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Abstract

We consider a repeated moral hazard problem, where both the principal and the wealth-constrained agent are risk-neutral. In each of two periods, the agent can exert unobservable effort, leading to success or failure. Incentives provided in the second period act as carrot and stick for the first period, so that the effort level induced in the second period is higher after a first-period success than after a failure. If renegotiation cannot be prevented, the principal may prefer a project with lower returns; i.e., a project may be "too good" to be financed or, similarly, an agent can be "overqualified."

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1 Introduction

This paper offers a new perspective on dynamic moral hazard problems. Consider a risk-neutral principal, who can hire a risk-neutral but wealth-constrained agent. The agent can exert unobservable effort, which increases the likelihood of success. In the one-shot problem, there is a well-known trade-off between effort incentives and rent extraction, which leads to a downward distorted effort level compared to the first-best solution. We extend the standard model by assuming that there is a second period, in which the principal can make an investment to continue the project and the agent can again exert unobservable effort. It turns out that there are several interesting insights that so far have escaped the literature on repeated moral hazard, which was focused on the case of risk-averse agents.

In particular, if the principal can commit not to renegotiate, the second period incentives can be used to partially circumvent the limited liability constraint. In the second period, the principal induces the agent to choose a particularly high effort level following a first-period success and a particularly low effort level following a first-period failure. The prospect of a higher second-period rent following a first-period success motivates the agent to exert more effort in the first period; i.e., rents in the second period act as reward and punishment for the first period. It should be emphasized that we assume no technological impact of a first-period success or failure on the second-period technology. Nevertheless, an optimal dynamic contract exhibits memory. Hence, if an outsider observed today a principal-agent pair that was successful and another identical pair that was not successful, he would be right to predict that the first pair also is more likely to succeed tomorrow. In other words, a serial correlation across periods, which we sometimes refer to as a "hot hand" effect, is generated endogenously, merely based on incentive considerations.[1]

Just as in the one-shot model, effort levels are distorted and not every project that would be installed in a first-best world will be pursued under moral hazard. It

[1] The term "having a hot hand" originated in basketball and means having a streak of successes that cannot be attributed to normal variation in performance. It seems to spectators that the probability of a success increases after a row of successes, even though the trials in question are independent; see Gilovich, Vallone, and Tversky (1985).

also is still the case that the principal will always prefer a project (or, equivalently, an agent) that yields a larger return in case of success (among otherwise identical projects or agents). Somewhat surprisingly, however, the latter observation is no longer true if renegotiation cannot be ruled out.

The "hot hand" effect implies that a principal would sometimes like to commit to terminate a project following a first-period failure, even though technologically the success probability of the second period is not affected by the first-period outcome. Yet, the threat to terminate may not be credible if renegotiation cannot be prevented. In this case, a new kind of inefficiency occurs, that to the best of our knowledge has not been identified in the repeated moral hazard literature so far: The principal might deliberately choose a project that is commonly known to yield smaller potential returns than another (otherwise identical) project that is also available. Similarly, she might deliberately hire an agent that is commonly known to be less qualified.

The reason that a project might be "too good" to be funded or an agent might be "overqualified" is the fact that the principal cannot resist the temptation to renegotiate if the potential return is too attractive, which is anticipated by the agent, whose incentives to work hard in the first period are dulled. In contrast, a less qualified agent or an agent working on a less attractive project may well be willing to exert more effort in the first period, because he knows that in case of a failure he will not get a second chance. Since the credible threat to terminate the project after a first-period failure improves first-period incentives, there are indeed parameter constellations under which a relatively bad project is funded, while a better project is not.

The literature on repeated moral hazard problems and renegotiation has different strands. Most papers consider repeated versions of the traditional moral hazard setting, where the agent is risk-averse and there is a trade-off between insurance and incentives.[2] In a pioneering paper, Rogerson (1985) considered a two-period moral hazard problem and showed that the optimal second-period incentives depend on the first-period outcome (i.e., the contract exhibits memory), even though

[2]For comprehensive surveys, see Chiappori, Macho, Rey, and Salanié (1994) and Bolton and Dewatripont (2005, ch. 10).

the periods are technologically independent. However, his result is driven by the consumption-smoothing motive of the risk-averse agent,[3] which is absent in our setting.

In moral hazard models with a risk-averse agent, renegotiation is an issue even in the one-shot problem, because after the agent has chosen an effort level, there is no need to expose him to further risk. Fudenberg and Tirole (1990), Ma (1991, 1994) and Matthews (1995, 2001) show that it depends on the details of the renegotiation game (specifically, who makes the renegotiation offer) whether or not effort incentives are reduced.[4] In contrast, in our framework there is scope for renegotiation only if the moral hazard problem is repeated, and the details of the renegotiation game are irrelevant for our results.

Although we consider a repeated moral hazard problem, it is interesting to note that our results are also related to the repeated adverse selection literature.[5] Specifically, in a seminal paper Dewatripont and Maskin (1995) consider a two-period model where the agent has private information about the quality of a project that he submits for funding. Ex ante, the principal would like to terminate bad projects after the first period in order to deter the agent from submitting them ("hard budget constraint"). Yet, at the beginning of the second period she is tempted to refinance them ("soft budget constraint"). The absence of commitment power thus enables bad projects to be funded. However, as has been pointed out by Kornai, Maskin, and Roland (2003, p. 1110), the principal would not finance a bad project if she knew the quality ex ante. In contrast, in our model a bad project may be funded, while a better project may not be funded, even though the quality is common knowledge.

In recent years, there has been a growing interest in repeated moral hazard mod-

[3]Cf. Malcomson and Spinnewyn (1988), Fudenberg, Holmström, and Milgrom (1990), and Rey and Salanié (1990).

[4]See also Hermalin and Katz (1991) and Dewatripont, Legros, and Matthews (2003), who consider observable but unverifiable effort.

[5]The fact that the one-shot moral hazard model with a risk-neutral but wealth-constrained agent has some similarities to the one-shot adverse selection model has already been noted by Laffont and Martimort (2002, p. 147).

els with limited liability to study long-term lender-borrower relationships. Contemporaneous work in this area includes Clementi and Hopenhayn (2006), De Marzo and Fishman (2007a, 2007b), and Biais, Mariotti, Rochet, and Villeneuve (2010).[6] These articles are concerned with the long-run dynamics of firm size and survival rates. It is analyzed how an entrepreneur is best induced to avoid large risks or to reveal private information about the cash flow, and whether the optimal investment and growth pattern can be implemented with standard financial contracts. For reasons of tractability, these complex dynamic models usually assume that the incentive problem of the entrepreneur/firm is a binary choice. In contrast, we study a simple model with only two periods but characterize the optimal sequence of effort levels when effort levels can be adjusted continuously.

The remainder of the paper is organized as follows. In Section 2.1, we introduce the one-shot moral hazard problem with a risk-neutral but wealth-constrained agent, which now is sometimes called "efficiency wage" model.[7] This model serves as a benchmark for the dynamic analysis. We then introduce the two-period model in Section 2.2.[8] In Section 3, we analyze the commitment scenario. In Section 4, it is assumed that renegotiation cannot be ruled out, which may lead to the "too good to be financed" (or "overqualification") effect. Finally, concluding remarks follow in Section 5. All proofs have been relegated to the appendix.

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[6] See also Fong and Li (2009) for a related analysis of relational contracts in an employment context.

[7] See Tirole (1999, p. 745) or Laffont and Martimort (2002, p. 174). Moreover, cf. the traditional efficiency wage literature (Shapiro and Stiglitz, 1984) and the literature on deferred compensation (Lazear, 1981; Akerlof and Katz, 1989), which are related but have a different focus. In related frameworks, Strausz (2006) studies auditing and Lewis and Sappington (2000) explore the role of private information about limited wealth.

[8] Dynamic models with risk-neutral agents, hidden actions, and wealth constraints include also Crémer (1995), Baliga and Sjöström (1998), Che and Yoo (2001), and Schmitz (2005). Yet, they rely on features (private information about productivity, observable yet unverifiable effort, common shocks, and technological relations between the periods, respectively) which are absent in the repeated (pure) moral hazard problem studied here. See also the unknown-quality model of Hirao (1993) and the binary-effort model of Bierbaum (2002), who compare short-term and long-term contracts. In related settings, Winter (2006) and Tamada and Tsai (2007) analyze sequential agency problems.

2 The model

2.1 The one-shot contracting problem

As a useful benchmark, let us first take a brief look at the one-shot moral-hazard problem that will be repeated twice in our full-fledged model. There are two parties, a principal and an agent, both of whom are risk-neutral. The agent has no resources of his own, so that all payments to the agent have to be nonnegative. The parties' reservation utilities are assumed to be zero. At some initial date 0, the principal can decide whether or not to pursue a project. If she installs the project, she offers a contract to the agent. Having accepted the contract, the agent exerts unobservable effort $e \in [0,1]$ at date 1. His disutility from exerting effort is given by $c(e)$. Finally, at date 2, either a success ($y = 1$) or a failure ($y = 0$) is realized, where the probability of success is normalized to equal the effort level, i.e. $\Pr\{y = 1|e\} = e$. The principal's verifiable return is given by yR.

Assumption 1. The effort cost function satisfies

a) $c' \geq 0$, $c'' \geq 0$, $c''' \geq 0$, and $c''(e) > 0$ for all $e > 0$,

b) $c(0) = 0$, $c'(0) = 0$, and $c'(1) \geq R$.

The first-best effort level e^{FB} maximizes the expected total surplus

$$S(e) := eR - c(e) \tag{1}$$

and is thus characterized by

$$S'(e^{FB}) = R - c'(e^{FB}) = 0. \tag{2}$$

The principal could attain the first-best effort level, but in order to do so she would have to leave all of her returns to the agent. Hence, the principal faces a trade-off between increasing the pie and getting a larger share for herself. In the second-best solution, the principal will not pay anything when no revenue is generated.[9] If t denotes the principal's transfer payment to the agent in case of

[9]This is a standard result. See e.g. Bolton and Dewatripont (2005, Section 4.1.2) for a simple textbook exposition of the one-shot moral hazard model with risk-neutrality and resource constraints. See also Innes (1990), Pitchford (1998), or Tirole (2001) for variants of this model.

success, the agent's expected payoff from exerting effort e is $et - c(e)$. If $t \leq R$, which will hold in the principal's optimal contract,[10] the agent's maximization problem has an interior solution characterized by $t = c'(e)$. Because of this one-to-one relationship between transfers set by the principal and the resulting effort levels, we can proceed as if the principal could directly set the effort level, and write the principal's problem in terms of effort levels. The principal thus maximizes her expected profit

$$P(e) := e(R - c'(e)), \tag{3}$$

hence the first-order condition that characterizes the second-best effort level e^{SB} is

$$P'(e^{SB}) = R - c'(e^{SB}) - e^{SB}c''(e^{SB}) = 0. \tag{4}$$

Our assumptions on the cost function guarantee that the function P is concave. We also define

$$A(e) := ec'(e) - c(e), \tag{5}$$

the agent's rent from a contract that leads him to choose effort e. By calculating the derivative $A'(e) = ec''(e)$ we see that A is a strictly increasing, convex, and nonnegative function. Hence, a higher implemented effort level yields higher rents for the agent. In order to reduce the agent's rent, the principal introduces a downward distortion of the induced effort level, $e^{SB} < e^{FB}$.

In the one-shot problem, the principal is willing to install the project whenever the installment cost is lower than $P(e^{SB})$, which is smaller than $S(e^{FB})$; i.e., not all projects that would be pursued in a first-best world will actually be installed. However, given the choice between two (otherwise identical) projects with possible returns R_g and $R_b < R_g$, the principal will never prefer the bad project that can yield R_b only.

2.2 The two-period model

Now we turn to the full-fledged two-period model. For simplicity, we neglect discounting. At date 0, the principal decides whether or not to install the project. To

[10] Note that offering a payment t larger than R would violate the principal's participation constraint.

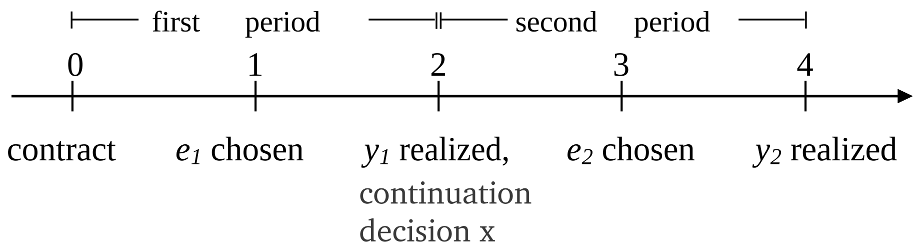

Figure 1: The sequence of events.

simplify the exposition, we assume that there are no installment costs at this date.[11] The principal makes a take-it-or-leave-it contract offer to the agent. Having accepted the offer, at date 1 the agent chooses an unobservable first-period effort level $e_1 \in [0, 1]$, incurring disutility $c(e_1)$. At date 2, the verifiable first-period return $y_1 R$ is realized, where $y_1 \in \{0, 1\}$ denotes failure or success, and $\Pr\{y_1 = 1|e_1\} = e_1$. The project may then be terminated ($x(y_1) = 0$) or continued ($x(y_1) = 1$), which is verifiable.[12] In order to continue the project, the principal must invest an amount $I_2 \leq S(e^{FB})$. In this case, at date 3 the agent chooses an unobservable second-period effort level $e_2(y_1) \in [0, 1]$. Finally, at date 4 the verifiable second-period return $y_2 R$ is realized, where $y_2 \in \{0, 1\}$ and $\Pr\{y_2 = 1|e_2(y_1)\} = e_2(y_1)$. Note that the two periods are independent; in particular, we do not assume any technological spillovers that would make a second-period success more likely after a first-period success. The sequence of events is illustrated in Figure 1.

The first-best benchmark solution. Assume for a moment that effort were verifiable. The principal would then continue the project regardless of the first-period outcome ($x(0) = x(1) = 1$), and she would implement the effort levels

[11] We thank an anonymous referee for suggesting this simplification. It is straightforward to extend the model to the case in which the principal incurs costs $I_1 > 0$ when she installs the project.

[12] We assume that it is too costly for the principal to replace the agent at date 2, because at that point in time the parties are "locked-in" (i.e., the relationship has undergone Williamson's (1985) "fundamental transformation"). For instance, hiring a new agent for the ongoing project might require specific training, which makes replacement unprofitable. See Spear and Wang (2005), Mylovanov and Schmitz (2008), and Kräkel and Schöttner (2010) for models in which replacement involves no costs. Our model could be extended to the case of costly replacement, but this would make the exposition less tractable without yielding additional economic insights.

$e_1 = e_2(0) = e_2(1) = e^{FB}$ with a straightforward forcing contract, leaving no rent to the agent.

Contracts when effort is unobservable. In the remainder of the paper, we assume again that effort levels are unobservable. We do not impose any ad hoc restrictions on the class of feasible contracts; i.e., there is complete contracting in the sense of Tirole (1999).

A contract specifies a continuation decision (which may be conditioned on the first period outcome) and transfer payments from the principal to the agent (which may be conditioned on the continuation decision and the first and second period outcomes). The transfer payments have to satisfy the limited liability constraint of the agent. The principal can also include recommended effort levels in the contract. The contractual terms must be such that it is in the agent's own self-interest to obey the recommendations (cf. Myerson, 1982); i.e., the recommendations must satisfy suitable incentive compatibility constraints.

Thus, a contract specifies for the possible first-period outcomes $y_1 \in \{0, 1\}$ the probability of continuation $x(y_1)$, the first-period transfer payments $t_1(y_1)$ to be made at date 2, and the second-period transfer payments $t_2(y_1, y_2)$ to be made at date 4 in case of continuation.[13] The limited liability constraints are given by

$$t_1(y_1) \geq 0 \tag{6}$$

for the first period and by

$$t_2(y_1, y_2) \geq 0 \tag{7}$$

for the second period. Note that the latter condition presupposes that the agent cannot be forced to pay back payments that he received in the past.[14] Finally,

[13] While it may well be optimal to randomize between continuation and termination, other kinds of randomization cannot occur. Stochastic transfer payments can always be replaced by their expected value, because both principal and agent are risk-neutral. This also includes transfer payments that depend on the randomization device that pins down the continuation decision. Moreover, it is straightforward to show that an optimal contract will never induce randomization over effort levels.

[14] Otherwise, the limited liability constraint would read $t_2(y_1, y_2) \geq -t_1(y_1)$. It turns out that our results would not change if we relaxed the limited liability constraint in this way. In fact, it would be without loss of generality to assume that all payments are made at date 4 only.

the contract specifies recommended effort levels e_1, $e_2(0)$, and $e_2(1)$. The incentive compatibility constraints for the second period are

$$e_2(y_1) \in \arg\max_{e \in [0,1]} e t_2(y_1, 1) + (1-e) t_2(y_1, 0) - c(e). \tag{8}$$

We denote the continuation payoff of the agent once the first period outcome is realized by

$$a(y_1) = t_1(y_1) + x(y_1)\Big[e_2(y_1) t_2(y_1, 1) + (1 - e_2(y_1)) t_2(y_1, 0) - c(e_2(y_1))\Big]. \tag{9}$$

The first-period incentive compatibility constraint is then given by

$$e_1 \in \arg\max_{e \in [0,1]} e a(1) + (1-e) a(0) - c(e). \tag{10}$$

We now show that the class of contracts that we need to consider can be simplified. In particular, we show that because only the difference between $t_2(y_2, 1)$ and $t_2(y_2, 0)$ matters for the agent's effort choice in the second period, contracts that reward a failure in the second period ($t_2(y_1, 0) > 0$) can be replaced by contracts that specify suitably larger payments at date 2. For any given transfer scheme (t_1, t_2) we define

$$\begin{aligned}
\tilde{t}_1(y_1) &= t_1(y_1) + t_2(y_1, 0) x(y_1), \\
\tilde{t}_2(y_1, 0) &= 0, \text{ and} \\
\tilde{t}_2(y_1, 1) &= \max\{t_2(y_1, 1) - t_2(y_1, 0), 0\}.
\end{aligned}$$

It is straightforward to check that the payments $(\tilde{t}_1, \tilde{t}_2)$ induce the same second period effort levels as (t_1, t_2), the same continuation payoffs $a(1)$ and $a(0)$, and therefore also the same first period effort levels. Moreover, they fulfill the limited liability requirements, and they lead to the same expected payoffs.[15] It is thus without loss of generality to restrict attention to a set C of contracts for the principal's optimization problem, where elements $\kappa \in C$ are given by $\kappa = (t_1, t_2, x, e_1, e_2)$ with

- $x : \{0, 1\} \to [0, 1]$,

[15] Note that a contract that satisfies only the weaker limited liability constraint $t_2(y_1, y_2) \geq -t_1(y_1)$ can be replaced by the scheme $(\tilde{t}_1, \tilde{t}_2)$ that consists of nonnegative payments only.

- $t_1 : \{0,1\} \to \mathbb{R}_{\geq 0}$, $t_2 : \{0,1\}^2 \to \mathbb{R}_{\geq 0}$, $t_2(y_1, 0) = 0$,
- $e_2 : \{0,1\} \to [0,1]$ with $e_2(y_1) \in \arg\max_{e\in[0,1]} et_2(y_1,1) - c(e)$, and
- $e_1 \in \arg\max_{e\in[0,1]} ea(1) + (1-e)a(0) - c(e)$.

Since the agent can always choose not to exert any effort at all, the limited liability constraint together with the incentive compatibility constraint ensures participation. Hence, all contracts in the set C satisfy the incentive compatibility and limited liability constraints and are accepted by the agent. If the principal offers a contract $\kappa = (t_1, t_2, x, e_1, e_2) \in C$, her expected profit is given by

$$\begin{aligned} \Pi(\kappa) \quad = \quad & e_1\Big(R - t_1(1) + x(1)\Big[e_2(1)(R - t_2(1,1)) - I_2\Big]\Big) \\ & + (1-e_1)\Big(-t_1(0) + x(0)\Big[e_2(0)(R - t_2(0,1)) - I_2\Big]\Big). \end{aligned} \tag{11}$$

In the solution of the optimization problem it will turn out that $t_1(0) = 0$; i.e., an agent will never be rewarded for a failure. A first-period success may be directly rewarded with a bonus payment $t_1(1)$, while a second-period success may be rewarded with a bonus $t_2(0,1)$ (following a first-period failure) or $t_2(1,1)$ (following a first-period success). As we will see, a first-period success will also be indirectly rewarded by the prospect of getting a larger bonus for a second-period success if it follows a first-period success, which will be a driving force behind our main results.

3 The full commitment case

In this section, we assume that the principal can commit not to renegotiate the contract that is written at date 0. In order to solve the full-fledged two-period model we first solve the one-period problem of finding the optimal continuation contract that leaves the agent with a certain payoff. While also being of independent interest, this result is then used to find the optimal continuation payoffs in the two-period problem. We denote by $\pi(a)$ the principal's maximum continuation payoff when she implements the expected second-period payoff a of the agent. Recall that the principal can implement any second-period effort level e_2 by setting $t_2(y_1, 1) = c'(e_2)$, sharing the second-period surplus $S(e_2) - I_2$ such that the agent gets $A(e_2)$

and the principal gets $P(e_2) - I_2$. In order to characterize the function π, we have to find the continuation contract (t_1, x, t_2, e_2) with $t_2 = c'(e_2)$ that maximizes the principal's payoff among those that implement a given expected payoff a of the agent. Before we can state the result, we need the following lemma and definition:

Lemma 1. *If $I_2 > 0$, then there is a unique effort level $\bar{e} > 0$ with*

$$S(\bar{e}) - I_2 = \frac{S'(\bar{e})}{A'(\bar{e})} A(\bar{e}). \tag{12}$$

If we define $\bar{e} = 0$ in case $I_2 = 0$, then the cut-off level $\bar{e}$ is a continuous and increasing function of I_2, with $\bar{e} = e^{SB}$ at $I_2 = P(e^{SB})$ and $\bar{e} = e^{FB}$ at $I_2 = S(e^{FB})$.

Proof. See the appendix. □

Because the right hand side of (12) is nonnegative, the net present value of a project with effort level $\bar{e}$ is also never negative. The so defined effort level $\bar{e}$ plays a role in implementing relatively low payoffs of the agent.

Lemma 2. *The following table shows the continuation contract that optimally implements a given continuation payoff a of the agent, and the resulting continuation payoff $\pi(a)$ of the principal:*

	t_1	e_2	x	$\pi(a)$
if $0 \leq a \leq A(\bar{e})$	0	$\bar{e}$	$\frac{a}{A(\bar{e})}$	$x(P(\bar{e}) - I_2)$
if $A(\bar{e}) < a < A(e^{FB})$	0	$A^{-1}(a)$	1	$P(e_2) - I_2$
if $A(e^{FB}) \leq a$	$a - A(e^{FB})$	e^{FB}	1	$S(e^{FB}) - I_2 - a$

The function $\pi(a)$ is concave and has the derivative $\pi'(a) = \frac{P'(e_2)}{A'(e_2)}$.

Proof. See the appendix. □

It becomes clear from the lemma that only projects with positive net present value and effort level equal to or greater than $\bar{e}$ will be implemented. Moreover, we see that as the agent's payoff a increases, the expected total surplus induced by the principal's optimal continuation contract weakly rises.

If the agent's payoff a is larger than $A(e^{FB}) = S(e^{FB})$, then the principal will implement $e_2 = e^{FB}$ and transfer the residuum $a - A(e^{FB})$ to the agent by making a positive payment t_1. Otherwise, there will be no such payment, since implementing a project with positive net present value is a better method to reward the agent than a direct transfer.

To see why a positive probability of termination is sometimes optimal for the principal, consider the case that I_2 is lower than $P(e^{SB})$, so that there exist effort levels that lead to a positive continuation payoff, while the required payoff a is so low that a project with effort level $e_2 = A^{-1}(a)$ would lead to a negative continuation payoff $P(e_2) - I_2 < 0$. In such a case, it is more profitable for the principal to implement a higher effort level with a positive payoff for herself and achieve the required a by adjusting the continuation probability x. The effort level $\bar{e}$ is the result of a trade-off between a larger continuation payoff $P(e) - I_2$ (which increases with e up to e^{SB}) and a lower probability of achieving this payoff ($x = \frac{a}{A(e)}$ decreases with e).

There is another case in which a positive probability of termination is optimal: Assume that I_2 is larger than $P(e^{SB})$, so that the principal's continuation payoff is negative for all effort levels, and a is so low that a project with effort level $e_2 = A^{-1}(a)$ would have a negative net present value. It is then more profitable for the principal to implement a higher effort level and scale the project down to achieve the required continuation payoff a. In this case, the implemented effort level $\bar{e}$ is larger than e^{SB}.

The following proposition characterizes the second-best solution of the two-period model under full commitment.

Proposition 1. *Assume that the principal can commit not to renegotiate. In the principal's optimal contract, the project is either always continued with some probability and the induced effort levels satisfy*

$$e^{FB} \geq e_2^C(1) > e_1^C > e^{SB} > e_2^C(0) > 0,$$

or the project is terminated after a failure and the effort levels satisfy

$$e^{FB} \geq e_2^T(1) > e_1^T \geq e^{SB}.$$

Proof. See the appendix. □

This proposition establishes the "hot hand" effect. Even though a success in the first period has no technological effect whatsoever on the likelihood of a success in the second period, the principal implements $e_2^C(1) > e^{SB} > e_2^C(0)$. Giving the agent in the second period particularly high incentives following a first-period success (and particularly low incentives following a failure) has desirable spillover effects on the first-period incentives: The agent works hard in the first period not only in order to get the direct reward $t_1(1)$, but also in order to enjoy a higher second-period rent. In fact, the direct first-period reward $t_1(1)$ will be positive only if the principal already induces $e_2^C(1) = e^{FB}$, so that implementing an even higher effort level following a first-period success would reduce the total surplus. Since giving the agent incentives in the first period is now cheaper than in the one-shot problem, the principal implements $e_1 > e^{SB}$.

In the next proposition, we explore the dependence of the optimal continuation decisions on the installment cost.

Proposition 2. *There exist cut-off levels I^C,I^T, and I^{TT}, where*

$$0 < I^C \leq I^T < P(e^{SB}) < I^{TT} \leq S(e^{FB}),$$

such that
a) if $I_2 \leq I^C$, then the project is always continued, $x(1) = x(0) = 1$.
b) if $I^T \geq I_2 > I^C$, then $x(1) = 1$ while $x(0) < 1$, i.e., the optimal contract leads with positive probability to termination after a failure.
c) if $I_2 > I^T$, then the project is terminated whenever the first period was a failure, $x(0) = 0$, and it is continued with $x(1) = 1$ after a success for $I_2 \leq I^{TT}$, and with some probability $x(1) \in (0, 1)$ for $I_2 > I^{TT}$.

Proof. See the appendix. □

While for low installment costs it is always beneficial for the principal to continue the project unconditionally, continuing the project after a first-period failure might not be in the principal's interest when her continuation costs I_2 are sufficiently large. Clearly, if I_2 is so large that $P(e_2^C(0)) < I_2$, the principal is worse off if she continues the project. Even if this inequality does not hold, it can still be optimal for the principal to commit to terminate the project at least with some probability, because doing so improves the agent's first-period incentives. As I_2 becomes large, it may also become optimal to terminate the project with a positive probability after a first-period success. To see why such a randomized decision $x(1) < 1$ may be beneficial for the principal, consider the case that I_2 is close to $S(e^{FB})$. Since the principal never installs a project with negative net present value, she will implement a very large effort level $e_2(1)$ close to e^{FB}. To implement such a large effort level she has to leave almost all of the second-period return to the agent while she bears the installment costs I_2. She will therefore scale the project down except in the case that the effect of the agent's large continuation payoff on the first-period effort level offsets the cost of setting $x(1) = 1$. This case occurs in the following example of a quadratic cost function, which shows that randomization does not have to occur in an optimal contract.

Lemma 3. *If the cost function is quadratic ($c(e) = \alpha e^2$), then in the optimal contract it is always true that $x(y_1) \in \{0, 1\}$ for $y_1 \in \{0, 1\}$.*

Proof. See the appendix. □

In the one-shot interaction, the most severe punishment available to the principal is not to pay anything to the agent. If a two-period contract can be signed, stronger incentives can be provided. The optimal contract displays memory; i.e., it does not coincide with contracts that ignore the information about the first period outcome. As it is beneficial for the principal to make use of the two-period structure, she will introduce certain "milestones" ($y_1 = 1$) that should be achieved by the agent, whenever this is possible.[16]

The inefficiencies exhibited by the second-best solution are of a similar nature as the inefficiencies we encountered in the one-shot model. There are downward

[16] See also Gershkov and Perry (2009), who address the value of midterm reviews for a tournament designer. A paper that takes this idea to the extreme is Che and Sakovicz (2004), in which a hold-up problem can be fully overcome in the limit if the parties monitor each other's investment more and more frequently and can base their behavior in the negotiations on the investment observed so far.

distortions of the effort levels compared with the first-best solution, and as a result there are projects that would be installed (and continued) in a first-best world, but that are not pursued (or at least not continued after a first-period failure) in the presence of moral hazard. However, it is still impossible for an investment opportunity to be "too good" to be pursued, as is stated in the following corollary.

Corollary 1. *Assume that the principal can commit not to renegotiate. If at date 0 the principal can choose between two (otherwise identical) projects with possible returns R_g and $R_b < R_g$, she will never prefer the bad project that can yield R_b only.*

Proof. See the appendix. □

4 Renegotiation and the "overqualification" effect

After the first period is over, the principal might want to modify the contractual arrangements, because at that point in time she would be best off under the optimal one-period contract as characterized in Section 2.1. In the following we assume that the principal cannot ex ante commit not to renegotiate the contract.[17] In our complete contracting framework, the principal can mimic the outcome of renegotiations in her original contract; i.e., we can confine our attention to renegotiation-proof contracts.[18]

Proposition 3. *Assume that the principal cannot commit not to renegotiate.*

a) If $P(e^{SB}) > I_2$, then the project is always continued, $x(0) = x(1) = 1$. The effort levels satisfy

$$e^{FB} \geq \bar{e}_2^C(1) > \bar{e}_1^C > \bar{e}_2^C(0) = e^{SB}.$$

[17] See Bolton and Dewatripont (2005) for extensive discussions of the assumption that renegotiation cannot be ruled out. See also Wang (2000) and Zhao (2006), who study renegotiation problems in more general frameworks.

[18] Note that, in particular, this means it is inconsequential how the renegotiation surplus would be split at date 2. The principal can achieve the same outcome that would be attained if she had all bargaining power in the renegotiation game by designing the appropriate renegotiation-proof contract at the outset.

b) If $P(e^{SB}) \leq I_2$, then the project is terminated whenever the first period was a failure, $x(0) = 0$, and the contract is the same as under full commitment.

Proof. See the appendix. $\square$

As we have seen in the previous section, if the project was continued under full commitment, the principal implemented a second-period effort level smaller than e^{SB} when the first period was a failure. The resulting smaller second-period rent acted as an indirect punishment of the wealth-constrained agent for the first-period failure. This is no longer possible if renegotiation cannot be ruled out, because at date 2 the principal would prefer to implement e^{SB} in order to maximize her second-period profit. While thus the "stick" is no longer available, the principal can still make use of the "carrot;" i.e., she can indirectly reward first-period effort by implementing an effort level larger than e^{SB} following a first-period success.[19] As a result, it is still cheaper for the principal to motivate the agent to exert first-period effort in the two-period model than in the one-shot benchmark model, so that $\bar{e}_1^C > e^{SB}$.

Just as in the full commitment regime, for sufficiently large investment costs I_2, the principal would be better off if she terminated the project whenever the first-period was a failure. However, if renegotiation cannot be ruled out, at date 2 the principal prefers to continue the project as long as she can make a positive second-period profit by doing so. Her threat to terminate the project after a first-period failure is no longer credible, unless her expected second-period profit in case of continuation would actually be negative.

In other words, the principal would like to commit to termination following a first-period failure, but she cannot do so. This observation has peculiar implications with regard to the project that the principal will choose at the outset, as is highlighted in Corollary 3 below. A new kind of inefficiency occurs, which we saw

[19] Note that the principal would like to reduce her promised payment $t_2(1,1)$ after a first-period success has occurred (in order to implement e^{SB} in the second period), but in this case there is no scope for mutually beneficial renegotiation. The agent would insist on the original contract, which gives him a larger rent.

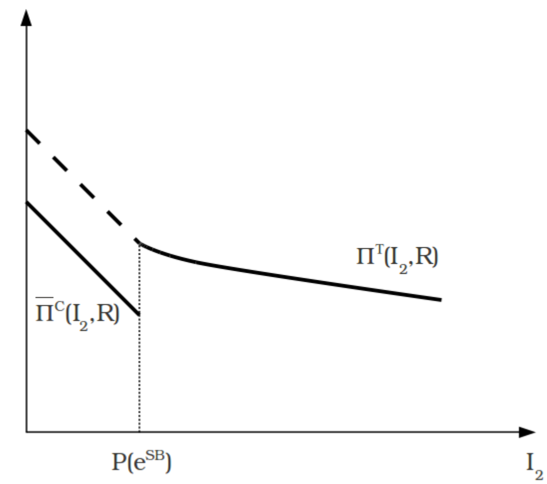

Figure 2:

This figure shows the jump in the principal's maximal payoff at $I_2 = P(e^{SB})$, where the termination contract becomes feasible. The dashed line shows the payoff with commitment.

neither in the well-known one-shot problem nor in the two-period model with full commitment.

Corollary 2. *Assume that the principal cannot commit not to renegotiate. For $I_2 < P(e^{SB})$ the principal's expected profit, denoted by $\bar{\Pi}^C(I_2, R)$, is decreasing in I_2. For $I_2 \geq P(e^{SB})$ it is denoted by $\Pi^T(I_2, R)$ and again decreasing in I_2. At $I_2 = P(e^{SB})$ there is an upward jump, which is bounded from below by $e^{SB}A(e^{SB})$, as illustrated in Figure 2.*

Proof. See the appendix. $\square$

Corollary 2 says that the principal can be better off if her continuation costs I_2 are increased, which may be surprising at first sight. Yet, this result follows immediately from the fact that the optimal contract with commitment is renegotiation-proof for $I_2 \geq P(e^{SB})$, while for smaller investment costs renegotiation-proofness is a binding constraint. Hence, the principal's expected profit makes an upward jump at $I_2 = P(e^{SB})$. This effect can be so strong that she would even prefer to have higher investment costs in both periods, or similarly, she would prefer to install a project that can only yield a smaller revenue R.

Corollary 3. *Assume that the principal cannot commit not to renegotiate. If at date 0 the principal can choose between two (otherwise identical) projects with possible returns R_g and $R_b < R_g$, she may prefer the bad project that can yield R_b only.*

Proof. See the appendix. $\square$

For example, let $c(e) = \frac{1}{2}e^2$, $I_2 = 0.12$, $R_b = 0.68$, and $R_g = 0.7$. It is straightforward to show that the principal's expected profit is $\Pi \approx 0.147$ if she installs the "good" project that can yield R_g, while it is $\Pi \approx 0.157$ if she installs the "bad" project that can yield R_b only (and is otherwise identical). Note that if there is a first-period installment cost $I_1 = 0.15$, this even means that while the principal would be willing to install the "bad" project, the "good" project would never be funded.

Intuitively, pursuing a bad project that can yield a relatively small return (or, similarly, hiring a less qualified agent who can generate only a small return or who requires higher investments by the principal) acts as a commitment device. The principal knows that if she chooses the more attractive alternative, then at date 2 she cannot resist the temptation to continue after a first-period failure. For this reason, a project can be just "too good" to be funded or an "overqualified" agent may not be hired.[20]

[20] Lewis and Sappington (1993) have also pointed out that employers will sometimes not hire applicants who are "overqualified," even when their salary expectations are modest. However, their model is quite different from ours; they consider an adverse selection problem with countervailing incentives due to type-dependent reservation utilities. Note that in our model a more productive agent might not be hired even if his reservation utility is not higher than the one of a less qualified agent. Similarly, Axelson and Bond (2010) also report a "talent scorned" effect in a model that is similar to ours. However, they endogenize the agent's outside option in the model, and the result that less qualified agents can be preferred is due to the fact that they have lower outside options.

5 Concluding remarks

In this paper, we have extended the literature on repeated moral hazard problems to cover hidden action models in which the agent is risk-neutral but wealth-constrained. We have compared the induced effort levels across periods and states. It has turned out that the optimal contract exhibits memory, even though the periods are technologically independent. Moreover, we have identified a novel kind of potential inefficiency that has escaped the previous literature.

The present contribution seems to be sufficiently simple to be used as a building block in more applied work. As has been pointed out in the introduction, our model shares some features with dynamic adverse selection models. It might thus be applied in fields which previously have been studied from the perspective of the literature on precontractual private information and soft budget constraints. Specifically, applications of our model could help to explain the funding of inferior projects (e.g., in the context of development aid), even if the project quality is commonly known. Our model could also be applied in the field of corporate finance, where moral hazard problems with risk-neutral but wealth-constrained agents are ubiquitous (see Tirole, 2005).

It is straightforward to relax several assumptions that were made to keep the exposition as clear as possible. For example, if it is required by an application, one might easily generalize the model by allowing different cost functions and different returns in the two periods. Moreover, one can dispense with the assumption that the principal has all bargaining power. Regardless of the bargaining protocol, the principal would only be willing to participate if her investment costs were covered. Hence, qualitatively our main findings would still be relevant. One could also consider the case in which the agent's wealth or his reservation utility may be positive. As long as the agent is not wealthy enough to "buy the firm," the effects highlighted in our model continue to be relevant.

Appendix

Proof of Lemma 1**.**

We define for $e > 0$ a function

$$F(e) := S(e) - I_2 - \frac{S'(e)A(e)}{A'(e)}, \tag{13}$$

which has the derivative

$$F'(e) = \frac{A(e)}{A'(e)^2}(-S''(e)A'(e) + S'(e)A''(e)). \tag{14}$$

Recall that for effort levels $e \leq e^{FB}$, the function S is increasing and concave, and A is positive, increasing, and strictly convex. Hence, F is strictly increasing for $0 < e \leq e^{FB}$. If $I_2 > 0$, then for sufficiently small effort levels e it holds that $S(e) < I_2$, and therefore $F(e) < 0$, while for $e = e^{FB}$ it holds that $F(e) = S(e^{FB}) - I_2 \geq 0$. Hence, there exists a unique zero $\bar{e} > 0$ of F. It follows immediately that $\bar{e} = e^{FB}$ for $I_2 = S(e^{FB})$. In addition it holds that

$$F(e) = P(e) - I_2 - \frac{S'(e)A(e) - A'(e)A(e)}{A'(e)} = P(e) - I_2 - \frac{P'(e)A(e)}{A'(e)}. \tag{15}$$

This equality also implies

$$P(\bar{e}) - I_2 = \frac{P'(\bar{e})A(\bar{e})}{A'(\bar{e})}, \tag{16}$$

which shows that $\bar{e} = e^{SB}$ if $I_2 = P(e^{SB})$.

Taking the derivative with respect to I_2 on both sides of equation (12) which defines $\bar{e}$ yields

$$\frac{\partial \bar{e}}{\partial I_2} = -\frac{A'^2(\bar{e})}{A(\bar{e})(S''(\bar{e})A'(\bar{e}) - A''(\bar{e})S'(\bar{e}))} > 0. \tag{17}$$

Hence, $\bar{e}$ is increasing in I_2. For $I_2 = 0$ it holds that $\lim_{e \to 0} F(e) = 0$, which implies that $\bar{e}$ approaches 0 as $I_2 \to 0$.

Proof of Lemma 2**.**

The principal solves

$$\begin{aligned} & \max_{t_1, e_2, x} x(P(e_2) - I_2) - t_1 \\ \text{s.t.} \quad & t_1 \geq 0, \\ & t_1 + xA(e_2) = a, \\ & x \in [0, 1]. \end{aligned} \tag{18}$$

We transform this problem by replacing t_1 by $a - xA(e_2)$:

$$\begin{aligned} &\max_{e_2,x} x(S(e_2) - I_2) - a \\ \text{s.t.} \quad & a \geq xA(e_2), \\ & x \in [0,1]. \end{aligned} \tag{19}$$

First, we consider the case $a \geq A(e^{FB})$. In this case, the required payoff of the agent is greater than the possible gross surplus $S(e^{FB}) = A(e^{FB})$ and can therefore only be achieved with a nonnegative transfer $t_1 = a - A(e_2)$. That is, the limited liability constraint cannot be binding, and performing the maximization in (19) without this constraint yields $x = 1$ and $e_2 = e^{FB}$. Since at these values the limited liability constraint is equal to $a \geq A(e^{FB})$, it follows that the limited liability constraint is binding if and only if $a \leq A(e^{FB})$.

For the case $a < A(e^{FB})$ it must therefore be true that $A(e_2)x = a$. If $a = 0$ then it is easy to see that $x = 0$ is optimal, with the effort level remaining unspecified. Since for $a > 0$ it holds that $x > 0$ and $A(e_2) > 0$, the limited liability constraint can be transformed to $x = \frac{a}{A(e_2)}$, and the constraint $x \leq 1$ becomes $A(e_2) \geq a$. Hence, for the case $a < A(e^{FB})$ we get the optimization problem

$$\begin{aligned} &\max_{e_2>0} \frac{S(e_2) - I_2}{A(e_2)} \\ s.t. \quad & A(e_2) \geq a. \end{aligned} \tag{20}$$

The Lagrangian for this problem is $\mathcal{L}(e_2, \lambda) = \frac{S(e_2)-I_2}{A(e_2)} + \lambda(A(e_2) - a)$ with $\lambda \geq 0$. In the optimum it holds that

$$\frac{S'(e_2)A(e_2)}{A'(e_2)} - (S(e_2) - I_2) = -\lambda A(e_2)^2 \tag{21}$$

and we have the complementary slackness condition $\lambda > 0 \Rightarrow A(e_2) = a$. The left-hand side of this equation vanishes at $e_2 = \bar{e}$, and it is shown in the proof of Lemma 1 that it is decreasing in e_2. Hence we either have that

$$\frac{S'(e_2)A(e_2)}{A'(e_2)} - (S(e_2) - I_2) < 0 \tag{22}$$

and $A(e_2) = a$, which is true if and only if $a > A(\bar{e})$, or we have that the effort level $e_2 = \bar{e}$ is implemented and the payoff is fine-tuned by adjusting the continuation probability $x = \frac{a}{A(\bar{e})}$. To summarize, the cheapest way for the principal to implement continuation payoff a is given by

- $x = 1$, $t_1 = a - A(e^{FB})$, $e_2 = e^{FB}$, with $\pi = S(e^{FB}) - I_2 - a$, if $a \geq A(e^{FB})$,
- $x = 1$, $t_1 = 0$, $e_2 = A^{-1}(a)$, with $\pi = P(e_2) - I_2$, if $A(e^{FB}) > a > A(\bar{e})$, and
- $x = a/A(\bar{e})$, $t_1 = 0$, $e_2 = \bar{e}$, with $\pi = x(P(e_2) - I_2)$, if $A(\bar{e}) \geq a \geq 0$.

It remains to show that the function π is continuously differentiable with weakly decreasing derivative $\pi'(a) = \frac{P'(e_2)}{A'(e_2)}$, which then implies concavity of π. For $a > A(e^{FB})$ we have $\pi'(a) = -1 = \frac{P'(e^{FB})}{A'(e^{FB})}$, and for $a < A(\bar{e})$ we have $\pi'(a) = \frac{P'(\bar{e})}{A'(\bar{e})}$. Because π is continuous, this includes $a = 0$ except for $I_2 = 0$. Both expressions are independent of a. For the intermediate case, $A(e^{FB}) > a > A(\bar{e})$, the derivative is $\pi'(a) = \frac{P'(e_2)}{A'(e_2)}$ with $e_2 = A^{-1}(a)$. It has the limits -1 as $a \to A(e^{FB})$ and $\frac{P'(\bar{e})}{A'(\bar{e})}$ as $a \to A(\bar{e})$, which due to continuity of π is sufficient for differentiability at the points $A(\bar{e})$ and $A(e^{FB})$. Moreover, on this interval we have

$$\pi''(a) = \frac{P''(e_2)A'(e_2) - P'(e_2)A''(e_2)}{A'(e_2)^3} = \frac{S''(e_2)A'(e_2) - S'(e_2)A''(e_2)}{A'(e_2)^3} < 0. \quad (23)$$

Proof of Proposition 1.

As shown in Lemma 2, no effort level greater than e^{FB} will be implemented, hence $e_2(1) \leq e^{FB}$. To show how the effort levels compare across periods and states, we have to solve the principal's maximization problem. Recall that $a(1)$ denotes the agent's continuation payoff in case of a success and $a(0)$ the agent's continuation payoff in case of a failure, so that in the first period the agent chooses an effort level $e_1 = \arg\max_e ea(1) + (1-e)a(0) - c(e)$. As described in Lemma 2, the principal can choose any pair of nonnegative continuation payoffs $a(0), a(1)$ and get the payoff $e_1(R + \pi(a(1))) + (1 - e_1)\pi(a(0))$. Because setting $a(1) \leq a(0)$ with $e_1 = 0$ is dominated by repeating the optimal one-period contract,[21] we can omit the constraint $a(1) \geq 0$ and use the first order condition $c'(e_1) = a(1) - a(0)$ to characterize the incentive compatible first-period effort level. Hence, we can state the principal's optimization problem in terms of e_1 and $a(0)$ as

$$\begin{aligned} &\max_{e_1, a(0)} e_1(R + \pi(c'(e_1) + a(0))) + (1 - e_1)\pi(a(0)) \quad (24) \\ s.t. \quad & a(0) \geq 0. \end{aligned}$$

[21] Unconditionally repeating the optimal one-period contract yields $2P(e^{SB}) - I_2$, while $e_1 = 0$ yields $P(e^{SB}) - I_2$ at best.

The Lagrangian for this problem is

$$\mathcal{L}(e_1, a(0), \lambda) = e_1(R + \pi(c'(e_1) + a(0))) + (1 - e_1)\pi(a(0)) + \lambda a(0),$$

with $\lambda \geq 0$. Recall that $A'(e) = ec''(e)$ to see that in the optimum it must hold that

$$R + \pi(a(1)) - \pi(a(0)) + A'(e_1)\pi'(a(1)) = 0, \tag{25}$$

and

$$e_1\pi'(a(1)) + (1 - e_1)\pi'(a(0)) = -\lambda, \tag{26}$$

with either $a(0) = 0$, which corresponds to the termination case in the proposition, or $a(0) > 0$ and $\lambda = 0$.

We start with using the first order conditions to show that in the optimum $e_1 < e_2(1)$. First, in the case $a(1) \leq A(e^{FB})$, note that

$$c'(e_1) = a(1) - a(0) \leq a(1) \leq e_2(1)c'(e_2(1)) - c(e_2(1)) < c'(e_2(1)).$$

Second, for the case $a(1) > A(e^{FB})$ we have $\pi'(a(1)) = -1$ and $\pi(a(1)) < 0$ (see Lemma 2), so that the first order condition (25) tells us that $A'(e_1) < R$. On the other hand, $A'(e_1) = e_1c''(e_1) \geq c'(e_1)$, because c'' is weakly increasing. Since for any $e_1 \geq e^{FB}$ it holds that $c'(e_1) \geq R$, it must in fact be true that $e_1 < e^{FB} = e_2(1)$.

Next, we show that $e_1 > e^{SB}$. Using the equality $a(1) - a(0) = c'(e_1)$ we can rewrite the first order condition (25) as follows:

$$P'(e_1) = a(0) + \pi(a(0)) - (a(1) + \pi(a(1))) - A'(e_1)(\pi'(a(1)) + 1).$$

Note that $\pi'(a(1)) \geq -1$ (see Lemma 2), and because the second period surplus rises in the implemented agent's payoff, we see that $P'(e_1) \leq 0$ and hence, $e_1 \geq e^{SB}$. Moreover, $e_1 = e^{SB}$ can only hold if $a(0) = 0$, $a(1) + \pi(a(1)) = 0$, and $\pi'(a) = -1$, which can only be true in the boundary case $I_2 = S(e^{FB})$.

It remains to be shown that, in case of continuation, $e_2(0) < e^{SB}$. From Lemma 2 we know that $\pi'(a) = \frac{P'(e_2)}{A'(e_2)}$ is decreasing in a, hence we have that $\pi'(a(1)) \leq \pi'(a(0))$. In the case that $a(0) > 0$ and $\lambda = 0$ it must be true that $\pi'(a(1))$ and $\pi'(a(0))$ have opposite signs for equation (26) to be fulfilled. Hence, it holds that $P'(e_2(1)) < 0 < P'(e_2(0))$, which implies $e_2(0) < e^{SB} < e_2(1)$.

Proof of Proposition 2.

First, we show that for installment costs smaller than $P(e^{SB})$ it holds that $x(1) = 1$ in the optimal contract, while for installment costs larger than $P(e^{SB})$ it holds that $x(0) = 0$. To see this, note that Lemma 2 implies that if $x(1) < 1$ then $e_2(1) = \bar{e}$ and that if $x(0) > 0$ then $e_2(0) \geq \bar{e}$. Moreover, Lemma 1 and Proposition 1 tell us that for $I_2 \leq P(e^{SB})$ it holds that $\bar{e} \leq e^{SB} < e_2(1)$, which contradicts $x(1) < 1$, while for $I_2 \geq P(e^{SB})$ it would hold that $\bar{e} \geq e^{SB} > e_2(0)$ in case $x(0) > 0$, which is a contradiction.

Next, we show that there exists a threshold $I^C > 0$ as in the proposition. To see what happens for very low installment costs $I_2 \to 0$, recall from the proof of Proposition 1 (equation 26), that an optimal contact must satisfy the condition $e_1\pi'(a(1)) + (1 - e_1)\pi'(a(0)) \leq 0$. Lemma 2 and Lemma 1 imply that if in the optimal contract $x(0) < 1$ then $\pi'(a(0)) = \frac{P'(\bar{e})}{A'(\bar{e})}$, so that $\pi'(a(0)) \to \infty$ as $I_2 \to 0$ while $\pi'(a(1)) \geq -1$. This shows that for sufficiently low levels of I_2 a contract with unconditional continuation ($x(0) = x(1) = 1$) is optimal.

Note that the effort levels induced by this unconditional continuation contract do not depend on I_2, which implies that the derivative of the principal's maximum profit with respect to I_2 is equal to -1 for low installment costs. In general, the maximum profit is decreasing and weakly convex in I_2. Consequently, there must exist an investment level $I^C > 0$ such that to always continue the project is optimal for all $I_2 \leq I^C$, but not for any $I_2 > I^C$.

Since we have already shown $x(0) = 0$ for all $I_2 \geq P(e^{SB})$, there must exist an investment level I^T with $I^C \leq I^T \leq P(e^{SB})$ such that a termination contract $(e_1^T, e_2^T, x^T, t_1^T, t_2^T)$ with $x^T(0) = 0$ is optimal for all $I_2 > I^T$. Next, consider the (possibly empty) range of installment costs between I^C and I^T for which the optimal contract features $x(0) \in (0, 1)$. Equation (26) in the proof of Proposition 1 tells us that the first period effort level induced by this contract is

$$e_1 = \frac{P'(\bar{e})}{P'(\bar{e}) - \pi'(a(1))A'(\bar{e})}. \tag{27}$$

Since at $I_2 = P(e^{SB})$ this condition reads $e_1 = 0$, but e_1 close to zero would contradict $e_1 > e^{SB}$, it must hold that $I^T < P(e^{SB})$.

Finally, we show existence of the threshold I^{TT}. If the agent's continuation payoff after a success, which is equal to $c'(e_1^T)$ in the termination contract, is smaller than $A(\bar{e})$, then the project is continued with probability $x^T(1) < 1$ only, else it is continued with probability $x^T(1) = 1$ (this is again Lemma 2). As proved in Lemma 1, the threshold $\bar{e}$ is increasing in I_2. Moreover, it is straightforward to show that e_1^T, which is implicitly characterized by

$$R + \pi(c'(e_1^T)) + A'(e_1^T)\pi'(c'(e_1^T)) = 0,$$

(see equation 25, with $a(0) = 0$), is decreasing in I_2. Consequently, there must exist a cut-off level $P(e^{SB}) < I^{TT} \leq S(e^{FB})$, such that for all $I_2 > I^{TT}$ it holds that $A(\bar{e}) > c'(e_1^T)$ and $x^T(1) < 1$, and for all $I_2 \leq I^{TT}$ it holds that $A(\bar{e}) \leq c'(e_1^T)$ and $x^T(1) = 1$.

Proof of Lemma 3**.**

First, note that for a quadratic cost function $c(e) = \alpha e^2$ our assumptions imply that $2\alpha \geq R$. For such a cost function, it holds that $e^{FB} = \frac{R}{2\alpha}$ and $S(e^{FB}) = \frac{R^2}{4\alpha}$, while $e^{SB} = \frac{R}{4\alpha}$ and $c'(e^{SB}) = \frac{R}{2}$. It is thus the case that $A(e^{FB}) \leq c'(e^{SB})$. Since the principal's optimal contract will always lead to a first-period effort level that exceeds e^{SB}, it must hold that $e_2(1) = e^{FB}$ and $x(1) = 1$ for all possible installment costs.

Next, assume that for some I_2 there was an optimal contract with $x(0) \in (0, 1)$. Going back to equation (27) in the proof of Proposition 2 we see that this contract would implement the first period effort level

$$e_1 = \frac{P'(\bar{e})}{S'(\bar{e})} = \frac{P(\bar{e}) - I_2}{S(\bar{e}) - I_2}, \tag{28}$$

where we used $\pi'(a(1)) = -1$ (see Lemma 2) for the first equality, and the definition of $\bar{e}$ in equation (12) together with equation (16) for the second. Taking into account the incentive constraint for e_1, $c'(e_1) = a(1) - a(0)$, we can rewrite the principal's profit from such a contract as

$$P(e_1) + e_1(S(e^{FB}) - I_2) - e_1 x(0)(S(\bar{e}) - I_2) + x(0)(P(\bar{e}) - I_2). \tag{29}$$

Plugging in the value for e_1, we see that it is equal to

$$P(e_1) + e_1(S(e^{FB}) - I_2) \leq \max_e P(e) + e_1(S(e^{FB}) - I_2) = \Pi^T(I_2, R), \tag{30}$$

where $\Pi^T(I_2, R)$ denotes the principal's payoff from a termination contract. Hence, a contract with $x(0) \in (0, 1)$ is never optimal.

Proof of Corollary 1**.**

Consider the optimal contract in the case of the bad project with return R_b. In the case of the good project with return $R_g > R_b$ the principal could simply offer the same contract. Then the agent's behavior would be the same, but the principal's expected profit would be strictly larger. By optimally adjusting the contract in the case of the good project, the principal's payoff can only improve.

Proof of Proposition 3**.**

The principal now has to take into consideration additional renegotiation-proofness constraints. First, we present the version of the renegotiation-proofness principle that applies here (cf. Hart and Tirole, 1988). Renegotiation-proofness is simple in our setting due to complete contracting and the fact that renegotiation can occur only between the two periods.[22] In particular, we do not need backward induction to define the set of renegotiation-proof contracts. Because the arguments are well known, we only sketch them here.

If there is a contract $\kappa = (e_1, e_2, x, t_1, t_2) \in C$ in place, then at date 2, when the outcome y_1 is realized, this contract would lead to a continuation payoff

$$a(y_1) = t_1(y_1) + x(y_1)\left(e_2(y_1)c'(e_2(y)) - c(e_2(y_1))\right)$$

for the agent and a continuation payoff

$$p(y_1) = -t_1(y_1) + x(y_1)\left(e_2(y_1)(R - c'(e_2(y)))\right)$$

for the principal. We could assume any renegotiation process that is described by a function that maps the current pair of continuation payoff $a(y_1), p(y_1)$ to a pair of expected payoffs $a^{RP}(y_1), p^{RP}(y_1)$ such that $a^{RP}(y_1) \geq a(y_1)$, $p^{RP}(y_1) \geq p(y_1)$,

[22] The only other points in time when new information arrives are at date 4, when y_2 realizes and only payments remain to be made, and when the continuation decision realizes. Note, however, that if an expected payoff (xa, xp) is Pareto-optimal, then the realized termination payoffs $(0, 0)$ or continuation payoffs (a, p) are Pareto-optimal as well.

and the pair $a^{RP}(y_1), p^{RP}(y_1)$ is Pareto-optimal in the set of attainable continuation payoffs. As an example for such a process, one can imagine that the agent (resp., the principal) makes a take-it-or-leave-it offer of a new continuation contract $(e_2', x', t_1', t_2',)$ with probability α (resp, $1-\alpha$), and the other party accepts or rejects. Clearly, the contract $\kappa = (e_1, e_2, t_1, t_2, x)$ is renegotiation-proof if and only if it already specifies a Pareto-optimal second-period outcome for both $y_1 \in \{0, 1\}$. If the contract κ is not renegotiation-proof, it will not lead to the specified effort levels. Instead, second period outcomes are determined by renegotiation, which is anticipated by the agent when he chooses the first-period effort level such that $c'(e_1^{RP}) = a^{RP}(1) - a^{RP}(0)$. The principal's payoff if the contract κ is written and renegotiated thus is

$$\Pi^{RP}(\kappa) = e_1^{RP}(R + p^{RP}(1)) + (1 - e_1^{RP})p^{RP}(0). \tag{31}$$

Let C denote the set of all possible contracts as defined in Section 2.2 and let C^{RP} denote the set of renegotiation-proof contracts. Furthermore, $\Pi(\kappa)$ denotes the principal's payoff if she can commit to the contract κ, and $\Pi^{RP}(\kappa)$ denotes the principal's payoff from a contract κ if there is renegotiation. The version of the renegotiation-proofness principle that applies in our framework says that

$$\max_{\kappa \in C} \Pi^{RP}(\kappa) = \max_{\kappa \in C^{RP}} \Pi(\kappa). \tag{32}$$

It follows by definition of renegotiation-proof contracts that $\Pi^{RP}(\kappa) = \Pi(\kappa)$ for all $\kappa \in C^{RP}$, and therefore $\max_{\kappa \in C} \Pi^{RP}(\kappa) \geq \max_{\kappa \in C^{RP}} \Pi(\kappa)$. The other direction follows because with complete contracts any Pareto-optimal allocation can be reached by a contract in C.[23] With renegotiation every contract $\kappa \in C$ leads to Pareto-optimal continuation payoffs $a^{RP}(y_1), p^{RP}(y_1)$, and a contract κ' that specifies the continuation payoffs $a'(y_1) = a^{RP}(y_1)$ and $p'(y_1) = p^{RP}(y_1)$ from the outset is then renegotiation-proof with $\Pi^{RP}(\kappa) = \Pi(\kappa')$.

[23]By working directly with the set C, we use the same initial simplifications to the set of contracts as in the full commitment case. The reason why we can do this is that all that matters for renegotiation are the continuation payoffs of the two parties, and the simplifications that were made to the set of contracts have the property that all possible continuation payoffs stay attainable with the reduced set of contracts.

Pareto optimal one-period outcomes can be found by maximizing the principal's payoff under the constraint that the agent gets at least a certain payoff, a problem that we already partially solved with Lemma 2. The Pareto frontier must consist of pairs $(a, \pi(a))$ of the form described in the lemma, but not all of these payoffs are indeed Pareto-optimal. The function π is increasing as long as $e_2 \leq e^{SB}$, and then decreasing. Consequently, all pairs $(a, \pi(a))$ with $e_2 \geq e^{SB}$ are Pareto-optimal, while all pairs with $e_2 < e^{SB}$ are Pareto-dominated.

Consider first the case $I_2 < P(e^{SB})$. In this case, $\bar{e} \leq e^{SB}$, and therefore of all continuation contracts described in Lemma 2 only those with $a \geq A(e^{SB})$ are renegotiation-proof. The principal solves

$$\max_{a(1),a(0)} e_1 \left(R + \pi(a(1))\right) + (1 - e_1)\pi(a(0)), \tag{33}$$

subject to $a(0) \geq A(e^{SB})$, and where e_1 is given by $c'(e_1) = a(1) - a(0)$.

This is solved by $a(0) = A(e^{SB})$ and $\bar{e}_2^C(0) = e^{SB}$ as well as $\bar{e}_1^C, \bar{e}_2^C(1)$ implicitly defined by $c'(\bar{e}_1^C) = a(1) - A(e^{SB})$ and

$$R + \pi(a(1)) - \pi(A(e^{SB})) + A'(\bar{e}_1^C)\pi'(a(1)) = 0.$$

The comparison of effort levels follows as before. We denote the principal's expected profit in the case of unconditional continuation by

$$\bar{\Pi}^C(I_2, R) = P(\bar{e}_1^C) + \bar{e}_1^C \left[S(\bar{e}_2^C(1)) - S(e^{SB})\right] + P(e^{SB}) - I_2. \tag{34}$$

To get this expression for the profit we used that $a(1) - c'(\bar{e}_1^C) - A(e^{SB}) = 0$ and that with unconditional continuation $a(y_1) + \pi(a(y_1)) = S(e_2(y_1)) - I_2$.

Consider next the case $I_2 \geq P(e^{SB})$. In this case, $\bar{e} \geq e^{SB}$, so that all continuation payoffs described in Lemma 2 are Pareto-optimal.

Since then $I_2 > I^T$, the termination contract characterized in the proof of Proposition 2 solves the principal's maximization problem. This contract is renegotiation-proof. The principal's profit in case of termination is

$$\Pi^T(I_2, R) = P(e_1^T) + e_1^T x^T(1)(S(e_2^T(1)) - I_2).$$

Proof of Corollary 2.

Let $\bar{\Pi}^C(I_2, R)$ and $\Pi^T(I_2, R)$ be the profit from a renegotiation-proof continuation contract and from a termination contract, resp., as defined in the proof of Proposition 3. The functions $\bar{\Pi}^C(I_2, R)$ and $\Pi^T(I_2, R)$ are continuous and decreasing in I_2 (with derivatives -1 and $-e_1^T$, resp.). At $I_2 = P(e^{SB}) > I^T$, we know that $\Pi^T(I_2, R) > \bar{\Pi}^C(I_2, R)$. Hence, at $I_2 = P(e^{SB})$ the principal's expected profit as characterized in Proposition 3 is discontinuous, and the size of the jump is given by

$$\begin{aligned} & \Pi^T(P(e^{SB}), R) - \bar{\Pi}^C(P(e^{SB}), R) \\ = \; & P(e_1^T) + e_1^T \left[S(e_2^T(1)) - P(e^{SB})\right] - \left(P(\bar{e}_1^C) + \bar{e}_1^C \left[S(\bar{e}_2^C(1)) - S(e^{SB})\right]\right) \\ > \; & P(\bar{e}_1^C) + \bar{e}_1^C \left[S(\bar{e}_2^C(1)) - P(e^{SB})\right] - \left(P(\bar{e}_1^C) + \bar{e}_1^C \left[S(\bar{e}_2^C(1)) - S(e^{SB})\right]\right) \\ > \; & e^{SB} A(e^{SB}). \end{aligned}$$

Proof of Corollary 3.

Fix R and $I_2 = P(e^{SB})$. Because $P(e^{SB})$ is increasing in R, Proposition 3 implies that any $R_b < R$ leads to the expected profit $\Pi^T(I_2, R_b)$, while any $R_g > R$ leads to the expected profit $\bar{\Pi}^C(I_2, R_g)$. Corollary 2 shows that $\Pi^T(I_2, R) > \bar{\Pi}^C(I_2, R) + e^{SB} A(e^{SB})$. Since $\Pi^T(I_2, R)$ and $\bar{\Pi}^C(I_2, R)$ are continuous in R, one can find an R_g slightly larger than R and an R_b slightly smaller than R, such that $\Pi^T(I_2, R_b) > \bar{\Pi}^C(I_2, R_g)$, i.e., the principal prefers R_b to R_g.

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