Uncertainty of policy recommendations for nonlinear econometric models: some empirical results

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UNCERTAINTY OF POLICY RECOMMENDATIONS FOR NONLINEAR ECONOMETRIC MODELS: SOME EMPIRICAL RESULTS

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ABSTRACT

A method for evaluating the reliability of policy recommendations derived from a linear dynamic structural econometric model in the framework of the linear quadratic control problem has been recently proposed by Friedmann (1980, 1981). The method analytically derives the asymptotic distribution of the estimated optimal policy and in particular the asymptotic standard errors of policy instruments, with respect to structural coefficients estimation errors. The use of analytic simulation and of Monte Carlo techniques allows to extend Friedmann's findings to medium and large size dynamic linear models and to nonlinear econometric models. Empirical results for some nonlinear models of national economies are reported in the paper.

1. INTRODUCTION

Some work has been carried out in the last few years for evaluating the uncertainty of policy actions, when derived from macroeconometric models. The common point of these researches is that, when the effect of a policy action is analyzed using a model with estimated parameters, the parameters errors involve, to some extent, uncertainty in the results.

After the seminal paper by Goldberger et al. (1961), Dhrymes (1973) derives explicit formulae for the asymptotic standard errors of reduced form coefficients (and therefore of impact multipliers) calculated from linear simultaneous equations systems in structural form.

Fair (1980b), and Bianchi et al. (1981a), using respectively a Monte Carlo and an analytic simulation approach, extend to nonlinear models the calculation of standard errors of the impact multipliers.

The uncertainty on dynamic multipliers is dealt with in Schmidt (1973), Brissimis and Gill (1978), and Gill and Brissimis (1979), who still present explicit formulae for the asymptotic standard errors in linear models.

Fair's (1980b) Monte Carlo method and the analytic simulation method, based on numerical differentiation, in Bianchi et al. (1981b) allow to extend the calculation of standard errors of dynamic multipliers to nonlinear structural models.

In a linear quadratic control framework, Friedmann (1980, 1981) derives explicit formulae to estimate the asymptotic covariance matrix associated with the estimated optimal policy instruments. This information can be "used to indicate the reliability of the econometric policy advice and to facilitate the comparison and combination of the policy recommendations derived from different econometric models" (Friedmann, 1981, p.415).

This paper aims at extending to nonlinear models the calculation of standard errors of optimal policy instruments.

Three methods will be briefly discussed and experimented with. The first is an analytic simulation method, based on numerical differentiation; in the particular case of linear models it is exactly equivalent to Friedmann's method, but it does not make use of explicit formulae. The second method is based on Monte Carlo drawings from the structural coefficients distribution; it is strictly related to the method used in Fair (1980b) to calculate the standard errors of multipliers. The third method is based on stochastic simulation over the sample period with re-estimation of the structural parameters.

2. THE PROBLEM DEFINITION

Let a nonlinear structural econometric model be represented as

$$f(y_t, y_{t-1}, x_t, z_t, a) = u_t; \qquad t=1,2,\dots,T \tag{2.1}$$

where:

$f = (f_1, f_2, \dots, f_m)'$ is a vector of functional operators,

continuously differentiable with respect to the elements of current and lagged y, x, z and a;

$y_t = (y_{1t}, y_{2t}, \dots, y_{mt})'$, y_{t-1}, $z_t = (z_{1t}, z_{2t}, \dots, z_{nt})'$ and $x_t = (x_{1t}, x_{2t}, \dots, x_{pt})'$ are vectors of current and lagged endogenous variables, exogenous variables and policy instruments, respectively;

$a = (a_1, a_2, \dots, a_s)'$ is the vector of the structural nonstochastic coefficients to be estimated (all the other known coefficients of the model are excluded from this vector and included in the functional operators);

$u_t = (u_{1t}, u_{2t}, \dots, u_{mt})'$ is the vector of structural stochastic disturbances (or error terms) at time t, having zero mean and independently and identically distributed over time, with finite contemporaneous covariance matrix (Σ) and independent of all the predetermined variables.

Let the model be used for an optimal control experiment over the time periods h+1, h+2, ..., h+r, conditional on the initial state, y_h; a welfare loss function for periods h+1, h+2, ..., h+r needs to be specified. In this paper, a quadratic function of some selected endogenous and control variables has been assumed:

$$w = \sum_{t=h+1}^{h+r} (y_t - y_t^*)' H_t (y_t - y_t^*) + \sum_{t=h+1}^{h+r} (x_t - x_t^*)' K_t (x_t - x_t^*) \tag{2.2}$$

where H_t and K_t are weights matrices, y_t^* and x_t^* are vectors of given targets and r is the planning horizon. Although the restrictions implicit in a quadratic welfare loss function are well known, (2.2) has been chosen because it allows the use of very simple minimization algorithms, like that proposed by

Holbrook (1974), whose application is straightforward when a FORTRAN simulation code is available for the model.

In a deterministic control framework ($u_t=0$ for each t; see Chow, 1975, p.280 and Kendrick, 1981, ch.3), the optimal policies are determined by minimizing the welfare loss function subject to the constraints (2.1) with $u_t=0$ for each t. Let $\bar{x}_t$, t=h+1, h+2, ..., h+r, be the relevant (unknown) values of the policy instruments. Performing the deterministic control experiment on a model with estimated coefficients ($\hat{a}$), the estimation errors in $\hat{a}$ will give rise to an uncertainty into the calculated optimal policy instruments ($\hat{x}_t$). We are interested in analyzing the "random vectors" $\hat{x}_t-\bar{x}_t$, starting from the information on the random vector a given by the estimation method which has been adopted for the structural model. In practice, we are looking for an estimate of the covariance matrix (standard errors, in particular) of the optimal policy instruments.

Three different methods can be used for this purpose. They will be referred to as:

- Analytic simulation on coefficients
- Monte Carlo on coefficients
- Stochastic simulation with re-estimation.

It must be noted that not only are there technical differences in the computational algorithms, but there are some basic conceptual differences among the methods.

Stochastic simulation with re-estimation tries to deal with the "small sample" distribution of the vectors $\hat{x}_t-\bar{x}_t$ directly.

Monte Carlo on coefficients starts from the estimate of the asymptotic covariance matrix of the structural coefficients, treats this matrix as an approximation to the small sample covariance matrix of the coefficients and derives the consequences of this assumption on the vectors $\hat{x}_t-\bar{x}_t$.

Also analytic simulation on coefficients starts from the estimated asymptotic covariance matrix of the structural coefficients and derives the asymptotic covariance matrix of the vectors $\hat{x}_t-\bar{x}_t$.

3. ANALYTIC SIMULATION ON COEFFICIENTS

This method, based on the numerical differentiation approach used in Bianchi et al. (1981a,b), is an extension to nonlinear models of the fully analytical method developed, for linear models, by Friedmann (1980, 1981); (see also Hughes-Hallet, 1979, for methods to evaluate the sensitivity of policy instruments to changes in the structural coefficients).

The method relies on the property, well known in large sample theory (see, for example, Rao, 1973, p.388), that asymptotic normality of sample statistics can be maintained through transformations, even nonlinear, provided they are continuously differentiable.

If the model is linear, the vectors $\hat{x}_t$ can be analytically expressed as functions of the restricted reduced form coefficients. On their turn, the restricted reduced form coefficients can be analytically expressed in terms of the structural form coefficients, and their asymptotic covariance matrix can be calculated using the methods by Goldberger et al. (1961), or Dhrymes (1973). Given the restricted reduced form covariance matrix, Friedmann's method derives analytically the asymptotic covariance matrix of the elements of the vectors $\hat{x}_t - \bar{x}_t$ in the control period.

The formal elegance of the method and the advantages which are usually connected with the use of analytical methods are, unfortunately, made to some extent useless by some computational burden related, in particular, to the dimensions of the covariance matrix of the restricted reduced form coefficients in overidentified models (see Calzolari, 1981). Additional complications would arise if the model was nonlinear, since an explicit linearization would be required. A straightforward procedure, relating the values of the optimal policy instruments to the structural coefficients, seems to be computationally simpler, even if the relationship is not explicit.

If we assume that, as the sample period length T increases, asymptotically

$$(3.1) \qquad \sqrt{T}\,(\hat{a} - a) \sim N(0,\Psi)$$

then, given the values of exogenous variables $z_{h+1}, z_{h+2}, \ldots, z_{h+r}$ and the starting values of endogenous variables y_h, asymptotically,

$$(3.2) \qquad \sqrt{T}\,\mathrm{vec}[\hat{x}_{h+1}-\bar{x}_{h+1};\hat{x}_{h+2}-\bar{x}_{h+2};\ldots;\hat{x}_{h+r}-\bar{x}_{h+r}] \sim N(0,G_{hr}\Psi G'_{hr})$$

where G_{hr} is the $(pr \times s)$ matrix of first order derivatives of

the elements of the vectors $\bar{x}_t$ with respect to the elements of a. If the computation is performed at point $\hat{a}$, and the available (consistent) estimate $\hat{\Psi}$ is used instead of the unknown Ψ, then $\hat{G}_{hr}\hat{\Psi}\hat{G}'_{hr}$ is a consistent estimate of $G_{hr}\Psi G'_{hr}$; the division by the sample period length, T, leads to the result we are looking for, the estimate of the covariance matrix of a multinormal distribution which approximates the small sample distribution of the random vectors $\hat{x}_t - \bar{x}_t$ over the control period.

Even for medium or large scale models there is usually no particular difficulty in computing the above derivatives with numerical methods (finite differences). The so obtained approximation to the covariance matrix is asymptotically exact if the policy instruments are continuously differentiable functions of the structural coefficients (see Rao, 1973, p.388) and if the estimated structural coefficients are consistent and asymptotically normally distributed. Given the kind of nonlinearities involved in most econometric models, the first condition should not be restrictive, even if the relations cannot be represented in explicit analytical terms. The second condition can be proved, under wide assumptions, in case of linear dynamic models (see, for example, Schmidt, 1976) or in case of nonlinear static models (Gallant, 1977). For the general case of nonlinear dynamic systems, formal proofs are not available, but the heuristic argumentations in Gallant (1977, pp.73-74) suggest that, even in the general case, it is reasonable to suppose that such a condition is satisfied; otherwise, the procedure should be considered approximated, not only for small samples, but even in the large sample case.

4. MONTE CARLO ON COEFFICIENTS

This method can be summarized as follows. Let $\hat{\Psi}/T$ be the available estimate of the covariance matrix of the structural coefficients $\hat{a}$.

1) A vector $\tilde{a}$ of pseudo random numbers, with mean $\hat{a}$ and covariance matrix $\hat{\Psi}/T$, is generated.
2) The pseudo-random coefficients vector $\tilde{a}$ replaces the original estimates $\hat{a}$ and the model is used for the deterministic optimal control experiment, obtaining the vectors of pseudo-random policy instruments $\tilde{x}_t$ at time $t=h+1$, $h+2$, ..., $h+r$.

The process is repeated from step 1 to 2 and the desired results follow from the computation of the sample variances of the elements of all the vectors $\tilde{x}_t$ computed in the various replications.

A difficulty may arise in the generation of the pseudo-random vectors $\tilde{a}$. The usual generation methods are, in fact, based on Choleski triangularization of the matrix $\hat{\Psi}/T$ (see Cooper and Fischer, 1974, or Nagar, 1969, for example) and this is possible only if such a matrix is positive definite. Unfortunately, this is not always the case. For example, when in a large scale model the length of the time series does not allow the application of system estimation methods, the matrix $\hat{\Psi}/T$ must be built block by

block (see, for example, Brundy and Jorgenson, 1971, p.215, for limited information instrumental variables estimates) and it is not necessarily positive definite. In this case, the generation of the pseudo-random coefficients vectors $\tilde{a}$ should pass through the generation of shorter vectors with full rank covariance matrix, involving some additional computational difficulties.

This problem clearly does not arise if only the diagonal blocks of the $\hat{\Psi}/T$ matrix are taken into account, as in the work of Cooper and Fischer (1974), and Fair (1980a). In our experiments it was always possible to use the complete matrix $\hat{\Psi}/T$.

5. STOCHASTIC SIMULATION WITH RE-ESTIMATION

This method can be summarized as follows. Let $\hat{\Sigma}$ be the available estimate of the covariance matrix of the structural disturbances.

1) T vectors of pseudo-random numbers, $\tilde{u}_t$, $t=1,2,...,T$, (each of which having multinormal distribution, zero means and covariance matrix equal to the available $\hat{\Sigma}$), are generated. The method by Nagar (1969) can be applied if $\hat{\Sigma}$ is positive definite; otherwise, the method by McCarthy (1972a) can be used.
2) The vectors $\tilde{u}_t$ are inserted into the model, where the structural coefficients are maintained fixed at their originally estimated values, and the model is solved over all

the sample period, obtaining for the endogenous variables the vectors $\tilde{y}_t$, t=1,2,...T.

3) The vectors $\tilde{y}_t$ are treated as a new set of observations of the endogenous variables and are used to re-estimate the model, thus obtaining a new vector, $\tilde{a}$, of pseudo-estimated structural coefficients.

4) The coefficients $\tilde{a}$ are inserted into the model to produce, via deterministic optimal control, the vectors of pseudo-random policy instruments $\tilde{x}_t$, at time t=h+1, h+2, ..., h+r.

The process is repeated from step 1 to 4 and the desired results follow from the computation of the sample variances of the elements of all the vectors $\tilde{x}_t$ computed in the various replications.

Some complications arise from the treatment of lagged endogenous variables both in the simulation phase (in fact, simulation can be static or dynamic) and in the re-estimation phase (the lagged endogenous variables can be maintained "static", i.e. fixed at some given value, like the historical value, or they can be "dynamic", i.e. their simulation value can be chosen). This problem is discussed in Schink (1971, pp.101-108), who suggests to avoid the static-dynamic and dynamic-static combinations; in all the experiments here performed the static-static combination has been adopted, and a few experiments performed with the dynamic-dynamic combination have led to sufficiently similar results.

This method is frequently used in the literature to derive small sample distributions of estimators for simultaneous

equation systems, when analytical methods are not available (see Mikhail, 1972). The main theoretical limitation is in the possible nonexistence of finite moments in the small sample distribution of the structural form or reduced form coefficients (these last directly related to optimal policy instruments); this topic is discussed, for example, in Dhrymes (1970, p.182), McCarthy (1972b, 1981) and Mariano (1980).

6. SOME EMPIRICAL RESULTS

Experiments have been performed on three different models. One of these is a small linear model, well known and widely used in the literature, which has been chosen to allow the comparison and reproduction of the results. The other two models are medium sized nonlinear models of national economies. They have been used here just as an experimental basis to test the efficiency of the above described algorithms and to compare their relative merits.

The first set of experiments is related to the linear Klein-I model. The model, described in Klein (1950), consists of 3 stochastic equations plus 3 identities; it includes 4 exogenous variables and 3 lagged endogenous variables; an additional income distribution equation has been inserted, as in Theil (1964). The sample estimation period is 1921-1941, with annual data, and the parameters estimates used for the experiments have been obtained by means of two stage least squares (2SLS).

The welfare loss function used in these experiments is the one

Table 1

Klein-I model. Initial estimates: 2SLS 1921-1941
Policy instruments and standard errors 1933-1936

		Pol.ins.	An.sim.	M.C.	Stoc.sim. & re-est. OLS	In.IV	LIVE
1933	W2	6.154	.215	.219	.164	.481	.472
	T	8.556	.515	.510	.411	.584	.485
	G	14.35	.981	.981	.600	1.23	1.11
1934	W2	6.239	.216	.220	.135	.284	.233
	T	8.373	.338	.337	.280	.328	.311
	G	14.01	.605	.605	.308	.513	.479
1935	W2	6.157	.271	.267	.131	.258	.204
	T	7.549	.279	.277	.215	.267	.282
	G	13.75	.426	.428	.242	.351	.350
1936	W2	6.355	.415	.398	.201	.399	.397
	T	6.943	.263	.267	.224	.258	.251
	G	13.23	.249	.260	.201	.249	.249

specified by Theil (1964, pp.80-81) as the sum of the quadratic deviations from desired values of three endogenous variables (consumption, investment and income distribution) and of three control variables (W2= government wage bill, T= business taxes and G= government expenditure) over a planning period of 4 years (1933-1936).

Table 1 displays the estimated optimal policies and their standard errors calculated with the three methods (plus two additional variants of stochastic simulation with re-estimation). Our experiments have been performed in an open loop framework. Therefore, analytic simulation, which is expected to be exactly equivalent to Friedmann's analytical method, supplies the same results as Friedmann (1981, p.425) for all the variables only in the first year, but not in the subsequent years.

The results related to stochastic simulation with re-estimation are slightly anomalous. While the starting values

of the parameters had been obtained by 2SLS, re-estimation after each stochastic simulation run has not been performed by 2SLS, but, for computational simplicity, by ordinary least squares (OLS) and by limited information instrumental variables, inefficient (In.IV) and efficient (LIVE) (in these last two cases starting from OLS, see Brundy and Jorgenson, 1971, 1974).

The results displayed in Table 2 are related to the nonlinear annual model of the Italian economy developed by ISPE (Istituto di Studi per la Programmazione Economica, Roma).

Table 2

ISPE model of Italian economy. Initial estimates: LIVE 1955-1976
Standard errors of policy instruments 1974-1976 (percentage)

		An.sim.	M.C.	Stoc.sim. & re-est. OLS	In.IV	LIVE
1974	AUXTAX	5.15	5.05	3.30	4.30	4.72
	AUXW	7.01	7.24	5.22	6.68	6.89
	ACSIM	11.6	10.1	8.46	11.0	10.8
	IES	10.9	9.92	7.15	8.85	8.62
1975	AUXTAX	7.84	7.88	6.05	7.61	7.58
	AUXW	6.15	6.91	5.86	7.60	7.66
	ACSIM	14.4	14.4	14.1	12.6	14.1
	IES	5.73	5.39	3.90	4.70	4.93
1976	AUXTAX	2.53	2.96	2.43	2.69	2.76
	AUXW	3.24	3.08	2.54	2.97	3.36
	ACSIM	7.10	6.70	5.14	5.79	6.57
	IES	8.36	7.95	6.80	8.01	7.93

The model, described in Sartori (1978), consists of 19 stochastic plus 15 definitional equations; there are 75 estimated coefficients. The initial estimates used for the experiments have been obtained by means of limited information instrumental variables efficient method (LIVE) over the sample period 1955-1976 (data are annual).

The welfare loss function has been specified as a weighted sum of quadratic deviations from desired values of four endogenous variables (price deflator for private consumption, gross output in the private sector, unemployment and foreign trade balance) and of four control variables over a planning horizon of 3 years (1974-1976). The four control variables are the following: AUXTAX= auxiliary variable to control direct taxes (neutral value =1), AUXW= auxiliary variable to control wages and salaries in the industrial sector (neutral value =1), ACSIM= social security contribution rate in manufacturing industry, IES= fixed investment in agriculture and public sectors. The standard errors are displayed as percentages of the values of the control variables.

It is clear from Table 2 that, for this model, most methods remain approximately equivalent.

The results displayed in Table 3 are related to a small-scale nonlinear version of the model of the Irish economy developed by the Central Bank of Ireland.
The model, described in Bradley and Kelleher (1978), consists of 44 equations, 12 of which are stochastic relations estimated over the sample period 1958-1975 (data are annual). The estimates used in the experiments have been obtained by means of limited information instrumental variables efficient method (LIVE).

Over a planning horizon of 3 years (1972-1974), the welfare loss function, as suggested in Bradley and O'Raifeartaigh (1981), has been specified as a weighted sum of quadratic deviations from desired values of the following six endogenous variables: balance of payments deficit, public sector borrowing requirements,

Table 3

Central Bank of Ireland model. Initial estimates: LIVE 1958-1975
Standard errors of policy instruments 1972-1974 (percentage)

		An.sim.	M.C.	Stoc.sim. & re-est. OLS	In.IV	LIVE
1972	G	1.72	1.51	1.69	3.02	2.98
	PER	2.55	2.58	2.70	3.19	3.13
	S	6.18	5.51	5.27	7.80	7.76
	TVAT	21.0	22.1	7.87	7.89	7.94
1973	G	3.63	3.52	3.06	4.16	4.02
	PER	2.77	2.85	2.89	2.86	2.85
	S	2.56	2.38	2.32	2.84	2.89
	TVAT	6.23	5.97	2.19	2.32	2.31
1974	G	3.39	3.28	2.81	3.43	3.22
	PER	.333	.387	.311	.422	.418
	S	2.35	2.45	2.04	2.67	2.63
	TVAT	3.59	2.93	3.33	3.82	3.93

reserves, real disposable personal income, unemployment rate and real GNP. The following four instruments are used: G= government expenditure on goods and services (real), PER= income tax personal allowances (nominal), S= government subsidies (nominal), TVAT= average value-added tax rate (percent).

7. CONCLUDING REMARKS

Some conclusions can be drawn from the analysis of the previous tables.

First of all, it is clear that the differences in the estimates of the standard errors for the three methods cannot be always ascribed to the limited number of replications in the stochastic simulation experiments. However, granted that the three methods differ from one another even from a conceptual point of view, the results for the re-estimation of coefficients method are, in most cases, substantially different from the others.

It is just the case to note here that the relative situation among the three methods seems to be reversed with respect to the analysis of forecast errors in Bianchi and Calzolari (1982). In that case, in fact, similarity was usually found between results of analytic simulation and of stochastic simulation with re-estimation. We would like to conclude this point by noting that the differences among the three methods would surely be worth a deeper investigation; moreover, it would be interesting to extend the analysis to the complete distribution of policy instruments, as well as to their joint distribution, rather than confining the analysis to the standard errors.

Finally, it must be pointed out that from a computational point of view the analytic simulation approach should be considered the most efficient method; however, its efficiency is to some extent reduced by the constraints on the choice of the tolerance in the minimization algorithm and of the variation of the coefficients in the computation of the derivatives. In fact, when a derivative is numerically computed, the tolerance in the minimization algorithm must be of some order of magnitude less than the change of the optimal values of instruments induced by the variation in each coefficient. This may force to use tolerance values smaller than otherwise required, thus increasing the number of iterations necessary to get the desired numerical solution.

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