Constraints on credit, consumer behaviour and the dynamics of wealth

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Abstract: This note develops a simple macro model where the pattern of wealth accumulation is determined by a credit multiplier and by the way households react to short run fluctuations. In this setup, long term wealth dynamics are eventually characterized by the presence of endogenous cycles.

Keywords: Credit constraints, Financial development, Consumer confidence, Endogenous business cycles, Nonlinear dynamics.

JEL classification: O41, E32, C61.

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1. Introduction

The relation between collateral requirements or credit constraints and business cycles is a relevant theme of discussion in today's macroeconomic analysis. This subject has gained visibility with the contributions of Bernanke and Gertler (1989) and Kyotaki and Moore (1997), and was further developed with the work of Aghion, Bacchetta and Banerjee (2001, 2004), Demirguç-Kunt and Levine (2001), Amable, Chatelain and Ralf (2004), Aghion, Angeletos, Banerjee and Manova (2005) and Caballé, Jarque and Michetti (2006), among others. The basic intuition underlying the previous references is that markets where firms face some degree of credit constraints are markets where investment is strongly pro-cyclical and, thus, main economic aggregates will be subject to amplified volatility that tends to persist over time. Basically, the lesson one withdraws from this literature is that cycles are more likely to be observed for specific levels of financial development than for others.[1] For instance, Caballé, Jarque and Michetti (2006), hereafter CJM, conclude that stability is found for low and high levels of financial development, while for intermediate levels endogenous business cycles dominate.

In this note, we recover the CJM model to present an alternative approach to the formation of endogenous cycles for intermediate levels of financial development. Basically, we consider a same scenario as the previous authors, but with two important changes: first, we consider physical capital as the unique production input (we ignore the country specific input with a constant supply assumed in the referred model); second, we introduce a mechanism through which households respond to short run wealth deviations from a potential wealth level. Furthermore, the analysis will be undertaken under an endogenous growth framework, in the sense that it assumes an AK production function.

This note is organized as follows. Section 2 presents the model's structure, section 3 discusses local dynamics; section 4 characterizes global dynamics and section 5 concludes.

2. The Structure of the Model

Consider a competitive economy populated by a large number of households and firms. Firms produce a tradable good under an AK production function, $y_t=Ak_t$, with

[1] The level of financial development is translated on the degree of constraints to credit.

A>0 a technology index and k_t, y_t the per capita levels of physical capital and output in moment t.[2] We assume that capital fully depreciates after one period and, hence, i_t=k_t, with i_t per capita investment. Households have the possibility to lend their financial resources directly to firms if the marginal productivity of capital (A) is above the economy's nominal interest rate (r); hereafter, we impose this constraint on parameters: A>r.

If the credit market is subject to some kind of imperfection, firms' financial resources (that we designate by wealth) will serve as collateral for the loans, and thus firms cannot borrow an amount over μw_t, with w_t the level of per capita wealth and μ a credit multiplier that reflects the degree of financial development of the economy. Households and firms agree on applying to the productive projects the largest amount of credit that can be subject to transaction, and thus investment in moment t corresponds to $i_t=(1+\mu)\cdot w_t$. Finally, the structure of the model is complete with a difference equation reflecting wealth dynamics,

$$w_{t+1} = y_t - r\mu w_t - c_t, \; w_0 \text{ given.} \tag{1}$$

Equation (1) states that wealth in moment t+1 corresponds to income in t, less the cost of debt and less the resources allocated to consumption (c_t is per capita consumption). In the CJM model, c_t corresponds to a constant fraction of income less debt payment. We generalize this assumption by considering that the marginal propensity to consume depends on the observable difference between effective levels of wealth and expected or potential wealth. In practice, agents react to business cycles by adopting the following rule: the higher the level of last period's wealth relatively to the benchmark level of wealth, the more optimistic households will be and, accordingly, the higher will be the share of consumption out of income. In other words, low (high) levels of observed accumulated wealth relatively to a benchmark level will imply a precautionary (confident) behaviour that is translated on a higher (lower) savings rate.

Formally, we consider $c_t = c\cdot(y_t - r\mu w_t)\cdot g(w_{t-1})$, $c\in(0,1)$ and $g(w_t)$ a positive, continuous and differentiable function, with g'>0. Consider w_t^* as the potential level of wealth; this is supposed to represent a wealth trend that grows at a same rate, γ, for all t.

[2] To simplify, assume that population does not grow.

Thus, function g will be such that $g(w_t)|_{w_t=w_t^*}=1$, $g(w_t)|_{w_t>w_t^*}>1$ and $g(w_t)|_{w_t<w_t^*}<1$.

The following functional form fulfils the required properties: $g(w_t)=\left(\frac{w_t}{w_t^*}\right)^a$, a>0.

The reduced form of the dynamic system is straightforward to obtain given the previous information,

$$w_{t+1}=[A+(A-r)\cdot\mu]\cdot\left[1-c\cdot\left(\frac{w_{t-1}}{w_{t-1}^*}\right)^a\right]\cdot w_t \tag{2}$$

Because production is subject to constant marginal returns, all relevant variables (k_t, y_t, i_t, c_t and w_t) grow at a constant positive rate in the steady state. Let this rate be γ, and thus we define variable $\hat{w}_t\equiv\frac{w_t}{(1+\gamma)^t}$ and constant $\hat{w}^*\equiv\frac{w_t^*}{(1+\gamma)^t}$. Effective wealth grows at a same rate as potential wealth in the steady state, but before this long run result is eventually accomplished, the growth rates might differ. Rewriting (2),

$$\hat{w}_{t+1}=\frac{A+(A-r)\cdot\mu}{1+\gamma}\cdot\left[1-c\cdot\left(\frac{\hat{w}_{t-1}}{\hat{w}^*}\right)^a\right]\cdot\hat{w}_t \tag{3}$$

Equation (3) has a unique equilibrium point: $\overline{w}=\left[\frac{1}{c}\cdot\left(1-\frac{1+\gamma}{A+(A-r)\cdot\mu}\right)\right]^{1/a}\cdot\hat{w}^*$.

This result allows us to state the following proposition,

Proposition 1*: The wealth model, with credit constraints and consumption reaction to deviations from last period's potential wealth, reveals that the higher is the level of financial development of the economy, the larger will also be the amount of accumulated wealth, in the steady state.*

Proof: Take the steady state expression for the wealth variable and compute derivative $\overline{w}_\mu=\frac{\partial\overline{w}}{\partial\mu}$. The computation gives,

$\overline{w}_\mu = \frac{\hat{w}^*}{a} \cdot \left[\frac{1}{c} \cdot \left(1 - \frac{1+\gamma}{A+(A-r)\cdot\mu} \right) \right]^{\frac{1-a}{a}} \cdot \left[\frac{1}{c} \cdot \frac{(1+\gamma)\cdot(A-r)}{[A+(A-r)\cdot\mu]^2} \right]$. Because this is a positive value, one infers that the accumulated level of wealth is positively correlated with financial development (measured by the credit multiplier parameter, μ)□

Note, relatively to the steady state value, that to guarantee $\overline{w} > 0$, the following inequality must hold: $A+(A-r)\cdot\mu > 1+\gamma$. This condition imposes a floor to the value of the credit parameter: $\mu > \frac{1+\gamma-A}{A-r}$ is the minimal requirement for the economy to be able to accumulate wealth.

3. Local Dynamics

In this section, we address the dynamics of equation (3) in the neighbourhood of point $\overline{w}$. This requires defining variables $\tilde{w}_t \equiv \hat{w}_t - \overline{w}$ and $\tilde{z}_t \equiv \hat{w}_{t-1} - \overline{w}$. With these variables, we turn equation (3) into a two equation system with two endogenous variables and just one time lag,

$$\begin{cases} \tilde{w}_{t+1} = \frac{A+(A-r)\cdot\mu}{1+\gamma} \cdot \left[1 - c \cdot \left(\frac{\tilde{z}_t + \overline{w}}{\hat{w}^*} \right)^a \right] \cdot (\tilde{w}_t + \overline{w}) - \overline{w} \\ \tilde{z}_{t+1} = \tilde{w}_t \end{cases} \quad (4)$$

Around the balanced growth path, system (4) takes the linearized form

$$\begin{bmatrix} \tilde{w}_{t+1} \\ \tilde{z}_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & -a \cdot \frac{A+(A-r)\cdot\mu-(1+\gamma)}{1+\gamma} \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \tilde{w}_t \\ \tilde{z}_t \end{bmatrix} \quad (5)$$

Note that the steady state values of variables $\tilde{w}_t$ and $\tilde{z}_t$ are, in both cases, 0. Proposition 2 synthesizes the local dynamics result.

***Proposition 2**: The wealth model under analysis is locally stable for* $\mu \in \left(\frac{1+\gamma-A}{A-r}; \frac{\left(\frac{1+a}{a}\right)\cdot(1+\gamma)-A}{A-r} \right)$*; when* $\mu = \frac{\left(\frac{1+a}{a}\right)\cdot(1+\gamma)-A}{A-r}$*, the system undergoes a Neimark-Sacker bifurcation.*

Proof: The Jacobian matrix in (5) has a positive determinant, $Det(J) = a \cdot \frac{A+(A-r)\cdot\mu-(1+\gamma)}{1+\gamma}$, and its trace is $Tr(J)=1$. Thus, stability conditions $1-Tr(J)+Det(J)>0$ and $1+Tr(J)+Det(J)>0$ are always satisfied. The only possible bifurcation occurs when the eigenvalues of the matrix are a pair of complex conjugate values with modulus equal to one, which is equivalent to say that $Det(J)=1$. The equality expression in the proposition is determined by solving this last condition in order to μ. Stability requires $1-Det(J)>0$☐

From proposition 1, we have concluded that the less constrained credit is, the larger is the amount of wealth the economy accumulates in the long run, while from proposition 2 one observes that there is a stability ceiling: if freedom to offer credit is too high, the guarantee that the steady state level of wealth is achieved vanishes. Therefore, one can interpret this theoretical structure as indicating both the advantages of financial development and of financial responsibility, in the sense that excessive credit may disrupt the financial system as agents fail to pay back the large amount of resources they have borrowed.

Local dynamics conceal meaningful features of the model. First, cycles appear to be absent. The theory on nonlinear dynamics points to the eventual presence of cycles after a bifurcation. In our concrete system, we should expect to effectively encounter a fixed point in the stability area identified in proposition 2, and a-periodic motion after the bifurcation and before instability truly sets in. This becomes evident with the global analysis of the following section. Second, the global analysis of this specific model will show that some points of stability are present in the locally unstable area, a result that can be used to justify a same kind of conclusion as the one in the CJM model: financial instability (cycles) occurs for intermediate levels of development, while stability is found for low and high values of the credit market development.

4. Global Dynamics

To address global dynamics consider an array of reasonable parameter values:[3] [A; c; γ; r; w^*; a]=[1; 0.75; 0.04; 0.03; 1; 0.7] and let us elect μ as the bifurcation parameter. In this particular case, the system is stable for 0.041<μ<1.573.

Figure 1 shows the bifurcation diagram; a bifurcation, that occurs for μ=1.573, separates an area of stability from an area where invariant cycles can be observed. After the region where endogenous cycles are evidenced, it follows a state where stability and instability alternate. Recall that $\tilde{w}_t$ is a variable that was modified twice: first, it was detrended and, then, adjusted to obtain a balanced growth path where the variable takes the value zero. In the long run, the original variable w_t grows exponentially, with a detrended value equal to $\overline{w}$.

*** Figure 1 here ***

To understand that this framework produces everlasting endogenous business cycles for specific values of the credit parameter, we present a time series of $\tilde{w}_t$ in figure 2. The Neimark-Sacker bifurcation, or Hopf bifurcation in discrete time, is able to generate a kind of dynamics that reproduces considerably well real world business cycles, in the sense that several consecutive periods of increasing wealth are followed by some periods where there is a slowdown on the growth of wealth, and so on.

*** Figure 2 here ***

To close the graphical presentation, and taking the same value of μ as in figure 2, we draw an attractor that reveals the long run relation between $\tilde{w}_{t-1}$ and $\tilde{w}_t$ (figure 3) and the basin of attraction that furnishes the set of initial points that allow for a convergence towards the long run attractor (figure 4).

*** Figures 3 and 4 here ***

5. Conclusions

[3] In particular, observe that the marginal propensity to consume corresponds to 75% of income, that the growth rate is 4% and that the interest rate is 3%.

Following the literature on credit constraints and business cycles, we have considered a basic AK endogenous growth model, where financial development is addressed through a credit multiplier and where economic agents take consumption decisions by weighting last period's difference between observed and potential wealth (if this difference is positive, consumers optimism rises and they will increase their consumption share of income; with a negative difference, households become less enthusiastic about consumption and they will prefer to increase the marginal propensity to save, as a precautionary measure).

The proposed setup is able to elucidate about two important points:

i) As the financial development level rises, per capita wealth, in the steady state, also increases;

ii) Fixed point stability is found for low levels of financial development (and, thus, low levels of accumulated wealth); an intermediate level of the credit multiplier allows to identify a-periodic cycles generated through a Neimark-Sacker bifurcation; and high levels of financial development are characterized by a scenario where stability and instability alternate.

The instability result for high levels of the credit multiplier adds a new feature relatively to the CJM model: too high credit multipliers, associated to loans with no collateral, imply a high risk in the credit market, in the sense that borrowers may not carry out their debt payment obligations. In this case, instability may be interpreted as a scenario of financial crisis that imposes the need to restore a certain ceiling on the level of available credit.

References

Aghion, P.; G. M. Angeletos; A. Banerjee and K. Manova (2005). “Volatility and Growth: Credit Constraints and Productivity-Enhancing Investment.” *NBER working paper* nº 11349.

Aghion, P.; P. Bacchetta and A. Banerjee (2001). “Currency Crises and Monetary Policy in an Economy with Credit Constraints.” *European Economic Review*, vol. 45, pp. 1121-1150.

Aghion, P.; P. Bacchetta and A. Banerjee (2004). “Financial Development and the Instability of Open Economies.” *Journal of Monetary Economics*, vol. 51, pp. 1077-1106.

Amable, B.; J. B. Chatelain and K. Ralf (2004). "Credit Rationing, Profit Accumulation and Economic Growth." *Economics Letters*, vol. 85, pp. 301-307.

Bernanke, B. and M. Gertler (1989). "Agency Costs, Net Worth, and Business Fluctuations." *American Economic Review*, vol. 79, pp. 14-31.

Caballé, J.; X. Jarque and E. Michetti (2006). "Chaotic Dynamics in Credit Constrained Emerging Economies." *Journal of Economic Dynamics and Control*, vol. 30, pp. 1261-1275.

Demirguç-Kunt, A. and R. Levine (2001). *Financial Structure and Economic Growth*. Cambridge, MA: MIT Press.

Kyotaki, N. and J. Moore (1997). "Credit Cycles." *Journal of Political Economy*, vol. 105, pp. 211-248.

Figures

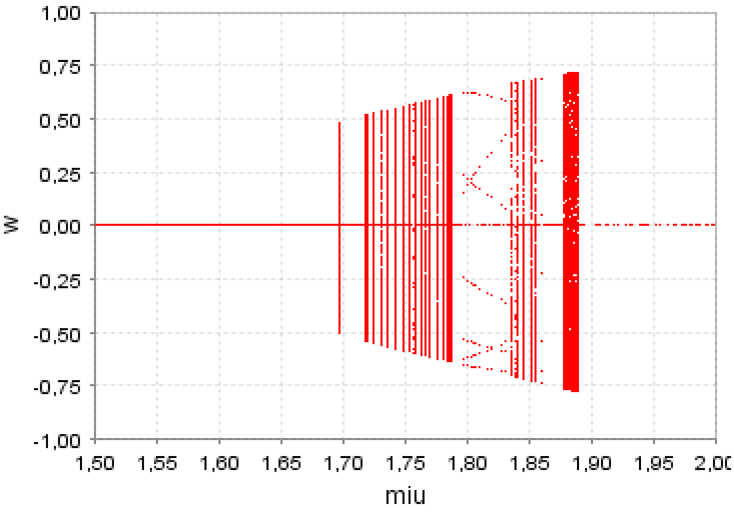

Figure 1 – Bifurcation diagram (1.5<μ<2).

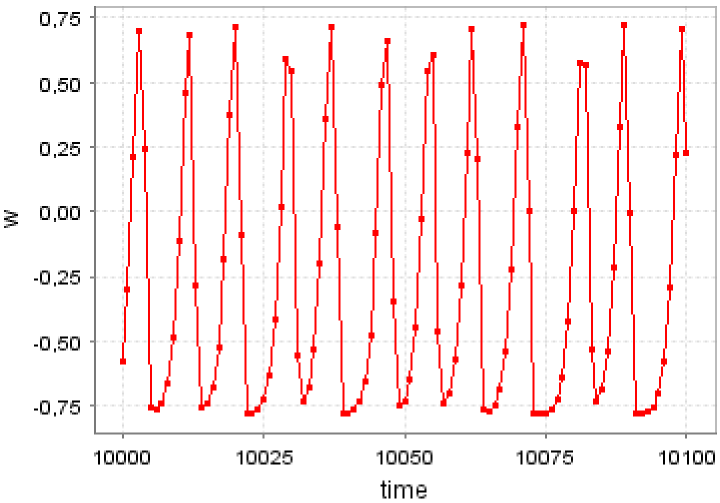

Figure 2 – Time series (μ=1.891).

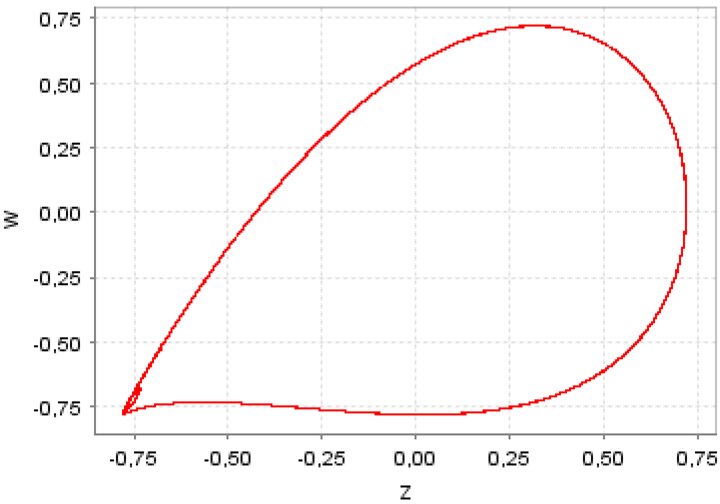

Figure 3 – Attracting set (μ=1.891).

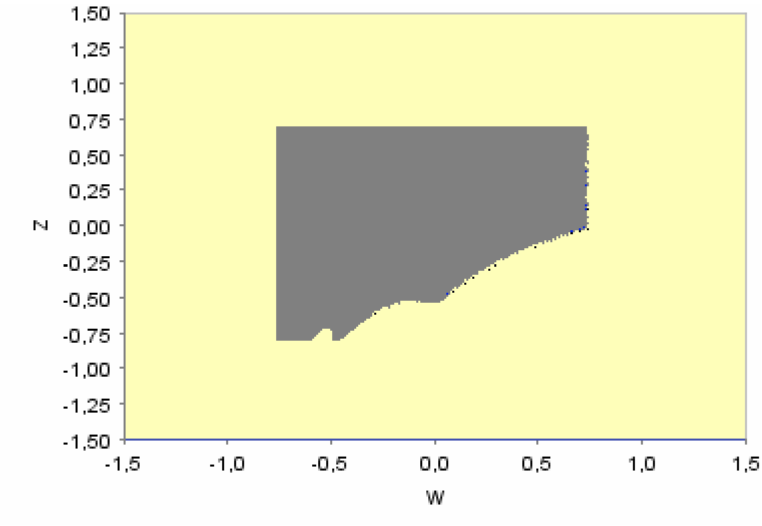

Figure 4 – Basin of attraction (μ=1.891).