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A Political Agency Model of Coattail Voting

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Abstract

This paper provides a theoretical model for the coattail effect, where a popular candidate for one branch of government attracts votes to candidates from the same political party for other branches of government. I assume a political agency framework with moral hazard in order to analyze the coattail effect in simultaneous presidential and congressional elections. I show that coattail voting is the outcome of the optimal reelection scheme adopted by a representative voter to motivate politicians’ efforts in a retrospective voting environment. I assume that an office-motivated politician (executive or member of congress) prefers her counterpart to be affiliated with the same political party. This correlation of incentives leads the voter to adopt a joint performance evaluation rule, which is conditioned on the politicians belonging to the same party or to different parties. Two-sided coattail effects then arise. On the one hand, an executive’s success props up, while failure drags down, her partisan ally in the congressional election, which implies a presidential coattail effect. On the other hand, the executive’s reelection itself is affected by a congress member’s performance, which results in a reverse coattail effect.

JEL classification: D72.

Keywords: Coattail voting; Presidential coattail effect; Reverse coattail effect; Simultaneous elections; Political Agency; Retrospective voting.
1. Introduction

The coattail effect is defined as the tendency of a popular candidate for one level of government to attract votes to candidates from the same political party for other levels of government. The presidential coattail effect, where a congressional voting decision is affected by an executive’s performance, has been a topic of frequent study in the empirical literature (see Miller 1955, Press 1958, Kaplowitz 1971, Calvert and Ferejohn 1983, Campbell 1986, Campbell and Summers 1990, Flemming 1995, Cohen et al. 2000, Mattei and Glasgow 2005, Gélineau and Remmer 2006, and Golder 2006, among many others). Other studies have reported evidence of reverse coattail effects, where popular lower-tier candidates prop up their parties’ candidates for higher levels of government (Ames 1994, Samuels 2000a, Samuels 2000b).¹

While a number of studies have identified and measured coattail effects, "there remains a great deal of uncertainty concerning the causal mechanism responsible for these effects."² Mondak and McCurley (1994) suggested that coattail effects arise mainly owing to "voters' reliance on a specific cognitive efficiency mechanism" and tested this claim empirically at the individual-voter level.³ There is, however, no formal model of coattail voting, to the best of my knowledge.

In this paper, coattail effects are explained within a retrospective voting model (i.e., a political agency model with moral hazard). In my framework, coattail voting arises as an outcome of the optimal implicit reward scheme that voters use to induce politicians’ efforts.

I consider a representative voter who has to elect an executive and a member of congress in simultaneous elections. Politicians want to be reelected, and are held accountable for their performance at the moment of election. The politicians therefore have incentives to satisfy the voter’s wishes. In addition, I assume that the politicians are loyal to their respective political parties: an executive prefers her partisan ally to win in the congressional election, and vice versa.⁴ Hence, the incentives of the executive and the congress member are correlated.

¹These publications provide evidence of reverse coattail effects in Brazil. Broockman (2009), however, found no evidence of reverse coattail effects in congressional district-level data from the US presidential elections between 1952 and 2004.
²Hogan (2005), p. 587.
⁴Several authors have made similar assumptions about politicians’ partisan preferences. According to Fox and Van Weelden (2010), the legislature ("overseer") may seek to damage the reputation of an executive from another party while seeking to protect the reputation of an executive from his own party. Brollo and Nannicini (2010) assumed that an executive wants to maximize "the political capital represented by aligned mayors" by increasing the likelihood that a municipality is run by a mayor aligned with the central government. Fréchette et al. (2008) assumed that the party leader’s objective is to maximize the reelection chances of the party’s
voter cares about the politicians’ performance, which is observable but not contractible. The voter evaluates the incumbents’ performance and votes accordingly. More precisely, the voter employs implicit evaluation rules when deciding whether to reward (reelect) politicians. Obviously, the voter can influence the politicians’ behavior through the choice of evaluation rules. In this paper, the space of possible evaluation rules is restricted to linear functions of performance.

I show that given the correlation between the two politicians’ incentives, the voter is better off adopting a joint performance evaluation rule (conditioned on the incumbents belonging to the same party or to different parties) rather than an individual-politician performance evaluation rule. In particular, the voter evaluates the performance of an executive and a congress member from the same party as a team. If the executive and congress member belong to different parties, then the voter compares their performances to create a competitive environment. This combination of coattail voting rules implies that improved performance increases a politician’s own reelection probability, while increasing/decreasing the reelection probability of that politician’s partisan ally/rival for the other office. Politicians therefore have an extra incentive to perform better, for the sake of their party as well as for themselves.

In equilibrium, the reelection outcomes for incumbents from the same party are therefore positively correlated: the voter tends to reward/punish one incumbent for the good/poor performance of the other incumbent. Two-sided coattail effects therefore arise. On the one hand, the executive’s performance affects the congress member’s reelection, which gives rise to a presidential coattail effect. On the other hand, the executive’s reelection depends on the congress member’s performance, which results in a reverse coattail effect.

The equilibrium reelection outcomes for incumbents from different parties are negatively correlated: the voter is more likely to punish one incumbent the better the performance of the other incumbent. In particular, an executive’s good performance drags down an incumbent congress member’s reelection chances and therefore props up the executive’s partisan ally in the congressional election, which implies a presidential coattail effect. In turn, the congress member’s success reduces the executive’s reelection chances and thus promotes for presidential office a candidate partisanly aligned with the congress member. As a result, a reverse coattail effect arises.

These results rest on the assumption of politicians’ partisan alignment; that is, it is assumed that executives and congress members prefer their partisan ally to win in the other incumbent politicians. In turn, Persico et al. (2007) modeled the hierarchy of party members as a "patron-client relationship," where a lower-tier politician (the "client") supports a higher-tier politician (the "patron") for promotion.
election. Coattail voting rules then serve as an extra tool to discipline the politicians. If the assumption of partisan alignment is relaxed, this effect vanishes and the voter no longer evaluates incumbents jointly. Instead, the voter uses a cutoff rule that each incumbent is reappointed only when her individual performance exceeds a critical threshold. There is then no coattail effect.

I turn now to the fundamental question of why political process is modeled as political agency. In addition to a sound theoretical framework, this approach has received considerable empirical support (see, for example, Peltzman 1992 and Besley and Case 1995a, 1995b, 2003). Besley (2006) provided an excellent introduction to political agency models and "emphasizes the empirical potential of these models in explaining real world policy choices." In a recent article in the New York Times, Glaeser pointed out that the "president ... is both our leader and our employee. We (the voters) chose him, our taxes pay his salary, and we can fire him in four years." The political agency approach may therefore be appropriate for modeling political interactions between politicians and voters. Even so, elected politicians can only be offered implicit incentive schemes; it is difficult to reward public policies with explicit contracts.


The results presented in this paper are also related to the literature on horizontal and vertical intergovernmental competition. Most analyses of horizontal competition are based on the assumption of interjurisdictional mobility of consumers, à la Tiebout (1956). In a similar vein, the literature on yardstick competition between jurisdictions started with the seminal work of Salmon (1987), to be followed by Besley and Case (1995a), Bordignon et al. (2004), Sand-Zantman (2004), Belleflamme and Hindriks (2005), Besley and Smart (2007), and others. The main assumption of this literature is that under decentralization, voters use

\[\text{\textsuperscript{5}}\text{Besley (2006), p. 3.}\]
comparative performance evaluation between different local governments to create yardstick competition.

The vertical-competition literature, on the other hand, assumes that "senior and junior governments provide similar or comparable services, and that office-holders in the government which is judged by citizens to be the more efficient supplier will increase their probability of getting the vote of these citizens" (Breton 1996, Breton and Fraschini 2003, Breton and Salmon 2001, Volden 2005, 2007). I follow these authors in assuming that voters compare the performance of local and regional governments, and are likely to reward the more efficient politicians with reelection. There is, however, an important difference between my research and the papers just cited. In the intergovernmental-competition literature, the result of the comparative performance evaluation is driven by either correlated shocks or interjurisdictional spillover. In my model, the joint performance evaluation arises from the fact that the politicians' incentives are correlated: each politician cares not only about her own reelection prospects but also about the success of other politicians affiliated with the same political party.

The remainder of the paper is organized as follows. Section 2 lays out a model. Section 3 proceeds with the formal analysis. Finally, Section 4 concludes the paper.

2. Model

Consider a representative voter who has to elect an executive $E$ and a congress member $C$ in simultaneous elections. The politicians running for the two offices belong to one of two political parties. I assume that there is exactly one candidate from each party – the incumbent and an opponent – in each election. The opponents are identical to the incumbents in all respects except party label. There is no ideological heterogeneity in politicians’ preferences.\(^8\) The participation constraints of the politicians are always satisfied.

While in office, each politician $i \in \{E, C\}$ has to implement a policy determined by her unobservable effort $a_i$. The set of efforts available to each politician is taken to be a non-degenerate interval $[0, \bar{a}] \subset \mathbb{R}$. I assume that the performance of politician $i$, $p_i$, is observed with an independent, unobservable noise $\varepsilon_i$:

$$p_i = a_i + \varepsilon_i,$$

---

7Breton and Salmon (2001), p. 139.

8Since there is no ideological component, it is convenient to consider a single representative voter in this framework.
where \( \varepsilon_i \sim N(0, \sigma^2) \).^9,10,11

The reward of politician \( i \) is denoted by \( \Pi_i(a_i) \). Effort is costly, and the standard convex cost function \( \frac{a_i^2}{2} \) is assumed here.\(^12\) The executive and the congress member independently choose effort levels \( a_i \) to maximize their utility, which is given by

\[
\Pi_i(a_i) - \frac{a_i^2}{2}.
\]

The function \( \Pi_i(a_i) \) will be defined explicitly in Subsection 2.1.

The voter cares about the politicians’ performances according to a linear utility function

\[ p_E + p_C. \]

I assume that the voter applies retrospective reappointment rules to reelect the incumbents, i.e., the voter bases the reappointment decision on the politicians’ performances \( p_E \) and \( p_C \).

I denote the state variable by \( \phi \in \{S, D\} \). Here, \( \phi = S \) corresponds to the case where the executive \( E \) and congress member \( C \) are members of the same party, and \( \phi = D \) corresponds to the case where \( E \) and \( C \) are affiliated with different parties.\(^13\)

This is a sequential political agency game between politicians (the executive and the congress member) and a representative voter. The timing of events is as follows. First, the incumbents are drawn randomly, and a state \( \phi \in \{S, D\} \) is realized. Second, the voter commits to the reappointment rules to be used in the coming elections. Third, the politicians

\(^9\)An extended version of the model is available upon request, where the two noise terms \( \varepsilon_E \) and \( \varepsilon_C \) are correlated and follow a bivariate normal distribution. I want to concentrate, however, on the case where the voter introduces joint performance evaluation owing to the correlation between politicians’ incentives rather than the correlation between shocks. The latter topic has been widely studied in the context of team evaluation in contract theory (for an overview, see Bolton and Dewatripont 2005) and in the literature on yardstick competition (see the references on yardstick competition in the Introduction).

\(^10\)One can assume that policy outcomes are determined by effort and ability (rather than by effort and noise). The results are qualitatively the same if politicians choose their efforts before knowing their abilities. Otherwise, one has to solve an asymmetric information model. This extension is left for future research.

\(^11\)Alternatively, the voter might not be able to distinguish between the politicians’ performances, and therefore would observe just their aggregate performance \( p = a_E + a_C + \varepsilon \). In that case the politicians would face a free-riding problem, as each of them contributes a costly effort to an aggregate output. The analysis of this alternative framework is left for future research.

\(^12\)An extended version of the model is available upon request, where the cost of policy implementation for an executive and a congress member from the same party is different from that for politicians from rival parties (e.g., because of synergy). The results of this extended model are qualitatively the same.

\(^13\)Another way to interpret the state variable \( \phi \) is in terms of unified and divided government. \( \phi = S \) then corresponds to the case of unified government, and \( \phi = D \) corresponds to the case of divided government. For the literature on divided government, see Alesina and Rosenthal (1996), Chari et al. (1997), Degan and Merlo (2011), Fiorina (1996), Ingberman and Villani (1993), and Jacobson (1990), among many others.
exert efforts $a_E$ and $a_C$. Finally, nature chooses noises $\varepsilon_E$ and $\varepsilon_C$, and the politicians’ performances $p_E$ and $p_C$ are observed. Both elections take place simultaneously and the voter applies the selected reappointment rules to reward or punish the incumbents.\footnote{This model can be extended to several periods. I want to concentrate, however, on voters’ motives for coattail voting rather than on dynamic political agency. A static model suffices for this task.}

The politicians’ preferences are described in the following subsection. The paper then turns to the voter’s problem and defines an equilibrium concept.

### 2.1. Politicians

The politicians’ preferences are as follows. First, the executive $E$ and congress member $C$ want to be reelected. Moreover, $E$ wants to improve her party’s representation in the legislature. If $C$ and $E$ belong to the same party, then $E$ prefers $C$ to be reelected. Otherwise, $E$ wants a new congress member (from her own party) to be elected for the next term. Likewise, $C$ wants to improve his party’s chances of winning the presidential election. Thus, $C$ wants $E$ to be reelected if the two are members of the same party, and wants the opponent to be appointed if $E$ is from the rival party. The value of holding office is normalized to 1. The values which $E$ and $C$ associate with their parties’ winning the other election are denoted by $\lambda_E$ and $\lambda_C$, respectively. The probability of winning election $i \in \{E, C\}$ is denoted by $\Pr_i(\cdot)$. Therefore, politician $i$ has the following reward function $\Pi_i : [0, \pi]^2 \rightarrow \mathbb{R}$, which depends continuously on both politicians’ efforts:

$$
\Pi_i(a_i, a_j, \phi) = \begin{cases} 
\Pr_i(a_i, a_j) + \lambda_i \Pr_j(a_i, a_j) & \text{if } \phi = S, \\
\Pr_i(a_i, a_j) + \lambda_i (1 - \Pr_j(a_i, a_j)) & \text{if } \phi = D,
\end{cases}
$$

where $i, j \in \{E, C\}$ and $j \neq i$. The preferences stated above reflect the politicians’ allegiance to their respective parties; individual politicians care about their party’s overall representation in the executive and legislative branches of government, and not just their own reelection prospects.\footnote{Alternatively, these preferences could arise because the executive and the congress member have to interact while in office. Each prefers to work with a member of her own party rather than with a rival.} \footnote{The literature has emphasized the role of parties as a coordination device, such that the parties’ leaders can coordinate the strategies of their members (Morelli 2004). So a way to interpret the preferences of the incumbents stated here is to say that the parties’ leaders coordinate their members’ actions by assigning $\lambda$s.} Still, it is a reasonable assumption here that a politician values her own office more than her party’s representation in the other office, i.e., $0 \leq \lambda_i \leq 1$.\footnote{In other words, politician $i$ does not mind reducing her reelection chances by 1% in exchange for increasing her ally’s election probability by $\frac{1}{\lambda_i} \% \geq 1\%$.} I call $\lambda_i$ the degree

$$
\text{where } i, j \in \{E, C\} \text{ and } j \neq i. \text{ The preferences stated above reflect the politicians’ allegiance to their respective parties; individual politicians care about their party’s overall representation in the executive and legislative branches of government, and not just their own reelection prospects.}^{15,16} \text{ Still, it is a reasonable assumption here that a politician values her own office more than her party’s representation in the other office, i.e., } 0 \leq \lambda_i \leq 1.^{17} \text{ I call } \lambda_i \text{ the degree
of politician i’s loyalty to her party (or the strength of her partisan alignment).\textsuperscript{18}

2.2. Representative Voter

The politicians’ performances $p_E$ and $p_C$ (but not their composition between effort and noise) are observed but are not contractible. It is difficult to reward public policies with explicit contracts. It is more natural to use implicit incentive contracting in this situation.

The voter observes the politicians’ performances $p_E$ and $p_C$, and in the elections rewards incumbents according to their performance; i.e., the voter reappoints incumbents who have shown "good" results. If an incumbent is thrown out of office, an opponent from the rival party is elected.

Obviously, the voter can influence the politicians’ behavior through the choice of evaluation rules. Intuitively, since politicians care about each other’s reelection chances, the reward rules should allow joint performance evaluation. Under joint performance evaluation, the voter conditions politician i’s reelection on her own performance $p_i$ (giving her an incentive to perform well, since she wants to be reelected) and on the performance $p_j$ of politician $j$ (giving an incentive to politician $j$, since he cares about i’s reelection chances).

The functional space of the performance evaluation rules is restricted to linear joint evaluation rules $(\beta_E, b_E)$ and $(\beta_C, b_C)$. $\beta_E$ and $\beta_C$ are the slopes of the executive’s and the congress member’s performance evaluation rules, respectively, and $b_E$ and $b_C$ are the corresponding intercepts; $\beta_E, \beta_C, b_E, b_C \in \mathbb{R}$, $|\beta_E \beta_C| \leq 1$.\textsuperscript{19} Under rules $(\beta_i, b_i)$, $i \in \{E, C\}$, the probability of $i$ being reelected to office is

$$Pr_i(a_i, a_j) = P(\{p_i(a_i) + \beta_i p_j(a_j) \geq b_i\})$$

with $i, j \in \{E, C\}$ and $j \neq i$. Figure 1 depicts the possible outcomes for $E$ and $C$ under rules $(\beta_E, b_E)$ and $(\beta_C, b_C)$ in the two-dimensional space of the observed performances $p_E$ and $p_C$. Note that it is required that $|\beta_E \beta_C| \leq 1$, so that the line $p_E + \beta_E p_C = b_E$ is steeper than the line $p_C + \beta_C p_E = b_C$. Otherwise, as one can see from Figure 1, an executive and

\textsuperscript{18} An extended version of the model is available upon request, where the strength of partisan alignment $\lambda_i$ might vary between states. If there is some preference for incumbents over unknown candidates, then $\lambda_i^S \geq \lambda_i^D$. (This case reflects the idea that an executive or a congress member might prefer an incumbent ally to an unknown ally for the other office.) If politicians prefer newcomers, then $\lambda_i^S < \lambda_i^D$. (In this case an executive or a congress member would like a new ally (a newcomer) to be elected for the other office.) The results of this extended model are qualitatively the same.

\textsuperscript{19} Linear evaluation rules allow a closed-form solution in this framework. The analysis of general evaluation rules is left for future research.
a congress member with poor performance would be reelected, while politicians with better performance would not.

Note that under linear rules \((\beta_E, b_E)\) and \((\beta_C, b_C)\), \(E\) is reelected when \(\varepsilon_E + \beta_E \varepsilon_C \geq b_E - a_E - \beta_E a_C\), where \(\varepsilon_E + \beta_E \varepsilon_C \sim N(0, (1 + \beta_E^2) \sigma^2)\). In turn, \(C\) is reelected when \(\varepsilon_C + \beta_C \varepsilon_E \geq b_C - a_C - \beta_C a_E\), where \(\varepsilon_C + \beta_C \varepsilon_E \sim N(0, (1 + \beta_C^2) \sigma^2)\). The two reelection outcomes are independent when \(\beta_E = 0\) and \(\beta_C = 0\), positively correlated when \(\beta_E > 0\) and \(\beta_C > 0\), and negatively correlated when \(\beta_E < 0\) and \(\beta_C < 0\). Throughout the rest of the paper, \(F\) will be used to denote the normal distribution function, and \(f\) for the corresponding density.

### 2.3. Equilibrium Concept

I search for a subgame perfect equilibrium by analyzing the game backwards. First, I solve for the politicians’ efforts \(a_E^\phi\) and \(a_C^\phi\) under rules \((\beta_E, b_E)\) and \((\beta_C, b_C)\) in each state \(\phi\). Second, I examine the voter’s choice of evaluation rules \(\left(\beta_E^\phi, b_E^\phi\right)\) and \(\left(\beta_C^\phi, b_C^\phi\right)\) for each state \(\phi\). Two definitions will now be introduced.

Given linear performance evaluation rules \((\beta_E, b_E)\) and \((\beta_C, b_C)\), the equilibrium in effort strategies is a profile of efforts \(\left(a_E^\phi, a_C^\phi\right)\) such that

\[
\Pi_i \left(a_i^\phi, a_j^\phi, \phi\right) - \frac{a_i^2}{2} \geq \Pi_i \left(a_i, a_j^\phi, \phi\right) - \frac{a_i^2}{2} \text{ for each } a_i \in [0, \bar{a}],
\]

where \(i, j \in \{E, C\}, i \neq j\).

An equilibrium in rule strategies is defined as a tuple \(\left(\beta_E^\phi, b_E^\phi, \beta_C^\phi, b_C^\phi\right)\) such that

\[
a_E^\phi \left(\beta_E^\phi, b_E^\phi, \beta_C^\phi, b_C^\phi\right) + a_C^\phi \left(\beta_E^\phi, b_E^\phi, \beta_C^\phi, b_C^\phi\right) = \max_{\beta_E, b_E, \beta_C, b_C} a_E^\phi \left(\beta_E, b_E, \beta_C, b_C\right) + a_C^\phi \left(\beta_E, b_E, \beta_C, b_C\right),
\]

where \(\left(a_E^\phi(\cdot), a_C^\phi(\cdot)\right)\) is an equilibrium in effort strategies.

### 2.4. Intuition

Before proceeding with the formal analysis, some intuitive considerations will be discussed. The incumbents care about each other’s reelection chances, which provides the voter with an additional tool to discipline them. The voter then uses joint performance evaluation to increase the politicians’ accountability. Intuitively, the voter rewards an incumbent from one branch of government not only for her own performance but also for the performance of the incumbent from the other branch of government. That joint evaluation gives extra incentives
for the latter incumbent to perform better, since he cares about the reelection prospects of the former incumbent.

Such an evaluation strategy of the voter leads to coattail effects. Intuitively, an executive wants a congress member from the same party to be elected. The executive will perform better if her own performance increases the reelection chances of an allied incumbent congress member and decreases those of a rival incumbent congress member. The voter uses this correlation of incentives, and rewards (reelects) the incumbent congress member for the executive's good performance if the politicians are members of the same party. However, the voter punishes the incumbent congress member for the executive's good performance if the politicians are affiliated with different parties. In this case, an opponent candidate (from the same party as the executive) is elected in the congressional election. Note that a presidential coattail effect arises here. The good performance of an incumbent executive leads not only to her own reelection but also to the election of a congress member from the same party, who "rides on the executive's coattails". If an incumbent executive shows poor performance in office, her reelection chances decrease, and likewise the election chances of a congress member from the same party. In this case there is a negative presidential coattail effect.

The same intuition (in reverse) shows the emergence of a reverse coattail effect. Since a congress member wants a partisan ally to be elected for presidential office, he has an extra incentive to exert higher effort if the reelection chances of an allied executive increase and those of a rival executive decrease with an increase in his performance. The voter knows this and is more likely to reelect an incumbent executive affiliated with the same party as the congress member if the congress member performs well. If the incumbents belong to different parties, then the voter tends to punish the incumbent executive for good performance by the congress member, which leads to the election of a challenger (affiliated with the same party as the congress member) for executive office. The executive thus may either benefit or suffer from a reverse coattail effect because her reelection is affected by the congress member's performance. Good performance by a congress member props up, while poor performance drags down, a candidate from the same political party for executive office.

Note that these coattail effects arise because of the correlation of politicians' incentives such that each politician prefers her partisan ally to win the other election. Relaxing this assumption results in a prediction of no coattail effects. Indeed, if politicians care just about their own reelection, the voter will reward them only for their own performance, so no coattail voting arises.
3. Analysis

In this section, the game is analyzed backwards to find a subgame perfect equilibrium. First, an equilibrium in effort strategies is characterized, and then an equilibrium in rule strategies.

3.1. Equilibrium in Effort Strategies

The voter uses evaluation rules \((\beta_i, b_i)\), \(i, j \in \{E, C\}\). Under these rules, politician \(i\)'s utility is

\[
\Pi_i (a_i, a_j, \phi) = \frac{a_i^2}{2} \quad \text{if } \phi = S,
\]

\[
\Pi_i (a_i, a_j, \phi) = P (\{p_i (a_i) + \beta_j p_j (a_j) \geq b_i\}) + \lambda_i P (\{p_j (a_j) + \beta_j p_i (a_i) \geq b_j\}) - \frac{a_i^2}{2} \quad \text{if } \phi = D.
\]

Politician \(i\) chooses an effort \(a_i\) before observing a realization of the noise, and takes the voter’s expectations as given. The proposition below establishes the existence of an equilibrium in effort strategies. The continuity properties of the politicians’ best response functions and Brouwer’s Fixed Point Theorem are used to obtain the result. Proofs of this and other propositions are given in the Appendix.

**Proposition 1.** Under linear performance evaluation rules \((\beta_E, b_E)\) and \((\beta_C, b_C)\) with \(|\beta_E\beta_C| \leq 1\), there exists an equilibrium in effort strategies \((a^*_E, a^*_C)\) if the following second-order conditions are satisfied:

\[
\begin{aligned}
&-f'_{\xi_i + \beta_i \xi_j} (b_i - a^\phi_i - \beta \alpha^\phi_j) - \lambda_i \beta^2_{ij} f'_{\xi_j + \beta_j \xi_i} (b_j - a^\phi_j - \beta \alpha^\phi_i) - 1 < 0 \quad \text{if } \phi = S, \\
&-f'_{\xi_i + \beta_i \xi_j} (b_i - a^\phi_i - \beta \alpha^\phi_j) + \lambda_i \beta^2_{ij} f_{\xi_j + \beta_j \xi_i} (b_j - a^\phi_j - \beta \alpha^\phi_i) - 1 < 0 \quad \text{if } \phi = D,
\end{aligned}
\]

where \(i, j \in \{E, C\}, i \neq j\). This equilibrium is defined implicitly by

\[
\begin{aligned}
&f_{\xi_i + \beta_i \xi_j} (b_i - a^\phi_i - \beta_i \alpha^\phi_j) + \lambda_i \beta_{ij} f_{\xi_j + \beta_j \xi_i} (b_j - a^\phi_j - \beta_j \alpha^\phi_i) - a^\phi_i = 0 \quad \text{if } \phi = S, \\
f_{\xi_i + \beta_i \xi_j} (b_i - a^\phi_i - \beta_i \alpha^\phi_j) - \lambda_i \beta_{ij} f_{\xi_j + \beta_j \xi_i} (b_j - a^\phi_j - \beta_j \alpha^\phi_i) - a^\phi_i = 0 \quad \text{if } \phi = D.
\end{aligned}
\]

Figure 2 depicts the politicians’ best response functions in states \(S\) and \(D\) for three scenarios: independent reelection outcomes with \(\beta_E = 0\) and \(\beta_C = 0\) (black), positively correlated reelection outcomes with \(\beta_E > 0\) and \(\beta_C > 0\) (red), and negatively correlated reelection outcomes with \(\beta_E < 0\) and \(\beta_C < 0\) (blue), while \(b_E\) and \(b_C\) are fixed. Note that for independent reelection outcomes (black), the best responses are flat in both states (since each politician’s reelection depends only on her own effort). For positively correlated reelection
outcomes (red), the best responses shift upwards if the politicians are members of the same party \((\phi = S)\) and downwards if the politicians are affiliated with different parties \((\phi = D)\). Intuitively, for positively correlated reelection outcomes, a politician has an extra incentive to exert effort if \(\phi = S\) (to increase her partisan ally’s reelection chances) and less incentive if \(\phi = D\) (to avoid helping her partisan rival get reelected). Finally, for negatively correlated reelection outcomes (blue), the best responses shift downwards if the politicians belong to the same party \((\phi = S)\) and upwards if the politicians are affiliated with rival parties \((\phi = D)\).

In this scenario, a politician does not want to damage her partisan ally’s reelection prospects, and so exerts a lower effort if \(\phi = S\). However, if \(\phi = D\), the politician has an extra incentive to work harder in order to reduce her partisan rival’s reelection chances.

Note that in the case of positively correlated reelection outcomes there is a free-riding effect between partisan allies \((\phi = S)\). Intuitively, politician \(i\) might prefer to exert a lower effort (and reduce the cost of that effort) if her partisan ally \(j\) is performing well enough to improve \(i\)’s reelection prospects. In fact, \(i\) "rides on the other incumbent’s coattails".

3.2. Equilibrium in Rule Strategies

I turn now to the voter’s choice of the evaluation rules \((\beta_E, b_E)\) and \((\beta_C, b_C)\). Maximizing \(a_E^{\phi} + a_C^{\phi}\) with respect to \(\beta_E, b_E, \beta_C\) and \(b_C\) yields an equilibrium in rule strategies \(\left(\beta_E^{\phi}, b_E^{\phi}, \beta_C^{\phi}, b_C^{\phi}\right)\). The results are summarized in the following proposition.

**Proposition 2.** There exists an equilibrium in rule strategies \(\left(\beta_E^{\phi}, b_E^{\phi}, \beta_C^{\phi}, b_C^{\phi}\right)\) given by

\[
\left(\beta_i^{\phi}, b_i^{\phi}\right) = \begin{cases} 
(\lambda_j, a_i^* + \lambda_j a_j^*) & \text{if } \phi = S, \\
(-\lambda_j, a_i^* - \lambda_j a_j^*) & \text{if } \phi = D, 
\end{cases}
\]  

(3.2)

where \(i, j \in \{E, C\}, i \neq j\). The politicians’ equilibrium efforts \(a_i^*\) in each state are equal to

\[
a_i^* = \frac{1}{\sqrt{2\pi}\sigma} \left( \frac{1}{\sqrt{1 + \lambda_j^2}} + \frac{\lambda_i^2}{\sqrt{1 + \lambda_i^2}} \right). 
\]  

(3.3)

It is important to point out that the second-order conditions (3.1) hold for \(\left(\beta_E^{\phi}, b_E^{\phi}, \beta_C^{\phi}, b_C^{\phi}\right)\). The equilibrium in effort strategies described in Subsection 3.1 is therefore well defined.

The voter is rational, and so realizes that the only alternative to reelecting incumbents is to vote for opponents from rival parties. The politicians’ performances are additively separable in effort and noise, and all politicians behave in the same way irrespective of the noise. If elected, an opponent \(i\) will exert an equilibrium effort \(a_i^*\), which maximizes
her expected utility. Thus, the voter compares the incumbents’ performances with their opponents’ expected performances and votes accordingly. That is why, in equilibrium,

\[ b_i^\phi = a_i^* + \beta^\phi_i a_j^* . \]

According to Proposition 2, if politician \( j \) is loyal to his political party (i.e., \( \lambda_j \neq 0 \)), the voter adopts a coattail voting rule to reelect politician \( i \). The probability of \( i \) being reelected to office under this rule is equal to

\[
Pr_i(a_i, a_j, \phi) = \begin{cases} 
  P \left( \left\{ p_i(a_i) + \lambda_j p_j(a_j) \geq a_i^* + \lambda_j a_j^* \right\} \right) & \text{if } \phi = S, \\
  P \left( \left\{ p_i(a_i) - \lambda_j p_j(a_j) \geq a_i^* - \lambda_j a_j^* \right\} \right) & \text{if } \phi = D.
\end{cases}
\]

Intuitively, the incentives of an executive and a congress member are correlated, because they care about the overall representation of their party in both branches of government. The voter therefore rewards politicians jointly rather than separately.

If the politicians belong to the same political party (\( \phi = S \)), then the voter uses a coattail voting rule under which the reelection of politician \( i \) is positively correlated with the performance of politician \( j \) (\( \beta^\phi_i > 0 \)). As a result, the voter evaluates the performance of politicians from the same party as a team and tends to reward incumbents from a well-performing party and punish incumbents from a badly performing party. However, if the politicians belong to different parties (\( \phi = D \)), the voter uses a coattail voting rule under which the reelection of politician \( i \) is negatively correlated with the performance of politician \( j \) (\( \beta^\phi_i < 0 \)). As a result, the voter compares the performance of one politician with that of the other, creating a competitive environment between the parties. In this scenario, the voter tends to reward the incumbent from the better-performing party, while punishing the incumbent from the worse-performing party. In sum, owing to the correlation between the executive’s and congress member’s incentives such that they care about their party’s chances of holding office, the voter is better off adopting party performance evaluation rather than individual performance evaluation.

This leads to two-sided coattail effects. On the one hand, an executive’s good performance props up, while poor performance drags down, a congress member candidate from the same party. A presidential coattail effect then arises. On the other hand, the executive’s own reelection depends on the congress member’s performance, which gives rise to a reverse coattail effect. Indeed, successful performance by a congress member advances the election of his partisan ally for executive office, whereas a congress member’s failures hinder it.

Note that the intensity of the coattail effects depends on the strength of the politicians’ partisan alignment. The more loyal the executive is to her political party (the higher \( \lambda_E \)
is), the more correlated the optimal reward scheme for the congress member is with the executive’s performance (positively if $\phi = S$ or negatively if $\phi = D$). The presidential coattail effect therefore becomes more intense. Analogously, the greater the partisan alignment of the congress member (the higher $\lambda_C$ is), the more correlated the executive’s reelection is with the congress member’s performance. So the reverse coattail effect becomes stronger.

If the politicians care equally about their own reelection chances and their party’s election chances, then the best reward schemes are perfectly correlated: incumbents from the same party are always reelected or dismissed together. In the case of incumbents from different parties, reelection of one implies dismissal of the other.

The less loyal the politicians are to their political parties, the less correlated are their incentives. As a result, the voter adopts less correlated reelection rules in equilibrium, and the coattail effects lessen. If politician $j$ is not at all loyal to his political party ($\lambda_j = 0$), then the optimal rule for reappointing politician $i$ is a simple cutoff rule: politician $i$ is reappointed only if her observed performance exceeds a critical threshold given by the equilibrium effort in this office. That is, the probability of $i$ being reelected to office depends only on $i$’s performance:

$$Pr_i(a_i) = P \{p_i(a_i) \geq a_i^* \}.$$  

Intuitively, when politicians care only about their own reelection prospects, the voter is better off rewarding politicians’ individual performances rather than the party’s performance. So the coattail effects vanish. Indeed, if an executive cares only about her own reelection ($\lambda_E = 0$), then her performance does not affect a congress member’s reelection chances, and the presidential coattail effect disappears. In turn, the performance of a nonpartisan congress member ($\lambda_C = 0$) has no impact on an executive’s reelection. So a reverse coattail effect does not arise if the incumbent congress member cares only about his own reelection prospects.

How do the equilibrium efforts $a_i^*$ in (3.3) depend on the values of the parameters? First, a larger variance $\sigma^2$ of the noise decreases the politicians’ efforts. Intuitively, more randomness in the observed performances $p_E$ and $p_C$ makes the reelection probabilities less sensitive to effort, reducing the politicians’ incentives. Second, if politician $i$’s partisan alignment $\lambda_i$ is strengthened, the equilibrium effort of politician $i$, $a_i^*$, increases while that of politician $j$, $a_j^*$, decreases. Intuitively, the more politician $i$ cares about her ally’s appointment to office $j$, the more incentive she has to perform better. However, this weakens politician $j$’s incentive to exert effort, because his reelection becomes less sensitive to his own effort. Note that partisan executive and congress member exert the same equilibrium effort as politicians from different parties. The reason is that the politicians’ preferences are symmetric between the
states. The voter therefore adopts symmetric strategies, and the politicians exert the same equilibrium effort regardless of the state.

3.3. Equilibrium Election Probabilities

In this subsection I calculate the equilibrium probabilities of election of partisan aligned candidates and of election of candidates affiliated with different parties. The probability that candidates from the same party are elected in the state $\phi$ is denoted by $P_{\phi S}$, and the probability that candidates from different parties are elected in the state $\phi$ by $P_{\phi D}$. I establish the following result.

**Proposition 3.** The equilibrium probability of election of partisan aligned candidates is given by

$$P_{\phi S} = \frac{1}{2} + \frac{1}{\pi} \left( \arctan \lambda_E + \arctan \lambda_C \right), \quad \phi \in \{S, D\},$$

where $\arctan(\cdot)$ is the arctangent function. The equilibrium probability of election of candidates affiliated with different parties is given by

$$P_{\phi D} = \frac{1}{2} - \frac{1}{\pi} \left( \arctan \lambda_E + \arctan \lambda_C \right), \quad \phi \in \{S, D\}.$$

Note that, independently of the incumbents’ party labels, the election of partisan aligned candidates is more likely than that of candidates affiliated with different political parties. Indeed, the probability of election of candidates from different parties is never greater than $\frac{1}{2}$: $P_{\phi D} \in [0, \frac{1}{2}]$. Intuitively, if the politicians belong to the same party ($\phi = S$), the voter adopts a coattail voting rule under which the incumbents’ reelection outcomes are positively correlated: good performance by one incumbent tends to prop up, while poor performance drags down, her incumbent partisan ally in the other election. As a result, the incumbents are more likely to be reelected together or dismissed together than they are to receive opposite rewards. Thus, partisan aligned candidates are more likely to be elected in both branches of government. If the incumbents are members of different parties ($\phi = D$), then the voter uses a coattail voting rule under which their reelection outcomes are negatively correlated: good performance by one incumbent increases, while poor performance decreases, the opponent’s chances of winning in the other election. Thus, it is more likely that one incumbent will be dismissed while the other is reelected, and, again, partisan aligned candidates are more likely to be elected in both branches of government. To confirm this intuition, the politicians’ reelection outcomes under equilibrium rules $\beta_E^\phi$ and $\beta_C^\phi$ in the two-dimensional space of performances $p_E$ and $p_C$ are depicted in Figure 3. The density function of the joint distribution of $p_E$ and $p_C$ is symmetric around $(a^{\phi}_E, a^{\phi}_C)$. 

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The probability of election of politicians from the same party, $P_{S \phi S}$, increases with $\lambda_E$ and $\lambda_C$. This probability takes its maximum value of 1 when $\lambda_E = \lambda_C = 1$, and its minimum value of $\frac{1}{2}$ when $\lambda_E = \lambda_C = 0$. Intuitively, the more aligned politicians are with their parties, the more correlated (positively if $\phi = S$ or negatively if $\phi = D$) the optimal performance evaluation rules are. The election of partisan aligned candidates is more probable in both states, as explained above, so stronger party alignment just increases the probability of this outcome. Note, furthermore, that this probability does not vary between states, owing to the symmetry of the politicians' preferences (which in turn implies symmetry of the performance evaluation rules that the voter adopts in equilibrium).

### 3.4. Discussion

The results above show that coattail voting is in fact a tool that the voter uses to discipline partisan politicians. The model generates both presidential and reverse coattail effects. Since an executive wants her party to be represented in the legislature, in order to give her an extra incentive the voter adopts a coattail voting rule such that the executive’s good performance promotes the election of her partisan ally in the congressional election. As a result, a presidential coattail effect arises. In turn, the congress member’s partisan preferences lead to a reverse coattail effect. The voter rewards or punishes the executive for the congress member’s performance in order to incentivize the latter, who wants to increase his party’s chances of winning presidential office.

I must stress that, in the model, coattail effects arise only if politicians are aligned with their political parties (in the sense that they want their party to win in both branches of government). So it is important to examine the assumption of partisan preferences of politicians at different levels of government. The literature on the allocation of intergovernmental transfers provides support for this assumption. According to Cox and McCubbins (1986), incumbents may use intergovernmental transfers to increase their reelection probability by allocating funds to districts with their supporters. An alignment between the two levels of government thus increases the amount of transfers because the central government favors its partisan allies and penalizes its partisan rivals. The recent empirical literature provides evidence in favor of this hypothesis (Arulampalam et al. 2009 (India), Brollo and Nannicini 2010 (Brazil), Larcinese et al. 2006 (United States), Rozovitch and Weiss 1993 (Israel), Solé-Ollé and Sorribas-Navarro 2008 (Spain), and Veiga and Pinho 2007 (Portugal)). Therefore the assumption of partisan preferences of politicians is more than reasonable. On the one hand, lower-tier politicians prefer their partisan ally to take executive office in the hope of receiving
more generous transfers. On the other hand, an executive allocates more funds to districts governed by her partisan allies exactly because she wants lower-tier politicians to be aligned with the central government. And the executive wants this "either because local politicians are important opinion leaders and they may turn to be useful allies in the next presidential campaign, or because they may engage in rent-seeking activities for the President."  

4. Conclusion

This paper has studied coattail effects in simultaneous presidential and congressional elections. In a political agency model with moral hazard, coattail voting is an additional tool that voters use to motivate politicians’ efforts.

The politicians’ incentives are assumed to be correlated, as an executive/congress member prefers her counterpart (the congress member/executive) to be affiliated with the same political party. A representative voter is therefore better off adopting a joint performance evaluation rule rather than an individual performance evaluation rule when deciding whether to reward the incumbents. Under a joint rule, I have shown that the reelection outcomes for politicians from the same party will be positively correlated and the reelection outcomes for politicians from different parties will be negatively correlated. Two-sided coattail effects therefore result. On the one hand, a presidential coattail effect arises, as the executive’s success/failure props up/drags down a candidate from the same party in the congressional election. On the other hand, the executive’s reelection itself depends on the congress member’s performance, which implies a reverse coattail effect.

I have focused on single-task policies. However, in reality, public policies pursue many goals. So it would be of interest to study coattail voting under the more realistic assumption of a multiple-task policy, where the problem of effort allocation among tasks can create policy trade-offs. One can also add an adverse selection problem by assuming that a politician’s performance is determined both by effort and by her privately known ability. These tasks are left for future research.

\[^{20}\text{Brollo and Nannicini (2010), p. 6.}\]
Appendix

A. Proof of Proposition 1

Under linear performance evaluation rules \((\beta, b_i)\), the probability of \(i\) being reelected to office is

\[
Pr_i(a_i, a_j) = P \left( \varepsilon_i + \beta_i \varepsilon_j \geq b_i - a_i - \beta_i a_j \right) = 1 - F_{\varepsilon_i + \beta_i \varepsilon_j} (b_i - a_i - \beta_i a_j),
\]

where the noises \(\varepsilon_i\) and \(\varepsilon_j\) \((i, j \in \{E, C\}, i \neq j)\) are independent normally distributed random variables, and so, by the convolution formula, \(\varepsilon_i + \beta_i \varepsilon_j \sim N \left( 0, (1 + \beta_i^2) \sigma^2 \right)\). Politician \(i\)'s utility is

\[
\Pi_i (a_i, a_j, \phi) - \frac{a_i^2}{2} = \begin{cases} 
1 - F_{\varepsilon_i + \beta_i \varepsilon_j} (b_i - a_i - \beta_i a_j) + \lambda_i \left( 1 - F_{\varepsilon_j + \beta_j \varepsilon_i} (b_j - a_j - \beta_j a_i) \right) - \frac{a_i^2}{2} & \text{if } \phi = S, \\
1 - F_{\varepsilon_i + \beta_i \varepsilon_j} (b_i - a_i - \beta_i a_j) + \lambda_i F_{\varepsilon_j + \beta_j \varepsilon_i} (b_j - a_j - \beta_j a_i) - \frac{a_i^2}{2} & \text{if } \phi = D.
\end{cases}
\]

The first-order conditions with respect to the actual effort \(a_i\), taking \((\beta, b_i)\) and \((\beta_j, b_j)\) as given, are

\[
\begin{align*}
&f_{\varepsilon_i + \beta_i \varepsilon_j} (b_i - a_i - \beta_i a_j) + \lambda_i \beta_j f_{\varepsilon_j + \beta_j \varepsilon_i} (b_j - a_j - \beta_j a_i) - a_i = 0 & \text{if } \phi = S, \\
&f_{\varepsilon_i + \beta_i \varepsilon_j} (b_i - a_i - \beta_i a_j) - \lambda_i \beta_j f_{\varepsilon_j + \beta_j \varepsilon_i} (b_j - a_j - \beta_j a_i) - a_i = 0 & \text{if } \phi = D.
\end{align*}
\]

The second-order conditions are

\[
\begin{align*}
&-f'_{\varepsilon_i + \beta_i \varepsilon_j} (b_i - a_i - \beta_i a_j) - \lambda_i \beta_j^2 f_{\varepsilon_j + \beta_j \varepsilon_i} (b_j - a_j - \beta_j a_i) - 1 < 0 & \text{if } \phi = S, \\
&-f'_{\varepsilon_i + \beta_i \varepsilon_j} (b_i - a_i - \beta_i a_j) + \lambda_i \beta_j^2 f_{\varepsilon_j + \beta_j \varepsilon_i} (b_j - a_j - \beta_j a_i) - 1 < 0 & \text{if } \phi = D.
\end{align*}
\]

I define the best response functions by \(R_i : [0, \overline{a}] \rightarrow [0, \overline{a}]\) such that

\[
R_i^\phi (a_j) = \arg \max_{a_i \in [0, \overline{a}]} \Pi_i (a_i, a_j, \phi) - \frac{a_i^2}{2}.
\]

The best response functions are then determined implicitly by the first-order conditions

\[
\begin{align*}
&f_{\varepsilon_i + \beta_i \varepsilon_j} (b_i - R_i^\phi (a_j) - \beta_i a_j) + \lambda_i \beta_j f_{\varepsilon_j + \beta_j \varepsilon_i} (b_j - a_j - \beta_j R_i^\phi (a_j)) - R_i^\phi (a_j) = 0 & \text{if } \phi = S, \\
&f_{\varepsilon_i + \beta_i \varepsilon_j} (b_i - R_i^\phi (a_j) - \beta_i a_j) - \lambda_i \beta_j f_{\varepsilon_j + \beta_j \varepsilon_i} (b_j - a_j - \beta_j R_i^\phi (a_j)) - R_i^\phi (a_j) = 0 & \text{if } \phi = D.
\end{align*}
\]

Since \(\Pi_i (a_i, a_j, \phi) - \frac{a_i^2}{2}\) is continuous, \(R_i^\phi (a_j)\) is continuous. Therefore, a composite function \(R_i^\phi \circ R_j^\phi : [0, \overline{a}] \rightarrow [0, \overline{a}]\) (defined as \(R_i^\phi \circ R_j^\phi (a_i) = R_i^\phi \left( R_j^\phi (a_i) \right)\)) is a continuous function.
from $[0, \overline{a}]$ into itself, where $[0, \overline{a}]$ is a nonempty, compact, convex set. Then, by Brouwer’s Fixed Point Theorem, $R^\phi_i \circ R^\phi_j$ has a fixed point; that is, there exists $a^\phi_i \in [0, \overline{a}]$ such that $a^\phi_i = (R^\phi_i \circ R^\phi_j)(a^\phi_j)$. So there exists a profile $(a^\phi_E, a^\phi_C)$ such that $a^\phi_E = R^\phi_E(a^\phi_C)$ and $a^\phi_C = R^\phi_C(a^\phi_E)$. This implies that $(a^\phi_E, a^\phi_C)$ is such that

$$\Pi_i(a^\phi_i, a^\phi_j, \phi) - \frac{a^\phi_i^2}{2} = \max_{a_i \in [0, \overline{a}]} \Pi_i(a_i, a^\phi_j, \phi) - \frac{a_i^2}{2},$$

where $i, j \in \{E, C\}$, $i \neq j$. Thus, $(a^\phi_E, a^\phi_C)$ is an equilibrium in effort strategies if it satisfies the second-order conditions (A.1), which completes the proof.

**B. Proof of Proposition 2**

The voter chooses $(\beta_E, b_E)$ and $(\beta_C, b_C)$ to maximize

$$a^\phi_E + a^\phi_C = \begin{cases} 
(1 + \lambda_C \beta_E) f_{\epsilon E + \beta_E \epsilon C} (b_E - a^\phi_E - \beta_E a^\phi_C) + (1 + \lambda_E \beta_C) f_{\epsilon C + \beta_C \epsilon E} (b_C - a^\phi_C - \beta_C a^\phi_E) & \text{if } \phi = S, \\
(1 - \lambda_C \beta_E) f_{\epsilon E + \beta_E \epsilon C} (b_E - a^\phi_E - \beta_E a^\phi_C) + (1 - \lambda_E \beta_C) f_{\epsilon C + \beta_C \epsilon E} (b_C - a^\phi_C - \beta_C a^\phi_E) & \text{if } \phi = D. 
\end{cases}$$

One can show that the values

$$b_E = a^\phi_E + \beta_E a^\phi_C, \quad b_C = a^\phi_C + \beta_C a^\phi_E$$

maximize $a^\phi_E + a^\phi_C$ in the state $\phi = S$ if $1 + \lambda_j \beta_i \geq 0$ and maximize $a^\phi_E + a^\phi_C$ in the state $\phi = D$ if $1 - \lambda_j \beta_i \geq 0$, $i, j \in \{E, C\}$, $i \neq j$. This yields

$$a^\phi_E + a^\phi_C = \begin{cases} 
\frac{1}{\sqrt{2\pi \sigma}} \left( \frac{1 + \lambda_C \beta_E}{\sqrt{1 + \beta_E^2}} + \frac{1 + \lambda_E \beta_C}{\sqrt{1 + \beta_C^2}} \right) & \text{if } \phi = S, \\
\frac{1}{\sqrt{2\pi \sigma}} \left( \frac{1 - \lambda_C \beta_E}{\sqrt{1 + \beta_E^2}} + \frac{1 - \lambda_E \beta_C}{\sqrt{1 + \beta_C^2}} \right) & \text{if } \phi = D. 
\end{cases}$$

Maximizing $a^\phi_E + a^\phi_C$ with respect to $\beta_E$ and $\beta_C$ yields the slopes of the equilibrium performance evaluation rules

$$\beta^\phi_i = \begin{cases} 
\lambda_j & \text{if } \phi = S, \\
-\lambda_j & \text{if } \phi = D,
\end{cases}$$

where $i, j \in \{E, C\}$, $i \neq j$. Note that in the state $\phi = S$, the condition $1 + \lambda_j \beta_i^S \geq 0$ is satisfied, and in the state $\phi = D$, the condition $1 - \lambda_j \beta_i^D \geq 0$ is satisfied. Therefore the
intercepts of the equilibrium performance evaluation rules are given by

\[ b_i^\phi = \begin{cases} 
  a_i^* + \lambda_j a_j^* & \text{if } \phi = S, \\
  a_i^* - \lambda_j a_j^* & \text{if } \phi = D,
\end{cases} \]

where \( a_i^* = a_i^\phi \left( \beta_E^\phi, b_E^\phi, \beta_C^\phi, b_C^\phi \right) \) is the politicians’ equilibrium efforts, which do not depend on the current state \( \phi \). The rest of the proof is straightforward.

**C. Proof of Proposition 3**

The reelection of an incumbent \( i \) is determined by a random variable \( \varepsilon_i + \beta_i \varepsilon_j \sim N \left( 0, (1 + \beta_i^2) \sigma^2 \right) \), \( i, j \in \{ E, C \}, i \neq j \). The density function of a bivariate normal distribution of random variables \( \varepsilon_E + \beta_E \varepsilon_C \) and \( \varepsilon_C + \beta_C \varepsilon_E \), denoted by \( f_{\varepsilon_E + \beta_E \varepsilon_C, \varepsilon_C + \beta_C \varepsilon_E} (x, y) \), is

\[ f_{\varepsilon_E + \beta_E \varepsilon_C, \varepsilon_C + \beta_C \varepsilon_E} (x, y) = \frac{1}{2\pi \sigma^2 \sqrt{(\beta_E \beta_C - 1)^2}} \exp \left\{ \frac{-(x - y \beta_E)^2 + (y - x \beta_C)^2}{2\sigma^2 (\beta_E \beta_C - 1)^2} \right\}. \]

In the state \( \phi = S \), partisanly aligned candidates are elected either when both incumbents are reappointed or when neither of them is reappointed (so, opponents from the rival party are elected). Denote by \( p_i^* = a_i^* + \varepsilon_i \) the performance of politician \( i \) in equilibrium. The equilibrium election probabilities in the state \( \phi = S \) are given by

\[ P_{SS} = P \left( \{ p_E + \lambda_C p_C^* \geq a_E^* + \lambda_C a_C^* \} \cap \{ p_C + \lambda_E p_E^* \geq a_C^* + \lambda_E a_E^* \} \right) + \\
  P \left( \{ p_E + \lambda_C p_C^* < a_E^* + \lambda_C a_C^* \} \cap \{ p_C + \lambda_E p_E^* < a_C^* + \lambda_E a_E^* \} \right) = \\
  P \left( \{ \varepsilon_E + \lambda_C \varepsilon_C \geq 0 \} \cap \{ \varepsilon_C + \lambda_E \varepsilon_E \geq 0 \} \right) + \\
  P \left( \{ \varepsilon_E + \lambda_C \varepsilon_C < 0 \} \cap \{ \varepsilon_C + \lambda_E \varepsilon_E < 0 \} \right) = \\
  \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f_{\varepsilon_E + \lambda_C \varepsilon_C, \varepsilon_C + \lambda_E \varepsilon_E} (x, y) \, dx \, dy + \int_{0}^{0} \int_{0}^{0} f_{\varepsilon_E + \lambda_C \varepsilon_C, \varepsilon_C + \lambda_E \varepsilon_E} (x, y) \, dx \, dy = \\
  \frac{1}{2} + \frac{1}{\pi} \left( \arctan \lambda_E + \arctan \lambda_C \right), \]

\[ P_{SD} = 1 - P_{SS} = \frac{1}{2} - \frac{1}{\pi} \left( \arctan \lambda_E + \arctan \lambda_C \right). \]

In the state \( \phi = D \), candidates from different parties are elected when both incumbents are reappointed or when neither of them is reappointed. The equilibrium election probabilities
in the state $\phi = D$ are given by

\[
P_{DD} = P\left(\{p_E^* - \lambda_C a_C^* \geq a_E^* - \lambda_C a_C^*\} \cap \{p_C^* - \lambda_E p_E^* \geq a_C^* - \lambda_E a_E^*\}\right) + \\
P\left(\{p_E^* - \lambda_C a_C^* < a_E^* - \lambda_C a_C^*\} \cap \{p_C^* - \lambda_E p_E^* < a_C^* - \lambda_E a_E^*\}\right) = \\
P\left(\{\varepsilon_E - \lambda_C \varepsilon_C \geq 0\} \cap \{\varepsilon_C - \lambda_E \varepsilon_E \geq 0\}\right) + \\
P\left(\{\varepsilon_E - \lambda_C \varepsilon_C < 0\} \cap \{\varepsilon_C - \lambda_E \varepsilon_E < 0\}\right) = \\
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{\varepsilon_E - \lambda_C \varepsilon_C, \varepsilon_C - \lambda_E \varepsilon_E}(x,y) \, dx \, dy + \\
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{\varepsilon_E - \lambda_C \varepsilon_C, \varepsilon_C - \lambda_E \varepsilon_E}(x,y) \, dx \, dy = \\
\frac{1}{2} - \frac{1}{\pi} (\arctan \lambda_E + \arctan \lambda_C),
\]

\[
P_{DS} = 1 - P_{DD} = \frac{1}{2} + \frac{1}{\pi} (\arctan \lambda_E + \arctan \lambda_C),
\]

where $\arctan(\cdot)$ is the arctangent function.

References


Figure 1: Executive $E$’s and congressman $C$’s reelection outcomes under linear rules $(\beta_E, b_E)$ and $(\beta_C, b_C)$ in the two-dimensional space of performances $p_E$ and $p_C$.

Figure 2: Best response functions of politicians $i$ and $j$ for independent reelections (black), positively correlated reelections (red) and negatively correlated reelections (blue) in states $\phi = S$ and $\phi = D$. 
Figure 3: Politicians’ reelection outcomes under equilibrium rules \((\beta^\phi_E, b^\phi_E)\) and \((\beta^\phi_C, b^\phi_C)\) in states \(\phi = S\) and \(\phi = D\).