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Abstract

This paper studies optimal monetary and fiscal policy in a small open economy. Two forces in the economy impose orthogonal restrictions on financing costs across governments. The first force requires constant financing costs across governments to have time consistent optimal policy of hours. The second force always asks for time-varying financing costs across governments in order to have time consistency optimal policy of consumption and real money balances. Thus, optimal monetary and fiscal policy is time inconsistent. However, if preferences (and/or productivity) satisfy certain conditions, the former force disappears and optimal monetary and fiscal policy becomes time consistent. The results hold with both flexible exchange rate regimes and fixed exchange rate regimes. The latter indicates that a credible fixed exchange rate regime does not help render optimal policy time consistent.

Keywords: Time consistency; Optimal monetary and fiscal policy; Small open economy.

JEL classification: E52; E61; E62.

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1 Introduction

This paper analyzes the time consistency property of optimal monetary and fiscal policy (hereafter OMFP) in a small open economy, where the meaning of OMFP follows the tradition of Lucas and Stokey (1983) and Chari et al. (1991) in the sense that the Ramsey government maximizes the utility of households by choosing the least distortional monetary and fiscal policy.[1] In line with the literature, whether OMFP is time consistent depends on whether a government could use policy instruments to influence its successor's policy choices in such a way that the successor will follow the policy continuation of real allocations [Alvarez et al. (2004) among others].[2] The discussion on time consistency could be traced back to Kydland and Prescott (1977) and Barro and Gordon (1983), which have shown that when it has the opportunity to reoptimize monetary and fiscal policy, the Ramsey government will have the incentive to renege on those policies made by the previous governments.

This paper is motivated by several observations. First, the time consistency issue of OMFP is empirically relevant. It has been argued that time inconsistency is an important reason for inflation bias [Kydland and Prescott (1977), Barro and Gordon (1983)] and financial crises because of the associated self-fulfilling multiple equilibria [Chari et al. (1998), Albanesi et al. (2003a)]. For example, Albanesi and Christiano (2001) argue that the dramatic output drops in several Asian countries during the 1998 Asian crisis are due to time inconsistency. Besides, the 2010 sovereign debt crisis in Greece provides one more example indicating the importance and empirical relevance of the time consistency issue in a small open economy.

Second, whether OMFP is time consistent is also an important theoretical question and thus widely studied. In the general equilibrium framework, the literature has identified one commitment technology to guarantee time consistency in a closed economy: an appropriate maturity structure of public debt. Lucas and Stokey (1983) show that the Ramsey gov-

[1] The forward-looking component (not necessarily stochastic) has a non-negligible impact on OMFP here. The condition rules out the cases in which OMFP is solely determined by the contemporaneous state, for example, the ones discussed in Albanesi and Christiano (2001) and Albanesi et al. (2003b).

[2] In line with Alvarez et al. (2004), policy instruments are defined as multiple-period bond holding positions; and policy choices are defined as nominal interest rates and tax rates in the case of flexible exchange rate regimes and as tax rates in the case of fixed exchange rate regimes.

ernment in a closed economy can choose the right maturity structure of real public debt to neutralize the renege incentive of the next government; and as a result, optimal fiscal policy will be time consistent.[3] Building on the work of Persson et al. (1987) and Calvo and Obstfeld (1990), both Alvarez et al. (2004) and Persson et al. (2006) impose additional restrictions to the model in Lucas and Stokey (1983) to neutralize the incentive of using surprising inflation to finance public spending, a phenomenon that was formally analyzed in Calvo (1978); and then show OMFP is time consistent in such a closed economy.

Third, it is still, however, an open question whether OMFP is time consistent in a small open economy for several reasons. First, monetary policy in a small open economy is affected by the choice of the foreign exchange rate regime (hereafter FERR). Second, a small open economy has conflicting features with respect to time consistency, comparing to the closed economy studied in Alvarez et al. (2004) and Persson et al. (2006). For example, the government of a small open economy does not have any control over real interest rates; which means that the government has fewer policy choices (of its successor) to influence. According to Tinbergen (1956), this feature will help render optimal policy time consistent. On the other hand, real bonds with different maturity dates become ineffective, which implies that the government has fewer policy instruments to use. This feature will decrease the government's ability of rendering time consistent OMFP. The net effect on time consistency is thus an open question.

This paper fills the gap by extending the discussion of time consistent OMFP to a small open economy with perfect capital mobility. In this paper, we follow the literature by looking for the sufficient condition for time consistency, i.e., whether there exists a maturity structure of public and external debt such that the Ramsey outcome is invariant to an ex post reoptimization. When the government chooses a flexible FERR, time consistency of OMFP depends how the labor-leisure choices affect the financing costs (Lagrange multipliers) across governments. When the labor-leisure choices impose the constant financing costs requirement, OMFP is time inconsistent. This is because the economy always asks for the

[3]Different types of commitment technology have been proposed, such as rules [Kydland and Prescott (1977)], reputation [Backus and Driffill (1985), Tabellini (1985)], "conservative" central banker [Rogoff (1985)], incentive contracts with inflation targets [Svensson (1997)].

time-varying financing costs across governments in order to have time consistent optimal policy of consumption and real money balances. In this case, there does not exist a maturity structure that could simultaneously satisfy both requirements.

However, when preferences and/or productivity processes satisfy certain conditions, the labor-leisure choices will not impose the constant financing costs requirement, and OMFP will be time consistent. In this case, there exist many maturity structures of external and public debt (both real and nominal), including ones over finite horizons, capable of rendering OMFP time consistent. This is because the $t = 0$ government always has more policy instruments (bonds of different maturity dates) than the $t = 1$ government has policy choices (nominal interest rates and labor income tax rates).

The same qualitative results hold with a fixed FERR. In this case, the monetary economy effectively reduces to a real economy and the aforementioned two forces still exist. Thus, unless preferences and/or productivity processes satisfy certain conditions, the labor market imposes a different restriction on financing costs from the good market. As a result, OMFP is time inconsistent even if the fixed FERR itself is credible. Alternatively, the government does not have sufficient policy instruments to influence the policy choices of the next government, as argued in Persson and Svensson (1986). This is a new and interesting result. It extends the existing understanding about the relationship between a credible fixed FERR and time consistency: according to Kydland and Prescott (1977), rules such as a credible fixed FERR will assure time consistency of monetary policy.

This paper is organized as follows: Section 2 discuss time consistency of OMFP with a flexible FERR. Section 3 discusses the same problem with a fixed FERR. Section 4 presents the relation between our results to those in the literature. And Section 5 concludes.

2 Time Inconsistency under A Flexible FERR

The recent finding in Persson et al. (2006) and Alvarez et al. (2004) shows that the time inconsistent incentive could be neutralized in a general equilibrium model of a closed economy. We check whether a similar conclusion, the time-inconsistent incentive in a small open economy with the flexible FERR could be neutralized by choosing the right maturity structure of public debt, can be obtained.

2.1 The Model

The model is a perfect foresight model, which is simple but sufficient to illustrate the main point of the paper. In addition, the model is a small open economy version of that in Persson et al. (2006). In order to be as close as possible to models in the literature, we assume that the government does not impose import/export duties or subsidies.

2.1.1 Households

In this economy, households are given the price of consumption good, p_t, the present value in period 0 of goods in period t, q_t, the labor income tax rate, τ_t, and the nominal interest rate i_{t+1}. A representative household chooses the time profile of consumption, $c_t, t \geq 0$, real money balances, $m_{t+1}, t \geq 1$, and hours, $h_t, t \geq 0$, to maximize its lifetime utility:

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t, m_t, h_t\right),$$

where β denotes the subjective discount factor, which weights the consumption bundles over time. The household is subject to its period budget constraints and the no-Ponzi game condition, both of which will be given below. Here we restrict the discussion to the case in which the period utility function is separable, $u(c, m, h) = u(c) + v(m) + g(h)$. The separable preferences assumption is arbitrary. We introduce this assumption in order to discuss the time consistency problem in a simple model and our practice is in line with the literature [Persson et al. (2006)]. In addition to usual concavity assumption of the utility function, we further assume that $u_{mt} \geq 0$. In other words, nominal interest rates, i_t, are always non-negative.

We assume Svensson timing of markets: the good market meets before the financial market [Svensson (1985)] in one period. Thus, real money balances are defined as $m_t = \frac{M_{t-1}}{p_t}$, where M denotes the nominal money balance. There are two reasons behind the use of Svensson timing. First, it introduces an inflation cost in the absence of price stickiness: when the price level increases, real money balances are reduced, and the household receives less utility from the given level of nominal money balances. This is in line with facts: Persson et al. (1996) show that high inflation has large and well-known social costs. Another rationale for the use of the beginning of period money balance is that inflation forces households to economize money thus bringing costs to holding money [Bailey (1956) and Tower (1971)]. The third rationale is that it gives the government more flexibility, compared to the use of Lucas timing, in choosing the maturity structure of public debt to guarantee time consistency.[4] In particular, both Persson et al. (2006) and Alvarez et al. (2004) show OMFP is time consistent in a closed economy. However, time consistency requires that nominal public debt in each maturity period be zero in Alvarez et al. (2004) because Lucas timing is assumed; while it does not have such a requirement in Persson et al. (2006) because Svensson timing is assumed.

The representative household's period budget constraint is given by:

$$\begin{aligned} & q_t\left[(1-\tau_t)w_t h_t + \frac{M_{t-1}}{p_t}\right] + \sum_{s=t}^{\infty} q_s\left({}_{t-1}b_s^{\mathrm{P}} + \frac{{}_{t-1}B_s^{\mathrm{P}}}{p_s}\right) \\ \geq \; & q_t\left(c_t + \frac{M_t}{p_t}\right) + \sum_{s=t+1}^{\infty} q_s\left({}_{t}b_s^{\mathrm{P}} + \frac{{}_{t}B_s^{\mathrm{P}}}{p_s}\right). \end{aligned} \tag{1}$$

The variable q_t denotes the price of net claims issued in period t, $\left({}_{t}b_s^{\mathrm{P}}\right)$. It immediately follows that we apply the same discount factors on both internal and external bonds. The superscript P means that the bond is held by the households. $\left({}_{t-1}b_s^{\mathrm{P}}\right)$ denotes the net claims by the domestic household when entering period t on the amount of goods to be delivered in period s. $\left({}_{t-1}B_s^{\mathrm{P}}\right)$ denotes the net claims on money to be delivered in period s. These bonds are real in this small open economy because their purchasing power will not change when the domestic price level changes. The sum $\sum_{s=t}^{\infty} q_s\left({}_{t-1}b_s^{\mathrm{P}} + \frac{{}_{t-1}B_s^{\mathrm{P}}}{p_s}\right)$ denotes the representative household's initial bond holding position.

The no-Ponzi game condition for the representative household is given by:

$$\lim_{j\to\infty}\left[q_{t+j}\frac{M_{t+j}}{p_{t+j}} + \sum_{s=t+j+1}^{\infty} q_s\left({}_{t+j}b_s^{\mathrm{P}} + \frac{{}_{t+j}B_s^{\mathrm{P}}}{p_s}\right)\right] \geq 0, \qquad \forall t \geq 0. \tag{2}$$

[4]With Lucas timing, the financial market meets before the good market in the same period [Lucas and Stokey (1987)].

This condition has its usual meaning: the representative household has to keep non-negative financial assets in the limit. This condition must hold in each period. In this economy, nominal interest rates are defined as:

$$\frac{1}{1+i_{t+1}} = \frac{q_{t+1}/p_{t+1}}{q_t/p_t} \leq 1, t \geq 0. \tag{3}$$

Note that when the government commits to a fixed FERR, i_t will be exogenous determined. When the government commits to a flexible FERR, i_t could be chosen by the government.

Combining the period budget constraint and the no-Ponzi game condition, we write the inter-temporal budget constraint of the representative household, Eq. (4), as:

$$\sum_{t=0}^{\infty} q_t \left[(1-\tau_t)w_t h_t + \Pi_t\right] + \frac{M_{-1}}{p_0} + \sum_{t=0}^{\infty} q_t \left({}_{-1}b_t^{\mathrm{P}} + \frac{{}_{-1}B_t^{\mathrm{P}}}{p_t}\right) = \sum_{t=0}^{\infty} q_t c_t + \sum_{t=1}^{\infty} q_t m_t i_t. \tag{4}$$

As a standard result in the literature (the derivation is available upon request), it can be shown that the time sequences for $\{c_t, m_{t+1}, h_t\}$ satisfying the constraints (1) and (2) are the same as those satisfying the single constraint (4). Thus, the representative household maximizes lifetime utility subject to the single constraint (4). Let λ denote the Lagrangian multiplier associated with Eq. (4). The optimality conditions for the domestic household are the single inter-temporal budget constraint (4) and

$$\beta^t u_{ct} = \lambda q_t, t \geq 0 \tag{5}$$

$$\tau_t = 1 + \frac{1}{w_t}\frac{u_{ht}}{u_{ct}}, t \geq 0 \tag{6}$$

$$i_{t+1} = \frac{u_{mt+1}}{u_{ct+1}}, t \geq 0. \tag{7}$$

All the optimality conditions have their usual meanings: Eq. (5) says that the marginal utility of consumption should equal the marginal cost of consumption; Eq. (6) shows that the introduction of labor income tax distorts the marginal rate of substitution between consumption and hours; and Eq. (7) states that there is cost to holding money.

2.1.2 Competitive firms

In each period, competitive firms use constant returns-to-scale technology in production:

$$y_t = z_t h_t, t \geq 0.$$

Here z_t denotes the total factor productivity. It can be either constant over time or time-varying, but not stochastic. One point worth mentioning is the possibility of zero output in this small open economy. There are several ways to rule out that possibility. One way is to assume that the net foreign asset accumulated by the representative household is not large enough so that the household will work at the given real wage rate in each period. Another way is to assume decreasing returns-to-scale technology as in Schmitt-Grohé and Uribe (2003). As a result, marginal product of labor at low levels of labor input is extremely high and the possibility of a corner solution is ruled out. However, in this case, firms will have non-zero profits which should be taxed. Such profit taxes will inevitably complicate the discussion of the time consistency of OMFP. To simplify the discussion, we assume the former way. In a closed economy, zero-output is not an issue because consumption, a part of output, is always positive with standard preferences.

We assume that output is sold in both domestic and international markets so that the "law of one price" for one tradable good holds in each period:

$$p_t = S_t p_t^{\star\star}, t \geq 0, \tag{8}$$

where the variable S_t denotes the nominal exchange rate at time t and the variable $p_t^{\star\star}$ denotes the world price at time t. Firms maximize profit, which is given by:

$$\Pi_t = z_t h_t - w_t h_t, t \geq 0. \tag{9}$$

The optimality condition for labor demand is given by:

$$w_t = z_t, t \geq 0. \tag{10}$$

Eq. (10) is a standard optimality condition in the firms' profit maximization problem.

2.1.3 The government

The government finances its expenditures, g_t by levying labor income taxes at the rate of τ_t, by printing money and by trading multi-period nominal and real bonds with both domestic households and international investors. In this paper, we focus on the scenario in which g_t is time-varying, an assumption maintained throughout the paper. The monetary/fiscal regime consists of plans for the policy instruments: money and bonds; and for the policy choices: nominal interest rates and labor income tax rates. Here we assume that lump-sum taxes are not available to the government, a standard assumption in the literature [Alvarez et al. (2004) and Persson et al. (2006)]. The period budget constraint of the government is thus given by:

$$\begin{aligned} q_t\left(g_t+\frac{M_{t-1}}{p_t}\right)+\sum_{s=t}^{\infty} q_s\left({}_{t-1}b_s^{\mathrm{G}}+\frac{{}_{t-1}B_s^{\mathrm{G}}}{p_s}\right) \;&\leq\; q_t\left(\tau_t w_t h_t+\frac{M_t}{p_t}\right) \\ &\quad+\sum_{s=t+1}^{\infty} q_s\left({}_{t}b_s^{\mathrm{G}}+\frac{{}_{t}B_s^{\mathrm{G}}}{p_s}\right). \end{aligned} \tag{11}$$

The superscript G means that the bond is issued by the government. The variable $\left({}_{t-1}b_s^{\mathrm{G}}\right)$ denotes total net claims on the amount of goods to be delivered by the government in period s. The variable $\left({}_{t-1}B_s^{\mathrm{G}}\right)$ denotes the net claims on money to be delivered by the government in period s.

The no-Ponzi game condition for the government is given by:

$$\lim_{j\to\infty}\left[q_{t+j}\frac{M_{t+j}}{p_{t+j}}+\sum_{s=t+j+1}^{\infty} q_s\left({}_{t+j}b_s^{\mathrm{G}}+\frac{{}_{t+j}B_s^{\mathrm{G}}}{p_s}\right)\right] \;\leq\; 0, \forall t\geq 0. \tag{12}$$

This condition rules out the possibility that the government borrows infinitely to finance its expenditures. The government's intertemporal budget constraint is given by:

$$\sum_{t=0}^{\infty} q_t\left({}_{-1}b_t^{\mathrm{G}}+\frac{{}_{-1}B_t^{\mathrm{G}}}{p_t}\right)+\frac{M_{-1}}{p_0} \;=\; \sum_{t=0}^{\infty} q_t\left(\tau_t w_t h_t-g_t\right)+\sum_{t=1}^{\infty} q_t i_t m_t. \tag{13}$$

2.1.4 International investors

International investors can always borrow and lend at a nominal interest rate of $i^{\star\star}$ in the international market. Due to assumption of perfect capital mobility, the uncovered interest rate parity condition holds:

$$(1+i_{t+1}) \quad = \quad \frac{S_{t+1}}{S_t}(1+i^{\star\star}) = \frac{1+i^{\star\star}}{1+\pi^{\star\star}}\frac{p_{t+1}}{p_t}, t \geq 0. \tag{14}$$

where $\pi^{\star\star}$ denote the inflation rate in the world economy.[5]. The second equality in Eq. (14) comes from the assumed purchasing power parity condition. In addition, in our simple model, we assume that

$$\beta\frac{1+i^{\star\star}}{1+\pi^{\star\star}} = 1, \tag{15}$$

Since $\frac{1+i^{\star\star}}{1+\pi^{\star\star}} = 1 + r^{\star\star}$ where $r^{\star\star}$ denotes the real interest rate, Eq. (15) is the non-stochastic steady state version of the standard Euler equation with respect to asset accumulation.

2.1.5 Competitive equilibrium

Definition 1 *A competitive equilibrium is defined as a sequence* $\{c_t, m_{t+1}, h_t, w_t, \Pi_t, q_{t+1}, p_{t+1}\}_{t=0}^{\infty}$*, a positive constant* λ*, and an initial price level* $p_0 > 0$*, satisfying Eqs. (3), (4), (5), (6), (7), (9), (10), (13), (14), given the initial asset conditions of* $\{M_{-1}, ({}_{-1}b_t^P), ({}_{-1}b_t^G)$*, and* $({}_{-1}B_t^P), ({}_{-1}B_t^G), \forall t \geq 0\}$*, the exogenous* $\{z_t, g_t\}_{t=0}^{\infty}$*, and a sequence of government policies* $\{\tau_t, i_{t+1}\}_{t=0}^{\infty}$

Equation (3) defines q_{t+1} given prices and nominal interest rates. Eqs. (4)–(7) solve the domestic household's utility maximization problem. Eqs. (9) and (10) solve the firms' profit maximization problem. Eq. (13) pins down the initial price level.[6] And Eq. (14) pins down the prices. Given Eq. (15) and the separable utility function assumption, we obtain the

[5]Even though here $i^{\star\star}$, $p^{\star\star}$, and $\pi^{\star\star}$ are assumed to constant over time here, our discussion will be the same when they are time-varying.

[6]In our simple model, we assume that the initial price level is uniquely determined.

following:

$$
\begin{aligned}
q_t &= \left(\frac{1+\pi^{\star\star}}{1+i^{\star\star}}\right)^t = \beta^t, t \geq 0, && (16)\\
u_{ct} &= \lambda q_t/\beta^t = \lambda \left(\frac{1+\pi^{\star\star}}{1+i^{\star\star}}\right)^t /\beta^t = \lambda, t \geq 0. && (17)
\end{aligned}
$$

Eqs. (16)-(17) essentially show that the discount factors will shrink at the rate of β and consumption is constant over time. We focus on this simple example because it is sufficient to serve our purpose in discussing time consistency.

Given the nature of the question discussed in the paper, the notation is generally complicated because of the use of maturity structures of public debt and the discussion on Ramsey equilibrium. It helps clarify the main differences between two similar models: the small open economy model in this paper and the closed economy model in Persson et al. (2006). First, a small open economy, by definition, does not have any influence on the world real interest rates, while the government in Persson et al. (2006) can affect the equilibrium real interests by its policy. This difference implies different policy choices and different policy instruments to governments, thus different time consistency property of OMFP.

Second, the small open economy in our model can borrow and lend in the international bond market to smooth consumption. Thus, the period resource constraint does not necessarily bind in each period in our small open economy model. On the contrary, the period resource constraint must bind in each period in the Persson et al. (2006) model. As a result, Persson et al. (2006) can use the binding period resource constraint to substitute leisure out in order to simplify the optimization problem; while we cannot. More importantly, because of this difference, the labor market does not impose any restriction in the discussion of the time consistency issue in a closed economy; and as a result, Persson et al. (2006) assume constant productivity without affecting their conclusion. However, it does impose a restriction in our small open economy model and whether productivity is constant or time-varying matters with respect to time consistency of OMFP in our model. We elaborate this point in detail in Section 2.3.1.

2.1.6 Intertemporal constraints

Using the optimality conditions, we can rewrite the government's intertemporal budget constraint containing only the initial price level, p_0, a constant, λ, real money balances, $\{m_{t+1}\}_{t=0}^{\infty}$, hours $\{h_t\}_{t=0}^{\infty}$, and the maturity structure it inherits from the last period, $\left({}_{-1}B_t^{\mathrm{G}}\right)$ and $\left({}_{-1}b_t^{\mathrm{G}}\right)$:

$$\frac{\lambda}{p_0}\left[\sum_{t=0}^{\infty} Q_t\left({}_{-1}B_t^{\mathrm{G}}\right) + M_{-1}\right] \quad = \quad \sum_{t=0}^{\infty}\beta^t\left[\lambda(z_t h_t - g_t - {}_{-1}b_t^{\mathrm{G}}) + u_{ht}h_t\right] + \sum_{t=1}^{\infty}\beta^t u_{mt}m_t, \tag{18}$$

where $Q_t = \prod_{j=1}^{t}\left(1+\frac{u_{mj}}{\lambda}\right)^{-1}$ and $Q_0 = 1$.

Similarly, combining the intertemporal budget constraint of the representative domestic household with that of the government, we obtain the intertemporal resource constraint for the whole small open economy:

$$\sum_{t=0}^{\infty}\beta^t\left[c_t + g_t + ({}_{-1}b_t^{\mathrm{F}}) - z_t h_t\right] \quad = \quad -\frac{1}{p_0}\sum_{t=0}^{\infty}Q_t({}_{-1}B_t^{\mathrm{F}}), \tag{19}$$

where ${}_{-1}B_t^{\mathrm{F}} = {}_{-1}B_t^{\mathrm{G}} - {}_{-1}B_t^{\mathrm{P}}$, and ${}_{-1}b_t^{\mathrm{F}} = {}_{-1}b_t^{\mathrm{G}} - {}_{-1}b_t^{\mathrm{P}}$. The superscript F means that the bond is traded with the international investors.

2.2 OMFP with Commitment

Proposition 1 *Under the condition that Ramsey governments commit to the announced policy, a Ramsey real allocation problem is to choose a constant λ [i.e., constant consumption, a point that can be seen from Eq. (17)], an initial price level p_0, and a sequence of $\{m_{t+1}, h_t\}_{t=0}^{\infty}$ to maximize the representative household's lifetime utility:*

$$u\left(c(\lambda), \frac{M_{-1}}{p_0}, h_0\right) + \sum_{t=1}^{\infty}\beta^t u\left(c(\lambda), m_t, h_t\right), \tag{20}$$

subject to Eqs. (18) and (19), given the initial money stock, M_{-1}, the initial real debt, $({}_{-1}b_t^{G})_{t=0}^{\infty}$, $({}_{-1}b_t^{F})_{t=0}^{\infty}$, the initial nominal debt, $({}_{-1}B_t^{G})_{t=0}^{\infty}$ and $({}_{-1}B_t^{F})_{t=0}^{\infty}$, and the exogenous *processes* $\{z_t, g_t\}_{t=0}^{\infty}$.

Proof The proof is standard. The key is to show that when governments commit to the announced policy, a sequence $\{c_t, m_{t+1}$, and $h_t\}_{t=0}^{\infty}$ satisfy Eqs. (3), (4), (5), (6), (7), (9), (10), (13), and (14), if and only they satisfy Eqs. (18) and (19).■

Let μ_0^{G} and μ_0^{E} be the Lagrange multipliers associated with Eq. (18), the $t = 0$ government's intertemporal budget constraint, and Eq. (19), the economy's intertemporal budget constraint, respectively. μ^{G} represents the marginal public financing cost while μ^{E} denotes the marginal external financing cost. Then the optimality condition with respect to λ is:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t u_{ct}\frac{\partial c}{\partial \lambda} &= \mu_0^{\mathrm{E}}\sum_{t=0}^{\infty}\beta^t\frac{\partial c}{\partial\lambda} - \mu_0^{\mathrm{G}}\left\{\sum_{t=0}^{\infty}\beta^t(z_t h_t - g_t - {}_{-1}b_t^{\mathrm{G}})\right\} + \frac{\mu_0^{\mathrm{E}}}{p_0}\sum_{t=1}^{\infty}({}_{-1}B_t^{\mathrm{F}})\frac{\partial Q_t}{\partial\lambda}\\ &+\frac{\mu_0^{\mathrm{G}}}{p_0}\left\{\left[\sum_{t=0}^{\infty}Q_t({}_{-1}B_t^{\mathrm{G}}) + M_{-1}\right] + \lambda\sum_{t=1}^{\infty}({}_{-1}B_t^{\mathrm{G}})\frac{\partial Q_t}{\partial\lambda}\right\}. \qquad (21)\end{aligned}$$

Eq. (21) is a single equation. The lefthand side represents the marginal cost in terms of utility due to an increase of λ. Intuitively, when it becomes more expensive to borrow to smooth consumption, the representative household will decrease its consumption in each period. The foregone discounted present value of utility due to the decrease in consumption is the marginal cost of the change in λ. The righthand side of Eq. (21) represents the corresponding marginal benefit, which contains four components: the first represents the increased discounted present value utility if the economy's intertemporal resource constraint is relaxed due to the decrease of consumption; the second represents the discounted present value disutility when the government's intertemporal budget constraint is relaxed for the same reason; the third component is the marginal benefit caused by the change in the discounted present value of outstanding external debt; and the last component comes from the associated change in the discounted present value of outstanding public debt.

The optimality condition with respect to m_t is:

$$u_{mt} = -\mu_0^{\mathrm{G}}(u_{mmt}m_t + u_{mt}) + \frac{\mu_0^{\mathrm{G}}\lambda}{\beta^t p_0}\sum_{s=t}^{\infty}({}_{-1}B_s^{\mathrm{G}})\frac{\partial Q_s}{\partial m_t} + \frac{\mu_0^{\mathrm{E}}}{\beta^t p_0}\sum_{s=t}^{\infty}({}_{-1}B_s^{\mathrm{F}})\frac{\partial Q_s}{\partial m_t}, t \geq 1. \qquad (22)$$

Eq. (22) denotes a system of equations. Its left-hand side represents the marginal cost in utility if real money balances decrease. The righthand side represents the corresponding marginal benefit in utility, which has three sources: the first source is the relaxing of the government's intertemporal budget constraint; the second is the change in the discounted present value public bonds due to the change in nominal interest rates; and the last source comes from the change in external financing due to the change in nominal interest rates.

The optimality condition with respect to h_t is:

$$-u_{ht} = \mu_0^{\mathrm{G}} \left(\lambda z_t + u_{hht} h_t + u_{ht}\right) + \mu_0^{\mathrm{E}} z_t, t \geq 0. \tag{23}$$

Eq. (23) also denotes a system of equations. It shows that optimal work hours is determined by equating the marginal benefit with the marginal cost. This optimality condition has the same components as the corresponding optimality condition in the closed economy. The only difference is that here the Lagrange multiplier in the second component of the right-hand side of Eq. (23) is the multiplier for the intertemporal budget constraint, while in the closed economy the corresponding multiplier is for the within-period resource constraint.

The optimality condition with respect to p_0 is:

$$u_{m0} M_{-1} = \mu_0^{\mathrm{G}} \lambda \left[\sum_{t=0}^{\infty} Q_t \left({}_{-1}B_t^{\mathrm{G}}\right) + M_{-1}\right] + \mu_0^{\mathrm{E}} \sum_{t=0}^{\infty} Q_t \left({}_{-1}B_t^{\mathrm{F}}\right). \tag{24}$$

Eq. (24) is a single equation. There is a marginal benefit in utility due to an increase in the price level since inflation reduces the outstanding nominal pubic debt and the external debt. This marginal benefit is given by the righthand side of Eq. (24). There is also an associated marginal cost in utility due to an increase in the price level since inflation erodes real money balances. In the equilibrium, this marginal cost exactly offsets the marginal benefit in equilibrium.

Definition 2 *A Ramsey equilibrium is defined as a choice of* (λ, p_0, $\{h_t\}_{t=0}^{\infty}$, $\{m_{t+1}\}_{t=0}^{\infty}$) *satisfying Eqs. (18), (19), (21), (22), (23), and (24), given the initial asset positions,* $\{{}_{-1}B_t^G, {}_{-1}B_t^F, {}_{-1}b_t^G, {}_{-1}b_t^F\}_{t=0}^{\infty}$ *and* M_{-1}*, and the exogenous processes* $\{z_t, g_t\}_{t=0}^{\infty}$.

2.3 OMFP with Discretion

We follow the same methodology as in the literature to discuss the time consistency property of OMFP. According to the methodology, the time consistency problem becomes whether the $t = 0$ government can find Lagrange multipliers, μ_1^{G} and μ_1^{E} and a profile $\{{}_0B_t^{\mathrm{G}}, {}_0B_t^{\mathrm{F}}, {}_0b_t^{\mathrm{G}}, {}_0b_t^{\mathrm{F}}\}_{t=1}^{\infty}$, such that the policy continuation of the t=0 government satisfy the optimality conditions of the $t = 1$ government [Lucas and Stokey (1983), Persson et al. (2006), and Alvarez et al. (2004)]. The policy continuation of the $t = 0$ government refers to the $t = 0$ government's optimal choices, $\lambda^*, h_t^*, m_{t+1}^*, p_1^*$.[7] The optimality conditions of the $t = 1$ government are the one-period ahead updated version of the optimality conditions of the $t = 0$ government.[8] If Lagrange multipliers and a profile exist, OMFP is time consistent; otherwise, it is time inconsistent.

In this section, we discuss the time consistency of OMFP when the government commits to a flexible FERR. Since the discussion of time consistency uses all the relevant optimality conditions, it does not matter the order of those conditions in our discussion. Given our model setup, we analyze the solution to the Lagrange multipliers first and then the profile. Note that Persson et al. (2006) discuss the profile first then the Lagrange multipliers. In addition, we assume the Ramsey equilibrium exists and is unique in our economy throughout the paper, an implicit assumption in those cited papers.

One thing worth emphasizing is that our discussion does not depend on whether issued bonds are denominated in domestic currency or in foreign currency (such as US$). The reason is as follows. (1) We can interpret real bonds denominated in US$ as real bonds denominated in domestic currency. To see this, note that $b_t^{\star\star} = \frac{B_t^{\star\star}}{p^{\star\star}} = \frac{B_t S}{p^{\star\star}} = \frac{B_t}{p_t} = b_t$, the first equality comes from the non-arbitrage condition in the foreign bond market, the second equality comes from the non-arbitrage condition across borders, the third equality comes from the law of one-price, and the last equality comes from the non-arbitrage condition in the domestic bond market. (2) We can also interpret nominal bonds denominated in US$ as real bonds denominated in local currency but inflated by the foreign price, $B_t^{\star\star} = b_t^{\star\star} p^{\star\star} = b_t p^{\star\star}$.

[7] To make the discussion as clear as possible, the policy continuation will be represented by a superscript of $*$.

[8] The optimality conditions of the $t = 1$ government are available upon request.

(3) Since $p^{\star\star}$ is exogenous and our discussion includes b_t, our result does not depend on whether the issued bonds are denominated in domestic currency or in US$.

2.3.1 Restriction on Financing Costs Imposed by the Labor Market

The one-period ahead version Eq. (23) is a system of linear equations with two unknowns - two Lagrange multipliers. It is convenient to use the version of Eq. (23) to discuss the solution to the two Lagrange multipliers, i.e, the restriction on the financing costs imposed by the labor market. Define

$$\Lambda_{t,s} = \begin{pmatrix} \lambda^* z_t + u^*_{hht} h^*_t + u^*_{ht} & z_t \\ \lambda^* z_s + u^*_{hhs} h^*_s + u^*_{hs} & z_s \end{pmatrix}, \forall t, s \geq 1 \text{ and } t \neq s.$$

We have the following Proposition:

Proposition 2 *In a small open economy with perfect capital mobility, Svensson timing of markets, and time varying productivity, both public and external marginal financing costs should be constant across governments in order to have time consistent optimal policy of hours when $\Lambda_{t,s}, \forall t, s \geq 1$ and $t \neq s$ is not singular. Both public and external marginal financing costs could vary across governments when $\Lambda_{t,s}, \forall t, s \geq 1$ and $t \neq s$ is singular.*

Proof Here is the scratch of proof. The optimality conditions respect to hours for the $t = 1$ government is given by:

$$\mu^{\mathrm{G}}_1 (\lambda^* z_t + u^*_{hht} h^*_t + u^*_{ht}) + \mu^{\mathrm{E}}_1 z_t = -u^*_{ht}, \forall t \geq 1. \tag{25}$$

Eq. (25) represents a system of linear equations with two unknowns, μ^{G}_1 and μ^{E}_1, and an infinite number of linear equations. Note that the $t = 0$ government has the similar optimality conditions

$$\mu^{\mathrm{G}}_0 (\lambda^* z_t + u^*_{hht} h^*_t + u^*_{ht}) + \mu^{\mathrm{E}}_0 z_t = -u^*_{ht}, \forall t \geq 1.$$

Thus, for any two periods, t and s, we have the following

$$\Lambda_{t,s}\begin{pmatrix}\mu_1^{\mathrm{G}}\\ \mu_1^{\mathrm{E}}\end{pmatrix}=\begin{pmatrix}-u_{ht}^*\\ -u_{hs}^*\end{pmatrix}\equiv\Lambda_{t,s}\begin{pmatrix}\mu_0^{\mathrm{G}}\\ \mu_0^{\mathrm{E}}\end{pmatrix}, \forall t,s\geq 1 \text{ and } t\neq s. \tag{26}$$

From Eq. (26), when the matrix $\Lambda_{t,s}$ is non-singular, the Lagrange multipliers are uniquely determined in order to have time consistency:

$$\mu_1^{\mathrm{G}}=\mu_0^{\mathrm{G}}=\hat{\mu}^{\mathrm{G}};\mu_1^{\mathrm{E}}=\mu_0^{\mathrm{E}}=\hat{\mu}^{\mathrm{E}}. \tag{27}$$

When the matrix $\Lambda_{t,s}$ is singular, the Lagrange multipliers are not uniquely determined by the labor-leisure choice. ■

By choosing the same Lagrange multipliers, we can guarantee that the policy continuation of hours will satisfy Eq. (25). The choice of constant Lagrange multipliers in Eq. (27) is a very strong result. Intuitively, the constant costs result comes from the special feature in the small open economy. According to Eqs. (23) and (25), optimal policy of hours in the Ramsey problem are determined by the period productivity and the Lagrange multipliers associated with the two intertemporal budget constrants. Given that the $t=0$ government and the $t=1$ government face the same flow of time-varying productivity, if the Lagrange multipliers are different, the $t=1$ government will deviate from the policy continuation of hours if $\Lambda_{t,s}, \forall t,s\geq 1$ and $t\neq s$ is not singular. In other words, governments want to keep the marginal financing costs constant over time in order to have time consistent optimal policy of hours. In the context of this small open economy, the constant marginal financing costs do not imply constant labor income tax rates (one for each period), because hours can change over time. The constant marginal financing costs also do not imply constant nominal interest rates because real money balances can change over time.

When the matrix $\Lambda_{t,s}$ is singular, the labor-leisure choices do not impose restrictions such that the Lagrange multipliers may not be uniquely determined by Eq. (25) alone. There are two sufficient conditions for $\Lambda_{t,s}$ to be singular, which is given by:

Proposition 3 *When separable preferences have $\frac{u_{hh}h}{u_h}=-\zeta$ and/or the total factor productivity is constant over time, $\Lambda_{t,s}, \forall t,s\geq 1$ and $t\neq s$ is singular.*

Proof The first sufficient condition for $\Lambda_{t,s}$ to be singular is a condition on preferences:

$$\frac{u_{hh}h}{u_h} = -\zeta, \tag{28}$$

where ζ is an integer. Under this condition, we have the following

$$\begin{aligned} -u_{ht}^* &= \mu_0^{\mathrm{G}}\left[\lambda^* z_t - (\zeta-1)\, u_{ht}^*\right] + \mu_0^{\mathrm{E}} z_t, \forall t \geq 1, \\ \Rightarrow u_{ht}^* &= \frac{\mu_0^{\mathrm{G}}\lambda^* z_t + \mu_0^{\mathrm{E}} z_t}{(\zeta-1)\,\mu_0^{\mathrm{G}} - 1}. \forall t \geq 1, \end{aligned}$$

Thus, we have:

$$\Lambda_{t,s} = \begin{pmatrix} \lambda^* z_t - \frac{(\zeta-1)\left(\mu_0^{\mathrm{G}}\lambda^* z_t + \mu_0^{\mathrm{E}} z_t\right)}{(\zeta-1)\mu_0^{\mathrm{G}}-1} & z_t \\ \lambda^* z_s - \frac{(\zeta-1)\left(\mu_0^{\mathrm{G}}\lambda^* z_s + \mu_0^{\mathrm{E}} z_s\right)}{(\zeta-1)\mu_0^{\mathrm{G}}-1} & z_s \end{pmatrix} \Rightarrow \left|\Lambda_{t,s}\right| = 0, \forall t, s \geq 1 \text{ and } t \neq s.$$

The second sufficient condition is a condition on productivity which is given by:

$$z_t \equiv z, \tag{29}$$

where z is a constant. With constant z, the system of equations reduces to one single equation and $\Lambda_{t,s}$ is clearly singular. It is worth noting that when $\Lambda_{t,s}$ is singular, the Lagrange multipliers are subject to one linear equation even though they are not uniquely determined.■

In the closed economy, the labor market does not impose any restrictions on the time consistency property of OMFP. To see this, note that the relevant optimality condition becomes

$$\mu_1^{\mathrm{G}}\left(u_{ct}^* z_t + u_{hht}^* h_t^* + u_{ht}^*\right) + \mu_{1t}^{\mathrm{E}} z_t = -u_{ht}^*, \forall t \geq 1. \tag{30}$$

where μ_{1t}^{E} denotes the Lagrange multiplier associated with the period resource constraint in period t. After the Ramsey government chooses the value for μ_1^{G}, μ_{1t}^{E} will adjust in such a way that policy continuation of hours will automatically satisfy Eq. (30). For this very reason, Persson et al. (2006) skip the discussion on the effect of the labor-leisure choice on the time consistency of OMFP.

2.3.2 Time Inconsistency with Non-singular $\Lambda_{t,s}$

When the financing costs are constant across governments, OMFP is not time consistent. The result is presented in the following proposition:

Proposition 4 *In a small open economy with perfect capital mobility, with separable preferences, with Svensson timing of markets, and a flexible FERR, OMFP is time inconsistent if $\Lambda_{t,s}$ is not singular. This result is independent of the time-varying government expenditure process and the initial asset position.*

Proof The key is to show that we cannot find a profile $\mathbf{X}_1 = \{{}_0B_t^{\mathrm{G}}, {}_0B_t^{\mathrm{F}}, {}_0b_t^{\mathrm{G}}, {}_0b_t^{\mathrm{F}}\}_{t=1}^{\infty}$ such that policy continuation of the $t = 0$ government will satisfy the optimality conditions of the $t = 1$ government's Ramsey problem. For that purpose, it is sufficient to show the non-existence of $\mathbf{X}_2 = \{{}_0B_2^{\mathrm{G}}, {}_0B_2^{\mathrm{F}}\}$. Two equations are crucial. One is the optimality condition with respect to consumption, and the other is the optimality condition with respect to money balances in period $t = 2$. With some manipulation, we obtain the following equations

$$\hat{\mu}^{\mathrm{E}} A_2 \left({}_0B_2^{\mathrm{F}}\right) + \hat{\mu}^{\mathrm{G}} \lambda^* A_2 \left({}_0B_2^{\mathrm{G}}\right) = \hat{D}_{31}, \tag{31}$$

$$\hat{\mu}^{\mathrm{E}} Q_2^1 \left({}_0B_2^{\mathrm{F}}\right) + \hat{\mu}^{\mathrm{G}} \lambda^* Q_2^1 \left({}_0B_2^{\mathrm{G}}\right) = \hat{D}_{32,2}, \tag{32}$$

where

$$
\begin{aligned}
Q_t^1 &= \prod_{j=2}^{t}\left(1+\frac{u_{mj}^*}{\lambda^*}\right)^{-1}, \\
A_t &= Q_t^1\left[\sum_{i=2}^{t}\frac{u_{mi}^*}{\lambda^*(\lambda^*+u_{mi}^*)}\right], \\
D_{31} &= p_1^*\frac{\lambda^*-\hat{\mu}^{\mathrm{E}}}{1-\beta}\left(\frac{\partial c}{\partial\lambda}\right)^* - p_1^*\hat{\mu}^{\mathrm{G}}\left[\sum_{t=1}^{\infty}\beta^{t-1}\frac{u_{ht}^*}{\lambda^*}h_t^* + \sum_{t=2}^{\infty}\beta^{t-1}\frac{u_{mt}^*}{\lambda^*}m_t^*\right], \\
D_{32,t} &= \hat{\mu}^{\mathrm{G}}m_t^*\beta^{t-1}p_1^*\left(\lambda^*+u_{mt}^*\right) + \frac{u_{mt}^*\left(1+\hat{\mu}^{\mathrm{G}}\right)\beta^{t-1}p_1^*\left(\lambda^*+u_{mt}^*\right)}{u_{mmt}^*}, \\
\hat{D}_{31} &= D_{31} - \sum_{t=3}^{\infty}\frac{A_t}{Q_t^1}\left(D_{32,t-1}-D_{32,t}\right), \\
\hat{D}_{32,2} &= D_{32,2} - D_{32,3}.
\end{aligned}
$$

Eq. (31) is the $t = 1$ government's optimality condition with respect to consumption and Eq. (32) is the $t = 1$ government's optimality condition with respect to real money balance in period $t = 2$. Note that under the condition that $\Lambda_{t,s}$ is not singular (which is the case here), Eq. (27) must hold. For this reason, we put a hat on top of the two Lagrange multipliers. Thus, there are two unknowns, ${}_0B_2^{\mathrm{G}}$ and ${}_0B_2^{\mathrm{F}}$, in two linear equations, Eqs. (31) and (32). The rest of the variables are functions of policy continuation and the pinned down Lagrange multipliers. One implicit assumption is that the values of all these terms are finite.

There is no solution to this system. To see this, simply check Eq. (31) and Eq. (32). it is clear that the solution to $\left({}_0B_2^{\mathrm{F}}\right)$ and $\left({}_0B_2^{\mathrm{G}}\right)$ exists only if the ratio of $\hat{D}_{31}/\hat{D}_{32,2}$ is the same as the ratio of A_2/Q_2^1. Since both ratios are functions of pre-determined variables, thus no solution exists and OMFP is time inconsistent.

Note that we have applied a "pre-determined argument": if two ratios are pre-determined, they are generally not equal. Even though this pre-determined argument is not standard, it has been used in the literature, for example, Uribe (2006).■

From Eqs. (31) and (32), consumption and real money balance in period $t = 2$ are determined by three components: the forward looking component, $\left(\hat{D}_{31} \text{ and } \hat{D}_{32,2}\right)$, the history component, $[\left({}_0B_2^{\mathrm{F}}\right)$ and $\left({}_0B_2^{\mathrm{G}}\right)]$, and the present component, $\left(\mu^{\mathrm{E}} \text{ and } \mu^{\mathrm{G}}\right)$. In our time-varying economy, it is generally true that the forward looking component to the $t = 1$

government is different from that to the $t = 0$ government. Hence, to have the $t = 1$ government to follow the $t = 0$ government's choice of consumption and real money balance in period $t = 2$, the $t = 0$ government must choose an appropriate combination of the history component and the present component. However, the history component imposes a singularity in the system of two equations, Eqs. (31) and (32) if the present component had been pinned down by the labor-leisure choices. Thus, to turn-around the singularity problem imposed by the history component, the present component must be time-varying. In other words, the good market and the money market ask that financing costs vary across government in order to have time consistency.

Put it together, when $\Lambda_{t,s}$ is not singular, there are two forces in this small open economy that are important with respect to time consistency of OMFP. One force, which arises from the labor market when $\Lambda_{t,s}$ is not singular, asks for constant external and public financing costs across government in order to have time consistency. The other force, which arises from the good market and the money market, asks for time-varying external and public financing costs across governments. They clearly contradict to each other. As a result, OMFP is time inconsistent.

2.3.3 Time Inconsistency with Singular $\Lambda_{t,s}$

When Lagrange multipliers are not uniquely determined by the time consistency requirement in the labor market, i.e., $\Lambda_{t,s}$ is singular, the force asking for constant financing costs disappears. Therefore, OMFP becomes time consistent and many maturity structures of public debt and external debt are capable of rendering OMFP time consistent. Proposition 5 summarizes this finding:

Proposition 5 *In a small open economy with perfect capital mobility, separable preferences, with Svensson timing of markets, and a flexible FERR, OMFP is time consistent if $\Lambda_{t,s}$ is singular. This result is independent of the time-varying government expenditure process, and the initial asset position of the government. Further, there are many maturity structures of bonds that are capable of rendering OMFP time consistent.*

Proof We prove Proposition 5 by construction as in the literature [Alvarez et al. (2004) and Persson et al. (2006)]. To proceed, we leave the Lagrange multipliers undetermined and make the following arbitrary assumptions to facilitate the discussion:

$$({}_0b_t^{\mathrm{F}}) = ({}_0\hat{b}_t^{\mathrm{F}}), t \geq 2; ({}_0b_t^{\mathrm{G}}) = ({}_0\hat{b}_t^{\mathrm{G}}), t \geq 1; ({}_0B_t^{\mathrm{G}}) = ({}_0\hat{B}_t^{\mathrm{G}}), t \geq 3. \tag{33}$$

where the variables $({}_0\hat{b}_t^{\mathrm{F}})$, $({}_0\hat{b}_t^{\mathrm{G}})$, and $({}_0\hat{B}_t^{\mathrm{G}})$ denote the values arbitrarily chosen for $({}_0b_t^{\mathrm{F}})$, $({}_0b_t^{\mathrm{G}})$, and $({}_0B_t^{\mathrm{G}})$, respectively. There are two layers of arbitrary assumptions here. First, the values for these bond holding positions are arbitrary. Second, the format of assumption (33) itself is arbitrary in the sense that we can interchange the superscript of G by F and vice versa across time.

The choice that the arbitrary values for $({}_0b_t^{\mathrm{F}})$ and $({}_0b_t^{\mathrm{G}})$ include all the values after $t = 2$ reflects that the maturity structure of real bonds is indeterminate (a result directly coming from the fact that real interest rates are exogenous to this small open economy). Further, the arbitrary values for $({}_0b_t^{\mathrm{G}})$ actually start at $t = 1$. This is due to the equivalence between $({}_0b_1^{\mathrm{G}})$ and $({}_0B_1^{\mathrm{G}})$ that occurs if the $t = 1$ government follows the policy continuation. This point will become evident in the following steps.

Assumption (33) is of interest because it applies to the case in which the $t = 0$ government has the maximum number of degrees of freedom in choosing the maturity structure of nominal bonds in order to render time consistent OMFP. In fact, this assumption says that the $t = 0$ government only needs to pay attention to two multiple-period public bond, and the government is free in choosing all other nominal public bonds.

We start with the optimality condition with respect to real money balances

$$\sum_{s=t}^{\infty} \mu_1^{\mathrm{E}} Q_s^1 \left({}_0B_s^{\mathrm{F}}\right) + \sum_{s=t}^{\infty} \mu_1^{\mathrm{G}} \lambda^* Q_s^1 \left({}_0B_s^{\mathrm{G}}\right) \quad = \quad \tilde{D}_{32,t}, t \geq 2 \tag{34}$$

Here we replace the $\{\hat{\mu}^{\mathrm{E}}, \hat{\mu}^{\mathrm{G}}\}$ in $D_{32,t}$ with $\{\mu_1^{\mathrm{E}}, \mu_1^{\mathrm{G}}\}$ to obtain $\tilde{D}_{32,t}$. Given the undetermined Lagrange multipliers, subtracting Eq. (34) held at $t = S$ from the equation (34) held at $t = S + 1$ produces the following equation involving $({}_0B_S^{\mathrm{F}})$ and $({}_0B_S^{\mathrm{G}})$:

$$\mu_1^{\mathrm{E}} Q_S^1 ({}_0B_S^{\mathrm{F}}) + \mu_1^{\mathrm{G}} \lambda Q_S^1 ({}_0B_S^{\mathrm{G}}) \quad = \quad \tilde{D}_{32,S} - \tilde{D}_{32,S+1}, S \geq 2. \tag{35}$$

Eq. (35) is the only generic restriction on the maturity structure of nominal bonds. It shows that if the government wants to decrease government financing by one unit, it must increase external financing by $\mu^{\mathrm{G}}\lambda/\mu^{\mathrm{E}}$ units. The economic interpretation of Eq. (35) is that once the $t = 1$ government follows the policy continuation, it does not care about the particular source of financing. Eq. (35) and assumption (33) are sufficient to pin down $({}_0B_S^{\mathrm{F}})$ and $({}_0B_S^{\mathrm{G}})$ for any $S \geq 3$ as functions of the undetermined Lagrange multipliers.

We then rewrite Eqs. (21) and (22) at $t = 2$ as

$$\begin{aligned}
\mu_1^{\mathrm{E}} A_2 \left({}_0B_2^{\mathrm{F}}\right) + \mu_1^{\mathrm{G}}\lambda^* A_2 \left({}_0B_2^{\mathrm{G}}\right) &= \tilde{D}_{31}, &(36)\\
\mu_1^{\mathrm{E}} Q_2^1 \left({}_0B_2^{\mathrm{F}}\right) + \mu_1^{\mathrm{G}}\lambda^* Q_2^1 \left({}_0B_2^{\mathrm{G}}\right) &= \tilde{D}_{32,2} - \tilde{D}_{32,3}, &(37)
\end{aligned}$$

Here we replace the $\{\hat{\mu}^{\mathrm{E}}, \hat{\mu}^{\mathrm{G}}\}$ in $\hat{D}_{31}$ with $\{\mu_1^{\mathrm{E}}, \mu_1^{\mathrm{G}}\}$ to obtain $\tilde{D}_{31}$. We can get rid of $({}_0B_2^{\mathrm{G}})$ and $({}_0B_2^{\mathrm{F}})$ to obtain one equation with the two undetermined Lagrange multipliers:

$$0 = Q_2^1 \tilde{D}_{31} - A_2 \left(\tilde{D}_{32,2} - \tilde{D}_{32,3}\right), \tag{38}$$

Note that $\tilde{D}_{31}$, $\tilde{D}_{32,2}$, and $\tilde{D}_{32,3}$ are functions of the undetermined Lagrange multipliers. Eq. (38) with the linear restriction from the labor market are two linear equations with two unknowns, the undetermined Lagrange multipliers. It is straightforward to see that there is no singularity issue here and we can thus solve for the Lagrange multipliers.

There are three more conditions to use. They are the government intertemporal budget constraint, the economy intertemporal resource constraint, and the optimality condition with respect to the initial price level. For convenience, we list them below

$$\begin{aligned}
\sum_{t=1}^{\infty} Q_t^1 \left({}_0B_t^{\mathrm{G}}\right) + p_1^* \sum_{t=1}^{\infty} \beta^t \left({}_0b_t^{\mathrm{G}}\right) &= D_{39} &(39)\\
\sum_{t=1}^{\infty} Q_t^1 \left({}_0B_t^{\mathrm{F}}\right) + p_1^* \sum_{t=1}^{\infty} \beta^t \left({}_0b_t^{\mathrm{F}}\right) &= D_{40} &(40)\\
\mu_1^{\mathrm{E}} \sum_{t=1}^{\infty} Q_t^1 \left({}_0B_t^{\mathrm{F}}\right) + \mu_1^{\mathrm{G}}\lambda^* \sum_{t=1}^{\infty} Q_t^1 \left({}_0B_t^{\mathrm{G}}\right) &= u_{m1}^* M_0^* - \mu_1^{\mathrm{G}}\lambda^* M_0^*, &(41)
\end{aligned}$$

where

$$
\begin{aligned}
D_{39} &= p_1^* \sum_{t=1}^{\infty} \beta^{t-1} \left[(z_t h_t^* - g_t) + \frac{u_{ht}^*}{\lambda^*} h_t^* \right] + p_1^* \sum_{t=2}^{\infty} \beta^{t-1} \frac{u_{mt}^*}{\lambda^*} m_t^* - M_0 \\
D_{40} &= p_1^* \sum_{t=1}^{\infty} \beta^{t-1} \left[z_t h_t^* - c(\lambda^*) - g_t \right].
\end{aligned}
$$

After $({}_0B_S^{\mathrm{F}})$ and $({}_0B_S^{\mathrm{G}})$ for all $S \geq 2$ and Lagrange multipliers are solved, Eq. (39) is an equation in one unknown $({}_0B_1^{\mathrm{G}})$ given assumption (33). This construction of $({}_0B_1^{\mathrm{G}})$ shows that there is one-to-one relation between $({}_0B_1^{\mathrm{G}})$ and $({}_0b_1^{\mathrm{G}})$. The $t = 1$ government has a degree of freedom to choose one of these two, and once the value for $({}_0b_1^{\mathrm{G}})$ is chosen, a corresponding value for $({}_0B_1^{\mathrm{G}})$ is determined by following the above steps.

We plug the solutions of $({}_0B_S^{\mathrm{F}}), S \geq 2$ and $({}_0B_S^{\mathrm{G}}), S \geq 1$, into Eq. (41), and solve for $({}_0B_1^{\mathrm{F}})$. The $t = 0$ government has to choose $({}_0B_1^{\mathrm{F}})$ in such a way that, under assumption (33), the benefit of surprise inflation is completely neutralized by the cost of surprise inflation. Finally, we plug all the solved optimal bond holding positions into Eq. (40) to solve for $({}_0b_1^{\mathrm{F}})$. This shows that under assumption (33), the choice of $({}_0b_1^{\mathrm{F}})$ has to satisfy the intertemporal budget constraint of the $t = 1$ economy.

It can be shown that the constructed maturity structure of bonds is consistent with the $t = 0$ government's intertemporal budget constraint and with the $t = 0$ economy's intertemporal budget constraint.[9] Thus, we construct a maturity structure of bonds and a solution to the Lagrange multipliers under arbitrary assumption (33). With the constructed maturity structure of bonds and the solution to the Lagrange multipliers, OMFP is time consistent because the policy continuation satisfies the optimality conditions of the $t = 1$ government. Since assumption (33) is arbitrary, we can change the values in that assumption to construct different maturity structures of bonds and different solutions for the Lagrange multipliers, which will together make the OMFP time consistent.■

One related question is about the horizon of the bonds. The question is of interest because for the given the general solution, any supportive maturity structure of bonds will potentially be over the infinite horizon. However, this finding cannot be useful from the

[9]The proof is available upon request.

central bankers' point of view because it is impossible to design and organize a maturity structure of bonds over the infinite horizon. Here is the result:

Corollary 1 *When OMFP is time consistent, the time horizon of both public bonds and external bonds to guarantee time consistent OMFP can be finite.*

Proof To see this, assume the $t = 0$ government inherits nominal bonds over finite horizon of ι periods, where $\iota \geq 3$. Thus the optimality conditions with respect to any m_s, where $s \geq \iota + 1$, can be rewritten as:

$$u_{ms}\left(1 + \mu^{\mathrm{G}}\right) = -\mu^{\mathrm{G}} m_s u_{mms}, s \geq \iota + 1 \tag{42}$$

From Eq. (42), all the m_s, where $s \geq \iota+1$, are the same. Thus, the $t = 0$ government can set all the public and external bonds matured after the $\iota+1$ period at zero. It is straightforward to show that these zero bonds satisfy the $t = 1$ government's optimality conditions with respect to real money balance, Eq. (34), for all the periods after the $\iota + 1$ period. For the rest bonds, the $t = 0$ government can follow the above steps to find a maturity structure of bonds to render the OMFP time consistent.■

3 Time Inconsistency under A Fixed FERR

The research following Kydland and Prescott (1977) and Barro and Gordon (1983) argues that a credible fixed FERR helps solve the time inconsistent problem while a flexible FERR is likely to render policy time-inconsistent. One relevant question this paper answers is whether such a claim holds in a small open economy version of those closed economy model in Lucas and Stokey (1983), Alvarez et al. (2004) and Persson et al. (2006).

When the government commits to the fixed FERR, i.e., $S_t \equiv S$, the monetary economy effectively becomes a real economy. There are several significant changes. First, $i_t \equiv i^{\star\star}$, a feature meaning that the government loses its control over monetary policy, as predicted by the impossible trinity theorem established in the Mundell-Fleming model. Second, $p_t \equiv Sp^{\star\star}$, a feature implying that p_0 is not a choice variable in the corresponding Ramsey real allocation problem. Third, $Q_t \equiv (1+i^{\star\star})^{-t}$, a feature indicating that nominal bonds (external and public) of different maturity dates are not effective policy instruments any more. Put it differently, only the total bond holding position, $\sum_{t=0}^{\infty} Q_t \left({}_{-1}B_t^{\mathrm{G}}\right) = \sum_{t=0}^{\infty} (i^{\star\star})^{-t} \left({}_{-1}B_t^{\mathrm{G}}\right)$ and $\sum_{t=0}^{\infty} Q_t \left({}_{-1}B_t^{\mathrm{F}}\right) = \sum_{t=0}^{\infty} (i^{\star\star})^{-t} \left({}_{-1}B_t^{\mathrm{F}}\right)$, matters in the discussion of time consistency.[10] Besides, when Q_t is fully determined by $i^{\star\star}$, it will be independent of λ and m_t.

Thus, the intertemporal budget constraints are rewritten as:

$$\lambda \mathbb{B}_t^{\mathrm{G}} = \sum_{t=0}^{\infty} \beta^t \left[\lambda(z_t h_t - g_t) + u_{ht} h_t\right] + \sum_{t=1}^{\infty} \beta^t u_{mt} m_t - \frac{\lambda M_{-1}}{p^{\star\star}}, \tag{43}$$

$$\mathbb{B}_t^{\mathrm{F}} = \sum_{t=0}^{\infty} \beta^t \left[c_t + g_t - z_t h_t\right], \tag{44}$$

where

$$\mathbb{B}_t^{\mathrm{G}} = \frac{1}{p^{\star\star}} \sum_{t=0}^{\infty} (i^{\star\star})^{-t} \left({}_{-1}B_t^{\mathrm{G}}\right) + \sum_{t=0}^{\infty} \beta^t ({}_{-1}b_t^{\mathrm{G}})$$

$$\mathbb{B}_t^{\mathrm{F}} = -\frac{1}{p^{\star\star}} \sum_{t=0}^{\infty} (i^{\star\star})^{-t} \left({}_{-1}B_t^{\mathrm{F}}\right) - \sum_{t=0}^{\infty} \beta^t ({}_{-1}b_t^{\mathrm{F}}).$$

The Ramsey real allocation problem under commitment becomes:

Proposition 6 *Under the condition that Ramsey governments commit to the announced policy, a Ramsey real allocation problem is to choose a constant λ and a sequence of $\{m_{t+1}, h_t\}_{t=0}^{\infty}$ to maximize the representative household's lifetime utility:*

$$u\left(c(\lambda), \frac{M_{-1}}{p^{\star\star}}, h_0\right) + \sum_{t=1}^{\infty} \beta^t u\left(c(\lambda), m_t, h_t\right),$$

subject to Eqs. (43) and (44), given the initial money stock, M_{-1}, the initial real debt, $({}_{-1}b_t^G)_{t=0}^{\infty}$, $({}_{-1}b_t^F)_{t=0}^{\infty}$, the initial nominal debt, $({}_{-1}B_t^G)_{t=0}^{\infty}$ and $({}_{-1}B_t^F)_{t=0}^{\infty}$, and the exogenous processes $\{z_t, g_t\}_{t=0}^{\infty}$.

Following the same procedures in the case of flexible FERR, we obtain the following result:

Proposition 7 *In a small open economy with perfect capital mobility, with separable preferences, with Svensson timing of markets, and a fixed FERR, OMFP is time inconsistent if*

[10] Again, those exogenous variables with $\star\star$ could be time-varying.

$\Lambda_{t,s}$ is not singular. This result is independent of the government expenditure process and the initial asset position.

Proof The proof is similar to that in the case of a flexible FERR. First, to have time consistency, we have Eq. (26); i.e., the labor market will ask for constant financing costs across governments if $\Lambda_{t,s}$ is not singular and will not ask for constant financing costs across governments if if $\Lambda_{t,s}$ is singular.

Second, the good market will still ask for time-varying financing costs across governments. To see this, note that the optimality condition with respect to λ for the $t = 1$ government is

$$\begin{aligned}\sum_{t=1}^{\infty}\beta^{t-1}u_{ct}\frac{\partial c}{\partial\lambda} &= \mu_1^{\mathrm{E}}\sum_{t=1}^{\infty}\beta^{t-1}\frac{\partial c}{\partial\lambda} - \mu_1^{\mathrm{G}}\sum_{t=1}^{\infty}\beta^{t-1}(z_t h_t - g_t -_0 b_t^{\mathrm{G}}) \\ &\quad + \frac{\mu_1^{\mathrm{G}}}{p^{\star\star}}\left[\sum_{t=1}^{\infty}(i^{\star\star})^{-t}\left({}_0B_t^{\mathrm{G}}\right) + M_{-1}\right]. \qquad (45)\end{aligned}$$

In Eq. (45), (1) β is a structural parameter, and $p^{\star\star}$, $i^{\star\star}$, z_t, and g_t are exogenous. (2) u_{ct}, h_t, $\frac{\partial c}{\partial\lambda}$ are evaluated at the policy continuation to have time consistency. (3) The sum of $\sum_{t=1}^{\infty}\beta^{t-1}({}_0b_t^{\mathrm{G}})$ and $\frac{1}{p^{\star\star}}\sum_{t=1}^{\infty}(i^{\star\star})^{-t}\left({}_0B_t^{\mathrm{G}}\right)$ is predetermined by the period budget constraint of the $t = 0$ government. To see this, just check the one-period ahead version of Eq. (43). It immediately follows that the sum is predetermined with the real allocations are evaluated at the policy continuation and the Lagrange multipliers are uniquelly determined. Thus, if $\Lambda_{t,s}$ is not singular, then Lagrange multipliers are uniquely determined by the labor-leisure choices and both sides of Eq. (45) are pre-determined. In this case, OMFP is time inconsistent.

Similarly, we can show that when $\Lambda_{t,s}$ is singular, OMFP will be time consistent.■

Intuitively, unless certain conditions such that $\Lambda_{t,s}$ is singular hold, the two main forces will remain when the FERR changes from a flexible FERR to a fixed FERR. The two forces impose opposite restrictions on financing costs, which leads to time consistency of OMFP. If we follow the literature by using the counting method initially proposed in Tinbergen (1956) and formally presented in Alvarez et al. (2004), the same main conclusion of time inconsistency will be obtained. In particular, when the FERR becomes a fixed FERR, the $t = 1$ government will not have control over i_t in each period. This means that the $t = 0$ government has one less policy choice in each period to influence. On the other hand, the nominal external and public bonds become ineffective policy instruments. This means that the $t = 0$ has two less policy instruments to use in each period. Given that OMFP is time inconsistent with a flexible FERR, OMFP will be time inconsistent when the FERR changes from a flexible FERR to a fixed FERR because the government loses more policy instruments than the decrease of policy choices during the change.

Note that with a fixed FERR, the monetary economy effectively becomes a real economy and OMFP essentially becomes optimal fiscal policy. It is thus not of surprise to get time-inconsistent optimal fiscal policy in this paper, the same conclusion obtained in Persson and Svensson (1986). There are several differences between this paper and Persson and Svensson (1986). First, Persson and Svensson (1986) focus on how the trade-off between tax distortions and wealth effects in each period affects the cost of public funds; while our paper focus on how the labor market and the good (money) market(s) impose the costs of public funds and external funds. Second, because of the different methods, this paper identify certain conditions under which optimal fiscal policy could be time consistent in a small open economy; while Persson and Svensson (1986) simply use the counting method and thus have ignored those conditions. Nevertheless, the main conclusion is the same: optimal fiscal policy is in general time inconsistent.

We summarized the result with a fixed FERR below:

Proposition 8 *It is generally impossible to have fixed FERR, perfect capital mobility, and time consistent OMFP at the same time in a small open economy.*

This finding is empirically relevant and has the potential in explaining the recent sovereign debt crisis in Greece. The crisis is the outcome of a burst of the fiscal problem accumulated in Greece over time, which is in line with Proposition 8. This is also a new result. Even though the monetary policy is time consistent (because monetary policy is not independent anymore) when the fixed FERR is credible, OMFP is time inconsistent because the government does not have sufficient policy instruments. Our new result complements the finding in Kydland and Prescott (1977) and Barro and Gordon (1983) in the following sense: credible rules may help render certain policy time consistent; but at the same time, they also lead to time inconsistency of other policies. Note that Proposition 8 is different from the impossible trinity theorem, which states that it is impossible to have the fixed FERR, perfect capital mobility, and an independent monetary policy at the same time. Instead, Proposition 8 states that OMFP is time inconsistent if the government commits to a fixed FERR and capital is perfectly mobile. Note that our conclusion is weaker because the result builds on that $\Lambda_{t,s}$ is not singular.

4 Relation to the Literature

If we define Lagrange multipliers as implicit policy instruments and bonds of different maturity dates as explicit policy instrument, time consistency of OMFP hinges on whether the $t = 0$ government has sufficient effective implicit policy instruments and more effective explicit policy instruments than the $t = 1$ government has policy choices. In turns out that the answer is generally no. The reason is that there are two forces in this economy and they generally impose opposite restrictions on the financing costs. The force from the labor market asks for constant financing costs across governments while the force from the good market and the money market asks for time-varying financing costs across governments. The $t = 0$ government could not find Lagrange multipliers to satisfy the opposite requirements due to two forces. As a result, OMFP is time inconsistent. When certain conditions are satisfied, the force asking for constant financing costs will disappears and OMFP will become time consistent. In this case, many maturity structures are capable of rendering OMFP time consistent becasue the $t = 0$ government always has more explicit policy instruments than the $t = 1$ government has policy choices.

The intuition is different from that in closed economies. In a closed economy, the labor-leisure choices do not impose any restriction on the time consistency of OMFP. This is because there is a Lagrange multiplier for the resource constraint in each period. These period multipliers will adjust in order to accommodate the fluctuations in the labor market. In other words, the government in a closed economy will always have sufficient effective implicit policy instruments. Moreover, OMFP is always time consistent because Ramsey governments always have a sufficient number of explicit policy instruments [Alvarez et al. (2004) and Persson et al. (2006)]. For more about the literature, please see these cited references.

5 Conclusion

This paper makes a small step in exploring the time consistency problem in a small open economy. We show that OMFP is generally time inconsistent, a result independent of FERRs. Furthermore, OMFP is time consistent only if preferences and/or productivity satisfy certain properties. Even though these identified sufficient conditions are new to the literature, they should be interpreted with caution. First, one sufficient condition, constant productivity, for time consistency is clearly too restrictive. Another sufficient condition, Eq. (29), show the time-consistency is partially determined by the preferences of households. It raises concern about the robustness of our results. This is because we obtain the findings on time consistency of OMFP under the assumption of separable preferences. When preferences are non-separable, the time-consistency problem will be analyzed in a complicated system, which is out of the scope of this paper.

In addition, we show in a simple model that the commitment to the fixed FERR does not help render OMFP time consistent, not to mention that the credibility of the fixed exchange rate commitment itself. This result clearly extends our understanding between the credibility of a fixed FERR and time consistency of OMFP.

Although intuitive, our results are still preliminary in the sense that they are far from useful for the quantitative analysis in this important field. Nevertheless, our results point out one important extension for future research to pursue Schmitt-Grohé and Uribe (2007)'s claim that "the most urgent step ... is to characterize credible policy ...". In particular, our results, combined with that in Persson and Svensson (1986), implies that future research about optimal monetary and fiscal policy in a small open economy should focus on time consistent discretionary policy rather than Ramsey policy under commitment.

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A Appendix - Not for Publication

A.1 Optimality conditions of the t=1 government

Define $Q_t = \prod_{j=1}^{t} \left(1 + \frac{u_{mj}}{\lambda}\right)^{-1}$. For convenience, the optimality conditions of the $t = 0$ government are given below. This first is the government budget constraint:

$$\frac{\lambda}{p_0}\left[\sum_{t=0}^{\infty} Q_t \left({}_{-1}B_t^{\mathrm{G}}\right) + M_{-1}\right] = \sum_{t=0}^{\infty} \beta^t \left[\lambda(z_t h_t - g_t -_{-1} b_t^{\mathrm{G}}) + u_{ht} h_t\right] + \sum_{t=1}^{\infty} \beta^t u_{mt} m_t, \tag{46}$$

The second is the economy budget constraint:

$$\sum_{t=0}^{\infty} \beta^t \left[c_t + g_t + ({}_{-1}b_t^{\mathrm{F}}) - z_t h_t\right] = -\frac{1}{p_0} \sum_{t=0}^{\infty} Q_t({}_{-1}B_t^{\mathrm{F}}), \tag{47}$$

The third is the first order condition with respect to consumption:

$$\begin{aligned}\sum_{t=0}^{\infty} \beta^t u_{ct} \frac{\partial c}{\partial \lambda} = {} & \mu_0^{\mathrm{E}} \sum_{t=0}^{\infty} \beta^t \frac{\partial c}{\partial \lambda} - \mu_0^{\mathrm{G}} \left\{\sum_{t=0}^{\infty} \beta^t (z_t h_t - g_t -_{-1} b_t^{\mathrm{G}})\right\} + \frac{\mu_0^{\mathrm{E}}}{p_0} \sum_{t=1}^{\infty} ({}_{-1}B_t^{\mathrm{F}}) \frac{\partial Q_t}{\partial \lambda} \\ & + \frac{\mu_0^{\mathrm{G}}}{p_0} \left\{\left[\sum_{t=0}^{\infty} Q_t({}_{-1}B_t^{\mathrm{G}}) + M_{-1}\right] + \lambda \sum_{t=1}^{\infty} ({}_{-1}B_t^{\mathrm{G}}) \frac{\partial Q_t}{\partial \lambda}\right\}. \end{aligned} \tag{48}$$

The fourth is the first order condition with respect to money balance:

$$u_{mt} = -\mu_0^{\mathrm{G}} (u_{mmt} m_t + u_{mt}) + \frac{\mu_0^{\mathrm{G}} \lambda}{\beta^t p_0} \sum_{s=t}^{\infty} ({}_{-1}B_s^{\mathrm{G}}) \frac{\partial Q_s}{\partial m_t} + \frac{\mu_0^{\mathrm{E}}}{\beta^t p_0} \sum_{s=t}^{\infty} ({}_{-1}B_s^{\mathrm{F}}) \frac{\partial Q_s}{\partial m_t}, t \geq 1. \tag{49}$$

The fifth is the first order condition with respect to hours:

$$-u_{ht} = \mu_0^{\mathrm{G}} (\lambda z_t + u_{hht} h_t + u_{ht}) + \mu_0^{\mathrm{E}} z_t, t \geq 0. \tag{50}$$

The last is the first order condition with respect to P_0:

$$u_{m0} M_{-1} = \mu_0^{\mathrm{G}} \lambda \left[\sum_{t=0}^{\infty} Q_t \left({}_{-1}B_t^{\mathrm{G}}\right) + M_{-1}\right] + \mu_0^{\mathrm{E}} \sum_{t=0}^{\infty} Q_t \left({}_{-1}B_t^{\mathrm{F}}\right). \tag{51}$$

A.2 Optimality conditions of the t=1 government

For convenience, the optimality conditions of the $t = 1$ government in the same order as in the case of the $t = 0$ government. However, we write them in a different format in order to facilitate the discussion on time consistency:

$$\sum_{t=1}^{\infty} Q_{1,t}\left({}_0B_t^{\mathrm{G}}\right) + p_1 \sum_{t=1}^{\infty} \beta^t \left({}_0b_t^{\mathrm{G}}\right) = D_{52} \tag{52}$$

$$\sum_{t=1}^{\infty} Q_{1,t}\left({}_0B_t^{\mathrm{F}}\right) + p_1 \sum_{t=1}^{\infty} \beta^t \left({}_0b_t^{\mathrm{F}}\right) = D_{53} \tag{53}$$

$$\sum_{t=2}^{\infty} \mu_1^{\mathrm{E}} A_{54,t}({}_0B_t^{\mathrm{F}}) + \sum_{t=2}^{\infty} \mu_1^{\mathrm{G}} \lambda A_{54,t}\left({}_0B_t^{\mathrm{G}}\right) = D_{54} \tag{54}$$

$$\sum_{s=t}^{\infty} \mu_1^{\mathrm{E}} Q_{1,s}({}_0B_s^{\mathrm{F}}) + \sum_{s=t}^{\infty} \mu_1^{\mathrm{G}} \lambda Q_{1,s}({}_0B_s^{\mathrm{G}}) = D_{55,t}, t \geq 2 \tag{55}$$

$$\mu_1^{\mathrm{G}}\left(\lambda z_t + u_{hht} h_t + u_{ht}\right) + \mu_1^{\mathrm{E}} z_t h_t = -u_{h_t}, t \geq 1 \tag{56}$$

$$\mu_1^{\mathrm{E}} \sum_{t=1}^{\infty} Q_{1,t}\left({}_0B_t^{\mathrm{F}}\right) + \mu_1^{\mathrm{G}} \lambda \sum_{t=1}^{\infty} Q_{1,t}\left({}_0B_t^{\mathrm{G}}\right) = u_{m1} M_0 - \mu_1^{\mathrm{G}} \lambda M_0, \tag{57}$$

where

$$\begin{aligned}
D_{52} &= p_1 \sum_{t=1}^{\infty} \beta^{t-1}\left[(z_t h_t - g_t) + \frac{u_{ht}}{\lambda} h_t\right] + p_1 \sum_{t=2}^{\infty} \beta^{t-1} \frac{u_{mt}}{\lambda} m_t - M_0 \\
D_{53} &= p_1 \sum_{t=1}^{\infty} \beta^{t-1}\left[z_t h_t - c(\lambda) - g_t\right] \\
A_{54,t} &= Q_{1,t}\left[\sum_{j=2}^{t} \frac{u_{mj}}{\lambda(\lambda + u_{mj})}\right] \\
D_{54} &= p_1 \frac{\lambda - \mu_1^{\mathrm{E}}}{1-\beta} \frac{\partial c}{\partial \lambda} - p_1 \mu_1^{\mathrm{G}}\left[\sum_{t=1}^{\infty} \beta^{t-1} \frac{u_{ht}}{\lambda} h_t + \sum_{t=2}^{\infty} \beta^{t-1} \frac{u_{mt}}{\lambda} m_t\right] \\
D_{55,t} &= \mu_1^{\mathrm{G}} m_t \beta^{t-1} p_1\left(\lambda + u_{mt}\right) + \frac{u_{mt}\left(1 + \mu_1^{\mathrm{G}}\right) \beta^{t-1} p_1\left(\lambda + u_{mt}\right)}{u_{mmt}}.
\end{aligned}$$

A.3 Liquidity constraint

Here we use some simplified notations, such as

$$b_0^G = \sum_{s=0}^{\infty} \beta^s ({}_{-1}b_s^G); b_1^G = \sum_{s=1}^{\infty} \beta^{s-1} ({}_0b_s^G);$$

$$b_0^F = \sum_{s=0}^{\infty} \beta^s ({}_{-1}b_s^F); b_1^F = \sum_{s=1}^{\infty} \beta^{s-1} ({}_0b_s^F).$$

The intertemporal budget constraint of the $t = 0$ government is given by

$$\sum_{s=0}^{\infty} Q_s \left(\frac{{}_{-1}B_s^G}{p_0}\right) + \frac{M_{-1}}{p_0} + b_0^G = \sum_{t=0}^{\infty} \beta^t \left[(z_t h_t - g_t) + \frac{u_{ht}}{\lambda} h_t\right] + \sum_{t=1}^{\infty} \beta^t \frac{u_{mt}}{\lambda} m_t$$

The intertemporal budget constraint of the $t = 1$ government is given by

$$\sum_{s=1}^{\infty} Q_{1,s} \left(\frac{{}_{-1}B_s^G}{p_1}\right) + \frac{M_0}{p_1} + b_1^G = \sum_{t=1}^{\infty} \beta^{t-1} \left[(z_t h_t - g_t) + \frac{u_{ht}}{\lambda} h_t\right] + \sum_{t=2}^{\infty} \beta^{t-1} \frac{u_{mt}}{\lambda} m_t$$

Here we want to show that given that the policy continuation satisfies the $t = 1$ government's budget constraint, the constructed maturity structure of bonds is consistent with the intertemporal budget constraint of the $t = 0$ government. Let's start with the $t = 0$ government's period budget constraint,

$$\begin{aligned}\sum_{s=0}^{\infty} Q_s \left(\frac{{}_{-1}B_s^G}{p_0}\right) + \frac{M_{-1}}{p_0} + b_0^G &= \left[(z_0 h_0 - g_0) + \frac{u_{h0}}{\lambda} h_0\right] \\ &\quad + \sum_{s=1}^{\infty} Q_s \left(\frac{{}_{0}B_s^G}{p_0}\right) + \frac{M_0}{p_0} + \beta b_1^G\end{aligned}$$

Substituting into the $t = 0$ government's intertemporal budget constraint:

$$\begin{aligned}\sum_{t=1}^{\infty} \beta^t \left[(z_t h_t - g_t) + \frac{u_{ht}}{\lambda} h_t\right] + \sum_{t=2}^{\infty} \beta^t \frac{u_{mt}}{\lambda} m_t &= \sum_{s=1}^{\infty} Q_s \left(\frac{{}_{0}B_s^G}{p_0}\right) + \frac{M_0}{p_0} \\ &\quad - \beta \frac{u_{m1}}{\lambda} m_1 + \beta b_1^G.\end{aligned}$$

Now given that $\frac{1}{1+i_1} = \frac{q_1/p_1}{q_0/p_0} = \frac{\beta p_0}{p_1}$, and $Q_t = Q_{1,t}/(1 + i_1)$, then we have:

$$\begin{aligned}&\sum_{t=1}^{\infty} \beta^{t-1} \left[(z_t h_t - g_t) + \frac{u_{ht}}{\lambda} h_t\right] + \sum_{t=2}^{\infty} \beta^{t-1} \frac{u_{mt}}{\lambda} m_t \\ = &\sum_{s=1}^{\infty} Q_s \left(\frac{{}_{0}B_s^G}{\beta p_0}\right) + \frac{M_0}{\beta p_0} - \frac{u_{m1}}{\lambda} m_1 + b_1^G \\ = &\sum_{s=1}^{\infty} Q_{1,s} \left(\frac{{}_{0}B_s^G}{p_1}\right) + (1 + i_1) m_1 - i_1 m_1 + b_1^G \\ = &\sum_{s=1}^{\infty} Q_{1,s} \left(\frac{{}_{0}B_s^G}{p_1}\right) + \frac{M_0}{p_1} + b_1^G\end{aligned}$$

This is the $t = 1$ government's intertemporal budget constraint. We simplify this budget constraint as follows

$$\sum_{s=1}^{\infty} Q_s \left(\frac{{}_{-1}B_s^G}{p_0}\right) + \frac{\beta M_0}{p_1} + b_1^G = \sum_{t=1}^{\infty} \beta^t \left[(z_t h_t - g_t) + \frac{u_{ht}}{\lambda} h_t\right] + \sum_{t=2}^{\infty} \beta^t \frac{u_{mt}}{\lambda} m_t$$

Similarly, it can be shown that the constructed maturity structure of bonds is consistent with the intertemporal budget constraint of the $t = 0$ economy.

A.4 What if households do not cooperate?

Several assumptions are crucial to obtain the time consistent result when productivity is constant. (a) Real interest rates are assumed to be exogenous. This assumption guarantees that there is no real time inconsistency from the change of real interest rates. It then follows that the maturity structure of real bonds is indeterminate. (b) Svensson timing is assumed: the beginning of period money enters the utility function so that there is a direct cost associated with surprise inflation. This Svensson timing mechanism gives governments freedom in choosing non-zero nominal bonds to smooth household consumption. It is thus the case that governments have far more nominal instruments to choose than the nominal choices they want to influence. And (c) domestic households are assumed to coordinate fully, which affords the government full control in choosing the level of external debt. In the next, we discuss the case when the assumption (c) does not hold.

Assumption (c) is strong and debatable: Domestic households make individual choice of the maturity structure of bonds. Since these individual choices are not based on optimization behavior, there is no guarantee that after aggregation, the realized maturity structures of $({}_0B_S^{\mathrm{P}})$ and $({}_0b_S^{\mathrm{P}})$ are the same as those implied in Theorem 4, especially when households respond after the government make the policy announcement. In this case, the only way to guarantee time consistent optimal choice is for domestic households to have complete cooperation. Since there is no clear mechanism in the model for household cooperation, it seems very unlikely that the required complete cooperation will obtain.

However, the government can force domestic households to be cooperative by using asymmetric taxes. The intuition is that, with asymmetric taxes, households give up their degrees of freedom in choosing the maturity structure of nominal bonds, which gives the government control over one or all of $\{{}_0B_S^{\mathrm{P}}\}_{S=2}^{\infty}$. This brings back to the time consistent optimal policy. Below we present an example that illustrates how to use asymmetric taxes to bring back time consistent optimal policy.

Suppose there are N households and that the maturity structure of nominal bonds for household j, where $j \in (1, 2, 3, ...N)$, is given by $\{{}_0B_S^j\}_{S=1}^{\infty}$. The $t = 0$ government randomly chooses one household k and collects asymmetric tax τ_2^j on household purchases of each unit of $\left({}_0B_2^j\right)$. The tax is asymmetric in the following sense

$$
\begin{aligned}
\tau_2^j &= x > 0, \text{ if } j \neq k \\
\tau_2^j &= 0, \text{ if } j = k.
\end{aligned}
$$

Any household $j \neq k$ will set $\left({}_0B_2^j\right) = 0$. This is the case because the household only cares about the discounted present value of its bond holding positions, and when it sets $\left({}_0B_2^j\right) \neq 0$, it has to pay a tax $x|{}_0B_2^j| > 0$, which lowers the present value of its asset positions and thus its utility. Other than $\left({}_0B_2^j\right)$, the household $j \neq k$ is free to choose any maturity structure it likes as long as its liquidity condition is satisfied. Household k is also free to choose $\left({}_0B_S^k\right)$ for $S \geq 3$ (note that $\left({}_0B_1^k\right)$ is chosen to satisfy the household's liquidity condition). Household k's choice of $\left({}_0B_2^k\right)$ will become clear below.

The arbitrary behavior of households leaves the $t = 0$ government the following restriction:

$$\left({}_0B_S^{\mathrm{P}}\right) = \left({}_0\tilde{B}_S^{\mathrm{P}}\right); S \geq 3.$$

This restriction occurs because $\left({}_0B_1^k\right)$ and $\left({}_0B_2^k\right)$ have not yet been determined. With this restriction, we get the government's choice of $\left({}_0B_S^{\mathrm{G}}\right)$ for $S \geq 3$ from Eq. (**??**). Thus, from Eqs. (36) and (37), the $t = 0$ government could choose a value for ${}_0B_2^{\mathrm{P}} ={}_0 B_2^{k\star}$ which will render optimal policy time consistent.

Returning now to household k, this household still has one degree of freedom in choosing $\left({}_0B_1^k\right)$ and $\left({}_0B_2^k\right)$. If household k chooses $\left({}_0B_2^k \neq_0 B_2^{k\star}\right)$, then optimal policy is time inconsistent and the household, along with other households, will achieve sub-optimal utility. If, instead, the household chooses $\left({}_0B_2^k =_0 B_2^{k\star}\right)$, then optimal policy is time consistent and the household, along with other households, obtains the optimal level of utility. As a result, household k gives up its degree of freedom by choosing $\left({}_0B_2^{k\star}\right)$. The $\left({}_0B_1^k\right)$ is chosen accordingly. It is straightforward to show that both the liquidity constraints of all households and the liquidity constraint of the aggregate economy are satisfied.

The above example shows how the government can use an asymmetric tax on the purchases of two-period nominal bonds in order to yield a time consistency outcome. This works because in equilibrium, the asymmetric tax τ_2^j will never be used and thus have no impact on the Ramsey equilibrium of this economy. The government can use two or more asymmetric taxes on multi-period nominal bonds to gain control over more $\left({}_0B_S^{\mathrm{P}}\right)$'s. This way, the government not only renders optimal policy time consistent, but also gains degrees of freedom in arranging the maturity structure of public debt, which gives the government greater ability to render optimal policy time consistent than its counterpart in the closed economy. In the closed economy, there is no role for these asymmetric taxes because households have to choose what the government supplies.