The Allocation of Investment across Vintages of Technology

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Abstract

This paper proposes a new mechanism that explains continued investment in older-vintage technology which rests on complementarity between *long-lived* and *short-lived* vintage-specific capital. The main result is a threshold condition that relates the rate of vintage-specific technological progress ($\hat{q}$) to two investment patterns: if $\hat{q}$ is above the threshold, all investment is allocated to the newest-vintage technology; otherwise, firms direct part of their investment to older-vintage technologies. The evidence supports our model's empirically testable implications: as $\hat{q}$ declines, investment is allocated more toward older-vintage technology; and equipment-price changes depend on capital's heterogeneous rates of depreciation.

1 Introduction

How much investment in old-fashioned equipment should be allocated instead to state-of-the-art equipment? This study answers this question under the framework

[*]As his last student, I express my deep sorrow for the death of Gary Saxonhouse. This paper is based on a chapter of my Ph.D. dissertation submitted to the University of Michigan. I am grateful to my advisers, John Laitner, Dmitriy Stolyarov, Miles Kimball, Gary Saxonhouse, and Jan Svejnar for their guidance and encouragement. I also thank Hiroshi Ohashi and Tsuyoshi Nakamura for providing their data on steel furnaces, Kozo Kiyota, Yasuyuki Todo, Takanobu Nakajima, and participants in the seminars at the University of Michigan, Kansai University, Yokohama National University, the University of Tokyo, Aoyama Gakuin University, and Osaka University for their helpful comments.

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of conventional vintage growth models by assuming two types of complementary capital in each vintage production function. The model shows that the optimal allocation depends on the trade-offs between the magnitude of the remaining stocks of the old-fashioned complementary capital and the relative advantages of the frontier technology.

This paper's model has two key elements: (i) it is a vintage growth model in which a certain technology is built into each unit of capital; and (ii) each vintage production function consists of two kinds of complementary capital that have different rates of depreciation.[1] Both types of investment are irreversible. The main idea is intuitive; if one type of capital depreciates more slowly (*long-lived*) than the other (*short-lived*), then investing in short-lived capital with an old-fashioned technology is sometimes rationalized in order to exploit the existing stock of complementary long-lived capital with that vintage technology. Long-lived capital is possibly intangible capital such as knowledge, software, and system capital, structures or networks depending on context, while short-lived capital is probably equipment in most instances.

The main result is a threshold condition that relates the rate of vintage-specific technological progress ($\hat{q}$) to two investment patterns: (i) if the rate of technological progress is above the threshold (the product of the long-lived capital's share and the difference in the rates of depreciation of the two capital types—which shows the relative importance of the remaining stock of long-lived capital), then all new investment will concentrate on the two types of capital with the frontier technology; (ii) otherwise, a part of new investment will be allocated to short-lived capital with older-vintage technology to exploit the existing stock of old-fashioned long-lived capital. The result can be tested empirically. For example, the model predicts that the rate of productivity growth should be negatively correlated with the relative size of investment in old vintages. The evidence supports this; the lower the technological progress of industries, the higher the intensity of maintenance and repair, which is a proxy for investment in vintage technology.

Another important result is that the price-changes (obsolescence) of vintage short-lived capital depend not only on the technological progress, but also on the difference in the rates of depreciation of the two capital types. In particular, if the rate of the technological progress is below the threshold, then the prices of old-fashioned

[1] In this study, "depreciation" refers solely to *physical depreciation*, and excludes *obsolescence*, which is explicitly treated as endogenously determined price changes in the following analysis.

short-lived capital remain unchanged even when the rate of technological progress is positive. A strand of literature (e.g., Gordon (1990), Hulten (1992), Greenwood, Hercowitz, and Krusell (1997), and Cummins and Violante (2002)) uses vintage models to measure the embodied technological progress from quality-adjusted equipment prices. My model implies that such estimates may be biased downwards by about 1.5%, making the rate of technological progress even faster than previously thought.

Related literature describes how other types of mechanisms can make it optimal to remain using obsolete technology: (i) vintage-specific human capital that is acquired through learning-by-doing (Chari and Hopenhayn (1991), Parente (1994), and Jovanovic and Nyarko (1996)); and (ii) complementarity in production *across* vintages (Jovanovic (2009)). These papers describe the mechanisms by which persistent use of obsolete technology can plausibly arise, but do not explore their investment mechanisms and empirically testable implications. The present paper, by contrast, provides the investment mechanisms, and puts the proposed explanation to empirical test.

Another strand of related literature analyzes Solow-type vintage growth models with two capital types (Greenwood, Hercowitz, and Krusell (1997), Gort, Greenwood, and Rupert (1999) and Laitner and Stolyarov (2003)). These models incorporate capital heterogeneity, but do not provide an explanation for investment in old vintage technology. The model presented here features investment allocation across and within vintages while comprehending the prevalent properties of Solow-type growth models.

The rest of the paper is organized as follows. Section 2 presents the model's framework and a characterization of a balanced growth path. Section 3 compares the model's prediction with empirical data such as investment patterns and changes in prices. Section 4 discusses examples of long-lived capital. Then Section 5 concludes the paper.

2 The Model

The model has two key elements: (i) each vintage of capital works with a separate production function that has a vintage-specific productivity level; and (ii) each vintage production function consists of two kinds of vintage compatible capital with different rates of depreciation. Apart from the second element, all assumptions are essentially identical to those of Solow (1960)'s.

We assume that the economy is competitive, and firms have perfect foresight and are rational. Each unit of capital is designed for a vintage-specific (v) technology that has an individual production function with a specific productivity level, q_v. At time $t \geq 0$, vintage $v \in [0, t]$ technology is available. Each vintage production technology requires three complementary inputs: two types of *vintage-specific* capital, A (*long-lived*) and B (*short-lived*), and *vintage-nonspecific* labor, L. A and B depreciate at the rates δ^A and δ^B where $\delta^A \leq \delta^B$. Let capital's subscript v denote a specific vintage v technology that is *embodied* in each type of capital, while L_v expresses the amount of labor that is *employed* for a vintage v. Each vintage-specific production function is of the Cobb–Douglas form of:

$$Y_v(t) = q_v A_v(t)^\alpha B_v(t)^\beta L_v(t)^{1-\alpha-\beta}, \tag{1}$$

where $Y_v(t)$ is output at the current time t produced by the vintage v technology, and α and β are constant shares of two capital types.[2] The frontier technology's productivity level (q_t) and labor supply (L) grow at constant rates, $\hat{q} > 0$ and $\hat{L} \geq 0$, where hat ($\hat{\ }$) denotes the time derivative of the natural log of the argument.

Assume that each output produced by a vintage-specific production function is homogeneous and equal to one constant physical unit over time. The aggregate homogeneous output can be defined as:

$$Y(t) \equiv \int_0^t Y_v(t)\, dv. \tag{2}$$

The aggregate homogeneous output is divisible into consumption and two types of irreversible capital investment. Investment in a unit of capital with any vintage requires one unit of homogeneous output. A fixed portion ($\sigma \in (0, 1)$) of aggregate output is allocated to investment, and each type of capital is freely disposable. In the following analysis, the time index (t) is dropped to simplify the exposition.

The distinctive feature of the current model is that it uses a different mechanism from existing models to explain the persistent use of old technology. The current model assumes complementarity of two types of capital within the same vintage tech-

[2]In the model presented here, we omit Hicks-neutral technological progress that affects all vintages of production, because the omission does not change the main results. Chapter 3 in Aruga (2006) shows the case when the neutral technological progress is also embedded.

nology, while existing models hinge on vintage human capital acquired by learning-by-doing (Chari and Hopenhayn (1991), Parente (1994), and Jovanovic and Nyarko (1996)) or complementarity of capital across vintages (Jovanovic (2009)).

2.1 Aggregation across Vintages

The model's simple structure makes it possible to aggregate the separated vintage production functions into a simple aggregate production function as in Solow (1960). Now suppose $A_v, B_v, L_v > 0$ for some v. Then, under the competitive market assumption, a firm's profit maximization conditions in terms of two capital types and labor are:

$$MPA_v = \alpha \frac{Y_v}{A_v} = P_v^A R_v^A, \tag{3}$$

$$MPB_v = \beta \frac{Y_v}{B_v} = P_v^B R_v^B, \text{ and} \tag{4}$$

$$MPL = (1 - \alpha - \beta) \frac{Y_v}{L_v} = W, \tag{5}$$

where MPX_v, P_v^X, and R_v^X are the marginal products, the prices in units of homogeneous output, and the rates of return of specific types of vintage capital, where $X \in \{A, B\}$. MPL and W are the marginal product of labor and the wage. MPL and W do not have vintage subscripts because labor is vintage-nonspecific. Note that:

$$R_v^X - \delta^X + \hat{P_v^X} = r \; \forall \; v \in [0, t] \tag{6}$$

where r is the interest rate, because holding any type of capital of any vintage must be identical for investors net of the depreciation (δ^X) and the obsolescence ($\hat{P_v^X}$). Note also that $P_v^X \in [0, 1]$ because each type of capital is freely disposable and investment in capital types with existing vintage technology is always possible.

Define the aggregate inputs to be the summation of marginal-product-weighted inputs relative to those of the frontier technology such that:

$$X \equiv \int_0^t \frac{MPX_v}{MPX_t} X_v \, dv = \int_0^t \frac{P_v^X R_v^X}{P_t^X R_t^X} X_v \, dv = \int_0^t \frac{Y_v / X_v}{Y_t / X_t} X_v \, dv = \frac{X_t}{Y_t} Y \tag{7}$$

where $X \in \{A, B\}$, and

$$L \equiv \int_0^t L_v \, dv = \int_0^t \frac{Y_v/L_v}{Y_t/L_t} L_v \, dv = \frac{L_t}{Y_t} Y. \tag{8}$$

Note that when rates of return (R_v^X) are unique across vintages, the defined aggregate input of that type simply shows the total values of that type in units of the price of frontier capital of that type. Then, using (1), (2), (7), and (8), aggregate output can be expressed as:

$$Y = \left[\frac{Y_t}{A_t} A\right]^{\alpha} \left[\frac{Y_t}{B_t} B\right]^{\beta} \left[\frac{Y_t}{L_t} L\right]^{1-\alpha-\beta} = q_t A^{\alpha} B^{\beta} L^{1-\alpha-\beta}. \tag{9}$$

Interestingly enough, the aggregate production function has the same form as (1) with frontier technology level q_t and the aggregate inputs.

Now using (1), (5), and (8), define aggregate consolidated capital as:

$$J \equiv \int_0^t J_v \, dv \equiv \int_0^t \left[q_v A_v^{\alpha} B_v^{\beta}\right]^{\frac{1}{\alpha+\beta}} \, dv, = \left[q_t A^{\alpha} B^{\beta}\right]^{\frac{1}{\alpha+\beta}} .$$

Using J_v and J, the labor allocation across vintages is given by:

$$L_v = \frac{J_v}{J} L.$$

In this way, the consolidated vintage capital (J_v), aggregate consolidated capital (J), and aggregate labor (L) determine L_v, and thus Y_v, MPX_v, and Y without specifying the prices of the various capital types.[3]

2.2 Balanced Growth Path

This subsection identifies the balanced growth path (BGP) of the model. The economy's BGP of interest is where *(i) the newest technology is continuously introduced, (ii) all the existing endogenous variables including the aggregate amounts defined by (7)–(9) grow at constant rates, and (iii) there is investment in both types of capital.* The first condition is from the typical observation in the real economies. The second condition follows the definition of BGP in growth literature. The last condition follows the discussion in Shell and Stiglitz (1967)–if firms only invest in one type, the

[3] The aggregate production function can be expressed as $Y = J^{\alpha+\beta} L^{1-\alpha-\beta}$, which has the same form as Solow (1960). J stands for Solow's *Jelly Capital.*

economy converges to the origin, which is not a rational BGP.

The previous subsection characterized the state of an economy including the labor allocation and the output distribution across vintages given the distribution of the two types of vintage capital. Given the state, the next step is to determine evolution of the economy: investment patterns in two dimensions, across vintages and capital types in a BGP.

The existence of two types of vintage compatible capital complicates the characterization of investment patterns and price distribution across vintages and capital types. Although Solow (1960)'s vintage growth model with a single type of vintage capital presumes that all new investment is concentrated on the capital that has the newest available vintage, there are other possibilities in the current model.[4] Suppose in the current model that, initially, the allocation of long-lived and short-lived capital with a specific vintage v is optimal such that the prices of two capital types are the same. Then, over time, the existing stock of the vintage long-lived capital becomes relatively abundant compared with that of the vintage short-lived capital without investment because of the difference in the rates of depreciation. This might result in a rise in the productivity and the price of the vintage short-lived capital. In a special case, investment in the vintage short-lived capital may become more attractive than that in the frontier technology.

As discussed above, investment may consist of the part of the mass of the frontier technology (i.e. introduction of new technology), X_t, and the part of distribution of the existing technologies (i.e. supplement of the depreciated capital), I_v. Figure 1 illustrates the relationship of these types of investment. Since the total amount of the investment is the fixed portion of the homogeneous output by assumption, the allocation of investment holds the following relationship:

$$\sigma Y = I^A + I^B = \int_0^t I_v^A \, dv + A_t + \int_0^t I_v^B \, dv + B_t. \tag{10}$$

Note that the prices of investment goods must be unity because otherwise there is no investment in that type.

In analyzing the investment patterns, the key is the relationships of capital prices

[4]There is no investment in old technology in Solow (1960)'s model because the capital that embodies the newest available vintage technology always has higher productivity than other obsolete vintage capital, given that there is only one type of vintage-specific capital and that the labor input is freely allocated across vintages.

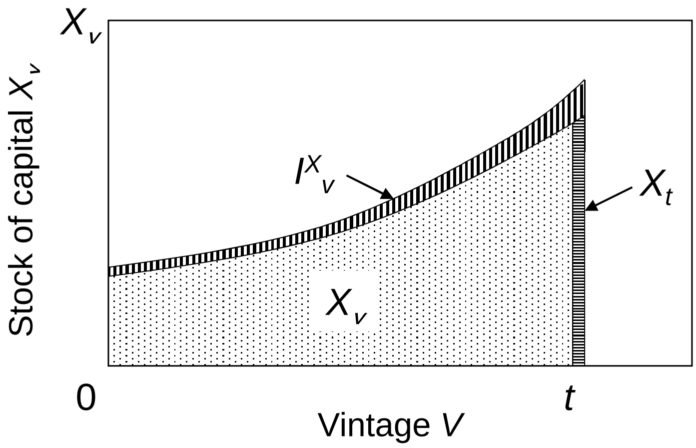

Figure 1: Allocation of investment in capital X. X_v, I_v^X, and X_t denote size of capital stock of X that embodies vintage technology v, investment in X that embodies existing vintage technology v, and investment in X that embodies the frontier technology t, respectively.

across types and vintages in (3) and (4). Consider long-lived capital with two different vintages, A_v and $A_{v'}$ where $v, v' \in [0, t]$, and $v \neq v'$. Because the interest rate r must be the same across vintages, from (3) and (6) we have:

$$\frac{Y_v}{P_v^A A_v} - \frac{Y_{v'}}{P_{v'}^A A_{v'}} = \frac{\hat{P_{v'}^A} - \hat{P_v^A}}{\alpha}. \tag{11}$$

Because both terms on the left-hand side of (11) grow at constant rates and the right-hand side is constant in a BGP, both sides must be zero. The same argument applies to short-lived capital (B). Therefore, using (6), for $X \in \{A, B\}$ and $\forall\, v, v' \in [0, t]$, we have:

$$\hat{P_v^X} = \hat{P_{v'}^X} = \hat{P^X}, \text{ and} \tag{12}$$

$$R_v^X = R_{v'}^X = R^X. \tag{13}$$

(1), (3)–(5), and (13) provide the relationships of prices across vintages as follows:

$$P_v^A = \left[\frac{q_v}{q_{v'}}\right]^{\frac{1}{\alpha+\beta}} \left[\frac{B_v/A_v}{B_{v'}/A_{v'}}\right]^{\frac{\beta}{\alpha+\beta}} P_{v'}^A, \text{ and} \tag{14}$$

$$P_v^B = \left[\frac{q_v}{q_{v'}}\right]^{\frac{1}{\alpha+\beta}} \left[\frac{B_v/A_v}{B_{v'}/A_{v'}}\right]^{-\frac{\alpha}{\alpha+\beta}} P_{v'}^B. \tag{15}$$

(14) and (15) provide:

$$\frac{q_v}{q_{v'}} = \left[\frac{P_v^A}{P_{v'}^A}\right]^\alpha \left[\frac{P_v^B}{P_{v'}^B}\right]^\beta. \tag{16}$$

(14) and (15) imply that, the lower the relative amount of a specific type of capital with a vintage, the higher the relative price of that type of capital with the vintage. Furthermore, the ratios of prices across vintages are proportional to the ratios of technology levels. (16) summarizes these relationships.

In a BGP, investment in a type of capital within an existing specific vintage must be continuous in order for the growth rate of the stock of that type of capital to be constant. Therefore, there will be four possible investment schemes regarding the existing capital types with a specific vintage (v) in a BGP: *there is positive continuous investment (a) only in A_v; (b) only in B_v; (c) neither in A_v nor B_v; and (d) both in A_v and B_v*. Furthermore, because (12) and (14) imply $\frac{B_v/A_v}{B_{v'}/A_{v'}}$ is constant $\forall\, v, v' \in [0, t]$ in a BGP, *investment schemes must be unique across vintages $v \in [0, t]$*.

Using (14)–(16) and the classification of these investment schemes , we characterize investment patterns across vintages in a BGP as follows.

Proposition 1 (Investment patterns across vintages of technology)**.** *In a BGP,*

(i) (Fast Case) *if $\hat{q} \geq \alpha(\delta^B - \delta^A)$, then the investment scheme is (c) $\forall\ v \in [0, t]$, where firms invest in neither A_v nor B_v; and*

(ii) (Slow Case)*otherwise, the investment scheme is (b) $\forall\ v \in [0, t]$, where firms continuously invest in B_v.*

Proof: See Appendix A.1.1.

In short, when technological progress is fast enough, there is no investment in capital types with old technologies. Otherwise, there will be investment in short-lived capital that embodies old technology in order to exploit excessive existing stock of long-lived capital. The threshold of the rate of technological progress is the product of long-lived capital's share (α) and the difference in the rates of depreciation of the two capital types ($\delta^B - \delta^A$). α shows the importance of long-lived capital in a production function, and ($\delta^B - \delta^A$) shows the rate of increase in the relative size of long-lived capital compared with short-lived capital. Intuitively, investment in obsolete short-lived capital will be made when the increase in relative size of the compatible long-lived capital (along with the rise in the price of short-lived capital) is fast enough compared with the vintage-specific technological progress.[5]

Next, given the investment scheme in a BGP, consider the allocation of the two capital types within vintages in a BGP , to simplify the exposition of the following analysis, define the aggregate effective labor, $N \equiv q_t^{1/(1-\alpha-\beta)}L$, and use lower case letters to express the aggregate amounts in units of effective labor: $a \equiv A/N$ and $b \equiv B/N$. Then, the profit maximization conditions and the laws of motion of capital types provide the allocation of capital types in a BGP as follows.

Proposition 2 (Allocation of capital types in a BGP)**.** *In a BGP, a and b have the following relationship from the profit maximization conditions:*

$$\beta a^{\alpha} b^{\beta-1} - \alpha a^{\alpha-1} b^{\beta} = \begin{cases} 0 & \text{when } \hat{q} \geq \alpha(\delta^B - \delta^A), \text{ and} \\ \delta^B - \left[\delta^A + \frac{\hat{q}}{\alpha}\right] & \text{otherwise,} \end{cases} \tag{17}$$

and a condition from the laws of motion:

$$\sigma a^{\alpha} b^{\beta} = \begin{cases} \left[\frac{\hat{q}+\alpha\delta^A+\beta\delta^B}{\alpha+\beta} + \hat{N}\right][a+b] & \text{when } \hat{q} \geq \alpha(\delta^B - \delta^A), \text{ and} \\ \left[\delta^A + \frac{\hat{q}}{\alpha}\right] a + \delta^B b + \hat{N}[a+b] & \text{otherwise,} \end{cases} \tag{18}$$

and there are unique, constant, and stable BGP values of $a = a^*$ and $b = b^*$ that satisfy conditions (17) and (18).

Proof: See Appendix A.1.2.

Figure 2 shows the possible relationships between a and b implied by (17) and (18), and equilibrium (BGP). The black circle and solid lines correspond to the Fast Case, and the white circle and dashed lines to the Slow Case. (17) is a straight line from the origin with the slope α/β in the Fast Case, while it is a convex curve above the straight line in the Slow Case. (18) is a quasi-triangle curve that passes through the origin. The curve of (18) in the Slow Case is more skewed to the a side than in the Fast Case because long-lived capital (A) is relatively more attractive when the interest rate is low.

The allocation of capital types and investment is determined by the combination of Propositions 1 and 2. In the Fast Case, the total investment—the product of

[5] Note that short-lived capital's share (β) does not enter the threshold. This is because the inequality relation between prices of different vintages of short-lived capital is independent of β, although β affects the relative levels of prices in Cobb–Douglas production technology. See equation (15) to confirm this point.

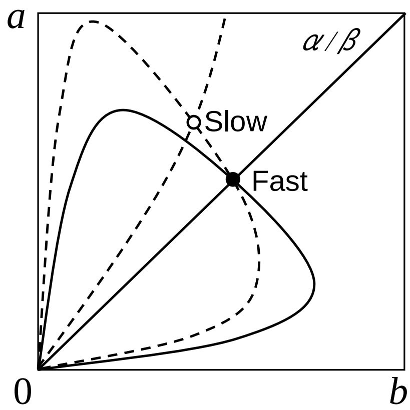

Figure 2: Relationship between a and b implied by (17), upward sloping curves from the profit maximization condition, and (18), quasi-triangle curves from the laws of motion. Solid and dashed lines correspond to the Fast and Slow Cases, respectively. Black and white circles show the BGP equilibrium in the Fast and Slow Cases.

the aggregate output (Y) and the exogenous saving rate (σ)–is simply divided into the frontier two types of capital in the proportions $\frac{\alpha}{\alpha+\beta}$ and $\frac{\beta}{\alpha+\beta}$. After the initial investment, the two types of capital with a specific vintage decline at the rates of depreciation. In the Slow Case, part of the total investment is allocated to the older short-lived capital such that their prices remain at exactly the same level of homogeneous output. The remaining part of the total investment is allocated to the frontier capital in proportion to the equilibrium values $\frac{a^*}{a^*+b^*}$ and $\frac{b^*}{a^*+b^*}$.

2.3 Properties of the Two Types of BGP

Table 1 summarizes the properties of the two BGP cases, which is obtained from the proofs of Propositions 1 and 2, and observed in Figure 2. In the Fast Case, the investment schemes of all the available vintages are (c), and all investment is allocated to the capital types with the frontier technologies, A_t and B_t. Both prices of the two capital types of a specific vintage decline exponentially over time as shown in Figure 3 (i). The decline in the prices of short-lived capital is slower than that of long-lived capital because short-lived capital with a vintage becomes relatively scarce compared with long-lived capital of that vintage over time. This is because their depreciation rates differ.

The ratios of investment in the frontier capital types, A_t/B_t, of market values of their vintage, $[P_v^A A_v]/[P_v^B B_v]$, and of the aggregate amounts of them, $A/B = a/b$, all

Table 1: Properties of the two cases of BGP.

BGP	(i) Fast Case	(ii) Slow Case
Technological progress ($\hat{q}$)	$\geq \alpha(\delta^B - \delta^A)$	$< \alpha(\delta^B - \delta^A)$
Investment scheme	(c)	(b)
Investment pattern	Frontier A/B only	Frontier A/B, and obsolete B
$\hat{P_v^A}^*$	$-[\hat{q} + \beta(\delta^B - \delta^A)]/(\alpha + \beta)$	$-\hat{q}/\alpha$
$\hat{P_v^B}^*$	$-[\hat{q} - \alpha(\delta^B - \delta^A)]/(\alpha + \beta)$	0 (Remains 1)
A_v/B_v	$> \alpha/\beta$	$> \alpha/\beta$
$[P_v^A A_v]/[P_v^B B_v]$	α/β	$> \alpha/\beta$
$A/B(= A_t/B_t)$	α/β	$> \alpha/\beta$

* Changes in vintage capital prices in the Fast Case are given by (14) and (15) applying $v' \to t$ with the condition $A_t/B_t = \alpha/\beta$ and $P_t^A = P_t^B = 1$. Those in the Slow Case are given by (16) with the condition $P_t^A = P_v^B = 1 \,\forall\, v$.

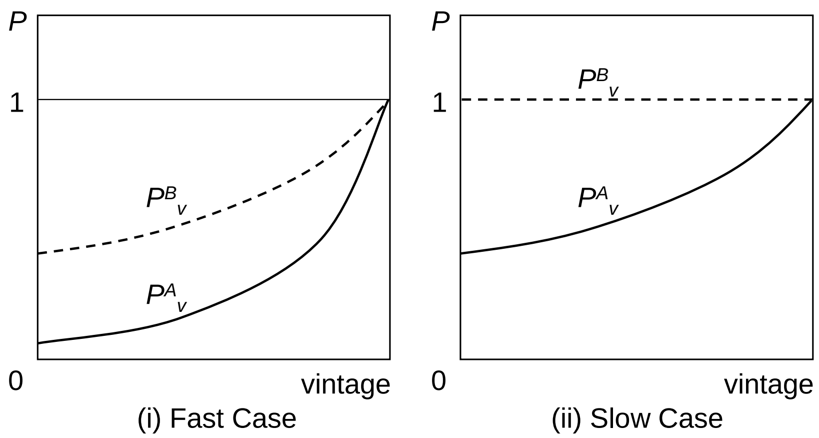

Figure 3: Prices of capital across vintages: (i) in a Fast Case, $\hat{q} \geq \alpha(\delta^B - \delta^A)$; and (ii) in a Slow Case $\hat{q} < \alpha(\delta^B - \delta^A)$.

keep α/β even when $\hat{q}$ changes. The reason is that prices of vintage capital types adjust such that they cancel out the difference in their rates of depreciation. Indeed, the total depreciation—the sum of obsolescence ($\hat{P_v^X}$) and physical depreciation (δ^X)—is $[\hat{q} + \alpha\delta^A + \beta\delta^B]/[\alpha + \beta]$ for both capital types. The allocations of labor skew toward newer technology as $\hat{q}$ increases.

In the Slow Case, investment is not only allocated to the frontier technology capital types, A_t and B_t, but also to short-lived capital with obsolete vintages, $B_v \, \forall \; v \in [0, t]$. This is because the marginal products of obsolete short-lived capital without investment are higher than those of the newest capital types, and thus there will be investment in short-lived capital with obsolete vintage technology. Therefore, while the prices of long-lived capital decline exponentially over time, the prices of short-lived capital across vintages remain at the same level as new output as shown in Figure 3 (ii).

The ratios of investment in the frontier capital types, A_t/B_t, of market values of vintage capital, $[P_v^A A_v]/[P_v^B B_v]$, and of the aggregate amounts, $A/B = a/b$, are all the same and larger than α/β. Unlike in the Fast Case, the ratio A/B rises as $\hat{q}$ declines, because a decline in $\hat{q}$ lowers the interest rate r that makes long-lived capital relatively more attractive. The distributions of short-lived capital as well as labor skew toward older technology as $\hat{q}$ declines.

Solow-type vintage growth models can be interpreted as specific cases of the current model. For example, the BGP of Laitner and Stolyarov (2003)'s model is a special case of the Fast Case in the present model; they assume a single rate of depreciation, $\delta^A = \delta^B$, which ensures the Fast Case ($\hat{q} \geq \alpha(\delta^B - \delta^A) = 0$) as long as the rate of technological progress is positive. The current model shows, however, that even when rates of depreciation differ, similar results to those in their model are observed in the Fast Case. The BGP of Shell and Stiglitz (1967)'s model is also a special case of the current model where there is no technological progress ($\hat{q} = 0$) and the rates of the depreciation of two capital types are the same ($\delta^A = \delta^B$).

3 Empirical Evidence and Relevancies

In the last section, the BGP analysis of the model reveals two distinct investment patterns depending on the relationships between the rate of technological progress and the threshold. This section shows how these results complement the existing literature by exploring two important empirically testable implications: the proportion of investment in older-vintage technologies depends negatively on the rate of technological progress; and the absolute rate of price changes in short-lived capital with a specific vintage depends on the difference in the rates of depreciation of two capital types as well as the rate of technological progress. These two points are tested by using the variations in data across industries and equipment. Furthermore, it is shown that the model is consistent with other empirical facts such as heterogeneity of capital lives.

3.1 Investment in Obsolete Capital

Although counterintuitive, investment in old-fashioned equipment—which is less productive than cutting-edge equipment—is observed in the real economy. For example, production of steam locomotives continued even after more efficient diesel locomotives were introduced.[6] Chari and Hopenhayn (1991), Parente (1994), Jovanovic and Nyarko (1996), and Jovanovic (2009) propose mechanisms that could generate investment in old technology, but did not explored the allocation of investment.

The current model provides quantitative predictions about the investment allocations of short-lived and long-lived capital across vintages. When the rate of technological progress is below the threshold, investment in short-lived capital of older-vintage is rationalized. In this case, the ratio of investment in short-lived capital of older-vintage to the total investment in short-lived capital (sum of older-vintage and

[6]See Figure 5 in Felli and Ortalo-Magne (1998). Other examples are found in cotton spinning technology (Saxonhouse and Wright (2000)), and steel furnace technology (Nakamura and Ohashi (2008)). Data in Nakamura and Ohashi (2008) show that the annual rates of decline in the capacity of open-hearth furnaces (OHFs) in Japan for 10 years after the introduction of more productive basic oxygen furnaces (BOFs) and for five years after the peak usage of OHFs were about 5% and 9% respectively, which are both much smaller than the rates of depreciation of "metalworking machines" in the U.S. official statistics, of 12%. This implies there had been investment in obsolete OHF technology after the new BOF technology became available.

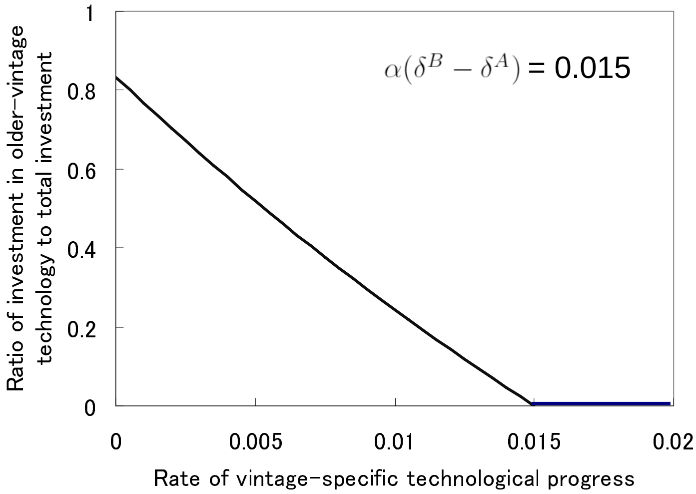

Figure 4: Relationship between the ratio of investment in short-lived capital that embodies older-vintage technology to the total investment in short-lived capital, and rate of vintage-specific technological progress. ($\alpha = 0.15, \beta = 0.25, \delta^A = 0, \delta^B = 0.1, \hat{L} = 0.02$)

newest-vintage) will be:[7]

$$\text{Ratio} = \frac{\int_0^t I_v^B dv}{\int_0^t I_v^B dv + B_t} = \frac{\delta^B - \delta^A - \frac{\hat{q}}{\alpha}}{\delta^B + \hat{N}}. \tag{19}$$

Figure 4 shows the relationship between the ratio of the investment and vintage-specific technological progress given by (19) when the threshold ($\alpha(\delta^B - \delta^A)$) is 0.015.[8] As can be seen, when technological progress is below the threshold, the smaller the $\hat{q}$, the larger the allocation of investment in older-vintage capital. When technological progress is above the threshold, there is no investment in older-vintage technology.

The empirical analysis here focuses on varieties of the production function at the industry level in order to compare Figure 4 and empirical data. We make five assumptions: (i) the economy is segregated at the industry level; (ii) all types of structures and equipment in an industry are homogeneous short-lived capital, and there is a type

[7] In a Slow Case BGP, $\hat{A}_v = -\delta^A$ because there is no investment in vintage A_v capital. Because rates of return are constant in a steady state, using (3), (4), and the changes in prices in Table 1, the growth rate of B_v is given by $\hat{B}_v = \hat{P_v^A} - \hat{P_v^B} + \hat{A}_v = -\frac{\hat{q}}{\alpha} - \delta^A$. Then, using (26) observe, $I_v^B = [\hat{B}_v + \delta^B]B_v = [\delta^B - \delta^A - \frac{\hat{q}}{\alpha}]B_v$. Thus, $\int_0^t I_v^B\, dv = [\delta^B - \delta^A - \frac{\hat{q}}{\alpha}]B$. On the other hand, from (10), (30), and (32), we have $\int_0^t I_v^B\, dv + B_t = I^B = [\hat{B} + \delta^B - \hat{P^B}]B = [\delta^B + \hat{N}]B$.

[8] In the following analysis, all time units are year.

of complementary, vintage-specific, and longer-lived capital;[9] (iii) maintenance and repair (MR) expenditure is proportional to investment in older-vintage short-lived capital; (iv) production functions of industries are homogeneous except for the rate of vintage-specific technological progress; and (v) each industry's multifactor productivity (MFP) growth is proportional to its vintage-specific technological progress.

We assume (iii) because there are no appropriate investment data that distinguish investment vintages. Firms try to keep using old technologies by MR and/or investment in new machinery with obsolete vintages. For example, replacement of tires or the muffler of obsolete automobiles may be considered as investment, while that of wiper blades may not be. Whether this kind of expenditure is considered as MR or capital investment depends on its magnitude.[10] These assumptions should be plausible for the purpose of checking the consistency of the model with empirical data without undermining the main messages of the model.

Figure 5 shows the relationships between MFP growth from 2005 to 2006 and the intensity of MR expenditure relative to capital investment in 86 U.S. manufacturing industries (NAICS four digit level) in 2006.[11] As can be seen, there is a statistically significant negative correlation between the relative MR expenditure and MFP growth.[12] This indicates that the less technological progress there is in an industry, the more investment there is towards older-vintage technology. The result is consistent with the presented model's unique prediction, which is not explored in existing models.

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[9] We discuss the examples of long-lived capital in Section 4.

[10] The U.S. Economic Census defines MR as "Included ... are payments made for all maintenance and repair work on buildings and equipment... Excluded from this item are extensive repairs or reconstruction that was capitalized, which is considered capital expenditures...".

McGrattan and Schmitz (1999) documents that data from 1961 to 1993 in Canada show that the size of MR expenditure on equipment/structure, reaches 50%/20% of the investment in equipment/structure, respectively, and MR can be a substitute for investment during recessions. In an extreme case, when the Canadian iron ore industry experiences a severe downturn, even equipment investment almost falls to zero, yet the industry still spent considerable expenditure on MR. Mullen and Williams (2004) develops a model that explains substitutability of MR and investment in the newest type of capital, however, their model does not provide predictions about investment in older technology.

[11] Data on MFP growth is obtained from BLS. Total expenditure on capital and MR expenditure are obtained from the 2007 U.S. Economic Census ("CEXTOT" and "PCHRPR").

[12] The correlation coefficient is -.34 and is significant at the 1% level.

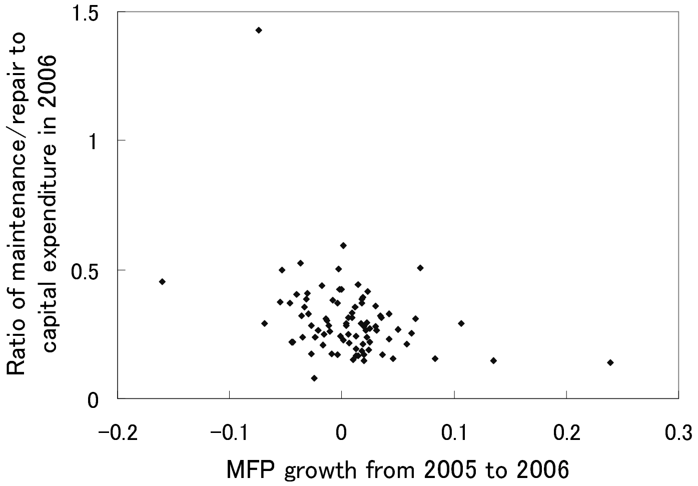

Figure 5: Negative relationships between multifactor productivity (MFP) growth from 2005 to 2006 and relative intensity of repair expenditure to capital investment in 2006 in 86 U.S. manufacturing industries (4-digit NAICS code). Source: BLS and 2007 Economic Census.

3.2 Changes in Capital Prices

The vintage growth models in the existing literature derive direct proportional relationships between the rate of vintage-specific technological progress and changes in equipment prices.[13] However, in the current model, this is not necessarily the case when there is a difference in the rates of depreciation of the two capital types. From Table 1, we observe:

$$\hat{P_v^B} = \begin{cases} -\frac{1}{\alpha+\beta}\hat{q} + \frac{\alpha}{\alpha+\beta}(\delta^B - \delta^A) & \text{when } \hat{q} \geq \alpha(\delta^B - \delta^A)\text{, and} \\ 0 & \text{otherwise.} \end{cases} \tag{20}$$

The current model predicts that, in the Fast Case, the changes in equipment prices depend not only on the rate of the technological progress, but also on the difference in the rates of depreciation between the short-lived and long-lived capital. In contrast, in the Slow Case, the price of short-lived capital remains at the output price, leaving the rates of technological progress and depreciation irrelevant.

Figure 6 shows the relationship between the changes in prices of short-lived capital ($\hat{P_v^B}$) and the rate of vintage-specific technological progress ($\hat{q}$) given by (20). When

[13]For example, Gordon (1990), Hulten (1992), Greenwood, Hercowitz, and Krusell (1997), and Cummins and Violante (2002).

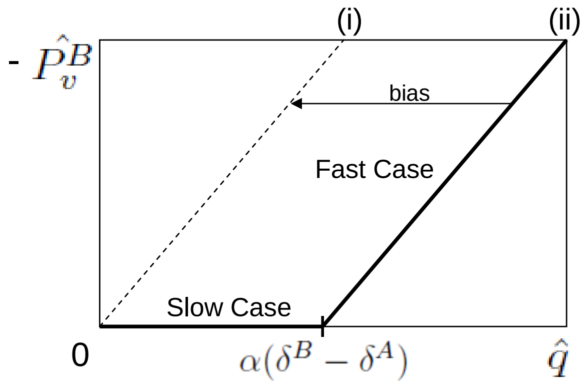

Figure 6: Changes in prices (i) when $\alpha(\delta^B - \delta^A) = 0$ (dashed line), and (ii) when $\alpha(\delta^B - \delta^A) > 0$ (solid line) given by equation (20).

there is no long-lived capital (i.e., $\alpha = 0$ or $\delta^A = \delta^B$), the threshold value ($\alpha(\delta^B - \delta^A)$) is zero. In this case, as (i) shows, there is a direct proportional relationship between the price changes in short-lived capital and the rate of technological progress as in the existing models listed in footnote 13. However, if there is long-lived capital (and thus the threshold value is positive, $\alpha(\delta^B - \delta^A) > 0$), the direct proportional relationship is biased downward to the size of the threshold as (ii) shows. This suggests that, in practice, the estimates of the rate of vintage-specific technological progress in the previous literature may be biased downward up to the size of the threshold, making the true rate even faster than previously thought.

The size of bias may be considerably large. As will be discussed in Section 4, intangible capital is a good candidate for long-lived complementary capital to ordinary physical capital. Suppose that at the aggregate level the share of intangible capital is 15% as suggested by Corrado, Hulten, and Sichel (2006); and the difference in the rates of depreciation of physical and intangible capital is 10%.[14] Then, the ad hoc threshold value is $\alpha(\delta^B - \delta^A) = 0.015$. That is, the actual rate of vintage-specific technological progress may be 1.5% higher than previous estimates.[15]

In order to examine the proposed relationships of (20), the following analysis focuses on well-documented price and depreciation data on various types of equip-

[14] Ideas do not physically depreciate, while the average of the physical rate of depreciation of private nonresidential equipment is about 11%, which is obtained by using the average of total depreciation in Fraumeni (1997) and obsolescence from Gordon (1990).

[15] For example, this size is about one-half of Greenwood, Hercowitz, and Krusell (1997)'s estimate of vintage-specific technological progress, 3.2%.

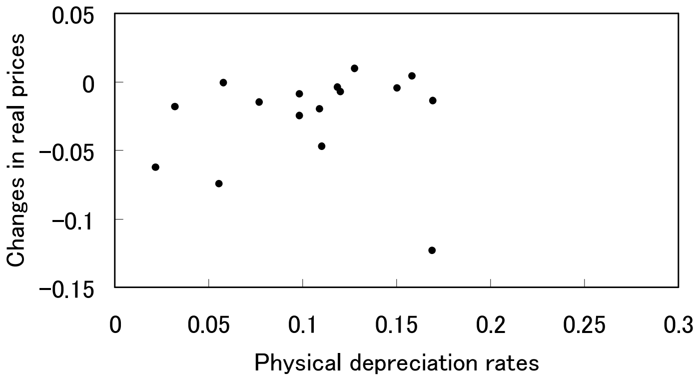

Figure 7: Relationships between the changes in equipment prices and physical rates of depreciation of 16 types of equipment in the U.S. 1947-1983. Source: Gordon (1990) and Bureau of Economic Analysis (BEA).

ment.[16] We make four assumptions to utilize the data: (i) the economy is segregated at the equipment level; (ii) each type of equipment works as the sole short-lived capital of equipment-specific production functions for the corresponding economy; (iii) all production functions utilize a kind of complementary, vintage-specific, and longer-lived capital; and (iv) parameters of the all equipment-specific production functions are the same except for equipment-specific δ^B. Under these assumptions, the equation (20) predicts that price changes and the depreciation rate of equipment will have positive relationships for production with the equipment in the Fast Case.

Figure 7 shows the relationships between the changes in equipment prices and physical depreciation rates of 16 types of equipment in the U.S. for 1947–1983.[17] As the model suggests, there is a statistically significant positive correlation between the price changes and rates of depreciation when the outlier that has the largest price-changes during the period is excluded.[18] Furthermore, as can be seen in Figure 7, changes in equipment prices seem upper-bounded at zero as the Slow Case of (20) suggests. These relationships have never been explored and explained in the existing

[16] $\hat{q}$ cannot be used in addition to the price of equipment in this analysis because there is no estimate of $\hat{q}$ from independent data sources.

[17] The physical depreciation rates are obtained by subtracting the rate of changes in prices (Gordon (1990)) from the total depreciation (sum of obsolescence and physical depreciation) in "BEA Depreciation Estimates."

[18] The correlation coefficient is 0.56 and is significant at the 5% level. The outlier is "office, computing, and accounting machinery", which will be associated with a very high rate of technological progress ($\hat{q}$), breaking the assumption (iv), but consistent with (20).

literature.[19]

3.3 Heterogeneity of Capital Lives

The current model shows that the longevity—which varies inversely with total rates of depreciation (sum of physical depreciation and obsolescence)—of two capital types can be heterogeneous only in certain cases. As can be seen from Table 1, in the Fast Case where $\hat{q} \geq \alpha(\delta^B - \delta^A)$, the total rates of depreciation of short-lived and long-lived capital are the same:

$$\hat{P_v^A} - \delta^A = -\frac{\hat{q} + \alpha\delta^A + \beta\delta^B}{\alpha + \beta} = \hat{P_v^B} - \delta^B, \tag{21}$$

while otherwise (in the Slow Case) long-lived capital literally lives longer:

$$\hat{P_v^A} - \delta^A = -\frac{\hat{q}}{\alpha} - \delta^A > -\delta^B = \hat{P_v^B} - \delta^B. \tag{22}$$

Interestingly enough, heterogeneity of physical rates of depreciation yields the heterogeneity of lives only in the Slow Case.

The prediction in the Fast Case is consistent with technologies that develop very quickly. For example, although software does not physically depreciate while the omputer does, the estimates of total depreciation rates of computer and software are similar; they are .31 and .33 in Fraumeni (1997) and Corrado, Hulten, and Sichel (2006), respectively. The production technology consisting of computers (short-lived) and software (long-lived) has high rates of technological progress and they are complementary and vintage-specific. Therefore, even when the rates of physical depreciation of computer and software are different, the rates of their total depreciation can be the same as (21) predicts.

The prediction in the Slow Case (22) is consistent with the behaviors of systems and their components. While they are complementary and vintage-specific to some degree, data shows that the lives of systems are substantially longer than those of their components as presented in Table 2. This suggests that systems can be considered as long-lived capital. Indeed, as the model predicts, firms invest in new components for

[19] There is no significant correlation if we use total depreciation instead of physical depreciation. This is consistent with the concern expressed by Fraumeni (1997) that there is double counting of obsolescence in the BEA's official estimation of the capital stock.

Table 2: Service lives of systems and their components.

Technology	System	Component
Nuclear power	Plants (60 years)[b]	Nuclear fuels (4 years) [a]
Air transportation	Airframes (15–25 years) [c]	Engines (6 years)[c]
Land transportation	Trucks (14 years) [a]	Tires, etc. (3 years) [d]

[a] From Table 3 in Fraumeni (1997), Private, nonresidential equipment.
[b] From Office of Nuclear Reactor Regulation, U.S. Nulcear Regulatory Commission (2008).
[c] From Table 3 in Fraumeni (1997), Federal, National defense.
[d] From Table 3 in Fraumeni (1997), Durable goods owned by consumers.

old systems in order to keep using the existing system. For example, a large part of investment in nuclear fuel is for nuclear power plants with old generation technology that require different specifications from the newer type of plant.[20] A large system corresponds to a large long-lived capital's share (α), which probably makes it likely the technologies in Table 2 are for the Slow Case.

Previous vintage models do not provide a consistent explanation for the difference in the lives of capital types and the investment in old technology. Most of the existing vintage-growth literature assumes a single type of capital and no heterogeneity of its longevity. Although Laitner and Stolyarov (2003) study a model that assumes two vintage-specific types of capital, the behavior of these types including the realized lives are the same because they assume the same rates of physical depreciation of the types. Greenwood, Hercowitz, and Krusell (1997) and Gort, Greenwood, and Rupert (1999) assume a difference in depreciation rates between two types of capital but no vintage-specific complementarity between the types, which results in investment only in the frontier technology at any as in Solow (1960).

3.4 Other Empirical Relevancies

Two other empirical relevancies support the predictions of the model: high-tech investment patterns during boom and recession; and magnitude relations between the actual rate of technological progress and the proposed threshold. First, as shown in Figure 4, the model implies that acceleration in the rate of vintage-specific techno-

[20]The second generation nuclear power plants built in the 1970s are still in operation although newer and more efficient generation III technology was introduced in the 1990s (U.S. DOE Nuclear Energy Research Advisory Committee and the Generation IV International Forum (2002)).

logical progress can cause reallocation of investment towards modern capital at the aggregate level. This is consistent with investment booms that are concentrated in certain "high-tech" equipment. There is a widely accepted observation that the economic boom in the late 1990s coincided with the diffusion of information technology (IT).[21]

While typical growth models consider investment in IT equipment as a *source* of improvement in aggregate productivity, the current model provides a different viewpoint; the concentration of investment in IT equipment is a *result* of fast improvement in the frontier productivity level. When technological progress of an aggregate economy is fast, firms should concentrate on investing in the capital with the newest technology in order to benefit from the better technology. Otherwise, concentration of investment in high-tech equipment is not necessarily the best decision because older technology with the stock of know-how may be more productive.

Second, it is of interest whether an economy possibly experiences both the Slow Case and the Fast Case. The ad hoc threshold value of $\alpha(\delta^B - \delta^A) = 0.015$ derived in the previous section is comparable in size with the growth rates of labor productivity and multifactor productivity in the postwar U.S. economy. It is likely that the rate of vintage-specific technological progress fluctuates around the threshold value, especially at industry/firm/equipment levels because productivity growths at the disaggregate level typically have wider variations than at the aggregate level.

4 What is Long-lived Capital?

The key assumption of the proposed model is the existence of two types of vintage compatible and complementary capital with different rates of depreciation. By assuming physical capital as short-lived and without specifying what is long-lived capital, empirical data confirm the model's predictions—allocation of investment across vintages of technology, and changes in prices of equipment. These results indicate that the heterogeneity of capital depreciation cannot be negligible in economic analysis.

This section discusses the possible examples of long-lived capital. The model requires three main properties of long-lived capital: lower rate of physical depreciation, vintage-specificity, and complementarity to short-lived capital. We argue that intangible capital, such as know-how, software, and system capital is a promising

[21] See Oliner and Sichel (2003), and Jorgenson, Ho, and Stiroh (2007) for example.

candidate, while other possibilities such as structures and networks should also be appropriate depending on context.

4.1 Intangible Capital

Although intangible expenditure had been simply regarded as an intermediate input in the official economic statistics, recent literature has started considering it as capital stock in production (Hall (2001), Atkeson and Kehoe (2005), and McGrattan and Prescott (2005)). While various types of intangible capital are proposed in the literature,[22] in practice, the BEA has recently started including software (1999) in the official statistics and releasing R&D satellite accounts (2006). Corrado, Hulten, and Sichel (2006)—by considering the private sector's intangible expenditure that aims to increase future output of individual firms as investment—show in their growth accounting that intangible capital's income accounted for 15% of total income in the U.S. nonfarm business sector during the period of 2000–2003, while that of physical capital accounted for 25%. The importance of intangible capital rivals that of physical capital in the modern economy.

Several types of intangible capital possess the properties of long-lived capital. For example, suppose the CD drive (physical capital) in your PC crashes for some reason. Then, would you buy a new PC or merely replace the CD drive? If the change in computer technology occurs quickly enough, you would purchase a new PC because it has much better features. Or you would replace the CD drive to keep using the existing PC with the installed software (intangible capital) that is incompatible with newer types of PCs. In this case, both the PC and software are vintage-specific to some degree, they are complements within vintages, and software is longer-lived than PCs because software does not physically wear or tear.[23] A similar argument applies to the combination of equipment and equipment specific know-how or configuration. Once accustomed to a specific type of equipment, it is sometimes difficult to switch

[22] Although a complete list is still under discussion, a tentative list should include: software (Corrado, Hulten, and Sichel (2006)), R&D (Prucha and Nadiri (1996) and Corrado, Hulten, and Sichel (2006)), brand (McGrattan and Prescott (2005), Corrado, Hulten, and Sichel (2006)), organization (Atkeson and Kehoe (2005), McGrattan and Prescott (2005), Corrado, Hulten, and Sichel (2006)), monopoly franchise (Hall (2001), McGrattan and Prescott (2005)), firm-specific human capital (Laitner and Stolyarov (2003)), and product designs (Laitner and Stolyarov (2003)).

[23] This type of hardware/software combination should apply to audio (analog records, cassettes, compact discs, digital cassettes, and iPods) and video (video cassettes, laser discs, DVDs, and blue-ray discs) players.

to a new generation of that type.[24]

Additionally, production systems that integrate various components can be considered as intangible capital. A production system consists of many components, and the value of the whole system should be higher than the sum of the raw value of each consisting component, because assembling components requires design and labor input.[25] This difference in the value of a whole system and the sum of the values of its raw components can be considered as *system capital.* The system capital should last longer than its components because the system keeps its original ability as long as the components are properly maintained and repaired. This interpretation is consistent with the longevity data on several systems and their components as discussed in Section 3.[26]

4.2 Structures and Networks

Apart from intangible capital, structures are another type of long-lived capital. Suppose a railroad company operates railroad tracks (structures) designed for conventional trains across the country. If the company intends to introduce advanced bullet trains that require wider tracks for their speed, it has to invest in wider railroad tracks that are specifically designed for the new type of trains.[27] Because railroad tracks last longer than railroad equipment, they can be considered as long-lived and short-lived capital.[28] In many cases, the old train network will be used because the value of the stocks of the existing railroad tracks is large, which requires persistent investment in the older types of trains. Similar reasoning may apply to the introduction of electric vehicles (equipment, short-lived) Because they require new types of fuel stations

[24]The roles of intangible and physical capital may reverse depending on context. For example, consider the *Coca-Cola Company* that produces and sells Coca-Cola using its factories (tangible capital) and brand name (intangible capital). Suppose the depreciation rate of its brand name is 60% as suggested by Corrado, Hulten, and Sichel (2006), which far exceeds that of their factories, and the rate of development of new beverages is slow. Then, advertisements for Coca-Cola can be interpreted as an investment in obsolete shorter-lived intangible capital to keep using the obsolete existing stock of longer-lived factories.

[25]If a component is not built into a system, the component alone has no productivity.

[26]A similar argument may apply to the organization, its human capital combination, and more broadly the social system, its citizen combination.

[27]In Japan, *Shinkansen* networks were introduced in the 1960's by constructing their own new tracks in addition to the conventional train network.

[28]The lives of "railroad replacement tracks" and "other railroad structures" are 38 and 54 years, respectively, which is substantially longer than that of "railroad equipment", 28 years in Fraumeni (1997).

(structures, long-lived) that provide battery replacement services, plug-in charging, or hydrogen fuel instead of conventional gasoline or diesel fuel. The production of conventional vehicles will persist for a while because the conventional fuel supply facilities will last longer than conventional vehicles.

Another example of long-lived capital is communication networks, such as DSL or fiber-optic cable. Suppose you have an Internet connection via a 1 Mbps DSL system that uses a conventional metal line. When your DSL modem stops working, you have two options: replace it with a new DSL modem, or invest in a 100 Mbps fiber-optic cable and modem in order to use the new broadband technology. Network cables have a lower physical rate of depreciation compared with modems, networks are compatible with modems, and they are technology-specific.[29] In this way, networks and communication equipment can be considered as long-lived and short-lived capital, respectively.

5 Conclusion

This paper studied a model with a production function consisting of long-lived and short-lived vintage-specific compatible capital. Both types of investment are irreversible. The model predicts two distinctive investment patterns: (i) if the rate of technological progress is above a threshold, then all new investment is concentrated on the capital types that embody the frontier technology; otherwise, (ii) a part of the investment is allocated to obsolete short-lived capital to exploit existing obsolete long-lived capital. Intangible capital such as know-how, software, and system capital, structures and networks can be long-lived capital depending on the context. The short-lived capital is probably equipment in many cases.

As a consequence of the neo-classical assumptions of the model, the model not only comprehends existing vintage growth models, but also provides original quantitative implications: relative intensity of investment in old technology; and relationship between the depreciation rate and the obsolescence of equipment. Two empirical analyses and other empirical relevancies support the model's predictions with some additional but reasonable assumptions. The model with capital heterogeneity provides a rich set of explanations for several economic observations that have not been

[29]The lives of "communication" and "communication equipment" are 11 and 40 years, respectively, in Fraumeni (1997).

well studied, suggesting that economists should pay closer attention to capital heterogeneity.

Avenues for future research consist of both theoretical and empirical work. Theoretically important applications include characterizing transition dynamics and generalizing the production function. Transition dynamics of the model would expand its applicability in the real economy. Generalization of the production function (e.g., to CES) would improve the promise of the model.

Empirical applications include econometric analyses of growth accounting, investment patterns across vintages, and obsolescence and depreciation, across countries, industries, firms, and types of equipment. For these empirical analysis, it is indispensable to properly separate physical depreciation from obsolescence, and to identify the long-lived capital. These analyses that explicitly consider capital heterogeneity between physical and intangible capital should provide a better picture of the policy implications of economic growth and investment patterns in a modern knowledge economy.

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A Appendix

A.1 Proofs

A.1.1 Proposition 1 (Investment patterns across vintages of technology)

Suppose the investment scheme is (d) $\forall v \in [0,t]$, which requires $P_v^A = P_v^B = 1 \, \forall v \in [0,t]$ because the prices of investment goods must be unity. Then, the right-hand side of (16) is unity, which cannot be true when technological progress is positive. Therefore, the investment scheme cannot be (d) in a BGP.

Next, suppose the investment scheme is (a) $\forall v \in [0,t]$, which requires $P_v^A = 1 \, \forall v \in [0,t]$. In this case, B_t has the highest price among B capital with $P_v^B = 1$ and $\hat{P_v^B} = -\hat{q}/\beta$ from (16). Then from (3) and (4), $\hat{\left[\frac{B_v}{A_v}\right]} = \hat{\left[\frac{MPA_v}{MPB_v}\right]} = \hat{P_v^A} - \hat{P_v^B} = \hat{q}/\beta$, because R^Xs are constant in a BGP.[30] This requires disinvestment in A_v because $-[\delta^B - \delta^A] \leq 0 < \hat{q}/\beta$, which is not allowed by the assumption of investment irreversibility.

Next, suppose the investment scheme is (b) $\forall v \in [0,t]$, which requires $P_v^B = 1 \, \forall v \in [0,t]$. In this case, as in the case of (a) above, $P_t^A = 1$ and $\hat{P_v^A} = -\hat{q}/\alpha$, and thus $\hat{\left[\frac{B_v}{A_v}\right]} = -\hat{q}/\alpha$. When $-[\delta^B - \delta^A] \geq -\hat{q}/\alpha$, there is no positive investment in B_v, which contradicts the definition of investment scheme (b). Therefore, in a BGP with $\hat{q} \geq \alpha(\delta^B - \delta^A)$, the investment scheme must be (c) $\forall v \in [0,t]$.

Now, suppose the investment scheme is (c) $\forall v \in [0,t]$. There is no investment in vintage capital and thus all investment should concentrate on the frontier capital types, A_t and B_t, which implies $P_t^A = P_t^B = 1$. Furthermore, observe that B_t/A_t is constant because $(\alpha/\beta)(B_t/A_t) = R_t^A/R_t^B$ from (3) and (4). But this is impossible when $\hat{q} < \alpha(\delta^B - \delta^A)$, because (15) implies that $P_v^B \, \forall v \in [0,t]$ exceeds one given $P_t^B = 1$ and a constant value of A_t/B_t. Therefore, in a BGP with $\hat{q} < \alpha(\delta^B - \delta^A)$, the investment scheme must be (b) $\forall v \in [0,t]$. ■

[30] Constant growth of r and R_v^X, and (6) impose constant r and R^X in a BGP.

A.1.2 Proposition 2 (Allocation of capital types in a BGP)

Relationship from Profit Maximizations Conditions: By canceling r from (3), (4), and (6), we observe that:

$$\left[\frac{\beta}{P_v^B B_v} - \frac{\alpha}{P_v^A A_v}\right] Y_v = [\delta^B - \hat{P_v^B}] - [\delta^A - \hat{P_v^A}]. \tag{23}$$

Using $Y/L = Y_t/L_t$, $A/L = A_t/L_t$, and $B/L = B_t/L_t$ from (7) and (9), $P_t^A = P_t^B = 1$ and (12), and applying $v \to t$, rewrite (23) in units of effective labor as:

$$\beta a^{\alpha} b^{\beta-1} - \alpha a^{\alpha-1} b^{\beta} = [\delta^B - \hat{P^B}] - [\delta^A - \hat{P^A}]. \tag{24}$$

Condition from Aggregate Laws of Motion: The laws of motion of the capital types of each vintage are:

$$\dot{A}_v = I_v^A - \delta^A A_v, \text{and} \tag{25}$$

$$\dot{B}_v = I_v^B - \delta^B B_v. \tag{26}$$

Because $P_t^A = P_t^B = 1$ in a BGP, using (13), (7) can be expressed as:

$$A = \int_0^t P_v^A A_v \, dv, \text{and} \tag{27}$$

$$B = \int_0^t P_v^B B_v \, dv. \tag{28}$$

Using (25)–(28), we obtain the laws of motion of aggregate capital as follows:

$$\begin{aligned}\dot{A} &= \frac{\partial}{\partial t} \int_0^t P_v^A A_v \, dv \qquad (29)\\ &= \int_0^t [P_v^A A_v][\hat{P_v^A} + \hat{A}_v] \, dv + A_t \\ &= \left[\hat{P^A} - \delta^A\right] A + \int_0^t I_v^A \, dv + A_t \\ &= \left[\hat{P^A} - \delta^A\right] A + I^A,\end{aligned}$$

and

$$\dot{B} = \left[\hat{P^B} - \delta^B\right] B + I^B. \tag{30}$$

The sum of the laws of motion, (29) and (30), in units of effective labor become:

$$\dot{a} + \dot{b} = \sigma a^{\alpha} b^{\beta} - [\delta^A - \hat{P^A} + \hat{N}]a - [\delta^B - \hat{P^B} + \hat{N}]b. \tag{31}$$

Because A grows at a constant rate in a BGP by definition, (29) implies $\hat{I^A} = \hat{A}$. Similarly, $\hat{I^B} = \hat{B}$. Then, (10) implies $\hat{Y} = \hat{I^A} = \hat{I^B}$. Thus from (9):

$$\hat{A} = \hat{B} = \hat{Y} = \frac{\hat{q}}{1 - \alpha - \beta} + \hat{L} = \hat{N}. \tag{32}$$

Therefore, a and b are constant in a BGP.

Changes in Prices: When $\hat{q} < \alpha(\delta^B - \delta^A)$, because proposition 1 indicates that there is always investment in old B, (16) and proposition 1 provide:

$$\hat{P^A} = -\frac{\hat{q}}{\alpha}, \text{ and } \hat{P^B} = 0. \tag{33}$$

When $\hat{q} \geq \alpha(\delta^B - \delta^A)$, because proposition 1 indicates that B_t/A_t is constant, and applying $v' \to t$, (14) and (15) provide:

$$\hat{P^A} = -\frac{\hat{q} + \beta(\delta^B - \delta^A)}{\alpha + \beta}, \text{ and } \hat{P^B} = -\frac{\hat{q} - \alpha(\delta^B - \delta^A)}{\alpha + \beta}. \tag{34}$$

(24) and (31) can be expressed as (17) and (18) provided (33) and (34).

Uniqueness and Stability: In the Fast Case, the uniqueness and stability of the BGP can be easily confirmed by using the basic Solow model's approach with the relationship $a/b = \alpha/\beta$ from equation (17).

In the Slow Case, the relationship (24) can be expressed as:

$$a = f(b). \tag{35}$$

Because (35) implies $\dot{a} = f'(b)\dot{b}$, (31) can be expressed as:

$$\dot{b} = \frac{\sigma f(b)^{\alpha} b^{\beta} - [\delta^A - \hat{P^A} + \hat{N}]f(b) - [\delta^B - \hat{P^B} + \hat{N}]b}{f'(b) + 1}. \tag{36}$$

Clearly, $\dot{b} = 0$ when $b = 0$. Then, observe that the numerator of the right-hand side of (36) can be expressed as $\frac{b^2}{a\beta - b\alpha}[\{\sigma(\delta^B - \hat{P^B} + \hat{P^A} - \delta^A) + (\delta^A - \hat{P^A} + \hat{N})\alpha - (\delta^B - \hat{P^B} + \hat{N})\beta\}\frac{a}{b} - (\delta^A - \hat{P^A} + \hat{N})(\frac{a}{b})^2\beta + (\delta^B - \hat{P^B} + \hat{N})\alpha]$. The value of the term inside the square brackets is positive when $\left[\frac{a}{b}\right]_{b\to+0} = \frac{\alpha}{\beta}$ and negative when $\left[\frac{a}{b}\right]_{b\to\infty} \to \infty$. Because (36) is continuous and smooth, there is at least one set of $a^*$and b^* such that $\frac{a^*}{b^*} > \frac{\alpha}{\beta}$, $b^* > 0$ and the value of the term inside the brackets is zero ($\dot{b} = 0$). At b^*, (17) implies $a^* > 0$ and $\dot{a} = 0$. Observe that the first series of the Taylor approximation of the summarized law of motion of capital (36) at b^* is $\dot{b} \approx \frac{(\alpha+\beta-1)\{\beta(\delta^A - \hat{P^A} + \hat{N})(a^*/b^*) + \alpha(\delta^B - \hat{P^B} + \hat{N})(b^*/a^*)\}}{2\alpha\beta + \beta(1-\beta)(a^*/b^*) + \alpha(1-\alpha)(b^*/a^*)}(b - b^*)$, where the coefficient is negative. Therefore, at a^* and b^*, the economy is stable and $b^* > 0$ will be a unique solution. ■