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Orlando Gomes

Escola Superior de Comunicação Social - Instituto Politécnico de  
Lisboa

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# Deterministic Randomness in a Model of Finance and Growth

Orlando Gomes\*

Escola Superior de Comunicação Social [Instituto Politécnico de Lisboa] and  
Unidade de Investigação em Desenvolvimento Empresarial [UNIDE/ISCTE].

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**Abstract:** Following the literature on growth, cycles and financial development, this paper develops an endogenous growth model where the source of endogenous business cycles relates to the allocation of credit between productive investment and consumption. An important role is given to consumer sentiment, because this determines the willingness of households in terms of demand for credit; in particular, optimistic beliefs about the economy's macro performance deviate financial resources from investment in favour of consumption. The dynamic analysis indicates that Neimark-Sacker and flip bifurcations eventually separate stable and unstable manifolds, and as a result a region of nonlinear motion is generated: cycles of various periodicities and chaotic motion characterize the behaviour of the long run time paths of accumulated wealth, output and consumption.

**Keywords:** Financial development, Endogenous business cycles, Endogenous growth, Credit to consumption, Local bifurcations, Nonlinear dynamics, Chaos.

**JEL classification:** O16, E32, C62.

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\* Orlando Gomes; address: Escola Superior de Comunicação Social, Campus de Benfica do IPL, 1549-014 Lisbon, Portugal. Phone number: + 351 93 342 09 15; fax: + 351 217 162 540. E-mail: [ogomes@escs.ipl.pt](mailto:ogomes@escs.ipl.pt).

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## 1. Introduction

Borrowing constraints have a relevant impact on growth and cycles. This evidence has been studied, both in theoretical and empirical grounds, by a large group of economists, from which we can highlight the contributions of Bernanke and Gertler (1989), Kyotaki and Moore (1997), Levine (1997, 2005), Aghion, Banerjee and Piketty (1999) and Amable, Chatelain and Ralf (2004), among others. Recently, some authors have pointed to the idea that, under meaningful and reasonable assumptions, a prototype growth-finance model is able to generate endogenous business cycles; this is the path followed by Aghion, Bacchetta and Banerjee (2004) and Caballé, Jarque and Michetti (2006). Mostly in this last paper, the idea is to establish a link between credit constraints or the level of financial development of an economy and the literature on deterministic cycles first addressed in the early 1980s [e.g., Benhabib and Day (1981), Day (1982), Boldrin and Montrucchio (1986), Deneckere and Pelikan (1986)] and relaunched with the work by Christiano and Harrison (1999), who adapt a deterministic version of the real business cycles model (RBC) to a scenario of endogenous fluctuations by including in the setup an externality over the production of physical goods that allows to consider an aggregate production function exhibiting increasing returns to scale.

Endogenous cycles have been a strong source of motivation for recent macroeconomic literature. Several directions are being followed. See, for instance, Schmitt-Grohé (2000) and Guo and Lansing (2002), who also focus on the RBC setup, Boldrin, Nishimura, Shigoka and Yano (2001), Mitra, Nishimura and Sorger (2005), and related literature, who search for extreme conditions in which competitive markets generate nonlinear motion, Cellarier (2006), who introduces a learning mechanism into the growth setup to trigger chaotic motion, and Cazavillan, Lloyd-Braga and Pintus (1998), Aloï, Dixon and Lloyd-Braga (2000), Lloyd-Braga, Nourry and Venditti (2006), and related literature, where the search for endogenous cycles is based on the OLG framework in the tradition of Grandmont (1985). For a survey on nonlinear dynamics in macroeconomics see Gomes (2006).

In this paper, we follow on the footsteps of the work on financial constraints and endogenous cycles, by proposing a model of endogenous growth (of the AK type) that considers not only a constraint over credit, but also two alternatives concerning the allocation of credit. To a representative agent is attributed the possibility to choose

between borrowing to invest or borrowing to consume. The shares of credit directed to one or to the other use are assumed as constant values if the economy performs as expected (i.e., if the accumulated level of wealth follows a predefined trend). Deviations from this expected performance are accounted by the representative agent, who rises the share of credit to consumption when levels of wealth are above the predicted outcome. In other words, consumer sentiment counts in what concerns how the economy splits the available credit into the possible utilizations.

The previous ingredients allow to develop a growth model where, from a local dynamic analysis point of view, bifurcations separating regions of stability and instability are observable, and from a global dynamics perspective we find, for some arrays of parameter values, the presence of cyclical and chaotic motion characterizing the time evolution of the economic aggregates we consider: wealth (the central variable for which the analysis is conducted) and, also, output, capital, investment, consumption and even the share of credit allocated to each available use (since all these variables are dependent on the path of wealth).

The remainder of the paper is organized as follows. Section 2 describes the model. Sections 3 and 4 respect to the stability analysis, which is undertaken both locally and globally. Section 5 presents some final comments.

## **2. The Model: Financing Production vs Financing Consumption**

Consider an endogenous growth framework, where output is given by a simple AK production function,  $y_t = Ak_t$ , with  $A > 0$  a technological index and  $y_t$  and  $k_t$  representing income and physical capital, respectively, in a given time moment  $t$ . In this economy, population does not grow.

Imposing the assumption that capital fully depreciates after each time period, investment will be equal to the amount of capital, i.e.,  $i_t = k_t$ . In this economy, there is a financial sector that allows private agents to borrow intertemporally; the agents (in the case, we assume a representative agent) may resort to credit in order to finance contemporaneous production and consumption, and over these loans interests have to be paid in subsequent time periods. Let  $b_t$  be the total amount of financial resources that may be borrowed in period  $t$ . A fraction of these resources is borrowed to invest in the production of final goods,  $v_t \cdot b_t$ , with  $0 \leq v_t \leq 1$ ; hence,  $1 - v_t$  will correspond to the share of financial resources respecting to credit to consumption.

Assuming that the representative agent makes use of the total borrowing capabilities, the production function can be presented as  $y_t = A \cdot (w_t + v_t \cdot b_t)$ , that is, the resources available to invest are the existing level of wealth ( $w_t$ ), plus the financial resources that may be borrowed through the financial sector. Wealth dynamics are characterized by rule (1),

$$w_{t+1} = y_t - r \cdot b_t - [c_t - (1 - v_t) \cdot b_t], \quad w_0 \text{ given.} \quad (1)$$

In difference equation (1),  $r$  is the nominal interest rate and, thus, wealth in the following period corresponds to today's income less interest payment and less the resources diverted from income to consumption. Total consumption,  $c_t$ , will be a sum of two terms: first, a fixed amount of available wealth and, second, the financial resources borrowed to consume. Letting  $c$  be the marginal propensity to consume out of disposable income, consumption is given by  $c_t = c \cdot (y_t - r \cdot b_t) + (1 - v_t) \cdot b_t$ .

Information asymmetry problems will imply a constraint on credit that corresponds to a linear function of wealth:  $b_t = \mu \cdot w_t$ . Parameter  $\mu > 0$  represents the level of financial development of the economy or, in other words, it can be thought as a credit multiplier. Finally, we take the hypothesis that the share of credit allocated to consumption or production varies according to the consumer sentiment about the path followed by macro aggregates. Our assumption is that in periods of recession credit to consumption falls, while expansions are characterized by relatively higher levels of credit to consumption. Formally, we consider  $1 - v_t = m \cdot f\left(\frac{w_{t-1}}{w_{t-1}^*}\right)$ , with  $m > 0$  the share

of credit directed to consumption when the economy's effective level of wealth equals the potential level of wealth. Variable  $w_t^*$  corresponds to the potential level of wealth. There is a time lag in the previous expression because we assume that the agent's reaction to short run economic performance is not immediate; behaviour is adjusted according to last period's economic results. Function  $f$  is a continuous and differentiable function that obeys to the following conditions:  $f' > 0$ ,  $f(0) = 0$  and  $f(1) = 1$ . To simplify computation, we take an explicit functional form:  $f\left(\frac{w_t}{w_t^*}\right) = \left(\frac{w_t}{w_t^*}\right)^\sigma$ , with  $\sigma > 0$ .

Recalling that we are working with an endogenous growth setup, variables output, capital, investment, consumption and wealth grow, in the steady state, at a same constant and positive rate. Let this rate be  $\gamma$ . Note also that  $w_t^*$  represents the wealth

trend, and therefore it is supposed to grow at rate  $\gamma$  not only in the steady state but in all time moments.

The previous set of features allows for rewriting equation (1) as

$$w_{t+1} = (1-c) \cdot \left\{ \left[ A + \mu \cdot (A-r) \right] - A \cdot m \cdot \mu \cdot \left( \frac{w_{t-1}}{w_{t-1}^*} \right)^\sigma \right\} \cdot w_t \quad (2)$$

or, considering a wealth variable that does not grow in the steady state,  $\hat{w}_t \equiv \frac{w_t}{(1+\gamma)^t}$ ,

$$\hat{w}_{t+1} = \frac{1-c}{1+\gamma} \cdot \left\{ \left[ A + \mu \cdot (A-r) \right] - A \cdot m \cdot \mu \cdot \left( \frac{\hat{w}_{t-1}}{\hat{w}_{t-1}^*} \right)^\sigma \right\} \cdot \hat{w}_t \quad (3)$$

with  $\hat{w}^* \equiv \frac{w_t^*}{(1+\gamma)^t}$  a constant value.

In our framework, we implicitly consider  $A > r$ , a condition that guarantees that investing in production is preferable than investing in financial assets (the marginal productivity is higher than the interest rate).

A balanced growth path is easily determined by solving (3) under condition  $\bar{w} \equiv \hat{w}_{t+1} = \hat{w}_t = \hat{w}_{t-1}$ . A unique steady state point exists:

$$\bar{w} = \left[ \frac{(1-c) \cdot [A + \mu \cdot (A-r)] - (1+\gamma)}{(1-c) \cdot A \cdot m \cdot \mu} \right]^{1/\sigma} \cdot \hat{w}^* \quad (4)$$

The requirement for a positive wealth level in the long term imposes an upper bound on the economy's growth rate,  $\gamma < (1-c) \cdot [A + \mu \cdot (A-r)] - 1$ . A second boundary condition is  $1 - \bar{v} < 1$ , which is equivalent to  $\gamma > (1-c) \cdot (A - \mu \cdot r) - 1$ . The double inequality just derived allows for inferring that the economy's growth rate will be located inside an interval that is delimited by two values that depend on the marginal propensity to consume, on the technology level, on the degree of financial development and on the interest rate.

We now present a second version of the model, in which the economy's choice between financing production or financing consumption takes in consideration both previous accumulated wealth results and also contemporaneous ones. We shall see

along the following sections that this new assumption has profound effects over the dynamic behaviour of the model's endogenous variables. The share of consumption

loans is now given by  $1 - v_t = m \cdot \left( \frac{w_t}{w_t^*} \right)^\rho \cdot \left( \frac{w_{t-1}}{w_{t-1}^*} \right)^\sigma$ , with  $\rho \geq 0$ . The particular case  $\rho = 0$

take us back to the previous formulation. An equation similar to (3) is straightforward to obtain,

$$\hat{w}_{t+1} = \frac{1-c}{1+\gamma} \cdot \left\{ [A + \mu \cdot (A-r)] - A \cdot m \cdot \mu \cdot \left( \frac{\hat{w}_t}{\hat{w}^*} \right)^\rho \cdot \left( \frac{\hat{w}_{t-1}}{\hat{w}^*} \right)^\sigma \right\} \cdot \hat{w}_t \quad (5)$$

The new steady state is

$$\bar{w} = \left[ \frac{(1-c) \cdot [A + \mu \cdot (A-r)] - (1+\gamma)}{(1-c) \cdot A \cdot m \cdot \mu} \right]^{1/(\rho+\sigma)} \cdot \hat{w}^* \quad (6)$$

The boundaries on the economy's long run growth rate, which are derived from  $\bar{w} > 0$  and  $0 < \bar{v} < 1$ , are the same as in the first framework.

### 3. Local Dynamics in a Two-Dimensional Map

The one dimensional systems  $\hat{w}_{t+1} = g(\hat{w}_t, \hat{w}_{t-1})$  discussed in the previous section must be rearranged in order to be possible to proceed with the analysis of local dynamics. Let us define variables  $\tilde{w}_t \equiv \hat{w}_t - \bar{w}$  and  $\tilde{z}_t \equiv \hat{w}_{t-1} - \bar{w}$ . Making the proper substitutions, equations (3) and (5) give place to the following two dimensional systems,

$$\begin{cases} \tilde{w}_{t+1} = \frac{1-c}{1+\gamma} \cdot \left\{ [A + \mu \cdot (A-r)] - A \cdot m \cdot \mu \cdot \left( \frac{\tilde{z}_t + \bar{w}}{\hat{w}^*} \right)^\rho \cdot \left( \frac{\tilde{z}_t + \bar{w}}{\hat{w}^*} \right)^\sigma \right\} \cdot (\tilde{w}_t + \bar{w}) - \bar{w} \\ \tilde{z}_{t+1} = \tilde{w}_t \end{cases} \quad (7)$$

and

$$\begin{cases} \tilde{w}_{t+1} = \frac{1-c}{1+\gamma} \cdot \left\{ [A + \mu \cdot (A-r)] - A \cdot m \cdot \mu \cdot \left( \frac{\tilde{w}_t + \bar{w}}{\hat{w}^*} \right)^\rho \cdot \left( \frac{\tilde{z}_t + \bar{w}}{\hat{w}^*} \right)^\sigma \right\} \cdot (\tilde{w}_t + \bar{w}) - \bar{w} \\ \tilde{z}_{t+1} = \tilde{w}_t \end{cases} \quad (8)$$

We begin by looking at the dynamics underlying (7) in the vicinity of  $\bar{w}$ . Linearizing the system around this point, we get

$$\begin{bmatrix} \tilde{w}_{t+1} \\ \tilde{z}_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & -\sigma \cdot \theta \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \tilde{w}_t \\ \tilde{z}_t \end{bmatrix} \quad (9)$$

with  $\theta \equiv \frac{1-c}{1+\gamma} \cdot [A + \mu \cdot (A-r)] - 1$ . A positive steady state value for wealth requires

$\theta > 0$ . Thus, the determinant of the Jacobian matrix corresponds to a positive value:  $Det(J) = \sigma \cdot \theta > 0$ ; the trace is equal to 1. In figure 1, we display a line that translates the possible stability outcomes of the model's dynamics. We can regard that stability and instability are both admissible, for different values of parameters.<sup>1</sup>

Proposition 1 states the local dynamics result.

**Proposition 1.** *In the growth-finance model with a consumption credit share depending on the last period's level of wealth, local dynamics are characterized by the following conditions:*

- i) *If  $\sigma \cdot \theta > 1$ , then the system is locally unstable;*
- ii) *If  $\sigma \cdot \theta = 1$ , then a Neimark-Sacker bifurcation occurs (the eigenvalues of matrix  $J$  are a pair of complex conjugate values with modulus equal to one);*
- iii) *If  $\sigma \cdot \theta < 1$ , then the system is locally stable; here we can distinguish between a stable node ( $\sigma \cdot \theta \leq 1/4$ ) and a stable focus ( $1/4 < \sigma \cdot \theta < 1$ ).*

Any change on the values of parameters  $\sigma$ ,  $c$ ,  $\gamma$ ,  $A$ ,  $\mu$  and  $r$  may imply a transition from the area of stability to the region of instability (and vice-versa), along the line drawn in figure 1. The other parameters of the system, namely  $m$  and  $w_t^*$ , have no influence on the local dynamics result. Noticing that  $\frac{\partial Det(J)}{\partial \sigma} > 0$ ,  $\frac{\partial Det(J)}{\partial c} < 0$ ,

<sup>1</sup> All figures are presented in the end of the paper.



$\frac{\partial Det(J)}{\partial \gamma} < 0$ ,  $\frac{\partial Det(J)}{\partial A} > 0$ ,  $\frac{\partial Det(J)}{\partial \mu} > 0$  and  $\frac{\partial Det(J)}{\partial r} < 0$ , then we conclude that a stable outcome becomes more likely to occur in circumstances where the marginal propensity to consume, the economy's growth rate and the nominal interest rate rise, and when elasticity  $\sigma$ , the technological level and the degree of financial development fall.

Let us concentrate our attention on the parameter concerning the level of financial development. According to the equilibrium result in (4), we compute the derivative

$$\frac{\partial \bar{w}}{\partial \mu} = \frac{1}{\sigma} \cdot \frac{(1+\gamma) - (1-c) \cdot A}{\mu} \cdot \left[ \frac{(1-c) \cdot [A + \mu \cdot (A-r)] - (1+\gamma)}{(1-c) \cdot A \cdot m \cdot \mu} \right]^{(1-\sigma)/\sigma} \cdot \hat{w}^*$$

According to the previous result, the balanced growth path level of wealth grows with the level of financial development if  $\gamma > (1-c) \cdot A - 1$ , which is a true condition under the boundaries previously computed. Therefore, we can state that the level of wealth in the long term effectively rises with the level of financial development; nevertheless, the local dynamics analysis demonstrates that  $\mu$  cannot be too high, because then the convergence to the steady state ceases to occur. Thus, the optimal level of financial development is the one for which  $\mu$  is close to but below  $\left[ \frac{1+\sigma}{\sigma} \cdot \frac{1+\gamma}{1-c} - A \right] / (A-r)$ .<sup>2</sup>

The mentioned result may be interpreted in the following way: lower constraints on credit allow for a better intertemporal allocation of resources and therefore turn it possible to obtain a higher long run level of wealth; nevertheless, excessively low constraints on credit, that is, a high amount of loans without collateral requirements, namely in the presence of credit to consumption (non productive credit) can lead to a state where a stable outcome is absent, what can be interpreted as a situation where the financial sector loses its capability to maintain a credible credit system. Thus, financial development is a synonymous of potential to accumulate wealth, but financial irresponsibility (a too high amount of loans) can cause serious damage on the way finance may serve growth. Parameter  $\mu$  should be kept on the interval  $0 < \mu < \left[ \frac{1+\sigma}{\sigma} \cdot \frac{1+\gamma}{1-c} - A \right] / (A-r)$ , and the closest possible to its upper bound. Note that this boundary can be enlarged by a stronger rate of growth ( $\gamma$ ).

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<sup>2</sup> This value is obtained by solving  $Det(J)=1$  in order to  $\mu$ .

Alternatively to system (7), we can analyze system (8). In section 2, we have seen that from a steady state perspective there are no significant qualitative changes. A new parameter is introduced, but the effects we have just described regarding changes in parameter values are closely related to the first case. However, there are significant differences in what concerns local dynamics, since now we cannot draw stability outcomes through a vertical line as in the simplest case, where  $\rho=0$ . To confirm this, linearize (8) around steady state point (6). The matricial system is

$$\begin{bmatrix} \tilde{w}_{t+1} \\ \tilde{z}_{t+1} \end{bmatrix} = \begin{bmatrix} 1 - \rho \cdot \theta & -\sigma \cdot \theta \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \tilde{w}_t \\ \tilde{z}_t \end{bmatrix} \quad (10)$$

We can distinguish system (10) from system (9) only because the element in the first row and first column is no longer unity, but a value below 1. As a consequence, the line representing the set of possible dynamic outcomes will now be a negatively sloped line, which is straightforward to determine. The trace and determinant of the Jacobian matrix in (10) are, respectively,  $Tr(J)=1-\rho\cdot\theta$  and  $Det(J)=\sigma\cdot\theta$ . These expressions take us to a relation between the trace and the determinant, as follows,  $Det(J) = \frac{\sigma}{\rho} - \frac{\sigma}{\rho} \cdot Tr(J)$ .

This line is represented in figure 2.

Figure 2 displays two possible locations for the line describing local dynamics. First, note that the line stops when it reaches the horizontal axis. In this point, a zero determinant coincides with a trace equal to one. Second, the dynamics line is negatively sloped; the slope is, in absolute value, equal to  $\sigma/\rho$ . Third, a Neimark-Sacker and a flip bifurcation may occur depending on the value of the ratio  $\sigma/\rho$ ; note that a flip bifurcation will only occur if  $\sigma/\rho < 1/3$ , that is, when line  $Det(J) = -1 - Tr(J)$  is crossed before  $Det(J) = 1$ . Proposition 2 puts together the relevant stability conditions,

**Proposition 2.** *In the growth-finance model with a consumption credit share depending on sentiments based on today's and on last period's levels of wealth, local dynamics are characterized by the following conditions:*

i) For  $\sigma/\rho > 1/3$ ,

a) If  $\sigma \cdot \theta > 1$ , then the system is locally unstable;

b) If  $\sigma \cdot \theta = 1$ , then a Neimark-Sacker bifurcation occurs;

- c) If  $\sigma \cdot \theta < 1$ , then the system is locally stable;<sup>3</sup>
- ii) For  $\sigma/\rho < 1/3$ ,
- a) If  $\sigma \cdot \theta > 1$ , then the system is locally unstable;
- b) If  $\sigma \cdot \theta = 1$ , then a Neimark-Sacker bifurcation occurs;
- c) If  $\rho \cdot \theta - 2 < \sigma \cdot \theta < 1$ , then the system has a saddle-path stable local equilibrium;
- d) If  $\sigma \cdot \theta = \rho \cdot \theta - 2$ , then a flip bifurcation occurs;
- e) If  $\sigma \cdot \theta < \rho \cdot \theta - 2$ , then the system is locally stable.<sup>4</sup>

Proposition 2 elucidates that a richer set of results arises when considering that the representative agent is influenced both by today's and by last period's economic performance, when deciding how to allocate the available credit. Nevertheless, some fundamental results are true in both frameworks. Mainly, this is the case of the idea that a higher degree of financial development benefits the accumulation of wealth, until a given point where a bifurcation changes the qualitative nature of the equilibrium, giving place to an unstable outcome.

#### 4. The Graphical Analysis of Global Dynamics

The analysis of local bifurcations in section 3 has allowed solely to establish the frontiers between stability and instability. The study of global dynamics will reveal that stability areas are in fact the ones computed analytically and presented in propositions 1 and 2. Instability (understood as the divergence from the steady state point) will not, however, be observable immediately after the bifurcation; in the numerical examples that follow, the bifurcation gives place to cycles of various periodicities, totally a-periodic cycles and chaotic motion, before the dynamics become characterized by instability.

The finding of endogenous cycles, that can be found on other finance and growth models as discussed in the introduction, allows us to state that the present model is able to furnish an alternative source of fluctuations relatively to the ones generally discussed in the literature. In this case, it is the reaction of a representative agent to the ability of

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<sup>3</sup> Condition  $\sigma \leq \frac{1}{4} \cdot \frac{(1 - \rho \cdot \theta)^2}{\theta}$  implies node stability;  $\frac{1}{4} \cdot \frac{(1 - \rho \cdot \theta)^2}{\theta} < \sigma < 1$  refers to a stable focus.

<sup>4</sup> Stable node and stable focus cases continue to be distinguished by the same conditions as previously.

the economy to accumulate wealth, when deciding to which use to allocate credit, that generates endogenous business cycles.

We choose to work with the following indexes:  $\hat{w}^* = 1$  and  $A=3$ ; other parameters just take reasonable values,  $c=0.75$ ,  $\gamma=0.04$ ,  $r=0.03$  and  $m=1$ . Consider, as well,  $\mu=2$ . These parameter values satisfy the boundary condition that was derived in section 2, which limits the value of the growth rate:  $-0.265 < \gamma < 1.235$ . As bifurcation parameters in the analysis that follows, we choose to work with  $\sigma$  and  $\rho$ . Figure 3 presents a bifurcation diagram for the first model ( $\rho=0$ ). The Neimark-Sacker bifurcation is clearly present, giving place to a region of a-periodic cycles and chaos.<sup>5</sup>

Note that, in our example,  $\theta=1.149$ , and thus the bifurcation occurs at  $\sigma = 1/1.149 = 0.87$ . To the left of this point, according to proposition 1 (and looking at figure 3) stability prevails. The bifurcation diagram can, as well, be drawn for variable  $v_t$  (figure 4). We regard that cycles have a increasing amplitude and that there is the possibility of this variable assuming negative values; this circumstance means that not only all available credit is directed to consumption, but that also part of the current resources available to invest in production are deviated to credit to consumption; instability arises when the economic system becomes unable to sustain a situation where a progressively larger amount of resources are allocated to finance future consumption.

The interpretation of figures 3 and 4 is essentially that if agents give little importance to past deviations from the benchmark level of wealth ( $\sigma$  low), stability holds. When this relevance rises, cycles set in and instability will end up by prevailing. Bifurcation diagrams could be drawn as well for any other parameter, like the level of financial development. We would have diagrams similar to the ones in the presented figures, and the conclusion of the transition from stability to cycles and from these to instability would be the same as the one depicted in the local analysis: instability (and, before this state, a-periodic cycles) arise for a too high level of credit availability.

To illustrate further the cyclical nature of the results, we present through figures 5 and 6 the long term time series of wealth and an attracting set that defines the long run relation between  $w_t$  and  $v_t$ ; this is done for a value of  $\sigma$  for which chaotic behaviour is evident.

Finally, we can close the graphical analysis of this simplest case ( $\rho=0$ ) with a diagram that allows to identify rigorously the areas of chaotic motion. These are the

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<sup>5</sup> All the figures concerning global dynamics presented in this paper are drawn using IDMC software (interactive Dynamical Model Calculator). This is a free software program available at [www.dss.uniud.it/nonlinear](http://www.dss.uniud.it/nonlinear), and copyright of Marji Lines and Alfredo Medio.

ones for which at least one of the two Lyapunov characteristic exponents associated to our system is positive. Recall that Lyapunov exponents are a measure of exponential divergence of nearby orbits, that is, a measure of sensitive dependence on initial conditions, which is a well accepted property of chaotic systems. See figure 7. Note, in this figure, that for values of parameter  $\sigma$  around 0.85 to 1.03 one of the Lyapunov exponents stays equal to zero; this is the result one expects to find in the presence of quasi-periodic cycles, that is, cycles with no countable order but that are too regular to be considered chaos (there is not an absolute dependence on initial conditions). Such a result is the feature that commonly characterizes the state of a system immediately after a Neimark-Sacker bifurcation. Chaotic motion is undoubtedly present for values of  $\sigma$  near 1.1.

To illustrate our second version of the model ( $\rho > 0$ ), we draw a diagram revealing what kind of cycles one observes in the space of parameters  $(\sigma, \rho)$ . We confirm the analysis of section 3, in the sense that for relatively high values of  $\sigma$ , cycles arising from a Neimark-Sacker bifurcation are identified, while relatively high values of  $\rho$  imply cycles originating from a flip bifurcation (figure 8).

Let  $\sigma = 1$ . For this value, figure 9 respects to a bifurcation diagram of the wealth variable for different values of  $\rho$ , figures 10 and 11 draw time series and an attractor for a specific value of  $\rho$  and figure 12 relates to Lyapunov exponents. Chaotic motion and cycles of different periodicities are found for the wealth variable, and thus cyclical motion will also be present in the time trajectories of other economic aggregates, namely output, investment and consumption. Figure 11 reveals the higher sophistication of the second considered case, in the sense that a ‘stranger’ (less regular) attractor is obtained. Lyapunov exponents for different values of parameter  $\rho$  also show that quasi-periodicity (one exponent equal to zero), periodic cycles and stability (both exponents negative) and chaos (one positive exponent) coexist and alternate as we change the value of the parameter.

The graphical analysis of this section was useful in characterizing the model beyond local dynamics. The main new result is that an area of cycles and chaotic motion is identified after the region of stability and before instability. Thus, for some combinations of parameter values one is able to assert that the behaviour of economic agents, in the case concerning credit decisions, can produce a situation of self sustained cyclical motion, which is triggered by no monetary phenomena (Keynesian cycles) or exogenous shocks (RBC).

## 5. Final Remarks

We have developed a model of growth, cycles and financial development. A standard growth model involving a constraint on credit was assumed, and over this setup one has considered that part of the available credit is directed to productive investment, while a second share is destined to anticipate in time households' consumption. Under the assumption that the referred share varies with consumer sentiment, and that this is influenced by contemporaneous and past economic performance (more accurately, by the gap between effective and potential wealth in the present and in the previous periods), we were able to furnish an explanation for endogenous cycles.

Cycles of various periodicities and chaotic motion are observable for given combinations of parameter values, and we found that the way wealth gaps impact over credit allocation choices are one of the most relevant determinants of the stability results (alongside with the level of financial development, the state of technology, the interest rate, the savings rate and the economy's growth rate). From a local analysis point of view, one has concluded that both stability and instability can prevail, and that by varying some parameters' values a bifurcation (Neimark-Sacker or flip) is likely to occur. When we search for the confirmation of these results through an analysis of global dynamics, we are confronted with a region of cycles and chaos that follows the point of bifurcation, before instability (divergence to zero or infinity) becomes dominant.

From a policy point of view, the undertaken analysis is particularly important, in the sense that it can give some hints on how to balance the allocation of credit to consumption and to investment, in order to remain in the stability area, and therefore avoid the welfare costs of cyclical motion.

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## Figures

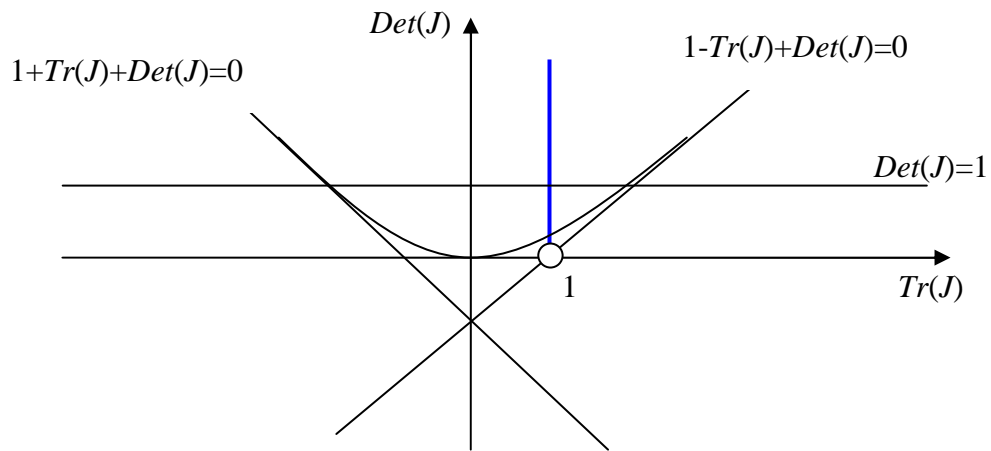


Figure 1 – Local dynamics;  $\rho=0$ .

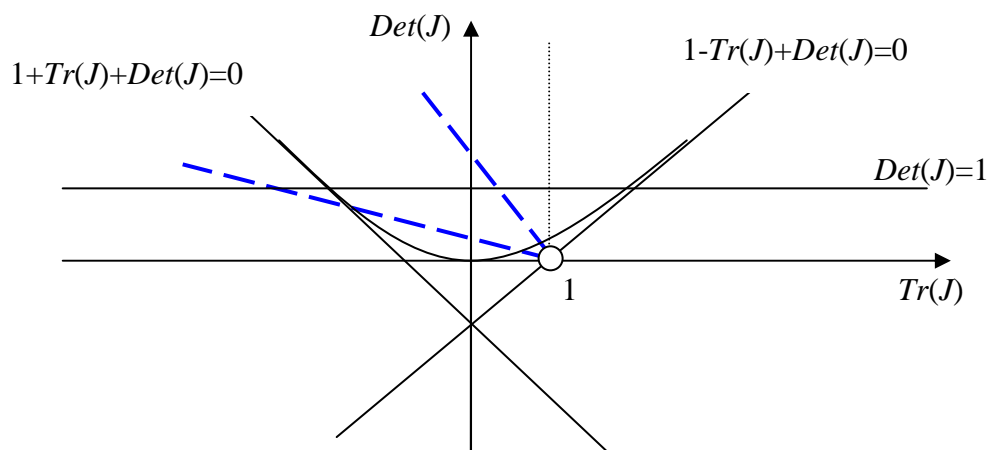


Figure 2 – Local dynamics;  $\rho>0$ .

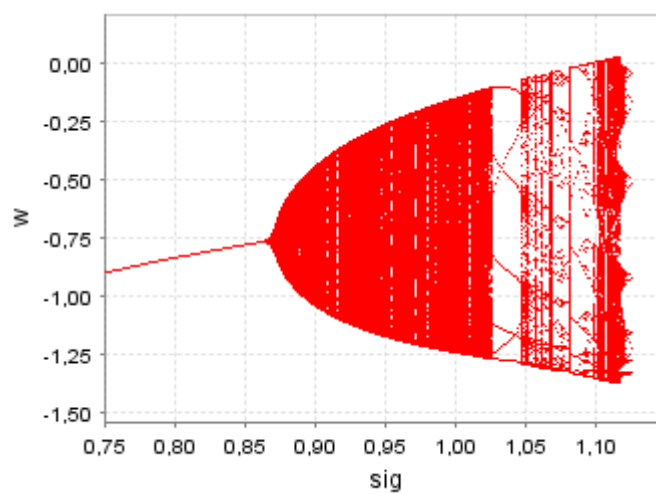
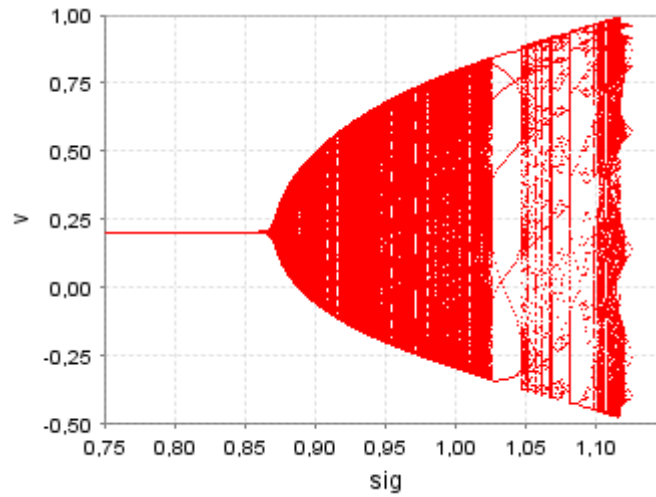
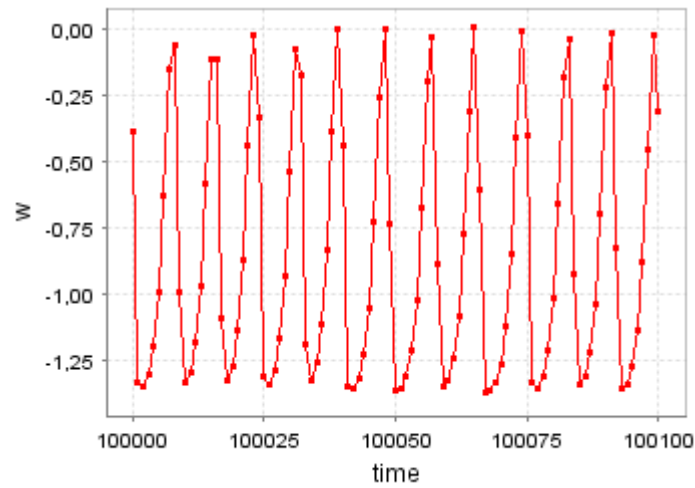


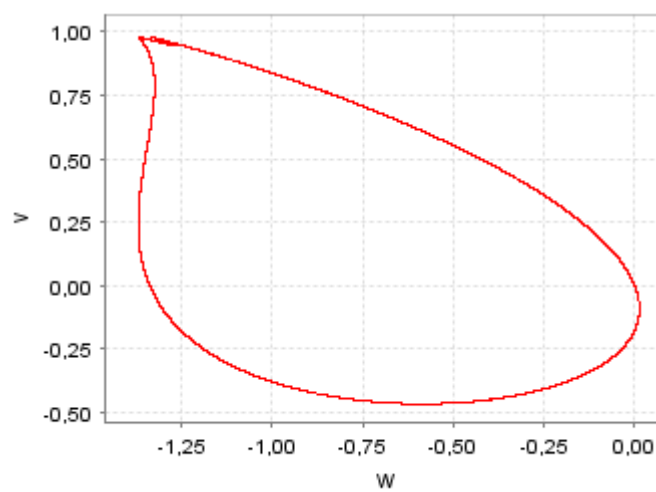
Figure 3 – Bifurcation diagram  $(\tilde{w}_t; \sigma)$ ;  $\rho=0$ .



**Figure 4 – Bifurcation diagram  $(v_i; \sigma)$ ;  $\rho=0$ .**



**Figure 5 – Long run time series of  $\tilde{w}_t$ ;  $\sigma=1.11$ ,  $\rho=0$ .**



**Figure 6 – Attractor  $(\tilde{w}_t; v_t)$ ;  $\sigma=1.11$ ,  $\rho=0$   
(the first 10.000 observations are excluded).**

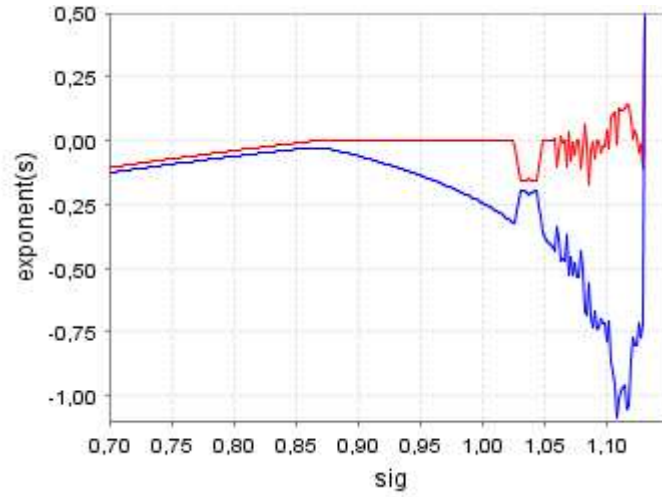


Figure 7 – Lyapunov characteristic exponents;  $\rho=0$ .

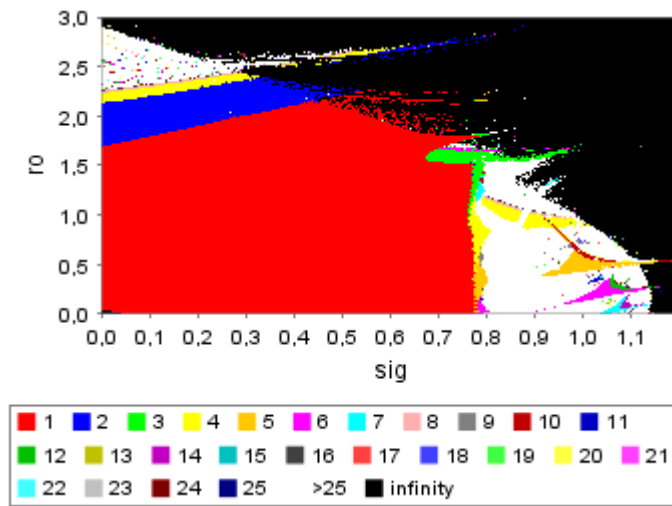


Figure 8 – Cycles in the space of parameters;  $\rho>0$ .

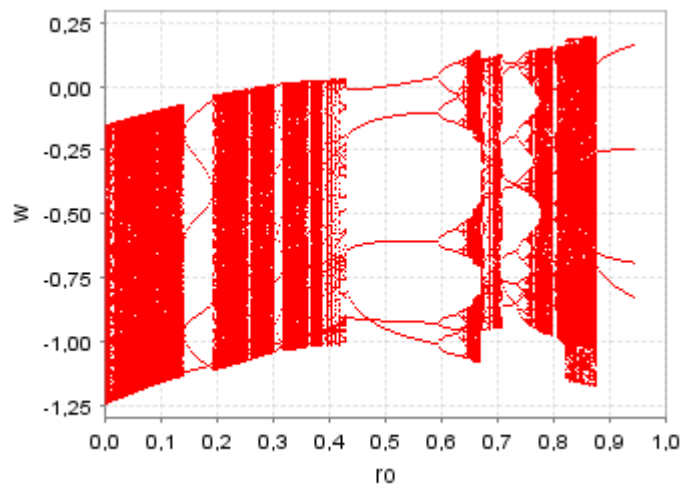


Figure 9 – Bifurcation diagram  $(\tilde{w}_i; \rho)$ ;  $\sigma=1$ , case  $\rho>0$ .

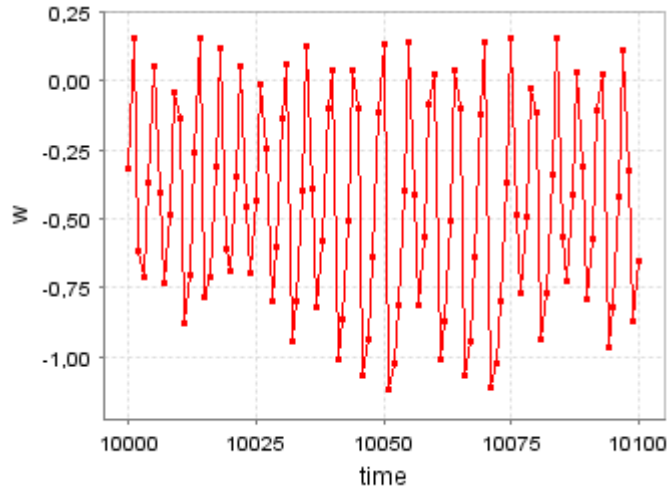


Figure 10 – Long run time trajectory of  $\tilde{w}_t$ ;  $\sigma=1, \rho=0.85$ .

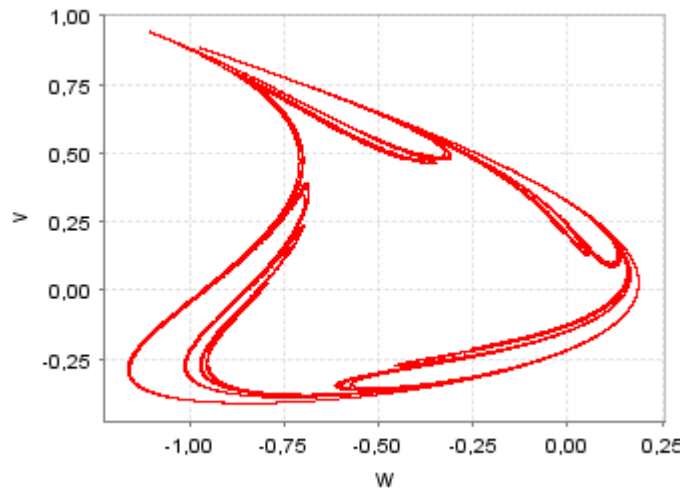


Figure 11 – Attractor  $(\tilde{w}_t; v_t)$ ;  $\sigma=1, \rho=0.85$   
(the first 10.000 observations are excluded).

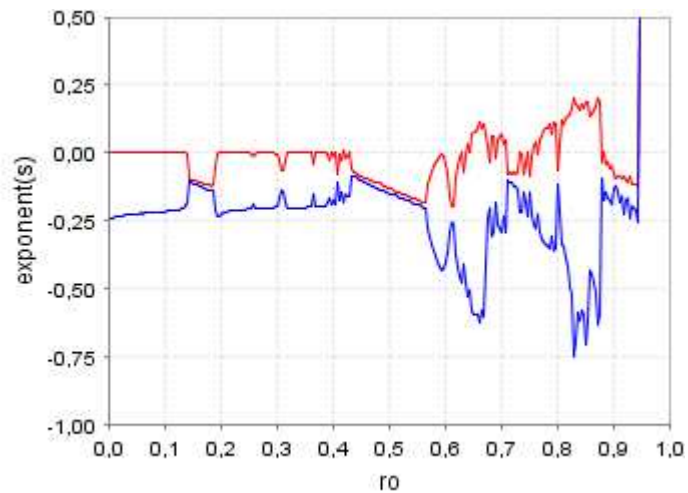


Figure 12 – Lyapunov characteristic exponents;  $\sigma=1$ , case  $\rho>0$ .