Vertical Integration versus Vertical Separation: An Equilibrium Model

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Abstract. In this paper, the role of strategic forces in vertical relationships is examined. Using a simple model of differentiated products with symmetric demands and costs, the Perfect equilibrium to a vertical integration-vertical separation game between manufacturers is determined. Given the assumptions of the model, I show that the manufacturer's decision whether to vertically integrate or to remain separate from its retailer depends on the degree of product differentiation. I show that when the products are poor substitutes, the only Perfect equilibrium is vertical integration by both manufacturers. As the products become closer substitutes, an additional Perfect equilibrium appears, both firms vertically separated. For manufacturers, the vertically separated equilibrium always Pareto dominates the vertical integration equilibrium when both equilibria exist.

Key Words. Vertical Integration. Vertical Separation. Differentiated Products.

I. Introduction

This paper extends the analysis of strategic considerations in vertical relationships by considering the decision between vertical integration and vertical separation on the part of manufacturers.[1] Vertical separation is defined as selling through an independent retailer, as opposed to selling directly to consumers. Using a simple model of differentiated products with symmetric demands and costs, I show that when the products are close substitutes there are multiple Perfect equilibria to a vertical integration-vertical separation game between manufacturers; both firms vertically separated and both firms vertically integrated. The vertically separated equilibrium always Pareto dominates the vertical integration result when multiple Perfect equilibria exist. When the products are relatively poor substitutes, the only Perfect equilibrium involves vertical integration by both firms. A mixed outcome, in which an integrated firm competes against an independent manufacturer and retailer is never a Perfect equilibrium in this model.

The intuition behind these results is the following. Manufacturers in the decision to raise their price to retailers face two conflicting forces. If they raise their price they exacerbate the double markup problem. That is, sales of the manufacturer's product are reduced below the desired level by an additional markup at the retail level if retailers have market power. However, manufacturers also obtain a strategic benefit from having other manufacturers raise their prices in response. This occurs if manufacturing and retail prices are strategic complements which is the case if the goods are substitutes and price competition exists at both stages. The relative strengths of these forces depends on the degree of product differentiation. When the products are poor substitutes, the adverse effect of the double markup problem overwhelms the benefit from having competitors also raise their price. In this case, vertical integration, which results in the manufacturing price being passed through at cost, is the dominant strategy for manufacturers. When the products are relatively close substitutes, the double markup problem is more than offset by the strategic benefit obtained from having the rival manufacturer and retailer raise their prices.

There are a number of papers which address similar issues in vertical control. Fershtman and Judd (1987) and Gal-Or (1991) emphasize the strategic nature of price competition when products are differentiated. Fershtman and Judd (1987) show that for the case of Bertrand-price competition with differentiated products, owners want their managers to set a high price to elicit higher prices from competing managers. Owners will therefore choose an incentive structure that overcompensates the manager for increasing price at the margin. Gal-Or (1991) finds that when products produced by manufacturers are only slightly differentiated, producers may benefit from the double marginalization which results from linear pricing. In her model, retailers are in perfectly elastic supply which implies that manufacturers can extract all profits from them. Gal-Or does not address the vertical integration-vertical separation decision. Rey and Stiglitz (1988), on the other hand, consider the strategic effect of exclusive territories on inter-brand competition (ie. competition among producers). In a model with many retailers, they show that in contrast to the case of perfect competition at the retail level, the imposition of exclusive territories eliminates intra-brand competition and therefore allows each retailer to enjoy some monopoly power over a fixed fraction of the final demand for his product. They show that exclusive territories may serve to facilitate collusion when imperfect competition exists among producers. They find that with exclusive territories joint profits are higher, but overall surplus is reduced.

More directly related to the analysis contained here, Bonanno and Vickers (1988) demonstrate in a duopolistic market that vertical separation is more profitable for both manufacturers than a vertically integrated equilibrium if franchise fees are used to extract the retailer's surplus (Proposition 1). They define vertical separation as the manufacturer raising its price above marginal cost and levying a franchise fee which extracts all profits from retailers. They assume there is perfect competition between potential retailers, which coupled with a franchise fee transfers all profits to the manufacturers. Thus, the double markup effect is inoperative in their model. Finally, Lin (1988) uses a model of zero-one demands and obtains two Nash equilibria in a vertical separation-vertical integration game between manufacturers. The Nash equilibria are vertical integration by both firms and vertical separation by both firms. This result will be contrasted with the results obtained here.

The paper is organized as follows. Section II outlines the simple model of differentiated products used in the analysis while section III outlines the game considered in the paper. Section IV derives the outcomes which are possible given the structure of the game; these are vertical integration by both firms, vertical separation by both firms, and a mixed outcome, one manufacturer vertically separated and one manufacturer vertically integrated, respectively. The section also contains a discussion of the strategic nature of vertical integration. Section V investigates the Perfect equilibrium to the vertical separation-vertical integration decision on the part of manufacturers. Section VI contrasts the results in this paper with those in Lin (1988).

II. The Model

The model involves a continuum of consumers of the same type with a utility function separable and linear in the numeraire good.[2] That is, a two sector model is assumed, one competitive, which acts as a numeraire, the other monopolistic, comprised of two firms.[3] The representative consumer maximizes $W \equiv U(q_1,q_2) - \Sigma p_i q_i$, $i=1,2$ where q_i is the amount of good i and p_i its price. Utility is assumed to be quadratic and strictly concave of the form, $U(q_1,q_2)=\alpha(q_1+q_2)-(\beta q_1^2+2\tau q_1 q_2+\beta q_2^2)/2$, where α and β are positive. Setting $p_1=\partial U/\partial q_1$ and $p_2=\partial U/\partial q_2$ yields demand functions which are linear which can be solved for the direct demand functions $q_1=a-bp_1+cp_2$, and $q_2=a-bp_2+cp_1$ where $a\equiv\alpha(\beta-\tau)/\sigma$, $c\equiv\tau/\sigma$, $b\equiv\beta/\sigma$, and $\sigma\equiv\beta^2-\tau^2$. A stability condition requires the restriction $\sigma\equiv\beta^2-\tau^2>0$.[4] This is also the region of quantity space where prices are positive. Given $\sigma>0$, this implies the restriction $(\beta-\tau)>0$ for the intercept a to be positive.[5] By assumption, $b>0$, and if the goods are substitutes an additional parameter restriction exists, $c>0$, since $\tau>0$ if the goods are substitutes.

Singh and Vives (1984) argue that when the intercepts of the demand functions are equal as is here, the sign and magnitude of the parameter $\theta=c^2/b^2$ characterizes the relationship between the goods. The goods are substitutes, complements, or independent depending on whether θ is greater, less than, or equal to 0. In this paper, only the cases of substitutes and independent goods will be considered, which means that θ is constrained to the interval [0,1].[6] That is, we are interested in characterizing differing degrees of product substitutability. A particular feature of this model is that the homogenous good Bertrand model emerges as a special case, when $\theta = 1$.

Independent retailers are assumed to choose p_i, the retail price to maximize, $\pi^R_i= (p_i-m_i-s_i)\, q_i$, for $i=1,2$, where m_i is the manufacturing price and s_i is the retail selling cost which has been set equal to zero with no loss in generality. Manufacturers, on the other hand, choose m_i, their manufacturing price, to maximize $\pi^M_i= (m_i-k_i)\, q_i$, $i=1,2$, where k_i, is average and marginal cost of production, which has also been set to zero, for convenience. It is also assumed that there are no fixed costs of production at either the retail or manufacturing stage.[7] Initially, I assume that franchise fees are not used.

In the next section, the type of game considered in this paper is presented.

III. The Timing of the Game

In this section, the decision faced by manufacturers in the setting envisioned here is illustrated. It is assumed that manufacturers are faced with the decision to vertically integrate or remain separate. The precise moves involved can be illustrated with the following timing line.

O------------------------------o---------------------------o------------------------O

Stage I	II	III	Payoffs
Manufacturers simultaneously choose VS, VI	Manufacturers that chose VS choose m_i	Retail prices are set	

Figure 1: Timing of the Game

It is important to discuss the implications of assuming the above sequence of decisions. The game described here involves manufacturers deciding to compete directly in the retail market or to sell through retailers. For the latter case, it is assumed that manufacturers choose their prices anticipating correctly the retail outcome. That is, vertically separated manufacturers choose their prices, simultaneously, assuming that retail prices will be set through Bertrand-Nash competition between independent retailers. Independent manufacturers know, therefore, that their manufacturing prices, already set above production cost, will be marked up again at the retail level given that no vertical restraints are used.

An important issue to decide is a sensible sequence of actions for the mixed outcome, that is competition between an integrated manufacturer and an independent retailer and manufacturer. In this paper, I argue that consistent with the vertical separation outcome, the manufacturing price of the separated firm should be chosen, prior to the retail competition. This is the same timing as found in Lin (1988). A game in which the retail prices for the integrated firm and independent retailer are set simultaneously with the manufacturing price of the independent manufacturer is not considered for the following reason. Simultaneous determination would imply retailers setting prices independent of their input prices, a rather odd situation. Moreover, given that its retailer would not respond to changes in manufacturing prices, this would imply that the optimal price for the manufacturer would be infinite, an unstable result.

IV. Vertical Integration, Vertical Separation and Mixed Outcome

1. Vertical Integration Outcome

Each integrated firm chooses a retail price to maximize total profits which can be expressed as $\pi^T_i = \pi^R_i + \pi^M_i$, i=1,2, where π^R_i is retail profits, and π^M_i is manufacturer profits. This can be written as follows: $\pi^T_i = p_i q_i$, since the manufacturing price drops out because it is indeterminate. The first order conditions to this problem yield the reaction functions for the respective integrated firms,
$R_1^I \equiv p_1=(a+cp_2)/2b$ and $R_2^I \equiv p_2=(a+cp_1)/2b$. These are depicted in figure 2 as the lines R1I for firm 1 and R2I for firm 2 respectively, which intersect at point I which is the retail Nash equilibrium for the two integrated firms.

[Figure 2 here]

The slopes and intercepts of the reaction functions depend on the degree of product differentiation with the reaction functions becoming perpendicular as the goods become more independent. Their solution yields the equilibrium retail price for the integrated firms, P_I*= a/(2b-c). Given that a>0, this implies a restriction for positive prices of c<2b. This is also the restriction that the reaction functions, also called best reply mappings, be contractions.[8] The outcome for the case of noncooperative vertically integrated firms is summarized below.

Table 1. Vertical Integration Outcome

P^I*= a/(2b-c)

q^I*= ab/(2b-c)

π^I*= $a^2b/(2b-c)^2$

For contrast, it is instructive to compare this outcome with one in which vertically integrated firms collude on price. For the collusive case, the outcome involves prices equal to P^{CI}=a/2(b-c), quantities equal to q^{CI}=a/2 and profits of $\pi^{CI}=a^2/4(b-c)$. Taking the difference $[P^I-P^{CI}]$ and simplifying yields the result that

P^I is greater (less) than P^{CI} according to whether -c/[2(2b-c)(b-c)] is greater (less) than zero. Given that c>0 for substitutes, the comparison depends on the signs of [(2b-c)] and (b-c). For the existence of a Nash equilibrium in price space, a necessary restriction is that c<2b. For a positive collusive price, c must be less than b, or given the definition of product differentiation, θ must be less than 1. Thus the noncooperative vertical integration price is less than the collusive vertical integration price. This implies there is an incentive for manufacturers to pursue higher retail prices.

2. Vertical Separation Outcome

In the vertically separated equilibrium both retailers are independent. Each retailer chooses its price to maximize, $\pi^R_i=(p_i-m_i)\, q_i$, for i=1,2, assuming Bertrand-Nash conjectures. Rearrangement of the first order conditions to this problem yield the retail reaction functions for the case of vertical separation; $R_1^{VS} \equiv p_1=(a+bm_1+cp_2)/2b$ and $R_2^{VS} \equiv p_2=(a+bm_2+cp_1)/2b$. These are depicted as the lines R1VS for retailer 1 and R2VS for retailer 2 in figure 2, intersecting at point S which is the Nash equilibrium for price competition between independent firms. Comparing the reaction functions for the vertically integrated equilibrium with the vertically separated case reveals that the retail prices will be higher for the separated case whenever m_1>0, m_2>0, where 0 refers to the average and marginal cost of production of the manufacturing firm, set at zero for convenience. The intercept for firm 1 is higher for the separated case, while the intercept for firm 2 is lower resulting in higher equilibrium retail prices. The solution to the reaction functions yields $P_1^{VS*}=[a(2b+c)+b(2bm_1+cm_2)]/D$ and $P_2^{VS*}=[a(2b+c)+b(2bm_2+cm_1)]/D$, where $D\equiv 4b^2-c^2$.

Each manufacturer is then assumed to choose m_i simultaneously, i=1,2 to maximize, $\pi^M_i = m_i\, q_i$, anticipating correctly that the retail outcome will be determined by Bertrand-Nash competition between retailers. Substituting the Bertrand-Nash prices into the demand functions yields the respective quantities, $q_1^{*VS}=[ab(ab+c)-Hbm_1+b^2cm_2]/D$ and $q_2^{*VS}=[ab(ab+c)-Hbm_2+b^2cm_1]/D$ where $D\equiv 4b^2-c^2$. The two first order conditions evaluated at the above quantities yield the following reaction functions in manufacturing space[9]; $m_1=[a(2b+c)+bcm_2]/2H$ and $m_2=[a(2b+c)+bcm_1]/2H$ where $H\equiv 2b^2-c^2$. Their solution yields the equilibrium manufacturing price, $m^{VS*}=a(2b+c)/(2H-bc)$.

The complete vertical separation equilibrium to the two stage game is listed in table 2.

Table 2: Vertical Separation Outcome

$m^{VS*} = a(2b+c)/(2H-bc)$

$P^{VS*} = 2a(H+b^2)/[(2b-c)(2H-bc)]$

$q^{VS*} = abH/[(2b-c)(2H-bc)]$

$\pi_M{}^{VS*} = a^2bH(2b+c)/[(2H-bc)^2(2b-c)]$

$\pi_R{}^{VS*} = a^2bH^2/[(2H-bc)^2(2b-c)^2]$

where $H \equiv 2b^2-c^2$

3. The Mixed Outcome Case: Integrated Firm and Independent Manufacturer and Retailer

The retail game in this case is an asymmetric duopoly. Equilibrium in the retail market involves the choice of p_1 by firm 1 to maximize integrated profits and the choice of p_2 by the non-integrated retail firm. The reaction functions for this outcome will intersect at a point like A in figure 2. To determine the equilibrium at the retail stage, the respective reaction functions are solved simultaneously to yield the asymmetric retail equilibrium, $P_1{}^{MI*}=[a(2b+c)+cbm_2]/D$ and $P_2{}^{MNI*}=[a(2b+c)+2b^2m_2]/D$, where $D \equiv 4b^2-c^2$. Both equilibrium prices are functions of demand parameters and the manufacturing price of the independent manufacturer. A simple comparison of shows that $P_1{}^{MI*}$ is less than $P_2{}^{MNI*}$ given a positive m_2, and $c<2b$. The second condition must be satisfied if a Nash equilibrium is to exist at this stage of the game.

In this model, the independent manufacturer chooses m_2 to maximize $\pi^M{}_2 = (m_2-k_2)\, q_2$, assuming, correctly, that the retail outcome will be determined by price competition between the independent retailer and the integrated firm. Evaluating q_2 at the asymmetric retail outcome, the first order condition can be solved to yield the optimal manufacturing price, $m_2*=[a(2b+c)]/2(2b^2-c^2)$. Once m_2* is substituted into $P_1{}^{MI*}$ and $P_2{}^{MNI*}$ the quantities and profits can be determined. The complete mixed outcome is presented below.

Table 3: The Mixed Outcome

$m_2^* = [a(2b+c)]/2H$

$P_1^{MI}{*}= [a(2H+cb)]/2H(2b-c)$

$P_2^{MNI}{*}= [a(H+b^2)]/H(2b-c)$

$q_1^{MI}{*}= [ab(2H+cb)]/2H(2b-c)$

$q_2^{MNI}{*}= [ab]/2(2b-c)$

$\pi_1^{MI}{*}= [a^2b(2H+cb)^2]/4H^2(2b-c)^2$

$\pi_2^{MNI}{*}= [a^2b(2b+c)]/4H(2b-c)$

$\pi_2^{RNI}{*}= [a^2b]/4(2b-c)^2$

where $H \equiv 2b^2-c^2$

Before the Perfect equilibrium for the complete game is considered, it is useful to examine the mixed outcome in more detail. In particular the strategic implications of a vertically integrated manufacturer competing against an independent retailer.

Proposition 1. For symmetric costs and demands; integration by one firm provides it with a strategic advantage for the range $0<\theta<1$. That is, the mixed equilibrium results in higher profits for the integrated firm than the combined profits of the independent retailer and independent manufacturer.
Proof: See Appendix.

Integration allows the firm to maintain a lower retail price when products are differentiated and these lower prices translate into higher sales for the integrating firm.[10] This proposition shows that for the specific demand and cost functions used, integration by one firm provides it with a strategic advantage. In this model, vertical integration is a credible commitment to a more aggressive retail pricing strategy. That is, eliminating the double markup allows the integrating firm to maintain a lower retail price while receiving a substantial pecuniary benefit from the higher retail price set by the independent retailer. The retail price set by the independent retailer involves a markup over the price set by the independent manufacturer.

V. The Perfect Equilibrium to the Complete Game

Using the associated profit functions, it is straightforward to determine the nature and existence of the Perfect equilibria to the game played between manufacturers. For the case of perfect substitutes, profits are zero for all firms, for the three respective outcomes. This is a natural result of Bertrand competition with identical goods. When the demands are independent for the two goods, each manufacturer is a monopolist. In this case, it is not clear whether the mixed outcome case applies, since strategic issues have little relevance when considering independent manufacturers. Without the mixed outcome case, we are left with two outcomes for each independent firm, vertical integration and vertical separation. The vertically integrated price is $P^{I}*=a/2b$ while the equilibrium quantity is $q^{I}*=a/2$ obtained in the usual manner. For the vertically separated case, the equilibrium price is $P^{VS}*=3a/4b$ while equilibrium quantity is $q^{VS}*=a/4$. This case is the double markup solution, since the retail price is obtained by marking up retailer costs, which is the monopoly manufacturing price.[11] It is easily shown that profits are higher for an integrated firm, $\pi^{I}=a^2/4b$, then for the total profits of the independent retailer and manufacturer, $(\pi_M{}^{VS}+\pi_R{}^{VS})= 3a^2/16b$, for the case of independent demands.

Of more general interest is the case of less than perfect substitutes, $0<\theta<1$.

Proposition 2. (a) There is a critical value of θ, $\theta*=.867$, such that for symmetric costs and demands: (i) there is a single Perfect equilibrium, both firms integrating, for the range of product differentiation $\theta<\theta*$; (ii) there are multiple Perfect equilibria, both firms integrating and both firms separated, for a range of $\theta*<\theta<1$. *(b)* When both Perfect equilibria exist, manufacturers earn higher profits in the Vertical Separation equilibrium than in the Vertical Integration equilibrium.

Proposition 2 is solved numerically to determine the nature and existence of the Perfect equilibria. Vertical Integration by both firms is always a Perfect equilibrium for $0<\theta<1$. Furthermore, Vertical Integration by both firms is the only Perfect equilibrium when the products are poor substitutes. The Mixed Outcome is not a Perfect Equilibrium in this symmetric model.[12] As the goods become closer substitutes, a second Perfect equilibrium, Vertical Separation by both firms appears. The Vertical Separation equilibrium only exists when the products are fairly close substitutes. Given this result, it is useful to compare manufacturer profits in the two equilibria.

For degrees of product differentiation greater than the critical θ* but less than 1, a search over the range $.867<\theta<1$ reveals that profits for manufacturers are higher under the Vertical Separation equilibrium than under the Vertical Integration equilibrium. The intuition for this result is as follows. As the goods become closer substitutes, the strategic effect plays a more important role than the double markup problem. The reason is that the percentage markup $(P^{VS*}-m^{VS*})/m^{VS*}$ shrinks as the goods become closer substitutes.[13] Thus, the adverse effect of the double markup problem is reduced. Given that higher retail prices move the outcome closer to the collusive outcome for the two firms, the Vertical Separation equilibrium is preferred to the Vertical Integration equilibrium.

The above results have been based on the assumption that manufacturers were restricted to a linear price schedule. As observed by Rey and Stiglitz, if the manufacturers anticipate the retail price equilibrium, it is possible for them to recover the retailers profits by the use of a franchise fee.[14] It is important to realize that manufacturers do not want to reduce the double marginalization which arises in the vertical separation outcome by charging retailers marginal cost for their supplies. This would eliminate the strategic effect which has been shown to raise the profits of manufacturers over the vertically integrated Nash equilibrium. I have shown, in the context of this model, that the only effect the introduction of franchise fees has is to make the second Nash equilibrium, Vertical Separation by both firms, appear at a lower degree of substitutability between the products.[15] The reason is that the vertical separation payoffs are now higher for both manufacturers reducing the incentive to vertically integrate, unilaterally, at all intermediate degrees of product differentiation.

VI. Conclusion

It is important to contrast these results with those in Lin (1988). One major difference is related to the type of product differentiation used in the models. Lin considers a model in which a randomly chosen consumer values the products of the two manufacturers at v_x and v_y, whose difference (v_x-v_y) is uniformly distributed on the interval [-0.5,0.5]. The number of consumers is normalized to 1. In his model, the consumer either buys one unit of the product from x or y, or does not buy at all.[16] The demand functions are given as $q_x = .5-p_x+p_y$; and $q_y = .5+p_x-p_y$, respectively. Lin's model would be described as a case of perfect substitutes in our framework. Thus, the two models of product differentiation not directly comparable. However, given the assumptions in Lin (1988), his model can be properly interpreted as a differentiated products model but of a different sort.[17] The second major difference is that the Lin model assumes that the total demand for the product is constant in the neighbourhood of the equilibrium. He asserts that "if demand is not sufficiently inelastic, our result may have to be altered."[18] This formulation gives a bias towards equilibria with higher prices since the adverse demand effect resulting from the double markup problem is absent. In contrast, our demand specification allows us to consider a range of demand elasticities.

Proofs:

Proposition 1. Proof: Since costs at the manufacturing level are zero, the comparison is between the revenues of the integrated firm $P_1^{MI}\, q_1^{MI}$ and revenues of the unintegrated retailer $P_2^{MNI}\, q_2^{MNI}$ which can be obtained from table 3 in the text. The ratio $P_1^{MI}\, q_1^{MI} / P_2^{MNI}\, q_2^{MNI}$ will be greater than 1 if the integrated firm earns higher profits than the combined unintegrated manufacturer and retailer. Taking the ratio and cancelling terms yields the condition $2(2b^2-c^2)(b^2-c^2) - c^2b^2>0$ for the proposition to hold. Substituting the definition of product differentiation, $\theta=c^2/b^2$, for c^2 and cancelling the term b^4 yields the condition, $(4-2\theta)> \theta/(1-\theta)$ which holds for any θ in the interval $0<\theta<1$. QED.

Proposition 2. Proof: The first step is to see whether the mixed outcome could be an equilibrium to the game. The condition $\pi_i^I/\pi_i^{MNI} > 1$ implies that if one manufacturer is integrated it will pay the other manufacturer to integrate. Thus if this condition holds the mixed outcome cannot be an equilibrium. Using the profit functions, performing the division and cancelling terms, yields the equivalent condition $4H > (2b-c)(2b+c)$, where $H \equiv 2b^2 - c^2$. Using the definition of product differentiation $\theta = c^2/b^2$, and substituting the expression $\theta^{1/2}b$ for c and θb^2 for c^2 in H yields the condition $4/3 > \theta$. Since $\theta \leq 1$, then the mixed outcome cannot be an equilibrium. This also implies that (VI,VI) will be a Perfect equilibrium for $0<\theta<1$. The last part of the proof involves a comparison between π_i^{MI} and π_i^{MS}, the integrated profit versus the manufacturer's profit under vertical separation. Vertical separation by both firms will also be an equilibrium if $\pi_i^{MI} / \pi_i^{MS} < 1$. Solving this numerically reveals that (VS,VS) will be a Nash equilibrium if $\theta > .867$. Thus $\theta^* = .867$. QED.

Notes:

1. For an excellent synthesis of the vertical integration literature, see M. Perry (1989).

2. This model is used by Dixit (1979) and is further developed by Singh and Vives (1984). The interpretation of the utility function as a continuum of consumers of the same type is due to Singh and Vives. A version of this model can also be found in Levitan and Shubik (1980). In that case, they consider the utility function as representing the preferences of consumers in the aggregate.

3. This specification allows for partial equilibrium analysis, since there are no income effects on the monopolistic sector.

4. If $\beta_1 = \beta_2 = \beta = \tau$, then $\sigma = 0$ and the utility function is not strictly concave. As σ approaches 0, the slope of the demand curves approach infinity.

5. Alternatively $a = \alpha(\beta - \tau)/\sigma = \alpha/(\beta + \tau)$ which is positive if goods are substitutes, $\tau > 0$ or independent $\tau = 0$.

6. The parameter θ is constrained to the interval (-1,1) because of the stability condition $\sigma \equiv \beta^2 - \tau^2 > 0$, which is the region of quantity space where prices are positive.

7. This is not an innocuous assumption and raises questions regarding product diversity. The introduction of fixed costs introduces a lower bound on price if the firms selling differentiated products are to exist.

8. See Friedman (1986, pp. 42-45) for a discussion of this point.

9. The reaction functions for manufacturers are effected in the same manner as the retail reaction functions for different degrees of product differentiation. As the demands for the two goods become more independent, the reaction functions approach the perpendicular.

10. Using Table 3 it is easy to show that $P_1^{MI}*<P_2^{MNI}*$, if -ab/2H is less than zero which is true since a,b,H are positive.

11. To obtain the double markup solution, note because there are no retail costs, the derived demand curve for manufacturers is given by m=p=(a-q)/b. Setting marginal revenue equal to marginal cost, which is zero, and solving, yields the optimal quantity for the manufacturer, q*=a/2. Substituting into the derived demand curve yields m*=a/2b. The retailer then sets its MR equal to its marginal cost, m*, and solves for the optimal quantity for the retailer, $q*_r$=a/4. Substituting into the retailer's demand curve yields p*=3a/4b.

12. This does not rule out the possibility that the Mixed Outcome might be a Perfect Equilibrium if there was some asymmetry in the model, either with respect to costs or demands.

13. Using Table 2, the percentage markup for the vertical separation outcome is $[(P^{VS*}-m^{VS*})/m^{VS*}]=[2(H+b^2)-(4b^2-c^2)]/[(2b-c)(2b+c)]$. Cancelling terms and substituting for $c^2=\theta b^2$ and $c=\theta^{1/2}b$, yields the equivalent expression, $(2-\theta)/(4-\theta)$. It is easily shown that this is decreasing in θ.

14. Rey and Stiglitz (1988, p. 565).

15. That is, the profits of retailers are added to the profits of manufacturers in the vertically separated and mixed outcomes. Specifically the critical θ^{**} is .594. For $\theta < \theta^{**}$, the only Nash equilibrium is vertical integration by both firms; for $\theta^{**}<\theta<1$, there are two Nash equilibria, (VS,VS) and (VI,VI) with the vertical separation equilibrium Pareto dominating the vertical integration equilibrium.

16. Lin (1988, p. 251) also assumes that "the minimum values of v_x and v_y in the consumer population are large enough to sustain equilibria in which all consumers buy."

17. An important issue to consider in the Lin model is that demand is not reduced as long as prices are equivalent. Demand is thus perfectly inelastic.

18. Lin (1988, p. 253).

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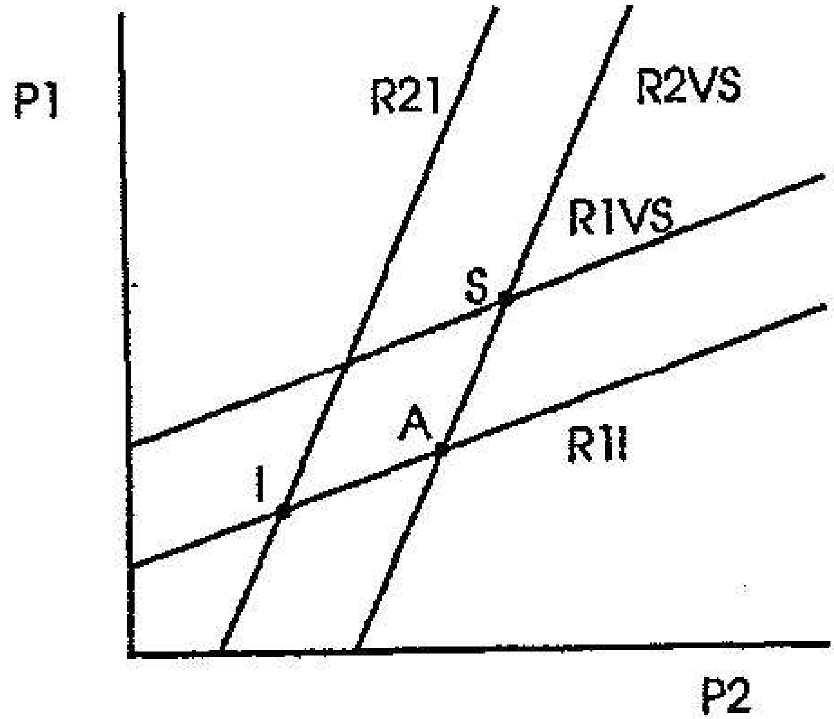

Fig. 2. Retail reaction functions vertical integration versus vertical separation.