Optimal Environmental Policy under Monopolistic Provision of Clean Technologies

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Abstract

In this paper, we characterize optimal environmental policy in a case where innovation in clean production technologies is developed and provided by a monopoly. Two policy instruments are considered: an emission tax on downstream polluting firms and an R&D subsidy for an upstream innovator in clean technologies. We find that (i) a higher emission tax may increase (decrease) R&D investment when the burden of the tax payment in the polluters’ marginal costs and the price-elasticity of the demand for polluting goods are rather small (large), (ii) the social optimum can be achieved by the combined implementation of an emission tax that is smaller than an ex-ante Pigouvian rate and a subsidy that is equal to the rate of emission reduction due to the new technology, and (iii) if the policy instrument is limited to the emission tax, the second-best tax rate lies between the first-best rate and the ex-ante Pigouvian rate. We test our model by numerical simulation and demonstrate the possibility of a type of “double dividend” due to the emission tax. Three extensions of the model are then considered: Cournot competition in the polluting industry, a subsidy to polluters who adopt the new technology, and technology spillovers.

keywords: Environmental taxes · R&D · Environmental damages · Patent

JEL Classification Q58 · Q55 · Q53 · L13 · L51

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1 **Introduction**

As an effective countermeasure against long-term environmental problems such as global warming, policymakers have become increasingly concerned with implementing technology policies to promote environmental innovations in addition to conventional regulation policies such as emission taxes, standards, and tradable permits to control pollution.\(^1\) Designing the optimal combination of the two types of policies is considered to be a difficult task because there is a bidirectional relationship between them: regulation policies affect environmental innovations as well as the level of pollution, and technology-promotion policies affect the level of pollution as well as environmental innovations.

The intellectual property rights system, including the patent system, also plays an important role in encouraging efforts to develop clean technology. Recently, in several developed countries, governments have tried to encourage patent applications in clean technology. For example, the United States Patent and Trademark Office (USPTO) expanded the Green Technology Pilot Program to promote the development of green technologies in 2009.\(^2\) In the UK, the Patent Office implemented a Green Channel for patent applications in 2009 that enabled applicants to request accelerated processing of their application if the invention had an environmental benefit.\(^3\) According to a study by UNEP, EPO, and ICTSD (2010), patenting rates (patent applications and granted patents) in selected clean energy technologies have increased at roughly 20 percent per annum since 1997, when the Kyoto Protocol was adopted. In addition, there has been a substantial concentration of environmental patents in Germany, Japan, and the USA, and 95-98% of industrial air and water pollution control technology has originated from machinery suppliers that are not engaged in polluting activities (Lanjow and Mody, 1996). Although patents encourage environmental innovations, they give an innovator monopoly power and the ability to charge prices much higher than their cost. Thus, governments may decide to assist in the diffusion and adoption of clean technologies.

In order to analyze the optimal design for a combination of regulation and technology policies, we construct a model of endogenous and monopolistic environmental innovations with perfect or imperfect competition in a polluting goods market. In line with the aforementioned trend on increased patenting, we assume that environmental innovations are made and patented by an upstream monopolist supplier who can license the developed clean production technologies to downstream polluting firms by charging appropriate royalties. This monopoly in the innovation sector implies that emission taxes alone will not provide sufficient incentives to develop and diffuse technologies. Thus, regulators should employ two policy instruments simultaneously: impose emission taxes to internalize environmental externalities, and provide R&D subsidies for the innovator to mitigate the underprovision of clean technology resulting from monopolistic innovations.

Within the above framework, we first show that introducing emission taxes is more likely to encourage innovation and diffusion of environmentally clean technologies if the demand for polluting goods

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\(^1\)Kneese and Schultze (1975) indicate that the effect of public policies on the development and spread of new technologies may be among the most important determinants of success or failure in environmental protection. Ulph and Ulph (2007) give examples in which policymakers recognize that tackling climate change will require a mix of both environmental and technology policies.


\(^3\)For information on the Green Channel for patent applications, see http://www.ipo.gov.uk/p-pn-green.htm.
is more inelastic and/or the tax burden on firms is smaller. This result is in contrast to the direct (or short-run Pigouvian) effect of taxes on emission reductions, which increases if the demand for polluting goods is more elastic and/or the tax burden becomes larger. In other words, the smaller the tax’s short-run (Pigouvian) effect in reducing emissions, the larger is its long-run (indirect) effect in reducing emissions through innovation and diffusion of environmentally clean technologies.

Second, we derive the first-best policy schemes for assuring a socially optimal allocation. The first-best policy combination of an emission tax and an R&D subsidy can completely remove two inefficiencies from a society: the overproduction of polluting goods and the underprovision of clean technologies. We find that the optimal tax rate, which we call a dynamic Pigouvian tax rate, is smaller than the ex-ante Pigouvian levels ignoring innovation, and that the optimal subsidy rate just equals the rate of improvement of emissions technologies through innovative investment. We also investigate the second-best case where the policy instrument is limited to emission taxes, and we compare its equilibrium to the first-best case. We find that the second-best tax lies between the dynamic and the ex-ante Pigouvian tax rate. Because the second-best case does not solve the inefficiency arising from the underprovision of new technology, the regulator must levy taxes above the dynamic Pigouvian level to increase the innovator’s incentives for R&D investment.

Third, by numerical simulation, we investigate the properties of the first-best and second-best policies and consider whether the optimal combination of an emission tax and an R&D subsidy has “double dividend” properties (i.e., whether the distortions caused by environmental externalities and monopolistic provision of clean technologies are completely corrected by recycling tax revenues into the R&D subsidy, with some amount left over to reduce distortions elsewhere in the economy). We find that the double dividend properties are more likely to hold true when the size of the market for polluting goods, marginal damages, and R&D efficiencies are smaller.

Finally, we provide three extensions of our model. First, we consider an oligopolistic market for polluting goods. Also in case of an imperfectly competitive goods market, introducing emission taxes encourages innovation and diffusion of environmentally clean technologies if the demand for polluting goods is more inelastic and/or the tax burden on firms is smaller. In addition, efficient allocation can be achieved by the appropriate policy combination of an emission tax and a subsidy for R&D unless the degree of competition is too low. Our second extension considers adoption subsidies as a technology policy instrument in place of R&D subsidies. We show that the optimal policy combination of an emission tax and an adoption subsidy offers exactly the same production and innovation incentives for polluting firms and for an innovating monopolist as does the policy combination of an emission tax and an R&D subsidy. Our third extension considers technology spillovers. As the degree of technology spillovers increase, equilibrium royalties and innovator’s incentives to engage in environmental R&D decrease. Therefore, to encourage R&D, the regulator should set higher taxes on emissions and higher subsidies on R&D than would be the case without technology spillovers.

This paper relates to the literature on the effects of different environmental policies on technological innovations in a perfectly competitive market for polluting goods (Magat, 1978; Downing and White, 1986; Milliman and Prince, 1989; Denicolò, 1999; Fischer et al., 2003; Fischer and Newell, 2008). This series of studies generally assumes that the market for polluting goods is perfectly competitive and that government cannot simultaneously employ more than one policy instrument. Our model is
largely based on the model developed by Denicolò (1999), who compares the effects of emission taxes and pollution permits on incentives for an upstream monopolistic R&D firm to invest in R&D. We consider emission taxes as regulation policies and subsidies for environmental R&D as technology policies under both perfect and imperfect competition in the polluting goods market, whereas Denicolò (1999) considers only regulation policy under a perfectly competitive polluting goods market.\(^4\)

This study also relates to the recent theoretical studies that explicitly consider an industry of abatement goods and services, a so-called “eco-industry” (David and Sinclair-Desgagné, 2005, 2010; Requate, 2005b; Canton et al., 2008; David et al., 2010; Perino, 2010). David and Sinclair-Desgagné (2005) investigate an imperfectly competitive eco-industry for abatement goods and find that the second-best pollution tax should be higher than in the case where there is no market power in the eco-industry. David and Sinclair-Desgagné (2010) also investigate the combination of taxing emissions on polluting firms and subsidizing eco-industry’s production of abatement goods can lead to the first-best.\(^5\) In addition, Canton et al. (2008) consider imperfect competition both at the level of an upstream eco-industry and that of a downstream polluting goods industry. They find that the optimal tax depends on the relative degree of market imperfection existing between the upstream and downstream industries. The difference between these studies and ours is the type of goods or technologies provided by the eco-industry. Specifically, these studies consider an eco-industry that provides abatement goods and services such as end-of-pipe abatement technologies. In contrast, we consider an eco-industry that provides cleaner production technologies (not abatement goods and services) that reduce unit emission coefficients for the downstream products.\(^6\) The difference plays a crucial role in characterizing the optimal policy.\(^7\)

The structure of the paper is as follows. In section 2 we present our basic model and derive the first-best and second-best environmental policies in a subgame-perfect Nash equilibrium. The innovation-inducing effect of an emission tax is also considered in this section. In section 3, we conduct numerical

\(^4\)Recently, constructing a two-sector (emitting and non-emitting) perfectly competitive model with three market failures (emissions, R&D spillovers, and learning spillovers), Fischer and Newell (2008) assess six different environmental and technology policies for reducing carbon dioxide emissions and promoting innovation of renewable energy. They find that an optimal combination of an emission price (tax) and subsidies for technology R&D can reduce emissions at a significantly lower cost than any single policy alone. Note that ours is a two-sector model (goods and innovation sectors) with three market failures (emissions, monopolistic R&D, and an imperfectly competitive goods market).

\(^5\)Specifically, David and Sinclair-Desgagné (2010) find that taxing emissions while subsidizing polluters’ abatement efforts cannot lead to the first-best, but the opposite occurs provided that the eco-industry’s output is subsidized. In contrast, our results indicate that the optimal policy combination of taxing emissions and subsidizing the adoption of cleaner technology by (downstream) polluting firms can also attain the first-best.

\(^6\)For example, in the case of CO₂ emissions, to the best of our knowledge, no end-of-pipe abatement technology exists.

\(^7\)This study also relates to the literature on the effects of environmental policies on strategic environmental R&D in imperfectly competitive markets for polluting goods, applying Cournot or Bertrand model in industrial organization literature (Katsoulacos and Xepapadeas, 1996; Ulph, 1997; Petakis and Xepapadeas, 1999; Montero, 2002). Our study is closely related to Katsoulacos and Xepapadeas (1996), who study optimal environmental policy designs consisting of emission taxes and subsidies on environmental R&D in the case of a pollution-generating duopoly with R&D spillovers. This series of studies assume that environmental R&D is undertaken independently by each polluting firm, and there exists no system of patent protection for developing cleaner technologies. In contrast, our study sharply distinguishes between a sector that develops new technologies, and a sector that uses them, such as in Parry (1995), Denicolò (1999), Requate (2005a), and Fischer and Newell (2008).
simulations to illustrate the nature of these equilibrium policies. In section 4, we consider several extensions of the basic model. Section 5 concludes the paper.

2 The Basic Model

We consider an economy with \( n \) identical firms producing a homogenous good with emissions, one R&D monopolist (hereinafter referred to as the innovator) that develops and provides a new, cleaner production technology, and a regulator (or government) that implements environmental policies.\(^8\) The timing of the game is as follows. In stage 1, the regulator sets environmental policy schemes (employing a combination of emission taxes and subsidies for environmental R&D or taxes only). In stage 2, given the regulator’s policy, the innovator invests in developing cleaner technology and sets royalty fees for licensing it. In stage 3, given established regulation and royalty fees, \( n \) firms decide whether to adopt (buy) the new technology and then compete in a perfectly competitive product market. The case of an oligopolistic product market is considered in section 4. The regulator is assumed to set policies before the innovator and all polluting firms act, which implies that it can set policies taking into account their effects on the behaviors of both the innovator and the polluting firms.

First, we derive the conditions for introducing an emission tax to induce environmental innovations. Second, we consider whether the regulator can attain an efficient allocation by setting appropriate policy schemes. We then investigate the second-best environmental tax in a case where the regulator does not have the option of implementing a subsidy. Finally, using numerical simulations, we investigate whether the first-best policies can be implemented under a deficit or surplus budget.

2.1 Firms

The model is solved by backward inductions. In stage 3, \( n \) identical firms producing polluting goods compete in a perfectly competitive market. We consider the competitive market as the market where firms compete, à la Bertrand, with constant marginal production costs denoted by \( c \geq 0 \).

Let \( X(P) \) and \( P(X) \) denote the demand and inverse demand functions, respectively, where \( P \) is the price of polluting goods and \( X \) is the aggregate output. We assume that \( P(X) \) is twice differentiable with \( P' < 0 \). As a result of Bertrand competition among identical firms with constant marginal costs, the price and the marginal cost of output are equalized. If, prior to production, firms choose not to adopt the cleaner technology developed by the innovator, the aggregate output is characterized by:

\[
P(X) = c + e(0)t,
\]

where \( e(\cdot) \) represents the emissions of one unit of output (i.e., the emission coefficient) and \( t \) is the emission tax set by the regulator. If firms adopt the new technology, aggregate output is characterized by:

\[
P(X) = c + e(z)t + r,
\]

where \( z \) and \( r \) represent the amount of R&D investment and the unit royalty fee of the new technology, respectively, both decided by the innovator. The function \( e \) is assumed to have: \( e > 0, e' < 0, \) and \( e'' > 0 \)

\(^8\)The model we develop is based on that discussed in Denicolò (1999).
for all \( z \geq 0 \). This assumption requires that the emission coefficient and the marginal improvement are decreasing in investment.

### 2.2 Innovator

Following Arrow’s (1962) classic analysis, in stage 2, the innovator sets its royalties \( r \) so as to maximize its revenues. Taking (1) and (2) into account, the innovator sets \( r \) such that firms are indifferent between adopting and not adopting the new technology. Thus, equating (1) and (2), we derive the optimal royalty fee set by the innovator: 

\[
\bar{r} = \left[ e(0) - e(z) \right] t.
\]

If \( r > \bar{r} \), then firms clearly choose not to adopt the innovator’s technology because they gain from using the old technology for free in the undercutting process of price competition. Conversely, if \( r < \bar{r} \), then the innovator always gains from increasing \( r \) to \( \bar{r} \). Thus, we have \( r = \bar{r} \).

Substituting \( \bar{r} \) into (2), we obtain the equilibrium output: 

\[
X = X(c + e(0)t),
\]

where \( X = P^{-1}(\cdot) \). Note that the equilibrium aggregate output is independent of R&D investment \( z \).

We next consider the innovator’s investment choices. The innovator’s profit is defined as

\[
\Pi = \max_z \left\{ \bar{r} X(c + e(0)t) - (1 - s)z \right\},
\]

where \( s \in [0, 1] \) represents the subsidy rate for investment in environmental R&D. The first-order condition of the above profit maximization is

\[
-e'(z) t X(c + e(0)t) = 1 - s,
\]

which implies that the marginal revenue of R&D investment (the left-hand side) equals the marginal cost of R&D (the right-hand side). \(^9\) The following comparative static results regarding the optimal investment are then obtained:

\[
\frac{dz}{dt} = \frac{-e'[X + e(0)tX]}{e''tX} \geq 0,
\]

\[
\frac{dz}{ds} = \frac{1}{e''tX} > 0.
\]

We see from the latter equation that the subsidy unambiguously spurs environmental innovations. However, the effect of taxes on R&D investment is ambiguous. To evaluate \( dz/dt \), we define the following:

\[
\sigma = \frac{e(z)t + r}{c + e(z)t + r} = \frac{e(0)t}{c + e(0)t} \in [0, 1], \quad \varepsilon_d = -\frac{X'(P)P}{X} \in [0, \infty),
\]

where \( \sigma \) represents the proportion of emission taxes and royalty charges in marginal cost and \( \varepsilon_d \) is the price elasticity of the demand for polluting goods. Using these definitions, we can rewrite (4) as follows:

\[
\frac{dz}{dt} = \frac{-e'[1 - \sigma \varepsilon_d]}{e''t} \geq 0.
\]

Thus, we obtain the following proposition.

**Proposition 1.** Raising the tax rate spurs environmental innovations if \( \sigma \varepsilon_d < 1 \).

\(^9\) The second-order condition is also satisfied.
An increase in tax heightens polluting firms’ incentive to adopt clean technology, and thus raises the equilibrium royalty fee set by the innovator. This is a positive effect of tax on innovation incentives. At the same time, however, an increase in tax reduces the demand for polluting goods. This is a negative effect of tax on innovation incentives.\footnote{The two competing effects of tax on environmental R&Ds are also shown in Ulph (1997). In his model, R&D investments are strategically engaged by each polluter (firm) and there is no system of patent protection for new technologies. It is shown that an increase in the emission tax has (i) a direct effect of increasing costs, which increases the incentive to invest in R&D in order to develop cost-saving pollution-abatement methods and (ii) an indirect effect of reducing product output that reduces the incentive to engage in R&D.} If the price elasticity of demand is large (small), an increase in tax considerably (barely) reduces the demands for the goods, and the negative effect is large (small). By the same token, the larger (smaller) proportion of emission-related charges in polluting firms’ marginal cost implies that an increase in the tax considerably (hardly) reduces their output. Thus, raising the tax rate spurs innovation when $\sigma$ and $\varepsilon_d$ are small.\footnote{Using the envelope theorem, we obtain $dP/dt = [e(0) - e(z)][X^2 + e(0)X'] \geq 0$, which has the same sign condition as that of $d\Pi/dt$. Thus, a tax increase is profitable for the innovator as long as the innovation-inducing effect is positive. Notice that if $t = 0$, then the innovator never has an incentive to engage in R&D.}

Proposition 1 may have an important implication concerning the relationship between the Pigouvian effect and the innovation-inducing effect of emission taxes. When the proportion of tax costs in the marginal cost and/or the price elasticity of demand are small, the innovation-inducing effects of the tax increase whereas its Pigouvian output-reduction effects diminish. In other words, the innovation-inducing effects of the tax are inversely related to its Pigouvian effects.

Because both $\sigma$ and $\varepsilon_d$ are generally considered to be small in a market for polluting goods, we hereafter assume that the innovation-inducing effect is always positive i.e., $\sigma\varepsilon_d < 1$ holds in equilibrium.\footnote{As an exception, in times of economic crises, some polluting goods are luxury goods for which the price-elasticity of the demand can be so high that $\sigma\varepsilon_d$ may become greater than unity. In such a case, raising the emission tax rate shrinks environmentally innovative investment and government may attempt to reduce tax rates to stimulate the economy and to encourage more investment.}

### 2.3 Regulator

The regulator’s objective is to maximize social welfare, as defined by:

$$SW = \int_0^X P(h)dh - cX - z - D(E),$$

(5)

where $E = e(z) \cdot X$ is the total amount emissions and $D(\cdot)$ is the environmental damage due to emissions. The damage function $D$ has the following linear properties: $D(E) \geq 0$, $D(0) = 0$, $D' > 0$, and $D'' = 0$ for all $E \geq 0$.

Before we solve for the subgame-perfect equilibrium of the game, it is useful to characterize the social optimum (efficient outcome) that is the solution to the welfare maximization problem by the
planer that directly controls \( X \) and \( z \). The first-order conditions of the problem are:

\[
\frac{\partial SW}{\partial X} = SW_X = 0 \iff P(X) = c + e(z)D', \quad (6)
\]

\[
\frac{\partial SW}{\partial z} = SW_z = 0 \iff -e'(z)XD' = 1, \quad (7)
\]

where equation (6) ((7)) represents the condition that the social marginal benefit and social marginal cost of output (investment) are equalized.

Next, we investigate the optimal regulation in a decentralized economy. The regulator sets the tax and subsidy rates so as to maximize social welfare. The first-order conditions of the problem are:

\[
\frac{\partial SW}{\partial t} = \frac{DX}{dt} \left[ e(0)t - e(z)D' \right] + \frac{dz}{dt} \left[ \frac{1-s}{t} D' - 1 \right] = 0 \quad (8)
\]

\[
\frac{\partial SW}{\partial s} = \frac{dz}{ds} \left[ \frac{1-s}{t} D' - 1 \right] = 0. \quad (9)
\]

**Proposition 2.** Efficient allocation can be achieved by the following first-best policy combination of emission tax \( t^{FB} \) and a subsidy for R&D \( s^{FB} \):

\[
t^{FB} = \frac{e(z^{FB})}{e(0)} D', \quad s^{FB} = \frac{e(0) - e(z^{FB})}{e(0)} = \frac{\Delta e}{e}.
\]

**Proof.** From (8) and (9), the first-order conditions are satisfied when \( t = \{e(z)D'\}/e(0) \) and \( s = \{e(0) - e(z)\}/e(0) \). Substituting these into (1) and (3) yields the same conditions as (6) and (7), respectively. Thus, it is proven that the policy combination of \( t^{FB} \) and \( s^{FB} \) leads to socially optimal output and innovation. \( \square \)

Intuitively, the above policy mix can remove two inefficiencies from society: the overproduction of goods attributed to environmental externalities and the underprovision of clean technologies attributed to a monopolistic innovation. First, to internalize environmental externalities, the regulator should impose the emission tax given by \( t^{FB} = e(z^{FB})D'/e(0) \). The intuition is simple: because the social marginal cost of production is \( c + e(z)D' \) (from eq. (6)), whereas the private marginal cost is \( c + e(0)t \) (from eq. (1)), government should equate them to attain an efficient allocation. Note that the tax rate is smaller than the marginal environmental damage \( D' \). In our model, \( D' \) can be considered as the *ex-ante Pigouvian tax rate* in the case with no innovation (i.e., the optimal level of tax when innovative activities are not considered \( z = 0 \)). The result that the optimal tax is smaller than the ex-ante Pigouvian tax is due to the innovation-inducing effect of the taxes. Therefore, the derived \( t^{FB} \) can be interpreted as a *dynamic Pigouvian tax*, including its innovation-inducing effect.

Second, the regulator should subsidize the innovator to encourage R&D investment. The first-best subsidy rate just equals the percentage changes in the emission coefficient resulting from R&D investment. For example, if the innovator succeeds in developing a technology that reduces the emission coefficient by 10%, the government should subsidize 10% of the innovator’s required investment costs.

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13 We assume that the social welfare function is concave in \( X \) and \( z \). Because \( SW_{XX} = P'(X) < 0 \) and \( SW_z = -e'XD' < 0 \), the assumption is equivalent to \( SW_{XX}SW_z - (SW_{Xz})^2 \geq 0 \), where \( SW_{Xz} = -e'D' \).

14 We also assume that the social welfare function is concave in \( t \) and \( s \).
Next, we consider the case in which the regulator cannot use the R&D subsidy instrument, and the only available policy tool is the emission tax. We now have the following proposition regarding the property of the second-best tax, \( t^{SB} \), where the superscript \( SB \) denotes the variable in the second-best case.

**Proposition 3.** If the policy instrument is limited to emission taxes, the second-best tax is greater than the first-best tax and is smaller than the ex-ante Pigouvian tax. Formally, \( t^{FB} < t^{SB} < D^0 \).

**Proof.** Substituting \( s = 0 \) into (8) and evaluating it in \( t = D^0 \) and \( t = t^{FB} \) yields:

\[
\frac{\partial SW}{\partial t} \bigg|_{t=D^0, s=0} = \frac{dX}{dt} \left[ (e(0) - e(z^{SB}))D^0 \right] < 0
\]

\[
\frac{\partial SW}{\partial t} \bigg|_{t=t^{FB}, s=0} = \frac{dX}{dt} \left[ (e(z^{FB}) - e(z^{SB}))D^0 \right] + \frac{dz}{dt} \left[ \frac{e(0)}{e(z^{FB})} - 1 \right] > 0.
\]

Thus, we obtain \( t^{FB} < t^{SB} < D^0 \) as long as \( SW \) is concave in \( t \). □

As in the case of the first-best policy, the second-best tax is smaller than the ex-ante Pigouvian tax \( D^0 \) (which ignores innovation) because of the innovation-inducing effects of the tax. However, the second-best tax is greater than the first-best tax because the inefficiency due to the underprovision of new technology remains uncorrected. The emission tax, besides its direct effect on polluters’ behavior, encourages higher innovation for clean technologies in an economy where there is initially underprovision of the new technology. Therefore, the second-best emission tax should be set above the dynamic Pigouvian rate to increase the innovator’s marginal revenue (incentive) for R&D investment. Conversely, if the regulator can use the combination of an emission tax and an R&D subsidy, a more efficient allocation can be achieved with a lower tax.

Substituting the derived first-best and second-best policies in the best response chosen by each agent in later stages, we obtain the equilibrium value of each endogenous variable in the first- and second-best cases, denoted by the superscripts \( FB \) and \( SB \), respectively.

**Corollary.** Comparing the equilibria in the first- and second-best cases, we have:

(a) \( X^{FB} > X^{SB}, z^{FB} > z^{SB}, e^{FB} < e^{SB}, \) and \( SW^{FB} > SW^{SB} \).

(b) A sufficient condition to hold \( E^{FB} < E^{SB} \) is \( d \left[ -\frac{e'(z)}{e(z)} \right] dz \geq 0 \).

**Proof.** See Appendix A. □

Corollary 1-(a) shows that in the case of the first-best policy combination, output and R&D investments are larger. However, it is ambiguous which of the total emissions in the two cases is greater. Corollary 1-(b) shows that if the marginal rate of improvement in the emission coefficient is non-diminishing in investment, then total emissions are necessarily lower in the first-best case than in the second-best case.
Finally, we define an environmental policy budget (denoted by $B$) as the budget resulting from implementing the policies. The budget is:

$$B^i = \left[ e(0) t^i X^i - s^i z^i \right], \quad i \in \{FB, SB\},$$

where the first term of the right-hand side represents the environmental tax revenues and the second term is the R&D subsidy payments. If $B^{FB} \geq 0$ holds in equilibrium, the first-best policies produce a kind of double dividend from environmental taxes, in the sense that the taxes not only internalize the environmental externalities but also correct the distortion of monopolistic provision of R&D in maintaining a fiscal surplus.\(^{15}\) In the next section, we specify several functional forms and engage in some numerical simulations.

### Table 1: Numerical example.

<table>
<thead>
<tr>
<th>Parameter values</th>
<th>$A = 20$</th>
<th>$c = 1$</th>
<th>$\alpha = 0.3$</th>
<th>$\beta = 0.5$</th>
<th>$e(0) = 1$</th>
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</table>

<table>
<thead>
<tr>
<th>Model results</th>
<th>$t$</th>
<th>$s$</th>
<th>$X$</th>
<th>$P$</th>
<th>$P_r$</th>
<th>$z$</th>
<th>$e$</th>
<th>$\Pi$</th>
<th>$E$</th>
<th>$SW$</th>
<th>$B$</th>
</tr>
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<tr>
<td>First-best</td>
<td>0.18</td>
<td>0.65</td>
<td>18.82</td>
<td>1.18</td>
<td>0.11</td>
<td>3.46</td>
<td>0.35</td>
<td>0.93</td>
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<td>174.69</td>
<td>1.10</td>
</tr>
<tr>
<td>Second-best</td>
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<td>0</td>
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<td>1.48</td>
<td>0.30</td>
<td>3.27</td>
<td>0.38</td>
<td>2.28</td>
<td>6.95</td>
<td>173.64</td>
<td>5.61</td>
</tr>
<tr>
<td>Without innovation</td>
<td>0.5</td>
<td>0</td>
<td>18.5</td>
<td>1.5</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>18.5</td>
<td>171.13</td>
<td>9.25</td>
</tr>
<tr>
<td>Laissez-faire</td>
<td>0</td>
<td>0</td>
<td>19</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>19</td>
<td>171</td>
<td>0</td>
</tr>
</tbody>
</table>

3 **A Numerical Example**

In this section, we conduct numerical simulation exercises to illustrate the nature of the first- and second-best environmental policies. We adopt the following functional forms:

$$P(X^i) = A - X^i, \quad e(z^i) = e(0) \exp[-\alpha z^i], \quad D(E^i) = \beta E^i,$$

for $i \in \{FB, SB\}$,

where the parameters $A$, $\alpha$, and $\beta$ are positive constants and represent the market size, the efficiency of R&D investment, and the marginal environmental damage, respectively. Both inverse demand and environmental damage functions are linear, and the emission coefficient declines exponentially with investment.\(^{16}\)

As a benchmark, we derive the equilibrium values of endogenous variables in the first-best, second-best, without innovation, and laissez-faire (no-policy) cases with the parameter values of $A = 20$, $c = 1$, $\alpha = 0.3$, $\beta = 0.5$, and $e(0) = 1$. The simulation results are presented in Table 1; they confirm results obtained in the previous section. We observe that the dynamic Pigouvian tax rate $t^{FB} = 0.18$ is smaller than both the second-best tax $t^{SB} = 0.48$ and the ex-ante Pigouvian tax $t|_{z=0} = \beta = 0.5$. We also find

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\(^{15}\)By definition, $s^{SB} = 0$; thus, $B^{SB} \geq 0$ holds true in the second-best case.

\(^{16}\)The explicit functional form of $e(z)$ is also used by Simpson and Bradford (1993). In this specification, the marginal rate of improvement is constant ($-e'/e = \alpha$) and is thus nondiminishing in $z$. The other forms of $e(z)$ and their properties are detailed by Ulph (1997).
that this policy combination reduces output less, induces innovation more, and reduces emissions more than the environmental policies of a single-policy instrument.\textsuperscript{17}

We then investigate the effects of changes in several parameters of the first- and second-best policies using numerical examples. In each panel of Figure 1, the value of all parameters except those on the horizontal axis are fixed at the values used in Table 1, and shaded regions indicate the regions in which no innovation occurs. We can see from any panel that the first-best tax is lower than the second-best tax, as Proposition 3 indicates.

The upper panel of Figure 1 represents the effects of changes in market size on equilibrium policies. In the shaded region where market size is extremely small, total output and thus the innovator’s revenues are so small that the innovator has no incentive to conduct R&D (i.e., $z = 0$). Thus, the optimal policy is a combination of $t = \beta$ and $s = 0$.\textsuperscript{18} In the region indicating positive R&D by the innovator, as market size increases, the first- and second-best taxes decrease because of the development and diffusion of cleaner technologies, and the first-best subsidy increases to remove the inefficiency from the innovator’s underinvestment. In other words, the regulator shifts its main policy from an emission tax to an R&D subsidy as the market size of polluting goods enlarges. The same intuition holds in the bottom panel of Figure 1; the regulator shifts its policy from an emission tax to a subsidy as R&D becomes more efficient. Note that no matter how large (high) the market size (the R&D efficiency) becomes, the optimal tax cannot reach zero because the innovator loses its incentive to innovate without a positive tax rate.

The middle panel of Figure 1 shows that in the region indicating positive R&D, the first-best subsidy increases as marginal environmental damage increases. However, the first-best tax is constant as the marginal damage increases, whereas the second-best tax increases. This indicates that it is more preferable to adjust the subsidy for environmental innovations than to adjust emission taxes following changes in the seriousness of environmental problems. Although tax and subsidy have similar

\textsuperscript{17}The result of $E_{FB} < E_{SB}$ is due to the specification of $e(z) = e(0) \exp\left( -\alpha z \right)$ with $d(e'/e)/dz = 0$.

\textsuperscript{18}Notice that in the shaded region, social welfare cannot be improved by setting a significantly higher subsidy to induces the innovator to conduct R&D investment.
emission-reducing effects, the difference between them is important. An increase in subsidy reduces total emissions without shrinking total output and consumer surplus (because $X$ is independent of $z$ and $s$), but an increase in the tax reduces total emissions by reducing output and consumer surplus.

Finally, we investigate the sign condition of environmental policy budget $B^{FB}$. As shown in the three panels of Figure 1, the environmental policy budget in the first-best case is more likely to be positive as the market size, the marginal damage, and R&D efficiency decrease (the region to the left of the $B^{FB} = 0$ line). In other words, in the region between $z' \geq 0$ and $B^{FB} \geq 0$, the first-best policy combination of an emission tax and an R&D subsidy has a kind of “double dividend” property, which slightly differs from the one introduced by Pearce (1991). Under the condition of $B^{FB} \geq 0$, distortions caused by environmental externalities and monopolistic provision of clean technologies are completely corrected by recycling the tax revenue into the R&D subsidy, with some amount left over to reduce distortions elsewhere in the economy.

4 Extensions

4.1 Under imperfect competition in the product market

Our first extension of the model is to consider imperfect competition in the product market. Here, we assume that $n$ homogenous firms compete in a Cournot fashion. The profits of firm $j$ ($j = 1, \cdots, n$) are defined as:

$$\pi^N_j = \max_{x_j} \left\{ P(x_j + (n-1)\bar{x})x_j - cx_j - e(0)tx_j \right\},$$

$$\pi^A_j = \max_{x_j} \left\{ P(x_j + (n-1)\bar{x})x_j - cx_j - e(z)tx_j - rx_j \right\},$$

where $\pi^N$ is the profit when the firm does not adopt clean technologies, and $\pi^A$ is the profit when the firm adopts them. The first-order conditions, evaluated in symmetric equilibrium, are respectively given by:

$$m(x, n) \equiv P(nx) + xp'(nx) = c + e(0)t \quad (13)$$

$$m(x, n) \equiv P(nx) + xp'(nx) = c + e(z)t + r, \quad (14)$$

respectively, where $m(x, n)$ represents the marginal revenue of each firm and is assumed to have ordinary properties of $m_x < 0, m_n < 0$.

As before, the innovator sets $r$ such that firms are indifferent between adopting and not adopting the new technologies, and all firms adopt the technology. Thus, equating (13) and (14), we have $r = [e(0) - e(z)]t$, which is the same as in the previous case of perfect competition. Substituting this into (14) yields $x = x(c + e(0)t, n)$, where $x_t \equiv \partial x/\partial t = e(0)/m_n < 0$ and $x_n \equiv \partial x/\partial n = -m_n/m_x < 0$. Total output $X$ is then obtained as $X = X(c + e(0)t, n) = nx$, where $X_t < 0$ and $X_n > 0$ hold. Note that the amount of R&D affects neither the output of each firm nor the total output of the industry.

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19For simplicity, we exclude the possibility of strategic adoption choices by each firm, i.e., all firms are assumed to simultaneously decide whether to adopt cleaner technologies.
The profits of the innovator are given by 
\[ P = \max_z \{(e(0) - e(z))t + x - (1 - s)z\}, \] as before. The first-order condition is:
\[ -e'(z)tnx(c + e(0)t, n) = 1 - s. \] (15)

Comparative statics of the above yields the following:
\[ \frac{dz}{dt} = \frac{-e'[x + tx]}{e't}, \quad \frac{dz}{ds} = \frac{1}{e't} > 0. \]

In order to identify the sign of \( \frac{dz}{dt} \), we define:
\[ \sigma = \frac{e(z) + r}{c + e(z)t + r} \in [0, 1], \quad \varepsilon_m = \frac{m_x}{m} \in [0, \infty) \]
where \( \sigma \) is again the proportion of emission taxes and royalty charges in marginal cost and \( \varepsilon_m \) is the elasticity of marginal revenue. We can therefore rewrite \( \frac{dz}{dt} \) as:
\[ \frac{dz}{dt} = \frac{-e'}{e't} \left( 1 - \frac{\sigma}{\varepsilon_m} \right). \]

**Proposition 4.** Under imperfect competition in product markets, raising emission tax spurs environmental innovations if \( \sigma < \varepsilon_m \).

Because the elasticity of marginal revenue (\( \varepsilon_m \)) is in general inversely related to the price elasticity of demand (\( \varepsilon_d \)), this result also implies that the innovation-inducing effect of the tax increases as the proportion of environmental variable costs in marginal costs and/or the price elasticity of demand become smaller.\(^{20}\)

Then, as before, we derive the first-best policies. The detailed derivation is provided in Appendix B.

**Proposition 5.** Given imperfect competition in product markets,

(a) if
\[ \hat{t} = \frac{e(\hat{z})}{e(0)}D' - \frac{\lambda}{e(0)} > 0, \quad \hat{s} = \frac{e(0) - e(\hat{z})}{e(0)} + \frac{\lambda}{e(0)D'} \in [0, 1], \]
where \( \lambda \equiv -XP'(X)/n > 0 \) holds in equilibrium, efficient allocation can be achieved by the policy combination of an emission tax (\( \hat{t} \)) and a subsidy for R&D (\( \hat{s} \)).

(b) if \( \hat{t} \leq 0 \) holds in equilibrium, then each firm has no incentive to adopt clean technologies and thus no environmental innovation occurs, regardless of how the regulator subsidizes the innovator’s R&D.

**Proof.** See Appendix B. \( \square \)

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\(^{20}\)For example, in the case of a linear demand such as \( P = A - bX \), we have \( \varepsilon_m = (n + 1)bx/(A - (n + 1)bx) \) and \( \varepsilon_d = (A - bnx)/bnx \). Thus, \( \varepsilon_d = 1/\varepsilon_m \) approximately holds if \( n \) is a sufficiently large number. In the case of demand with constant elasticity, as for \( P = AX^{-1/\alpha} \), we have \( \varepsilon_m = 1/\varepsilon_d \). In this case, Proposition 4 states the same sign condition as Proposition 1.
Because the total output is suppressed by oligopolists, the regulator has an incentive to subsidize their production as well as to tax their emissions. An optimal subsidy on output can be derived from (13) as \( \hat{\lambda} \). Thus, optimal tax should be reduced by \( \hat{\lambda}/e(0) \) from the optimal tax rule given by Proposition 2.\(^{21}\) This tax cut, however, diminishes the innovator’s marginal revenue of R&D. Thus, the optimal subsidy should be incremented by the rate that is proportional to \( \hat{\lambda}/e(0) \). As long as \( \hat{\lambda} > e(\hat{\lambda})D' \) and \( \hat{\delta} \) lies between 0 and 1, the policy combination of \( \hat{\lambda} \) and \( \hat{\delta} \) leads to a socially optimal innovation process and emission control even when the output market is imperfectly competitive.

However, if the output reduction caused by oligopolists is large, the tax-cum-output subsidy becomes negative. In this case, there is no demand for cleaner technologies, and thus no innovation takes place. Therefore, although government can induce innovations by setting positive emission tax rates, an efficient allocation is no longer achieved in such cases.

To derive a condition in which the above combination of \( \hat{\lambda} \) and \( \hat{\delta} \) can lead to efficient allocation, we perform the following simulation. With parameter values \( A = 10, c = 1, \alpha = 0.3, \beta = 0.5, \) and \( e(0) = 1 \), the optimal emission tax and R&D subsidy are calculated as

\[
\hat{\lambda} = 0.387 - \frac{8.613}{n}, \quad \hat{\delta} = 0.226 + \frac{17.226}{n}.
\]

Thus, as long as \( n > 22.256 \approx 23 \), \( \hat{\lambda} > 0 \) and \( \hat{\delta} \in [0, 1) \) hold in equilibrium. If \( n < 23 \), then \( \hat{\lambda} < 0 \) and there is no incentive for polluting firms to adopt (buy) cleaner technologies. The simulation result indicates that an efficient allocation can be achieved by the policy combination of \( \hat{\lambda} \) and \( \hat{\delta} \) unless the degree of competition in the market for polluting goods is sufficiently low.

### 4.2 Adoption subsidy

Our second extension of the model considers an alternative policy combination: tax emissions and subsidized adoption of cleaner technologies. The adoption subsidy here is the policy under which polluting firms receive a refund on their emission taxes if they adopt cleaner technologies. Hereafter, we show that all results obtained in the previous section hold if we assume polluting firms receive a subsidy for adopting new technologies instead of assuming that the innovator receives an R&D subsidy.

As in section 2, if firms choose not to adopt technologies developed by the innovator, aggregate output is characterized by \( P(X) = c + e(0)t \). If, however, firms choose to adopt the technologies and pay license fees, aggregate output is \( P(X) = c + e(\tilde{z})t + (1 - \xi)r \), where \( \xi \) represents the subsidy rate for license payments. Then, by equating the above two expressions, the equilibrium royalties are obtained as:

\[
\tilde{r} = \frac{\{e(0) - e(\tilde{z})\}t}{1 - \xi}.
\]

As a result, aggregate output here coincides with that derived in subsection 2.2. The profits of the innovator are then:

\[
\tilde{\Pi} = \max \left\{ \frac{\{e(0) - e(\tilde{z})\}t X - \tilde{z}}{1 - \xi} \right\}.
\]

\(^{21}\)In the models without environmental innovations, Ebert (1992) and Requate (2006) also show that if emissions are completely determined by output and oligopolistic firms do not have abatement technology, then an emission tax, that is just reduced by the output subsidy, can solely implement the social optimum. For more on this point, see Proposition 5 in Requate (2006, p.144).
The first-order condition of the above profit maximization is \(-e'(z)tX(e + e(0)t) = 1 - \zeta\), which is the same condition as equation (3) when \(\zeta = s\). Thus, in our model, the optimal policy combination of an emission tax and an adoption subsidy and the combination of an emission tax and an R&D subsidy offer exactly the same production and innovation incentives for polluting firms and the innovator.\(^{22}\)

### 4.3 Technology spillovers

Our last extension considers technology spillovers. Suppose there are spillover benefits of innovation for nonadopting firms. The firms decide whether to pay royalties to use the cleaner technology \(e(z)\) or to pay nothing for using the technology \((1 - \delta)e(0) + \delta e(z)\), which is a linear combination of new and old technologies. The parameter \(\delta \in [0, 1]\) then represents the degree of technology spillovers (or the degree of an innovation’s appropriatability). When \(\delta = 0\) (there are no technology spillovers), the firms cannot imitate the technologies, and the innovator can fully appropriate the innovation. When \(\delta = 1\), the innovation is effectively a pure public good and can be perfectly reproduced by imitation.

In this setting, the optimal royalty fee set by the innovator is obtained as

\[
\hat{r} = (1 - \delta)[e(0) - e(z)]t,
\]

which implies that greater technology spillovers induce a smaller royalty fee and thus a smaller incentive to engage in R&D.\(^{23}\) Therefore, to regulate less-clean production and to encourage R&D more, the regulator should set higher tax and subsidy rates than in the case without technology spillovers. This also implies that, taking some technology spillovers into account, the condition for achieving welfare maximum with a budget surplus (i.e., \(\bar{B} \geq 0\) in equilibrium) becomes more severe than in the case with no spillovers. Note that if the policy instrument is limited to an emission tax, the tax rate may be greater than the marginal environmental damages.

### 5 Concluding Remarks

We investigate optimal environmental policy schemes in an economy with endogenous and monopolistic innovations in environmental technologies. We show that the innovation-inducing effect of introducing emission taxes is greater if demand for polluting goods is less elastic and/or the tax burdens upon polluting firms are smaller. This finding is in sharp contrast to the output-reducing effect of an emission tax.

\(^{22}\)It is interesting to compare the above result with the results obtained by David and Sinclair-Desgagné (2010). Focusing on the imperfect competition in eco-industry of end-of-pipe abatement goods and services (not cleaner production technologies), they show that the combination of an emission tax and an adoption subsidy to downstream polluters cannot achieve the first-best allocations because the upstream innovators would thereby be allowed to raise prices indefinitely. The difference between our model and theirs is that our adoption subsidy is paid according to license payments, while their subsidy is paid according to the amount of abatement (i.e., \(sa\) in equation (8) of David and Sinclair-Desgagné (2010)). If the subsidy is paid according to outputs in our model, then the total output is characterized by \(P(X) = c + e(z)t + r + \zeta\) in the case of adoption. In that case, the upstream innovator raises royalty fees just by the subsidy \(\zeta\), and thus the combination of an emission tax and an adoption subsidy to downstream polluters cannot achieve the first-best.

\(^{23}\)In fact, the existence of technology spillovers has two effects on R&D investment: negative direct effects due to decreases in the royalty fee and positive indirect effects due to increases in outputs. The latter is considered to be very small, so technology spillovers reduce environmental R&D.
tax, which is greater if demand is more elastic and/or the tax burdens are larger. We also demonstrate that the first-best policy consists of imposing an emission tax that is smaller than the ex-ante Pigouvian tax rate and setting an R&D subsidy that equals the rate of improvement in the emission coefficient brought by environmental R&D. If the policy instrument is limited to a tax, the regulator should set the second-best tax rate higher than the first-best tax to increase incentives for engaging in innovative R&D. Furthermore, we conduct numerical simulations to investigate the first- and second-best policies and their budget conditions. Finally, we consider three extensions of our basic model: the case involving an oligopolistic market for polluting goods, involving adoption subsidies instead of R&D subsidies, and involving technology spillovers.

Our results have important policy implications for implementing environmental taxes. In many countries, industry or industrial associations oppose environmental taxes, mainly on two grounds. First, the low (short-term) price elasticity of demand for polluting goods such as energy and fuel consumptions means that introduction of small environmental taxes would have little effect in reducing output and pollution. Second, the considerably higher tax rate needed to reduce pollution would harm the international competitiveness of domestic industries. Our findings show that introducing or raising environmental taxes greatly encourages environmental innovation, as the price elasticity of polluting goods and/or the tax burden on firms are smaller. In other words, even if the output-reducing effect of a tax is small due to low price elasticity, introducing emission taxes effectively reduces pollution by inducing greater innovation.

Moreover, our results indicate the importance of recycling emission-tax revenues into subsidies for firms undertaking R&D or firms adopting cleaner technologies. An appropriately designed policy combination achieves a double dividend: distortions caused by the environmental externalities and by the monopolistic provision of clean technologies are corrected by recycling emission tax revenue into an R&D subsidy, with some amount left over to reduce distortions elsewhere in the economy. The result corresponds to that of EIEP (2000), where the effectiveness of the policy combination is confirmed by conducting simulation analyses of the Japanese economy.

Appendix A. Proof of Corollary 2

Because the equilibrium output is represented by \( X(c + \varepsilon(0)I) \), we immediately obtain \( X^{FB} > X^{SB} \) from \( t^{FB} < t^{SB} \). R&D investment in the first-best case \( z^{FB} \) is characterized by (7) (i.e., \(-e'(z^{FB})X^{FB}D^{0} = 1\)), whereas that in the second-best case \( z^{SB} \) is characterized by \(-e'(z^{SB})X^{SB}t^{SB} = 1\). Because \( X^{FB} > X^{SB} \) and \( D^{0} > t^{SB} \), we obtain \(-e'(z^{FB}) < -e'(z^{SB})\), which shows that \( z^{FB} > z^{SB} \).

The total emissions are represented by \( E' = e'(z)X' \). Using (7) and (3), we have:

\[
E^{FB} = \frac{1}{-e'(z^{FB})D^{0}}, \quad E^{SB} = \frac{1}{-e'(z^{SB})t^{SB}}.
\]

Because \( D^{0} > t^{SB} \) holds in equilibrium, \(-\frac{e'(z^{FB})}{e(z^{FB})} > -\frac{e'(z^{SB})}{e(z^{SB})}\) is sufficient for \( E^{FB} < E^{SB} \). Thus, from \( z^{FB} > z^{SB} \) we

\[\text{For more on this point, see OECD (2006).}\]

\[\text{EIEP (2000) shows that, in Japan, to reduce CO}_2\text{ emissions by 2% from 1990 levels by 2010, a tax rate of only $27 per ton of carbon is needed if the funds from a carbon tax are fully recycled into subsidies for investment in energy efficient technology, whereas a tax rate of $273~364 per ton of carbon is needed if the subsidies are absent.}\]

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show that \( d(-e'(z)/e(z))/dz \geq 0 \) (the nondecreasing marginal rate of improvement in emission coefficient) is the sufficient requirement for \( E^{FB} < E^{SB} \). □

Appendix B. Proof of Proposition 5

Social welfare is defined as \( SW = \int_0^\infty P(h)dh - n c x - z - F(e(z)n x) \). When the government directly controls both output, \( x \), and R&D investment, \( z \), the first-best social optimum is characterized by the following first-order conditions:

\[
\frac{\partial SW}{\partial x} = P(n x) - c - e(z) D' = 0, \tag{A.1}
\]

\[
\frac{\partial SW}{\partial z} = -1 - e'(z) n x D' = 0. \tag{A.2}
\]

In a decentralized economy, the regulator sets a tax rate \( t \) and a subsidy rate \( s \) so as to maximize the social welfare, with \( x \) and \( z \) determined by the market equilibrium conditions. Using (14) and (15), the first-order conditions are:

\[
\frac{\partial SW}{\partial t} = \frac{dx}{dt} n \left[ e(0) t - e(z) D' - \frac{X P'(X)}{n} \right] + \frac{dz}{dt} \left[ -1 + \frac{1 - s}{t} D' \right] = 0, \tag{A.3}
\]

\[
\frac{\partial SW}{\partial s} = \frac{dz}{ds} \left[ -1 + \frac{1 - s}{t} D' \right] = 0.
\]

Thus, the following optimal tax and subsidy rates are derived:

\[
\hat{t} = \frac{e(z)}{e(0)} D' + \frac{X P'(X)}{e(0) n}, \quad \hat{s} = \frac{e(0) - e(z)}{e(0) n} - \frac{X P'(X)}{e(0) n D'}.
\]

Substituting \( \hat{t} \) and \( \hat{s} \) into (14) and (15), respectively, characterizes the outputs and R&D investments in a decentralized economy, which coincide with the social optimum represented by (A.1) and (A.2). □

References


