesmann (2009, 2010) provide regression-based evidence for the importance of the quality of education measured by cognitive skills for economic growth. They argued that the quality of education causes growth. Hanushek and Kimko (2000) report that an increase of one standard deviation of labor force quality increases real per capita growth by 1.4 percentage points per year.[2] Hanushek and Woessmann (2009) found that relatively poor economic growth in Latin American countries can be explained by the poor quality of education. And Hanushek and Woesmann (2010) show that cognitive skills can account for growth differences between OECD countries. They found that "a one standard deviation increase in educational achievement, defined as a 100 test score points on the PISA scale, yields an average annual growth rate over 40 years that is 1.86 percentage points higher." Positive association of economic growth with cognitive skills is also confirmed in Barro (2010), Bosworth and Collins (2003), and Ciccone and Papaioannou (2009).

This paper too, focuses on the issue of quality of human capital.[3] The main objectives are to: (1) measure the contribution of changes in the quality of education to productivity growth; and (2) measure the extent of cross-country relative differences in productivity growth that are attributed to cross-country relative differences in the quality of human capital.

However, our paper differs from the above literature in many ways, but most importantly we do not derive our results from regression-based growth equations. Rather, we accomplish the objectives above by calibrating an endogenous growth model. In this model, the growth rates of the capital-output ratio, the stock of human capital, and labor affect the transitional dynamics. In addition and most importantly, growth is driven by the global stock of ideas, where a few technologically advanced developed countries in the sample lead research and knowledge efforts. Potentially, researchers' efforts are education-driven. Ideas in research and problem-solving issues are traded across the world and directly affect the quantity and efficiency of the production of goods and services.

The growth model that we calibrate is Jones's (2002) model, which is in a class of growth models. Here, human capital is also a key driver of economic growth, for example, Lucas (e.g., 1988, 2009), Aghion and Howitt (1992), Howitt (1998, 1999), and Grossman and Helpman (1991).

[2] Hanushek and Kimko (2000) referenced the cognitive skill measure reported in TIMMS, but did not use that data. Instead, they used six international tests of student achievement in mathematics and science that were conducted over the past thirty years. Four of these were administered by the International Association for the Evaluation of Educational Achievement (IEA), and two by International Assessment of Educational Progress (IAEP). They combined these test scores under certain assumptions to arrive at a normalized, weight-representing measure of cognitive skills, and hence the quality of labor.

[3] The quality of labor is an important explanatory variable in labor supply and economic growth literature, Lucas and Rapping (1969).

These are models, where long-run economic growth and technical progress are driven by innovations and ideas. The model is also related to Nelson and Phelps (1966), Romer (1990a), and Rebelo (1991), which are other examples of endogenous growth models, where knowledge proxy by either human capital innovations, or R&D, drive growth.

We modify Jones's (2002) model slightly to allow for the quality of human capital. The quality of human capital is derived from the quality of cognitive skills. Cognitive skills are measured by standardized test scores in mathematics and science (TIMMS).[4] Relative TIMMS scores are used to adjust the stock of human capital in every country in our sample. Essentially, the endogenous growth model in this paper allows for a causal relationship between cognitive skills as a measure of the quality of human capital, human capital, and growth.

The model is calibrated for thirty countries, which includes the G7, plus seven OECD countries, three Southeast Asian countries, and thirteen Middle East and North African countries (MENA). The choice of the sample is solely based on the availability of the data. We do not have data for South and Latin America, and Africa, which were analyzed in Hanushek and Woesmann (2009, 2010).

Our results are quantitatively different from the regression-based results reported in the regression-based literature cited above. We find partial evidence of positive association in our sample between the quality of human capital, measured by relative cognitive skills, and productivity growth. Only a few countries in our sample (about 8 out of 30) experienced an increase in the relative quality of human capital measured by cognitive skills over the past thirty years. Fewer developed countries experienced positive growth in productivity and an increase in the quality of human capital, hence a positive correlation. There is a large number of countries that enjoyed positive growth in productivity and at the same time a deterioration in the relative quality of human capital, hence a negative correlation. The majority of these are developed countries.

Most of the developing MENA countries experienced negative growth rates for productivity as well as deteriorations in the relative quality of human capital, hence a positive correlation. We also found that cross-country differences in the relative quality of human capital for a number of non-MENA countries were highly positively associated with cross-country differences in productivity growth.

The paper is organized as follows. Next, we present an endogenous growth model. Section 3 contains data measurements and calibration. Section 4 is a growth accounting. Conclusions are in section 5.

---

[4] TIMMS is Trends in International Mathematics and Science Study.

2. The growth model

We begin with Jones's (2002) endogenous growth model. In each economy j, output is produced by the following production function (we remove the subscript j from all equations for the time being):

$$Y_t = A_t^{\sigma} K_t^{\alpha} H_{Yt}^{1-\alpha}, \qquad (1)$$

where Y_t is total output produced at time t, A_t is the stock of ideas available in the economy, K_t is the stock of physical capital, H_{Yt} is the total quantity of human capital employed to produce output. The production function exhibits increasing returns to scale because $\sigma > 0$, but constant returns to scale in K_t and H_{Yt},

We modify the production function above to include oil and gas endowment in order to fit a number of oil-producing countries in our sample, where W_t is the rate of utilization of oil and gas, see Solwo and Wan (1976) and Stiglitz (1974). These countries have very high output per capita levels due to oil wealth. The production function still exhibits overall increasing returns to scale, but constant returns to scale in capital and labor, i.e., $\alpha + \beta = 1$.

$$Y_t = A^{\sigma} K_t^{\alpha} H_{Yt}^{\beta} W_t^{\omega}. \qquad (1`)$$

The growth rate of the stock of capital is given by:

$$\dot{K}_t = s_{Kt} Y_t - dK_t \quad K_0 > 0, \qquad (2)$$

where the variable S_{Kt} is the fraction of output that is invested, d is a constant, positive, and exogenous depreciation rate, and n is the constant population growth rate. Each country is populated with N identical, infinitely-lived agents.

The aggregate human capital employed to produce output is:

$$H_{Yt} = h_t L_{Yt}, \qquad (3)$$

where h_t is human capital per person multiplied by the total amount of labor. And h_t is produced by forgoing time in the labor force. The individual spends time in education, training, …etc. This time is l_h, so h_t is:

$$h_t = e^{\psi\, l_{ht}}, \quad \psi > 0; \quad 0 \le \xi \le 1. \qquad (4)$$

Equation (4) is like that of Mincer (1974), as shown in Bils and Klenow (2000), where ψ is the rate of return on education.

We extend the model to include the quality of human capital, ξ.

$$h_t = e^{\psi \xi l_{ht}}, \quad \psi > 0; \quad 0 \le \xi \le 1. \qquad (4`)$$

The parameter ξ measures the average relative cognitive skills of the country. We introduce this parameter as a measure of the quality of human capital, hence the quality-adjustment of human capital. The parameter l_{ht} is the length of time an individual spends in accumulating human capital.

We use the data for country score in mathematics and science, which are published by TIMSS, as a measure of cognitive skills and hence the quality of human capital. In effect, we are arguing that countries whose 4th and 8th grade students have systematically done relatively well in TIMMS mathematics and science scores over time, have better quality of human capital. Within the sample, we identify one country with the highest score $\bar{\xi}$. For each country j in our sample we choose the score ξ_j and deflate by the score of the leading country or group $\bar{\xi}$, i.e., $\xi = \frac{\xi_j}{\bar{\xi}}$. Essentially, it is a measure of the relative quality of human capital of country j. It turned out that Singapore maintained the lead in TIMMS scores allover the sample.

The final element in the production function of output is the stock of ideas, A_t. The countries in this model share ideas (there is no trade in goods and services in this model). Ideas created anywhere in the world are potentially available to be used in any other economy. It follows that A_t corresponds to the cumulative stock of ideas created anywhere in the world and is common to all economies.[5] New ideas are produced by research using:

$$\dot{A}_t = \delta H_{At}^{\lambda} A_t^{\phi}, \quad A_0 > 0, \qquad (5)$$

where H_{At} is effective world research effort, and is given by:

$$H_{At} = \sum_{i=1}^{M} h_{it}^{\theta} L_{Ait}. \qquad (6)$$

Note that here we have a subscript i. The index i refers to the economies i to M. Jones (2002) assumes that global research is the weighted sum of research conducted in the five advanced countries: US, UK, Germany, France and Japan (i.e., $M = 5$) and assumes that $\theta \ge 0$, which means that the quality of research is constant across these five countries. L_{Ai} is the number of researchers in country i.

[5] Jones (2002) articulates that he made the model more complicated by assuming ideas are not instantaneously available for use by other countries, but rather functions of some economic factors. He assumed that ideas must be learned before they can be used in production. He found this complication did not alter the final results.

The number of new ideas produced at any point in time depends on the number of researchers and existing stock of ideas. Jones (2002) allows $0 < \lambda \leq 1$ in equation (5) to capture the possibility of duplication in research, i.e., a doubling of the number of researchers produces less than a doubling of the number of ideas. Jones also assumes that $\phi < 1$. There is also a binding resource constraint on labor. Each economy is populated by N_t identical, infinitely-lived agents. The number of agents in each economy grows over time at a common and exogenous rate $n > 0$:

$$N_t = N_0 e^{nt}. \qquad N_0 > 0 \tag{7}$$

Each individual is endowed with one unit of time, which is used to produce goods, ideas, and human capital. Because the time spent in school is excluded from labor force data, the constraint is:

$$L_t = L_{At} + L_{Yt}, \tag{8}$$

where L_t denotes employment and $L_t = (1 - l_{ht})N_t$. Jones also defines $l_A = L_A / L$ as the fraction of the labor force that works on producing ideas (research intensity) and $l_Y = L_Y / L$ as the fraction of the labor force that works on producing goods.

The variables $s_K, l_A, l_Y,$ and l_h are referred to as allocations and may differ across countries.

Rewriting the production function in equation (1) in terms of output per worker, where y_t per worker, yields:

$$y_t = \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} l_{Yt}\, h_t\, A_t^{\frac{\sigma}{1-\alpha}}. \tag{9}$$

Under our modification for the oil-producing countries, the production function (1`) becomes:

$$y_t = \frac{K_t^{\alpha/\beta}}{Y_t^{\frac{1-\beta}{\beta}}} l_{Yt} h_t A_t^{\sigma/\beta} W_t^{\omega/\beta}, \qquad (9`)$$

Jones (2002) assumes that the stock of capital K_t and the technology A_t grow at constant rates, which requires H_A to also grow at a constant rate – recall these rates are different across countries in this paper – he decomposed output per worker as:

$$y_t = \left(\frac{s^*_{Kt}}{n + g_k + d} \right)^{\frac{\alpha}{1-\alpha}} l_{Yt}\ h_t \left(\frac{\delta}{g_A} \right)^{\frac{\gamma}{\lambda}} H^{*\gamma}_{At}, \qquad (10)$$

where $\gamma = (\sigma / 1 - \alpha)(\lambda / 1 - \phi)$. The notation $g_{(.)}$ denotes a constant growth rate, which is also different across countries. The asterisk on the variable denotes a quantity that is growing at a constant rate.

The first term in the equation is the capital-labor ratio, which grows at a constant rate and is proportional to the investment rate *a la* Solow (1956, 1957). The last term is derived from the fact that, when the stock of ideas grows at a constant rate, this stock can be inferred from the flow of research effort H_A. To drive it, divide both sides of equation (5) by A, get $\dot{A}_t / A_t$ = $\delta H^{\lambda}_{At} / A^{1-\phi}_t$, when the growth rate of A is constant, this equation can be solved such that A is proportional to H^{γ}_{At}. The term $\left(\frac{\delta}{g_A} \right)$ is the factor of proportionality, which depends on g_A.

When all variables grow at a constant and exponential rate forever, a stable balanced growth path is found. The allocations mentioned above are constant along that path. Then the growth rate of output per worker is proportional to the growth rate of global effective research H_A. And, h must be constant along a balanced growth path, growth in the effective number of researchers is driven by population growth, and a balanced growth path gives (see appendix):

$$g_y = \frac{\sigma}{1-\alpha} g_A = \gamma n. \qquad (11)$$

The equation says that the long-run per capita growth is ultimately tied to the world growth population: a greater global population means a greater number of researchers. These researchers produce more new ideas, which raise income around the world – a scale effect. For example, if the world population doubles, keeping all parameters and allocations constant, H^*_{At} is doubled. This raises the level of income for all countries in the world in the long run by a factor of 2^{γ}.

Contrary to all cross-country growth regressions, the covariance between per capita growth and population growth is > 0. Mankiw *et al.* (1992) interpret the negative covariance of per capita growth and population growth as reflecting the transition dynamics of the neoclassical growth model, i.e., a higher population growth reduces the steady-state capital-output ratio because more investments are required to maintain the existing capital-output ratio as population is growing. This effect is captured by the first term of equation (10). The last term in the equation comes from the fact that the stock of ideas grows at a constant rate (and the stock can be inferred from the flow).

For growth accounting, Jones (2002) derived the following equation:

$$\hat{y}_t = \frac{\alpha}{1-\alpha}(\hat{K}_t - \hat{Y}_t) + \hat{h}_t + \hat{l}_{Yt} + \left(\frac{\sigma}{1-\alpha}\hat{A}_t - \gamma n\right) + \gamma n, \quad (12)$$

where a hat on top of the variable denotes the growth rate between two points in time, in our case1980 and 2007. By adding and subtracting γn, this equation has the interpretation that the RHS terms (except the last) are zero in the steady state. These terms represent the transitional dynamic of growth. If an economy is close to its balanced growth path, the last term should account for the bulk of growth as in the Jones model. So the productivity growth transitional dynamic is a function of capital intensity, human capital growth, growth rate of labor allocated for the production of output, and excess ideas. Excess ideas is the growth rate of TFP in excess of population growth of the advanced G8 countries.

In this paper we derive different expressions for (10) and (12) for countries with natural resource endowments.

$$A_t = \frac{y_t^{\frac{\beta}{\sigma}}}{\frac{K_t^{\alpha/\sigma}}{Y_t^{\frac{1-\beta}{\sigma}}}\left(l_{Yt} h_t\right)^{\frac{\beta}{\sigma}} W_t^{\omega/\sigma}}. \quad (13)$$

Following Jones (2002) we assume that constant returns to scale is in K_t and H_{Yt} only, i.e., $\alpha + \beta = 1$. This yields output per capita level:

$$y_t = \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} l_{Yt}\, h_t\, A_t^{\frac{\sigma}{1-\alpha}} W_t^{\frac{\omega}{1-\alpha}}, \quad (14)$$

which is very similar to Jones (2002), and the corresponding level of ideas is:

$$A_t = \frac{y_t^{(1-\alpha)/\sigma}}{\left(\frac{K_t}{Y_t}\right)^{\alpha/\sigma}\left(l_{Yt} h_t\right)^{(1-\alpha)/\sigma} W_t^{\omega/\sigma}}. \quad (15)$$

And the equation which corresponds to (10) in the level is:

$$y_t = \left(\frac{s^*_{K_t}}{g_k + d + n}\right)^{\frac{\alpha}{1-\alpha}} l_{Y_t} h_t (g_A/\delta)^{\frac{\gamma}{\lambda}} H^{*\gamma}_{A_t} W_t^{\frac{\omega}{1-\alpha}}. \quad (16)$$

And that for the growth rate is:

$$\hat{y}_t = \frac{\alpha}{1-\alpha}(\hat{K}_t - \hat{Y}_t) + \hat{h}_t + \hat{l}_{Yt} + \left(\frac{\sigma}{1-\alpha}\hat{A}_t + \frac{\omega}{1-\alpha}\hat{W}_t - \gamma n\right) + \gamma n \quad (17)$$

Derivations are in Appendix 1.

3. Data, measurements and calibration

We calibrate equations (12) and (17) for 30 countries. Our sample of countries is based on the availability of the data. These countries are the G7, OECD countries (Australia, New Zealand, Austria, Denmark, Netherlands, Sweden, plus Turkey), three Asian countries (Hong Kong, South Korea, and Singapore), the Arab non-oil producing countries (Egypt, Jordan, Lebanon, Morocco, Syria, and Tunisia). Equation (17) is calibrated for the oil-producing countries only: Norway, Algeria, and the Gulf Cooperation Council (GCC) countries of Bahrain, Kuwait, Oman, Qatar, and Saudi Arabia.

The parameters required for the calibration are α , γ , ω , ψ , and ξ . The values of these parameters vary across countries j . The values of α and ω used in the calibration are the average values over the period 1980 to 2007. The share of capital, α , is computed from the national income accounts as the ratio of gross operating surplus to GDP.[6]

The rate of return to education ψ was calibrated to 0.07 for the US as in Jones (2002). For the rest of the countries in our sample, we estimate the values using an unbalanced panel regression. The World Bank publishes surveys of different dates for 73 countries in the world.[7] We estimate the following (white-corrected standard errors) regression $\psi_j = a + b\,h_j + c\,D_j + \varepsilon_i$, where h is human capital measured as in Barro and Lee (2010) by average years of schooling, and D is a country income dummy for high and low income.[8] This regression is stable in the parameters so we use it to generate returns on education, ψ , for the countries in our sample using $\psi_j = \hat{a} + \hat{b}\,h_j$, where the hat on the parameter denotes estimated values. We compute the return to education for 1980 and 2007. We find that the values range between 0.07 and 0.12, with higher returns associated with developing countries and smaller returns associated with developed countries.

The value of ω , the share of oil and gas in output, is compiled from the World Bank World Development Indicators. The rate of hydrocarbon utilization W_j is from the World Bank World Development Indicators.

For ξ_j , the quality of human capital we use average country score for tests in math and science as a measure of cognitive skills as a proxy. The data are published in TIMMS every four years for (1995, 1999, 2003, and 2007) for a

---

[6] It might be a measurement issue if investments in intangible capital are large and not included in measuring total effective capital, Prescott (1997).

[7] Data are taken from Psacharopoulos and Harry Anthony Patrinos (2002).

[8] Jorgenson and Fraumeni (1992) calculates an alternative stock of human capital, based on lifetime earnings. We do not have data for the MENA countries to compute that index.

number of countries, where 4th and 8th graders were given standardized tests in math and science. Hanushek and Kimko (2000) and Hanushek and Woesmann (2009, 2010) use regressions to estimate the trend in the quality of education. Their measures are different from ours. They use IEA and IAEP test scores as mentioned earlier. The explanatory variables include public expenditures on primary education / GDP ratio, population growth and pupil / teacher ratios consistent with Barro's specification of the education production function typically used in this literature.

We use TIMMS only. TIMMS data are not available for 1980. Our sample covers 1980 to 2007. To estimate TIMMS for 1980, we use a back-casting method to back-cast the country scores for every four years from 1980 to 1991. The model is an autoregressive model in the level since these scores do not change significantly over time but vary across different countries. We fit various cross section – time series specifications and found that one lagged values of ξ_{jt} has a coefficient of unity. We found that a random-effect unbalanced panel and white-correction fits the 84 different countries found in TIMMS best. We estimate the values for the years back to 1980 in intervals of four years.

Another significant difference between our analysis and the literature found mainly in Hanushek and Kimko (2000), and Hanushek and Woesmann (2009, 2010), is that they show a secular trend in the quality of education, highly associated with secular growth. To the contrary, TIMMS indicate a global decline in the test scores between 1993 and 2007. The extended data from 1980 to 2007 show that the mean TIMMS gap relative to Singapore (country j relative to Singapore) increased from 22 percent in 1980 to 24 percent in 2007. Figure 1 plots the data.

The crucial parameter γ is fully discussed in Jones (2002). He shows that the parameter is a collection of parameters from the production function of ideas and is difficult to calculate. By dividing both sides of the production function of ideas, Jones arrives at $\left(\dot{A}_t / A_t\right) = \delta\left(H_{At}^{\gamma} / A_t\right)^{1-\phi}$. Thus, productivity growth depends on the ratio of the quantity of human capital used in producing ideas to the level of productivity. The growth rate of productivity is I(0), while both H_{At} and A_t trend upward. Therefore, the parameters γ is important for de-trending the ratio H_A^{γ} / A such that productivity growth is I(0).[9]

Under the assumption that the growth rate of total factor productivity remains constant over the sample, the parameter γ_j is equal to the ratio of the growth rate of total factor productivity for country j, $\hat{A}_{jt}$, to the growth rate of the effective global research efforts, $\hat{H}_A$ (a hat over the variable denotes the

[9] It is a well known problem that we cannot measure TFP and output simultaneously. Another problem is the increasing share of services in GDP in developed countries because services output is difficult to measure. Productivity becomes difficult to measure and so is TFP. In developing countries, especially oil-rich countries, the size of service sector is even larger.

growth rate). We computed γ this way for each country. Jones (2002) also uses econometrics to estimate the value of γ. The range of values of estimated γ for the U.S. was found to be between 0.05 to 0.33. The difficulty surrounding the estimation of γ suggests that sensitivity analysis is probably the best strategy for the calibration.

The parameter σ is also unknown. It is unidentified because ideas are unobservable. Jones (2002) normalizes σ to be equal to $1-\alpha$. It means that total factor productivity A is measured in units of "Harrod-neutral productivity". Empirically, however, A is computed from inverting equation (9), and, for the oil-producing countries, from equation (13).

The labor allocated for the production of output l_Y is the total labor force – the number of researchers in the country.

The data are fully described in the data appendix.

4. Growth accounting

Jones (2002) calibrated the model for the US only. We calibrate the model for 30 countries. All our results are reported in Table 1. It reports the results of growth accounting, based on the calibration of equations (12) and (17). The table has 14 columns. Column one lists the countries. We begin with the G7 countries, followed by other OECD countries, then the Asian countries, followed by MENA countries. MENA countries are divided into Arab oil-producing and non-producing countries.

The second column reports the values of α_j, the share of capital. The crucial parameter γ_j, which is measured by $\hat{A}_{jt}/\hat{H}_A$, is listed in column three. Since A_{jt} varies with h_{jt} and the latter is adjusted for quality, we also adjust the value of γ_j and report γ_j^* in column four. Column five reports the value of ω_j, which only applies for the oil-producing countries in our sample. Column six reports the growth rate of real GDP per hour worked (except for Singapore which has no data for hours, so we use working age population). Real GDP is measured as PPP-adjusted constant price data, thus productivity growth.

Growth accounting begins in column seven, where we report the capital-output growth rate as a measure of capital intensity. Column eight reports the growth rate of labor allocated for the production of output. Columns nine and 10 report the stock of human capital and the quality-adjusted human capital growth rates, respectively. Columns 11 and 12 report the quality-unadjusted and adjusted excess idea growth rates. Similarly, columns 13 and 14 report the steady state growth rates, both unadjusted and adjusted for quality of human capital. Columns seven to 12 are the transition dynamics. The last columns are the steady state growth rates. The results are also reported graphically. The numbers in parentheses are percent contributions to productivity growth. We do not report the percent contributions for countries with negative productivity growth.

The growth rate of capital – output ratio (capital intensity) explains very little of productivity growth in the U.S. and the U.K. just like in Jones (2002). However, other G7 countries have contributions between 17 percent to 30 percent with Canada having the highest contribution from capital. The growth rate of human capital accounts for three percent of productivity growth in the US, much smaller than the 30 percent contribution reported in Jones (2002) over the period 1950-1993. When we adjust human capital for quality in column 10, the contribution doubles but is still much smaller than in Jones (2002). Obviously, the sample matters. The US supply of skills increased substantially during the period 1970 to 1980, by about 5.19 percent per year or by 52 percent in the decade, according to Autor *et. al* (1998). The supply of skills then fell and that coincided with our sample.

There is a significant difference in the contribution of capital intensity to productivity growth in Australia and New Zealand. It is 11 percent in Australia, and negative in New Zealand. This is consistent with the literature on growth in New Zealand and Australia. Razzak (2007) shows that capital intensity is a significant explanatory variables of the productivity differential between the two countries. Hall and Scobie (2005) argue that capital shallowness is a problem for New Zealand. In Australia, the stock of human capital growth rate explains about 22 percent of the productivity growth, and about 15 percent in New Zealand. Excess ideas, $\hat{A}^* - \gamma n$, adjusted for quality of human capital, where $\hat{A}^*$ is quality-adjusted TFP growth and γn is the effect of the growth of population in the G8 on generating new ideas, explain half of Australia's productivity growth, but a staggering three-quarters of New Zealand's growth. This suggests that three-quarters of the growth in productivity is TFP-driven in New Zealand. Also surprising is that world stock of ideas effects arising from γn are 12 and 17 percents in Australia and New Zealand, which are a lot higher than those of the G7. Given the similarities between the two countries, these differences are significant.

About 80 to 85 percent of the growth rate of productivity in the G7 and Europe comes from transitional dynamics, where the growth rates of human capital and excess ideas are the drivers of productivity growth. A very small contribution is found from the scale effect, i.e., world population, which is also similar to results from Jones (2002). The results for the G7 and Europe also exhibit large variations.

The Asian countries have different growth experience. They have the highest productivity growth in our sample, 3 and 4 percent in Hong Kong and Singapore, and about 5 percent in Korea. There are large contributions from capital (except for Singapore), human capital, its quality, and excess ideas. The latter explains about 40 percent of productivity growth in Hong Kong and South Korea and 70 percent in Singapore. More than 15 percent of Singapore's productivity growth, and 8 to 9 percents of Hong Kong and Korea, are also explained by the G8 population effect.

Pissarides and Veganzoes-Varoudakis (2005) studied the labor markets in the MENA countries. They found a lack of correlation between human capital and productivity growth and attributed this to either low quality human capital or to a misallocation. The latter involves labor diverted away from productive activity, (i.e., stood idle), concentrated in government jobs, or engaged in rent-seeking activities. They showed that government jobs dominate the labor market, where more than half of graduates are employed. They did not investigate the effect of the quality of labor.

Over the past three decades, MENA countries experienced a positive trend in human capital, but also suffered relatively low productivity growth rates. Figure 2 plots human capital measured by average years of schooling (Barro and Lee, 2010) for the years 1980 and 2007.[10] Clearly quantitatively, the growth rate has been impressive. Figure 3 is a scatter plot of growth rates of human capital and productivity, where productivity is the PPP-adjusted real GDP per hour. The trend might be positive, but the lack of correlation is obvious. The scatter points are further away from the 45 degree line. The correlation coefficient between the growth rate of the PPP-adjusted real GDP per hour and the growth rate of human capital stock, unadjusted for quality, for MENA countries is 0.053. It is -0.05 when we adjust human capital for quality. For the OECD, the correlations coefficient is 0.40.

Figure 4 summarizes the contributions of excess ideas to productivity growth. It is a scatter plot of excess ideas growth rates adjusted for quality of education $\hat{A}^* - \gamma n$, and productivity growth rates for the full sample.[11] Note that both axis have zero growth rates such that the graph has four quadrants. The association is impressive for the non-MENA countries, but the overall fit is quite strong along the 45-degree line.

Figure 5 scatter plots two quantities: the changes in relative education quality ξ_j on the horizontal axis and productivity growth on the vertical axis. The change in the quality of education is the relative change $\Delta\xi_j$, which is solved for from the model and is equal to $\Delta \ln(h_{jt}^* / h_{jt})$ divided by the number of years. MENA countries are represented by filled circles while the non-MENA countries (G7, European OECD, Turkey, and Asia) are represented by with unfilled circles. There are four quadrants in this graph. Note that MENA countries, except Egypt, Tunisia, and Oman, are all located in the southwest quadrant, where both growth rates are negative. Non-MENA countries are in the northeast quadrant, where both quantities are positive. Not a single

---

[10] Barro (1993), Romer (1990b) and Mankiw *et al.* (1992) used school enrollment, which is a flow variable to measure human capital. This measure was criticized in the literature (e.g., Levine and Renelt (1992)) as an inaccurate measure of the stock of human capital. Barro and Lee (1993, 2010) improved the measurement of human capital by computing stocks using country-level surveys and consensus data, but the data do not reflect "quality".

[11] Canada and France are not reported in TIMMS, so we do not have quality-adjusted variables.

MENA country is there. The graph indicates a partial positive association between productivity growth and quality of labor because the majority of countries in the sample fall in the northwest and southeast quadrants.

To be specific, countries falling in the northeast quadrant such as South Korea, Singapore, US, Italy, and Australia and countries that fall in the southwest quadrant like Saudi Arabia, Kuwait, Qatar, and Algeria (all of which are oil-producing MENA countries), confirm the hypothesis that quality of human capital and productivity growth are positively correlated. Countries falling in the northwest and southeast quadrants do not confirm such a positive correlation. In the northwest, we have four MENA countries, of which only one is among the oil-producing countries, Oman. The others are Egypt, Tunisia, and Morocco. Sweden and Norway have positive productivity growth and significant deterioration in the quality of education. They are followed by the Netherlands. New Zealand, Denmark, Austria, Germany, and the UK have near-zero change in their quality of education, but positive productivity growth. In the southeast quadrant, three MENA countries have improved their quality of education over the past thirty years but their productivity growth remained stagnant. These are Bahrain, Jordon, and Lebanon.

Finally, Figure 6 scatter plots the difference in the cross-country growth rates and the cross-country difference in the quality of human capital. We measure the relative growth rate of productivity by $\hat{y}^* - \hat{y}_j$, where $\hat{y}^*$ is the growth rate of real GDP per hour for Korea. Korea has the highest productivity growth in the sample. The level of per capita real GDP growth rate in 1950 was smaller than all other countries in our sample (see Maddison data online), except for Lebanon. We scatter plot $\hat{y}^* - \hat{y}_j$ against $\Delta\xi^* - \Delta\xi_j$, the relative change differential in the quality of education, where. Non-MENA countries are represented by filled circles and MENA countries are represented by unfilled circles. Again, first we observe that non-MENA countries are closest to the 45-degree line, i.e., close fit, except for Norway and Sweden, whose quality of education plummeted recently. MENA countries, except Oman, are further from the 45-degree line because South Korea's productivity growth exceeds theirs significantly, and the quality of education differentials are also very significant.

5. Conclusions

The growth literature emphasizes the role of human capital in driving technical progress. Empirical support for this assertion is strong in the literature. Adjusting human capital for quality has received less emphasis, however, even though some classic papers in macroeconomics included it in modeling (see for example Lucas and Rapping, 1969). Hanushek and Kimko (2000) and Hanushek and Woesmann (2009, 2010) focus entirely on measuring and analyzing the effect of quality of education on growth. They proxy quality with measures of cognitive skills. In this paper we follow them and measure the quality of human capital stock by cognitive skills. We use a different data source, Trends in International Mathematics and Science study (TIMMS), and methodology. Instead of growth regressions, we calibrate an endogenous growth model for thirty developed and developing countries over the period 1980 to 2007. We measure the contribution of changes in the quality of education to productivity growth and measure how much of the cross-country difference in productivity growth could be attributed to cross-country differences in the quality of human capital.

TIMMS data clearly reveal a world-wide decline in the relative quality of education – relative to Singapore which maintained leading scores in all TIMMS cognitive skills tests – over the period 1980 to 2007. We identified four groups of countries: one with positive productivity growth and positive changes in the relative quality of education, thus positive correlation and another with negative productivity growth and a decline in the relative quality of education, which implies a positive correlation. The first group consists only of OECD countries while the second group consists only of MENA Arab countries. The two groups made up one-third of the sample. The other two groups included countries with positive productivity growth and negative changes in the relative quality of education, and countries with negative productivity growth and positive changes in the relative quality of education. The former included OECD countries, and four MENA Arab countries (Egypt, Oman, Tunisia, and Morocco) while the latter includes three MENA Arab countries. These two groups made up the remaining two-thirds of the sample. We conclude that there is only partial evidence for the quality of human capital-growth hypothesis.

We examined the correlation between differences in productivity growth for each country and differences in the quality of education relative to Korea, which is the country with the highest productivity growth. We found that most of the OECD relative differences in productivity growth are highly correlated with relative quality of education, but this is not so for MENA Arab countries.

The model also revealed a few interesting things about productivity growth. There are significant cross-country differences in the quality-adjusted transitional dynamics. MENA Arab countries are very different from the rest of the world. With the exception of Egypt, Tunisia, and Oman, MENA countries had negative productivity growth. Within the G7, for example, Germany, France, and Italy are different from the rest.

Productivity growth in Canada, UK, Denmark, Sweden, Singapore, New Zealand, Egypt, and Tunisia, for example, is largely driven by the stock of ideas in excess of the steady state rate, TFP. This effect amounts to three-quarters of the growth in productivity. This is also true, but to a lesser degree, in countries like Japan, Australia, Austria, the Netherlands, and Norway. Human capital, on the other hand, is the main contributor to productivity growth in France and Germany. About three-quarters of growth could be attributed to quality-adjusted human capital in Germany. The MENA countries are far behind the OECD and Asia.

Given these results, MENA countries must look seriously at the experiences of Egypt, Tunisia, and Oman and take measures to enhance productivity growth. There are no shortages in capital intensity and labor. They have problems in efficiency, which might be related to problems in the quality of education. Bahrain, Jordon, and Lebanon have already increased the quality of their human capital, but are still unproductive.

Table 1 – Accounting for Growth with and without Quality of Human Capital 1980 – 2007

Transition Dynamics												Steady State	
Country					Output per hour	Capital Intensity	Labor Reallocation	Educational Attainment	Quality-Adjusted Educational Attainment	Excess Idea Growth	Excess Idea Growth Adjusted for Quality of Human Capital	Steady State Growth	Quality-Adjusted Steady State Growth
j	α [1]	γ [2]	γ^*	ω [3]	$\hat{y}$	$\frac{\alpha}{1-\alpha}(\hat{K}-\hat{Y})$	$\hat{l}_Y$	$\hat{h}$	$\hat{h}^*$	$\hat{A}-\gamma n$	$\hat{A}^*-\gamma n$	γn	γn^*
Canada[5]	0.34	0.36			**1.48**	**0.46**	**-0.02**	**0.58**		**0.38**		**0.08**	
					(100.0)	(31.06)	(-1.35)	(39.18)		(25.6)		(5.4)	
France[5]	0.38	0.42			**2.02**	**0.34**	**-0.01**	**1.52**		**0.14**		**0.03**	
					(100.00	(16.98)	(-0.36)	(75.01)		(6.88)		(1.49)	
Germany	0.28	0.04	0.07		**1.43**	**0.28**	**-0.05**	**1.03**	**0.92**	**0.13**	**0.23**	**0.03**	**0.05**
					(100.0)	(19.69)	(-3.25)	(72.23)	(63.94)	(9.31)	(16.12)	(2.02)	(3.50)
Italy	0.30	0.025	0.05		**1.43**	**0.41**	**0.00**	**0.92**	**0.84**	**0.08**	**0.15**	**0.02**	**0.03**
					(100.0)	(28.45)	(-0.20)	(64.78)	(58.93)	(5.74)	(10.54)	(1.24)	(2.29)
Japan	0.33	0.21	0.23		**2.24**	**0.64**	**-0.03**	**0.77**	**0.71**	**0.71**	**0.76**	**0.15**	**0.16**
					(100.0)	(28.77)	(-1.48)	(34.25)	(31.59)	(31.60)	(33.79)	(6.85)	(7.32)
UK	0.27	0.46	0.47		**2.16**	**0.09**	**-0.04**	**0.26**	**0.24**	**1.52**	**1.54**	**0.33**	**0.33**
					(100.0)	(4.07)	(-1.78)	(12.24)	(10.96)	(70.25)	(71.29)	(15.23)	(15.45)
USA	0.23	0.41	0.39		**1.88**	**0.19**	**-0.01**	**0.06**	**0.13**	**1.35**	**1.29**	**0.29**	**0.28**
					(100.0)	(9.94)	(-0.41)	(3.33)	(7.00)	(71.62)	(68.60)	(15.52)	(14.87)
Australia	0.29	0.28	0.30		1.74	0.19	-0.02	0.44	0.38	0.93	0.98	0.20	0.21
					(100.0)	(11.09)	(-1.37)	(25.17)	(22.10)	(53.51)	(56.04)	(11.60)	(12.15)
New Zealand	0.41	0.31	0.33		1.37	-0.07	-0.07	0.26	0.19	1.02	1.08	0.22	0.23
					(100.0)	(-5.18)	(-4.81)	(19.30)	(14.06)	(74.54)	(78.84)	(16.16)	(17.09)

Table 1 – Continued - Accounting for Growth with and without Quality of Human Capital 1980 – 2007

					Transition Dynamics							Steady State	
Country					Output per hour	Capital Intensity	Labor Reallocation	Educational Attainment	Quality-Adjusted Educational Attainment	Excess Idea Growth	Excess Idea Growth Adjusted for Quality of Human Capital	Steady State Growth	Quality-Adjusted Steady State Growth
j	α	γ	γ^*	ω	$\hat{y}$	$\frac{\alpha}{1-\alpha}(\hat{K}-\hat{Y})$	$\hat{l}_Y$	$\hat{h}$	$\hat{h}^*$	$\hat{A}-\gamma n$	$\hat{A}^*-\gamma n$	γn	γn^*
Austria	0.28	0.12	0.14		**1.15**	**0.17**	**-0.07**	**0.56**	**0.47**	**0.41**	**0.48**	**0.09**	**0.10**
					(100.0)	(14.50)	(-6.36)	(48.54)	(41.22)	(35.59)	(41.61)	(7.72)	(9.02)
Denmark	0.32	0.39	0.40		**2.13**	**0.35**	**-0.03**	**0.25**	**0.20**	**1.29**	**1.32**	**0.28**	**0.29**
					(100.0)	(16.21)	(-1.26)	(11.67)	(9.52)	(60.30)	(62.07)	(13.07)	(13.46)
Netherlands	0.28	0.06	0.09		**0.99**	**0.10**	**-0.02**	**0.72**	**0.58**	**0.18**	**0.29**	**0.04**	**0.06**
					(100.0)	(9.90)	(-2.19)	(72.36)	(58.72)	(18.17)	(29.38)	(3.94)	(6.37)
Norway	0.35	0.27	0.35	0.14	**2.41**	**-0.31**	**-0.02**	**1.16**	**0.84**	**0.87**	**1.14**	**0.19**	**0.25**
					(100.0)	(-12.70)	(-0.71)	(48.31)	(34.76)	(36.05)	(47.19)	(7.81)	(10.23)
Sweden	0.23	0.33	0.37		**1.57**	**-0.06**	**-0.07**	**0.50**	**0.23**	**0.99**	**1.21**	**0.21**	**0.26**
					(100.0)	(-3.95)	(-4.41)	(31.72)	(14.69)	(62.99)	(76.98)	(13.65)	(16.69)
Hong Kong	0.46	0.41	0.40		**3.03**	**0.92**	**-0.03**	**0.54**	**0.55**	**1.32**	**1.31**	**0.29**	**0.28**
					(100.0)	(30.20)	(-0.87)	(17.72)	(18.22)	(43.52)	(43.11)	(9.43)	(9.34)
Korea	0.45	0.57	0.57		**4.78**	**1.68**	**-0.06**	**0.88**	**0.89**	**1.87**	**1.86**	**0.40**	**0.40**
					(100.0)	(35.18)	(-1.20)	(18.51)	(18.67)	(39.04)	(38.90)	(8.46)	(8.43)
Singapore	0.40	0.91	0.91		**4.21**	**-0.27**	**-0.05**	**0.92**	**0.92**	**2.97**	**2.97**	**0.64**	**0.64**
					(100.0)	(-6.53)	(-1.24)	(21.95)	(21.95)	(70.53)	(70.53)	(15.29)	(15.29)
Turkey	0.40	-0.04	0.04		**1.82**	**0.77**	**-0.02**	**1.22**	**0.89**	**-0.13**	**0.14**	**-0.03**	**0.03**
					(100.0)	(42.60)	(-0.93)	(67.06)	(48.94)	(-7.18)	(7.72)	(-1.56)	(1.67)

Table 1 – Continued - Accounting for Growth with and without Quality of Human Capital 1980 – 2007

Transition Dynamics												Steady State	
Country					Output per hour	Capital Intensity	Labor Reallocation	Educational Attainment	Quality-Adjusted Educational Attainment	Excess Idea Growth	Excess Idea Growth Adjusted for Quality of Human Capital	Steady State Growth	Quality-Adjusted Steady State Growth
j	α	γ	γ^*	ω	$\hat{y}$	$\frac{\alpha}{1-\alpha}(\hat{K}-\hat{Y})$	$\hat{l}_Y$	$\hat{h}$	$\hat{h}^*$	$\hat{A}-\gamma n$	$\hat{A}^*-\gamma n$	γn	γn^*
Egypt	0.38	0.46	0.60		**3.01**	**-1.00**	**-0.004**	**1.58**	**1.03**	**2.00**	**2.46**	**0.43**	**0.53**
					(100.0)	(-33.1)	(-0.14)	(52.5)	(34.0)	(66.4)	(81.5)	(14.4)	(17.7)
Jordan	0.36	-0.86	-0.77		**-0.85**	**0.98**	**-0.06**	**1.62**	**1.29**	**-2.79**	**-2.52**	**-0.60**	**-0.55**
Lebanon	0.40	-1.38	-1.28		**-3.40**	**0.45**	**0.001**	**1.62**	**1.21**	**-4.49**	**-4.16**	**-0.97**	**-0.90**
Morocco	0.56	-0.56	-0.42		**0.16**	**1.32**	**-0.007**	**1.08**	**0.51**	**-1.83**	**-1.37**	**-0.40**	**-0.30**
					(100.0)	(842.09)	(-4.221)	(686.6)	(324.6)	(-1170.7)	(-873.25)	(-253.7)	(-189.3)
Syria	0.33	-0.24	-0.19		**-0.56**	**-0.29**	**0.001**	**0.54**	**0.35**	**-0.63**	**-0.48**	**-0.17**	**-0.13**
Tunisia	0.50	0.80	0.92		**2.40**	**-2.04**	**-0.028**	**0.97**	**0.51**	**2.92**	**3.31**	**0.56**	**0.65**
					(100.0)	(-85.05)	(-1.183)	(40.64)	(21.25)	(122.05)	(137.99)	(23.54)	(26.99)
Algeria	0.59	-1.23	-1.32	0.23	-2.11	-0.36	-0.003	1.70	1.07	-2.58	-2.06	-0.87	-0.75

Table 1 – Continued - Accounting for Growth with and without Quality of Human Capital 1980 – 2007

					Transition Dynamics							Steady State	
Country					Output per hour	Capital Intensity	Labor Reallocation	Educational Attainment	Quality-Adjusted Educational Attainment	Excess Idea Growth	Excess Idea Growth Adjusted for Quality of Human Capital	Steady State Growth	Quality-Adjusted Steady State Growth
j	α	γ	γ^*	ω	$\hat{y}$	$\frac{\alpha}{1-\alpha}(\hat{K}-\hat{Y})$	$\hat{l}_Y$	$\hat{h}$	$\hat{h}^*$	$\hat{A}-\gamma n$	$\hat{A}^*-\gamma n$	γn	γn^*
Bahrain	0.58	-1.23	-1.07	0.22	**-0.46**	**-1.22**	**0.001**	**1.22**	**0.90**	**0.53**	**0.79**	**-0.99**	**-0.93**
Kuwait	0.57	-0.14	-0.19	0.32	**-0.77**	**-1.83**	**0.000**	**0.42**	**0.21**	**0.54**	**0.71**	**0.10**	**0.14**
Oman	0.53	0.34	0.44	0.20	**1.81**	**-0.55**	**-0.001**	**1.07**	**0.68**	**1.04**	**1.36**	**0.24**	**0.31**
					(100.0)	(-30.21)	(-0.048)	(59.22)	(37.82)	(57.70)	(75.29)	(13.33)	(17.15)
Qatar	0.50	-0.20	-0.10	0.35	**-2.85**	**-0.74**	**-0.001**	**0.88**	**0.40**	**-2.83**	**-2.44**	**-0.16**	**-0.07**
Saudi Arabia	0.51	-1.67	-1.57	0.25	**-4.74**	**1.71**	**-0.001**	**0.95**	**0.56**	**-6.23**	**-5.91**	**-1.18**	**-1.11**

1. α is the share of capital in output computed from national income accounts, which is gross operating surplus to real GDP ratio averaged over the sample.
2. γ is $\Delta A_{jt} / \Delta H_{At}$, for country j.
3. ω is the share of oil and gas in output, which is total revenues to real GDP ratio, averaged over the sample.
4. Numbers in parentheses are percentages of the growth rate of output per working age population.
5. Canada and France are not reported in TIMMS therefore there are no quality-adjusted figures.
6. Singapore does not report hours-worked, hence output / working age person is used instead of output per hour.
7. Hat on the variables denotes growth rate, $(\ln x_{2007} - \ln x_{1980}) * 100 / number\ of\ years$.
8. Excess ideas $\frac{\sigma}{1-\alpha}\hat{A} - \gamma n$ in equations (12 and 19) has $\sigma / 1-\alpha = 1-\alpha / 1-\alpha = 1$.
9. The oil producers, Norway, Algeria, and the GCC countries excess idea and excess idea adjusted for quality in columns 11 and 12 are as in equation 17,

$$\frac{\sigma}{1-\alpha}\hat{A} + \frac{\omega}{1-\alpha}\hat{W} - \gamma n$$

Figure 1
Average TIMMS

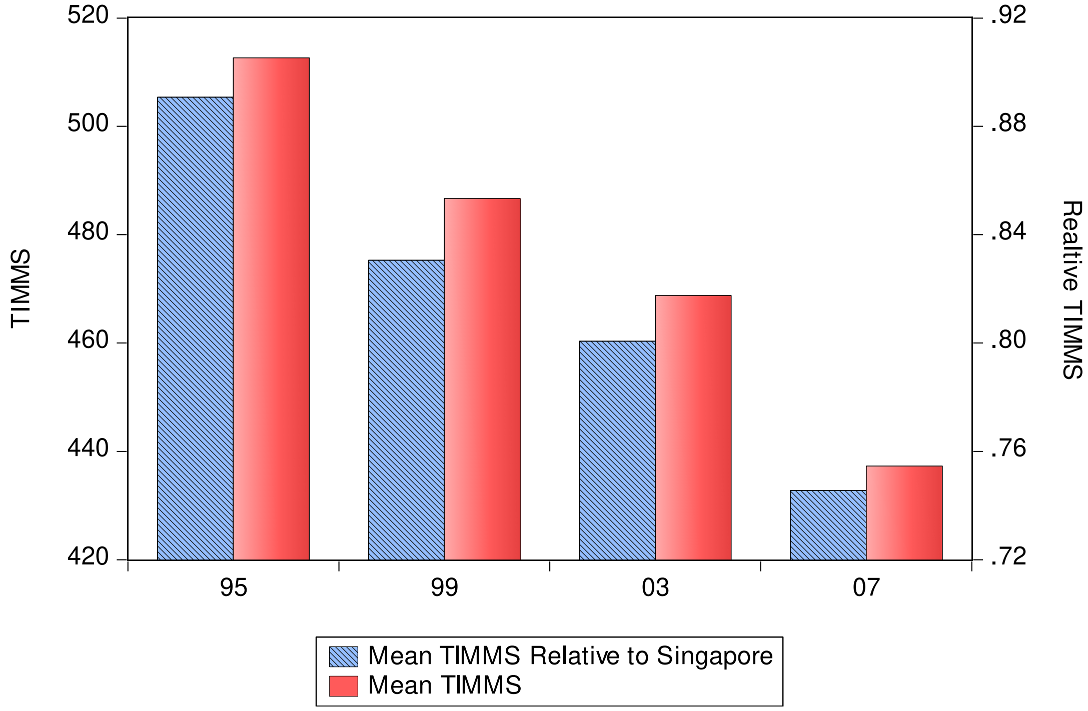


Figure 2
Human Capital for the MENA Countries

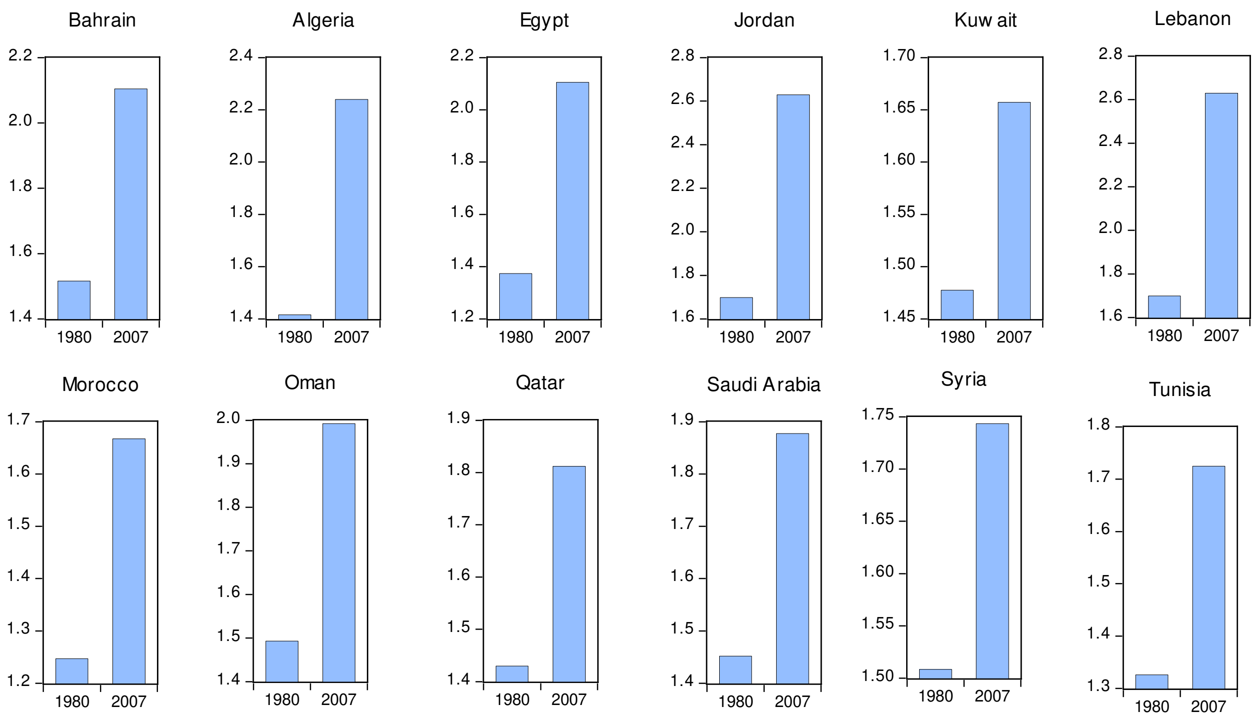


Figure 3
Growth Rate of Human Capital and Productivity for the MENA Countries
(1980-2007)

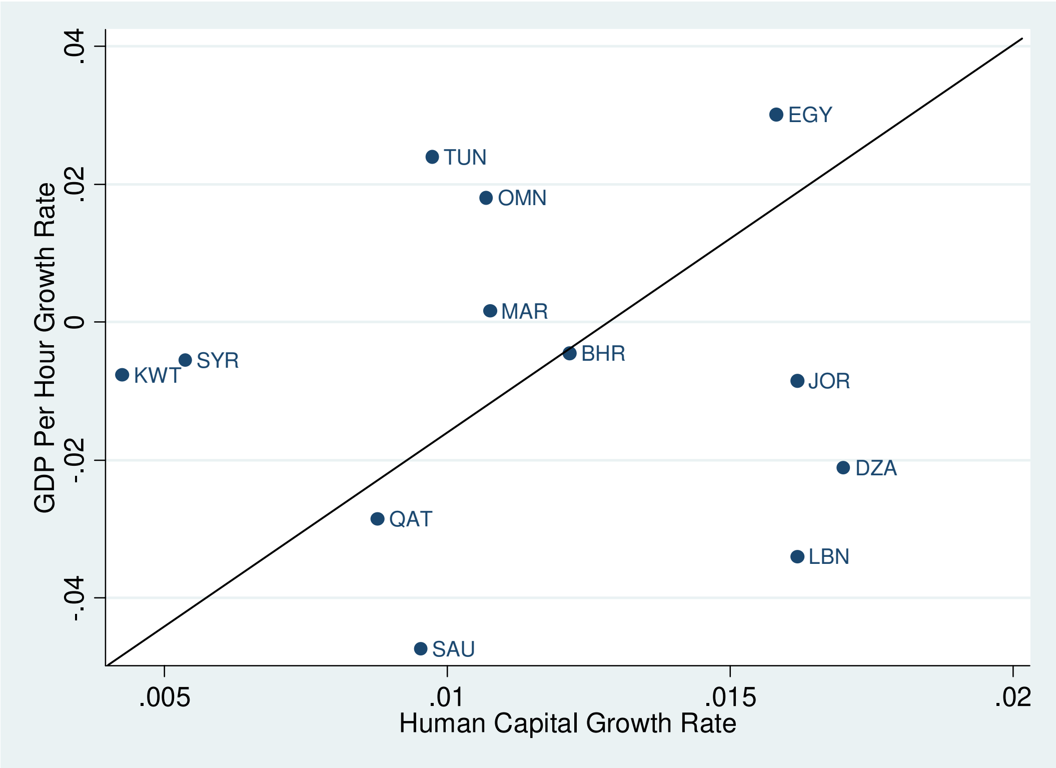


Figure 4
Excess Ideas and Productivity Growth
(1980-2007)

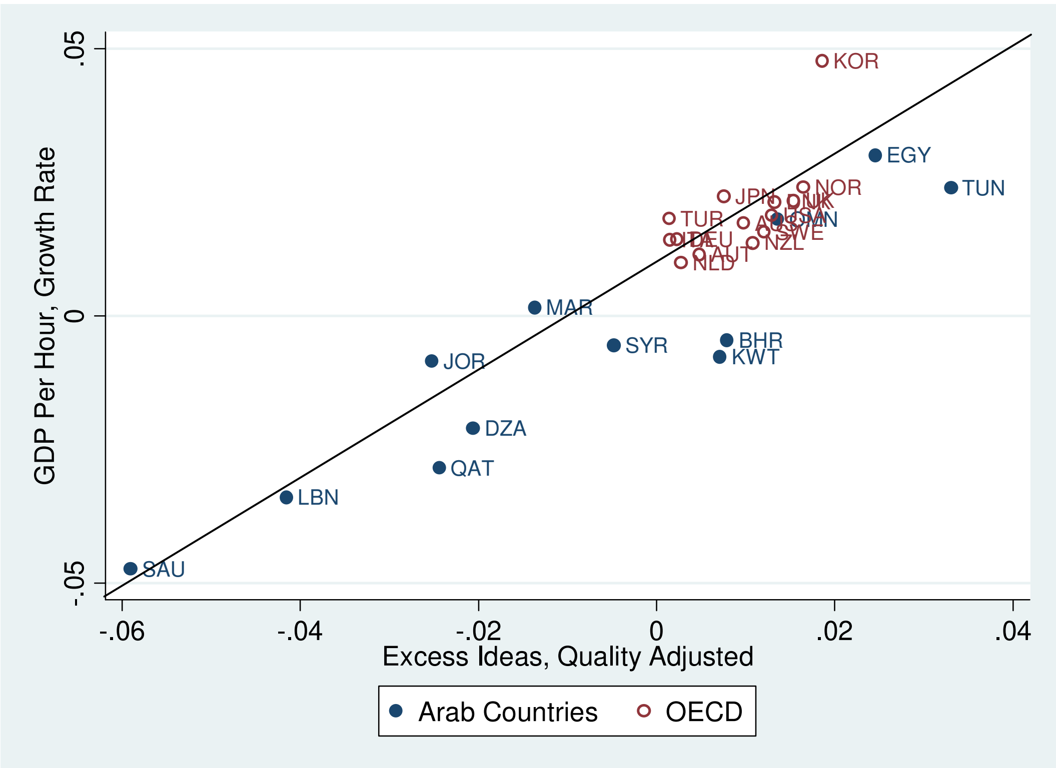


Figure 5
Quality of Education and Productivity Growth
(1980-2007)

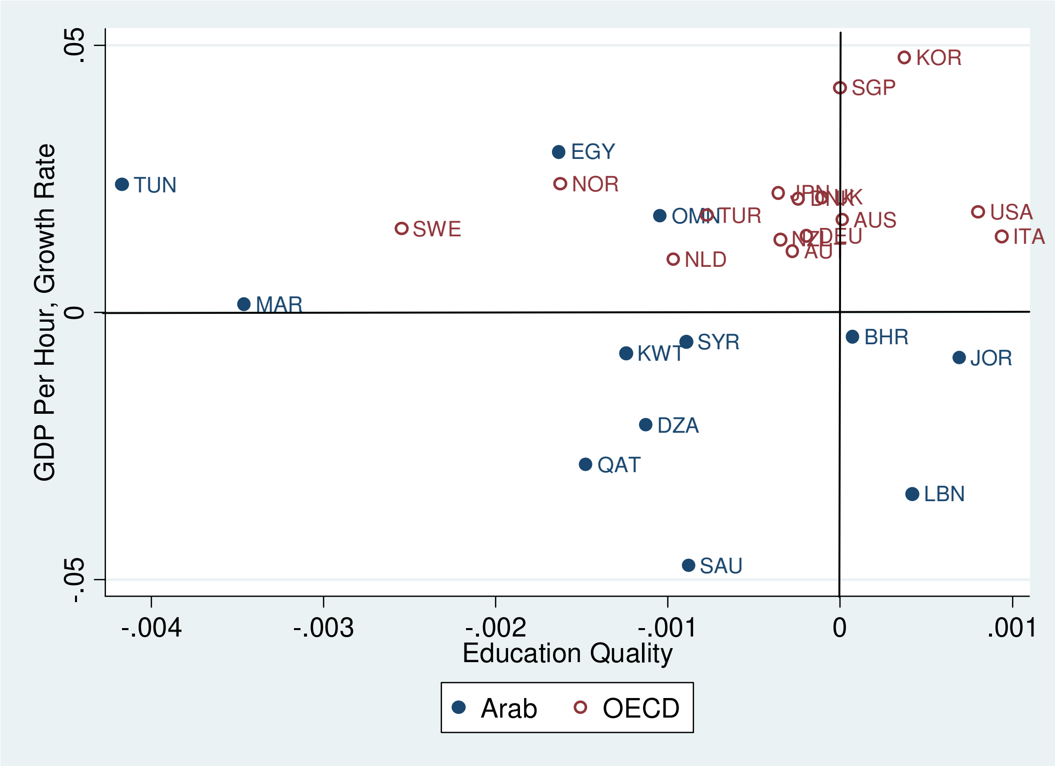
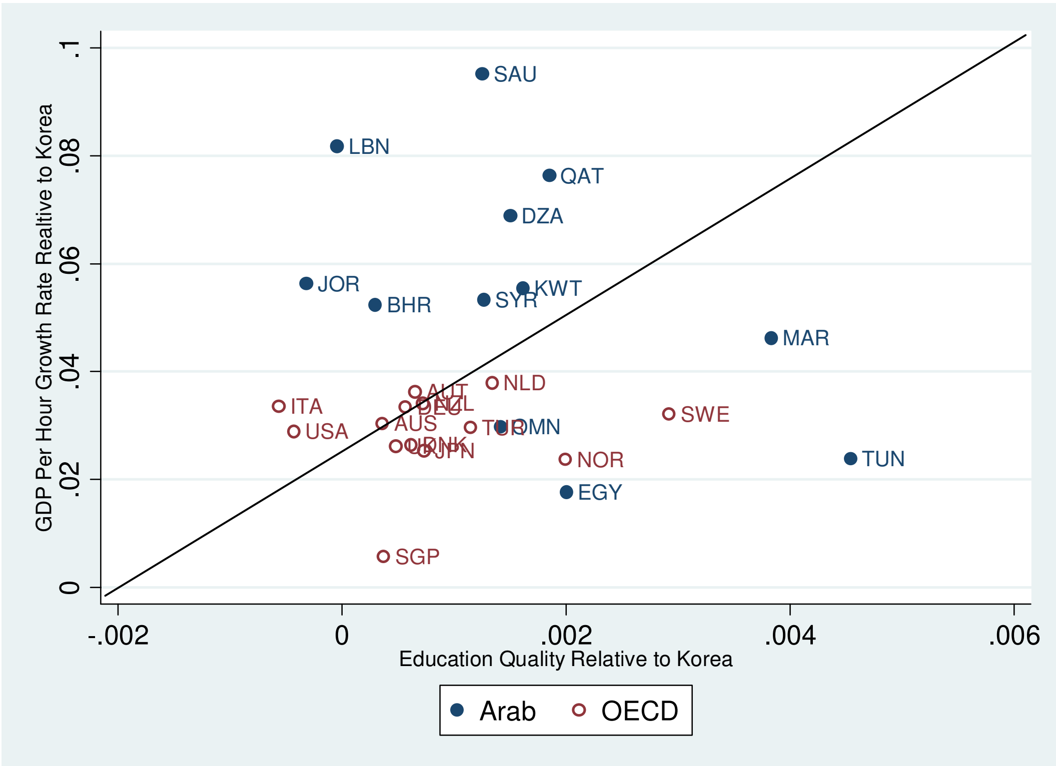


Figure 6
Relative Change in the Quality of
Education & Productivity Growth Differentials (1980-2007)



Appendix 1:

Deriving the model for the case of the alternative production function for the oil-producing countries.

Output per unit of labor is:

$$y_t = \frac{Y_t}{L_t} ;$$

Human capital used in the production of goods is:

$H_{Y_t} = h_t L_{Y_t}$; where h_t is the stock of human capital per person and L_t is labor used in the production of goods and services.

$$l_{Y_t} = \frac{L_{Y_t}}{L_t} ;$$

$$y_t = \frac{Y_t}{L_t} = \frac{Y_t}{L_{Y_t}} \cdot \frac{L_{Y_t}}{L_t} = \frac{A_t^{\sigma} K_t^{\alpha} H_{Y_t}^{\beta} W_t^{\omega}}{H_{Y_t} / h_t} l_{Y_t} = l_{Y_t} h_t A_t^{\sigma} K_t^{\alpha} H_{Y_t}^{\beta - 1} W_t^{\omega}$$

Or from $Y_t = A_t^{\sigma} K_t^{\alpha} H_{Y_t}^{\beta} W_t^{\omega}$ we obtain ; $H_{Y_t}^{\beta} = Y_t / A_t^{\sigma} K_t^{\alpha} W_t^{\omega}$ and $H_{Y_t} = Y_t^{1/\beta} / A_t^{\sigma/\beta} K_t^{\alpha/\beta} W_t^{\omega/\beta}$ then dividing the two we get: $H_{Y_t}^{\beta-1} = H_{Y_t}^{\beta} / H_{Y_t} = Y_t^{1-1/\beta} A_t^{\sigma/\beta - \sigma} K_t^{\alpha/\beta - \alpha} W_t^{\omega/\beta - \omega}$

After replacing this in the production function, we have;

$$y_t = l_{Y_t} h_t A_t^{\sigma} K_t^{\alpha} H_{Y_t}^{\beta-1} W_t^{\omega} = l_{Y_t} h_t A_t^{\sigma} K_t^{\alpha} \left(Y_t^{1-1/\beta} A_t^{\sigma/\beta-\sigma} K_t^{\alpha/\beta-\alpha} W_t^{\omega/\beta-\omega} \right) W_t^{\omega}$$

Simplification gives:

$$y_t = l_{Y_t} h_t A_t^{\sigma/\beta} K_t^{\alpha/\beta} Y_t^{1-1/\beta} W_t^{\omega/\beta} = K_t^{\alpha/\beta} Y_t^{1-1/\beta} l_{Y_t} h_t A_t^{\sigma/\beta} W_t^{\omega/\beta}$$

$$y_t = \frac{K_t^{\alpha/\beta}}{Y_t^{\frac{1-\beta}{\beta}}} l_{Y_t} h_t A_t^{\sigma/\beta} W_t^{\omega/\beta}$$

Then $A_t^{\sigma/\beta} = y_t / (\frac{K_t^{\alpha/\beta}}{Y_t^{\frac{1-\beta}{\beta}}} l_{Y_t} h_t W_t^{\omega/\beta})$, and

$$A_t = \frac{y_t^{\frac{\beta}{\sigma}}}{\frac{K_t^{\alpha/\sigma}}{Y_t^{\frac{1-\beta}{\sigma}}}\left(l_{Y_t} h_t\right)^{\frac{\beta}{\sigma}} W_t^{\omega/\sigma}}$$

If we assume $\alpha + \beta = 1$ only, we get:

$$y_t = \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} l_{Y_t} h_t A_t^{\frac{\sigma}{1-\alpha}} W_t^{\frac{\omega}{1-\alpha}} \quad \text{and}$$

$$A_t = \frac{y_t^{\frac{1-\alpha}{\sigma}}}{\left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{\sigma}} \left(l_{Y_t} h_t\right)^{\frac{1-\alpha}{\sigma}} W_t^{\omega/\sigma}}$$

The growth rate model:

From $\dot{A}_t = \delta H_{A_t}^{\lambda} A_t^{\phi}, A_0 > 0 \Rightarrow g_A = \frac{\dot{A}}{A_t} = \delta H_{A_t}^{\lambda} A_t^{\phi}$

Solving for $A_t = (g_A / \delta)^{\frac{1}{\phi - 1}} H_{A_t}^{\frac{\lambda}{\phi}}$.

$$y_t = \frac{K_t^{\alpha/\beta}}{Y_t^{\frac{1-\beta}{\beta}}} l_{Y_t} h_t A_t^{\sigma/\beta} W_t^{\omega/\beta} \, .$$

Solving for: $A_t^{\sigma/\beta} = A_t^{\frac{\sigma}{1-\alpha}} = (g_A / \delta)^{\frac{1}{(\phi-1)}\frac{\sigma}{(1-\alpha)}} H_{A_t}^{\frac{\lambda}{(\phi-1)}\frac{\sigma}{(1-\alpha)}}$

Letting $\gamma = \frac{\lambda}{(\phi - 1)} \frac{\sigma}{(1-\alpha)}$ as in Jones (2002) we get:

$$A_t^{\frac{\sigma}{1-\alpha}} = (g_A / \delta)^{\frac{\gamma}{\lambda}} H_{A_t}^{\gamma}$$

Let $g_k = d\ln\left(\frac{K_t}{L_t}\right)$, thus,

$$\left(\frac{\dot{K}_t}{K_t}\right) = d\ln(K_t) = d\left(\ln(K_t / L_t) + \ln(L_t)\right) = g_K + n \, ;$$

and from $\dot{K}_t = s_{K_t} Y_t - dK_t, K_0 > 0$

$\left(\frac{\dot{K}_t}{K_t}\right) = s_K\left(\frac{Y_t}{K_t}\right) - d$, which is equal to $g_K + n$ from above

So we can have:

$$\frac{K_t}{Y_t} = \frac{s_K}{g_K + d + n}$$

Asterisk denotes constant growth rate at steady state:

$$\frac{K_t}{Y_t} = \frac{s^*_{K_t}}{g_k + d + n}$$

Finally:

$$y_t = \left(\frac{s^*_{K_t}}{g_k + d + n}\right)^{\frac{\alpha}{1-\alpha}} l_{Y_t} h_t (g_A / \delta)^{\frac{\gamma}{\lambda}} H^{*\gamma}_{A_t} W_t^{\frac{\omega}{1-\alpha}}$$

The steady state relationship between productivity growth and growth in ideas given by equation (11) is easily seen from equation (9). When all variables grow at constant rates in the steady state, the growth rate of productivity given by g_y is equal to $\frac{\sigma}{1-\alpha} g_A$. From $A_t^{\frac{\sigma}{1-\alpha}} = (g_A / \delta)^{\frac{\gamma}{\lambda}} H^{\gamma}_{A_t}$, the steady state gives $\frac{\sigma}{1-\alpha} g_A = \gamma g_{H_A}$ since h must be constant along the steady state path, growth in the effective number of world researchers in the financial sector H_{A_t} is driven by population growth, so : $g_{H_A} = n$, hence equation 11.

Taking log of the output per capita equation $y_t = (\frac{K_t}{Y_t})^{\frac{\alpha}{1-\alpha}} l_{Y_t} h_t A_t^{\frac{\sigma}{1-\alpha}} W_t^{\frac{\omega}{1-\alpha}}$ under the assumption that $\alpha + \beta = 1$ gives:

$$\ln y_t = \frac{\alpha}{1-\alpha}\left(\ln K_t - \ln Y_t\right) + \ln(l_{Yt}) + \ln(h_t) + \frac{\sigma}{1-\alpha}\ln A_t + \frac{\omega}{1-\alpha}\ln(W_t)$$

And the derivative with respect to time is:

$\hat{y}_t = \frac{\alpha}{1-\alpha}(\hat{K}_t - \hat{Y}_t) + \hat{h}_t + \hat{l}_{Yt} + \left(\frac{\sigma}{1-\alpha}\hat{A}_t + \frac{\omega}{1-\alpha}\hat{W}_t\right)$, where a hat means the growth rate.

Appendix 2: Data

Our data are compiled from different sources. GDP per capita level is taken from Penn Table PWT 6.3, where data are in constant PPP 2005 prices. Population figures are taken from the World Bank Development Indicators. And hours data are from the International Labor Office. For MENA countries, hours are taken from Laabas and Razzak (2010), who estimated equilibrium average hours worked for the Arab countries by calibrating a neoclassical labor supply model for the period from 1990 to 2007. We use the estimates of hours for 2007 and use the same method to compute hours for 1980. Productivity is GDP per hour is then (GDP per capita times population) / hours. Productivity growth is calculated as the log difference between 1980 and 2007.

Capital stock is computed using PIM method, where the starting period (capital output ratio for 1960) is taken from Nehru Drashwar (1994) database.[12] Capital stock for the base year is GDP times the capital output ratio of that year. Perpetual Inventory Method is used to update capital stock data using investment data from the PWT 6.3 database. The depreciation rate is fixed at 5 percent. The initial capital stock is assumed to equal 2 times real GDP in the year 1960. The capital stock is in constant prices and PPP-adjusted. The depreciation rate is 5 percent.

The stock and production of hydrocarbons are from BP Statistical Review. BP website at http://www.bp.com/statisticalreview

Human capital is measured by the average years of schooling. Data for 1980 are taken from the Barro-Lee (2010) database. Then, data for 2007 are computed from a quasi PIM method proposed by Barro. The schooling and enrollment data are from the World Bank database.

Working age population and labor force data are from WDI of the World Bank.

The number of researchers data are from the OECD, World Development Indicators of the World Bank, and UNESCO. The global stock of ideas is produced by the G8 in this paper. These are the G7 plus South Korea, which has 6 percent of the world researchers.[13] In Jones (2002), ideas are produced by the G5.

Country codes: Canada (CAN); France (FRA); Germany (DEU); Italy (ITA); Japan (JPN); Australia (AUS); Austria (AUT); Denmark (DNK); Netherlands (NLD); Norway (NOR); Sweden (SWE); New Zealand (NZL); Turkey (TUR); Hong Kong (HKG); Korea (KOR); Singapore (SGP); Egypt (EGY); Jordon (JOR); Lebanon (LBN); Morocco (MAR); Syria (SYR); Tunisia (TUN); Algeria (DZA); Bahrain (BHR); Kuwait (KWT); Oman (OMN); Qatar (QTR); Saudi Arabia (SAU). Data are listed below.

[12] http://unstats.un.org/unsd/default.htm

[13] Russia has about 11 percent of the world researchers and should be included in the calibration, but Russia lacks the data required to calibrate the model.

COUNTRY	YEAR	Population	Working Age Population % of Population	Labor Force	Researchers	Rate of Return to Education	TIMMS	Average Years of Schooling	Average Weakly Hours	GDP	Capital Output Ratio	Rate of Hydrocarbon Utilization	Share of Hydrocarbon Revenues to GDP
Algeria	1980	18811202	0.50	4554319	1319	11.93	398.1	2.91	10.60	9.76E+10	4.66	0.01	0.23
Algeria	2007	33858168	0.67	14079508	16895	11.23	381.8	7.18	10.60	2.17E+11	4.35	0.03	0.23
Australia	1980	14692000	0.65	6753840	33635	8.05	511.3	10.29	22.52	2.95E+11	3.23		
Australia	2007	21072500	0.67	11079764	126070	7.97	513.5	11.87	22.25	7.65E+11	3.67		
Austria	1980	7553000	0.64	3243557	4837	8.23	517.9	7.34	20.41	1.58E+11	3.40		
Austria	2007	8300788	0.68	4252367	89458	8.10	515.5	9.32	21.61	2.99E+11	3.82		
Bahrain	1980	346997	0.63	135821.5	95	8.45	429.0	4.92	15.20	8.64E+09	1.89	0.01	0.22
Bahrain	2007	759560	0.71	355284.3	156	8.11	432.0	9.18	15.20	2E+10	1.49	0.14	0.22
Canada	1980	24593300	0.68	12141552	35833	11.01		9.70	23.69	5.36E+11	2.64		
Canada	2007	32935961	0.70	18359909	161887	10.91		11.23	23.35	1.19E+12	2.01		
Chile	1980	11181360	0.61	3768210	1053	8.26	409.7	6.97	23.42	8.84E+10	2.42		
Chile	2007	16636135	0.68	7271076	13426	8.08	403.8	9.71	24.78	3.06E+11	2.42		
Denmark	1980	5123000	0.65	2664040	7760	8.12	521.9	8.98	20.65	9.76E+10	3.14		
Denmark	2007	5461438	0.66	2931218	29660	8.07	520.0	9.87	20.24	1.87E+11	3.83		
Egypt	1980	44433037	0.54	13229573	10198	12.01	423.7	2.65	18.47	9.29E+10	1.74	0.07	0.12
Egypt	2007	80060540	0.63	25768389	49363	11.30	399.5	6.59	18.47	4.57E+11	1.12	0.15	0.12
France	1980	55110236	0.64	23897227	126848	11.37		5.96	22.89	1.09E+12	3.18		
France	2007	63681742	0.65	28405931	205813	11.00		9.88	19.40	1.89E+12	3.70		
Germany	1980	78303000	0.66	35296037	164216	8.17	527.6	8.13	19.62	1.57E+12	3.10		
Germany	2007	82266372	0.66	42221941	721712	7.97	526.5	11.84	18.32	2.58E+12	3.77		
Hong Kong	1980	5063100	0.68	2428274	772	8.18	555.3	7.95	29.91	7.9E+10	2.22		

COUNTRY	YEAR	Population	Working Age Population % of Population	Labor Force	Researchers	Rate of Return to Education	TIMMS	Average Years of Schooling	Average Weakly Hours	GDP	Capital Output Ratio	Rate of Hydrocarbon Utilization	Share of Hydrocarbon Revenues to GDP
Hong Kong	2007	6925900	0.74	3697997	27419	8.07	565.8	9.86	32.72	2.99E+11	2.97		
Italy	1980	56434000	0.64	22124202	62937	8.35	487.5	5.89	22.99	1.04E+12	3.34		
Italy	2007	59375289	0.66	24814163	90038	8.11	504.3	9.14	23.00	1.71E+12	4.31		
Japan	1980	116782000	0.67	56407490	477440	8.15	563.5	8.51	27.48	2.2E+12	3.59		
Japan	2007	127770750	0.66	66935002	1157570	8.00	560.0	11.26	22.49	3.91E+12	5.12		
Jordan	1980	2181000	0.47	470052.7	3840	11.58	441.8	4.58	21.03	1.3E+10	1.26		
Jordan	2007	5718855	0.61	1813156	42153	11.09	454.5	8.72	21.03	2.95E+10	2.02		
Korea	1980	38124000	0.62	15038682	30466	8.16	566.9	8.29	34.05	2.08E+11	2.61		
Korea	2007	48456000	0.72	24363373	421549	7.99	575.0	11.46	32.06	1.16E+12	4.55		
Kuwait	1980	1375000	0.58	460967.2	290	8.49	377.2	4.60	5.70	5.97E+10	2.28	0.01	0.32
Kuwait	2007	2662966	0.75	1402068	869	8.33	359.0	6.06	5.70	1.12E+11	1.57	0.01	0.32
Lebanon	1980	2784713	0.56	709734.7	126	11.58	423.3	4.58	21.03	3.29E+10	2.07		
Lebanon	2007	4162450	0.66	1390852	459	11.09	431.5	8.72	21.03	3.22E+10	2.48		
Morocco	1980	19566920	0.53	5841393	5394	12.32	372.4	1.79	16.88	7.67E+10	1.72		
Morocco	2007	31224136	0.65	11578298	31326	11.61	319.0	4.41	16.88	1.69E+11	2.27		
Netherlands	1980	14150000	0.66	6044931	25732	8.17	542.3	8.23	18.11	2.98E+11	3.30	0.05	0.04
Netherlands	2007	16381696	0.67	8801361	88723	8.02	529.0	10.80	18.01	5.63E+11	3.53	0.04	0.04
New Zealand	1980	3113000	0.63	1511660	2254	7.99	501.6	11.47	22.22	4.81E+10	3.38		

COUNTRY	YEAR	Population	Working Age Population % of Population	Labor Force	Researchers	Rate of Return to Education	TIMMS	Average Years of Schooling	Average Weakly Hours	GDP	Capital Output Ratio	Rate of Hydrocarbon Utilization	Share of Hydrocarbon Revenues to GDP
New Zealand	2007	4228300	0.67	2282934	43600	7.95	498.0	12.43	22.71	1.07E+11	3.28		
Norway	1980	4091000	0.63	2009181	10198	8.17	500.3	8.15	19.16	9.94E+10	4.37	0.03	0.14
Norway	2007	4709153	0.66	2556875	24710	7.95	476.5	12.33	18.02	2.28E+11	3.75	0.05	0.14
Oman	1980	1187441	0.52	336364.1	96	8.47	412.5	4.73	7.70	1.56E+10	1.39	0.04	0.20
Oman	2007	2726301	0.65	1026241	532	8.15	397.5	8.46	7.70	6.73E+10	1.22	0.03	0.20
Qatar	1980	229442	0.66	107486.4	93	8.54	326.1	4.19	10.50	1.87E+10	1.49	0.01	0.35
Qatar	2007	1137553	0.82	792937.6	818	8.24	304.0	7.22	10.50	1E+11	1.22	0.00	0.35
Saudi Arabia	1980	9604374	0.53	2481037	482	8.52	378.5	4.38	6.20	4.13E+11	1.02	0.02	0.25
Saudi Arabia	2007	24157431	0.64	8816526	4186	8.20	366.0	7.68	6.20	4.89E+11	1.59	0.01	0.25
Singapore	1980	2414000	0.68	1116395	1856	8.41	584.3	5.24		3.23E+10	3.82		
Singapore	2007	4588600	0.73	2450748	38255	8.15	586.5	8.47		2.05E+11	3.42		
Sweden	1980	8310000	0.64	4436677	23810	8.08	545.8	9.71	18.70	1.63E+11	3.20		
Sweden	2007	9148092	0.66	4931971	117109	7.99	507.5	11.50	20.37	3.02E+11	3.03		
Syria	1980	8971343	0.48	2112590	1345	11.79	436.0	3.49	26.10	1.95E+10	1.02	0.03	0.06
Syria	2007	20082697	0.61	6486626	3012	11.54	423.5	4.82	26.10	5.89E+10	0.87	0.04	0.06
Tunisia	1980	6384000	0.54	1844318	832	8.69	441.8	3.25	17.10	2.86E+10	3.06	0.02	0.04
Tunisia	2007	10225400	0.69	3716479	29987	8.29	377.5	6.58	17.10	1.03E+11	1.77	0.06	0.04
Turkey	1980	46161316	0.55	16226802	4198	11.80	453.5	3.41	20.77	1.9E+11	2.05		

COUNTRY	YEAR	Population	Working Age Population % of Population	Labor Force	Researchers	Rate of Return to Education	TIMMS	Average Years of Schooling	Average Weakly Hours	GDP	Capital Output Ratio	Rate of Hydrocarbon Utilization	Share of Hydrocarbon Revenues to GDP
Turkey	2007	73003736	0.67	24980513	119738	11.31	443.0	6.47	24.55	5.65E+11	2.80		
UK	1980	56314217	0.64	26555959	157503	8.16	540.9	8.27	21.84	9.39E+11	2.69		
UK	2007	61005113	0.66	31143860	504269	8.10	541.3	9.21	21.72	1.96E+12	2.87		
USA	1980	227225000	0.66	1.12E+08	764092	7.97	509.3	11.86	23.12	5.58E+12	2.75		
USA	2007	301290000	0.67	1.57E+08	1391351	7.96	524.0	12.09	23.13	1.29E+13	3.26		

References

Aghion, P. and P. Howitt, "A Model of Growth Through Creative Destruction," Econometrica, 1992, 60(2), 323-51.

Autor, D., A. Krueger, and L. Katz, "Computing Inequality: Have Computers Changed the Labor Market?" Quarterly Journal of Economics, CXIII, 1998, 1169-1213.

Barro, R. J. and Sala-i-Martin, X., Economic Growth, New York: McGraw-Hill, 1995.

Barro, R. J. and J.-W. Lee, "International Comparisons of Educational Attainment," Journal of Monetary Economics, December 1993, 32(3), 363-94.

Barro, R. and Jong-Wha Lee, "A New Data Set of Educational Attainment in the World, 1950–2010," NBER Working Paper No. 15902, April 2010.

Bils, M. and P. J. Klenow, "Does Schooling Cause Growth?" American Economic Review, 2000, 90(5), 1160-83.

Bosworth, B. and S. M. Collins, "The Empirics of Growth," Brookings Institution and Georgetown University, September 2003.

Ciccone, A. and E. Papaioannou, "Human Capital, the Structure of Production, and Growth, 2005, Barcelona, Universitat Pompeu Fabra.

Grossman, G. and E. Helpman, Innovation and Growth in the Global Economy, Cambridge, MA: MIT Press, (1991), 119.

Hall, J. and G. Scobie, "Capital Shallowness: A Problem for New Zealand," New Zealand Treasury Working Paper, 5/ 2005.

Hanushek, E. A. and D. D. Kimko, "Schooling, Labor-Force Quality and the Growth of Nations," American Economic Review, 2000, 90(5), 1184-1208.

Hanushek, E. A. and L. Woessmann, "Schooling, Cognitive Skills, and the Latin American Growth Puzzle," NBER, Cambridge MA, WP 15066, 2009.

Hanushek, E. A. and L. Woessmann, "How Much Educational Outcomes Matter in OECD Countries?" NBER, Cambridge MA, WP 16515, 2010.

Howitt, P. and P. Aghion, "Capital Accumulation and Innovation as Complementary Factors in Long-Run Growth," Journal of Economic Growth 1998, 3, 111-30.

Howitt, P., "Steady Endogenous Growth with Population and R&D Inputs Growing," Journal of Political Economy, 1999, 107(4), 715-30.

Jones, C. I., “Sources of U.S. Economic Growth in a World of Ideas,” American Economic Review, March 2002, 220-39.

Jorgenson, D. and B. M. Fraumeni, Investment in Education and U.S. Economic Growth,” Scandinavian Journal of Economics, 1992, 94, 51-70.

Laabas, B. and W. A. Razzak, “Taxes, Natural Resources Endowment and the Supply of Labor: New Evidence,” Arab Planning Institute Working Paper, Kuwait, 2010.

Levine, R. and D. Renelt, “A Sensetivity Analysis of Cross-Country Growth Regressions,” American Economic Review, September 1992, 82(4), 942-63.

Lucas, R. Jr., “On the Mechanics of Economic Development,” Journal of Monetary Economics, 22(1), 1988, 3-42.

Lucas, R. Jr. and L. A. Rapping, “Real Wages, Employment and Inflation,” Journal of Political Economy, 1969, 77, 721-754.

Lucas, R. Jr., “Ideas and Growth,” Economica 2009, 76, 1-19.

Mankiw, G., D. Romer, and D. Weil, “A Contribution to the Empirics of Economic Growth,” Quarterly Journal of Economics, May 1992, 107(2), 407-37.

Mincer, J., Schooling, Experience and Earnings, New York, Colombia University Press, 1974.

Nehru, V. and A. Dhareshwar, “New Estimates of total factor productivity growth for developing and industrial countries,” Policy Research Working Paper #1313, World Bank, 1994.

Nelson, R. and E. Phelps, “Investments in Humans, Technology Diffusion, and Economic Growth,” American Economic Review, May 1966, 56(2), 69-75.

Pissarides, C. and M A Veganzones-Varoudakis, “Labor Market and Economic Growth in the MENA Region,” World Bank Manuscript, January 2005.

Prescott, E. C., “Needed: A Theory of Total Factor Productivity,” Federal Reserve Bank of Minneapolis, 1997.

Psacharopoulos, G. and Harry Anthony Patrinos, “Returns to Investment in Education: A Further Update,”.2002, Policy research Working paper 2881. The World Bank.

W A Razzak, “Explaining the Gaps in Labour Productivity in Some Developed Countries,” Applied Econometrics and International Development, 2007, 7(2), 193-229.

Rebelo, S., "Long-Run Policy Analysis and Long-run Growth," Journal of Political Economy, June 1991, 99(3), 500-21.

Romer, P., "Endogenous Technological Change," Journal of Political Economy, October 1990a, Pt. 2 99(5), S71-S102.

Romer, P., "Human Capital and Growth: Theory and Evidence," Carnegie-Rochester Conference Series on Public Policy, Spring 1990b, 32, 251-86.

Solow, R. M., "A Contribution to the Theory of Economic Growth," Quarterly Journal of Economics, February 1956, 70 (1), 65-94.

Solow, R. M., "Technical Change and Aggregate Production Function," Review of Economics and Statistics, August 1957, 39(3), 312-20.

Solow, R. and F. Y. Wan, "Extraction Costs in the Theory of Exhaustible Resources," The Bell Journal of Economics, 1976, 7(2), 359-370.

Stiglitz, J., "Growth with Exhaustible Natural Resources: Efficient and Optimal Growth Paths," Review of Economic Studies, 1974, 123-137.

TIMMS, Trends in International Math and Science Study, 2009, Amsterdam. (http://nces.ed.gov/timss/)