Non-positive scaling factor in probability quantification methods: deriving consumer inflation perceptions and expectations in the whole euro area and Ireland

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Deriving consumer inflation perceptions and expectations in the whole euro area and Ireland

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Abstract

There are problems with using probability quantification methods when the scaling factor applied in those methods becomes non-positive. The way of adjusting them proposed in this note and verified empirically allows using them in such circumstances. The results for the euro area and Ireland suggest that the recent financial crisis made consumer inflation perception and expectations go down, however it did not create deflationary expectations in this groups of economic agents.

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1. Introduction

Probability quantification methods are widely used to derive numerical estimates of perceived and expected inflation on the basis of qualitative survey data. However, the use of those methods during the episodes of non-positive inflation rates is constrained. The note proposes an adjustment in the probability method allowing quantification of survey data in such circumstances. It also presents the results of applying it in the quantification of consumer inflation perception and expectations in the euro area and Ireland during the recent financial crisis.

2. Quantification of survey responses when the scaling factor is positive

There are two survey questions concerning price changes in the European Commission Consumer Survey carried out every month in the EU economies. The first question concerns inflation perception: *"In your opinion, is the price level now compared to that 12 months ago: (1) much higher; (2) moderately higher; (3) a little higher; (4) about the same; (5) lower; (6) difficult to say"*. The second one concerns inflation expectations: *"Given what is currently happening, do you believe that over the next 12 months prices will: (1) rise faster than at present, (2) rise at the same rate, (3) rise more slowly, (4) stay at their present level, (5) go down, (6) difficult to say"*.

Quantifying survey data of this kind with probability methods (e.g. Batchelor and Orr 1988), survey responses are scaled given the certain type of the distribution of perceived or expected inflation and sensitivity intervals surrounding point responses (see e.g. Łyziak 2010). Considering the question on future inflation, the scaling factor is given by the measure of perceived current inflation and can be represented either by the official inflation rate available to respondents when the survey is carried out or by the quantified outcome from the survey question on past inflation. For the question on perceived price changes the scaling factor can be defined as a moderate inflation rate, providing a benchmark in declaring the magnitude of perceived price increase (e.g. Batchelor and Orr 1988).

The quantification of perceived or expected inflation can be summarized in the set of the following equations:

$$a_{1t}^{\bullet} = \Pr\left(\pi_t^{\bullet} > sf_t^{\bullet} + s_t^{\bullet}\right) = \int_{sf_t^{\bullet}+s_t}^{\infty} f_t\left(\pi_t^{\bullet}\right) d\pi_t^{\bullet} \quad , \quad \bullet = e, p \tag{1}$$

$$a_{2t}^{\bullet} = \Pr\left(sf_t^{\bullet} - s_t^{\bullet} < \pi_t^{\bullet} < sf_t^{\bullet} + s_t^{\bullet}\right) = \int_{sf_t^{\bullet}-s_t^{\bullet}}^{sf_t^{\bullet}+s_t^{\bullet}} f_t\left(\pi_t^{\bullet}\right) d\pi_t^{\bullet} \quad , \quad \bullet = e, p \tag{2}$$

$$a_{3t}^{\bullet} = \Pr\left(l_t^{\bullet} < \pi_t^{\bullet} < sf_t^{\bullet} - s_t^{\bullet}\right) = \int_{l_t^{\bullet}}^{sf_t^{\bullet}-s_t^{\bullet}} f_t\left(\pi_t^{\bullet}\right) d\pi_t^{\bullet} \quad , \quad \bullet = e, p \tag{3}$$

$$b_t^{\bullet} = \Pr\left(-l_t^{\bullet} < \pi_t^{\bullet} < l_t^{\bullet}\right) = \int_{-l_t^{\bullet}}^{l_t^{\bullet}} f_t\left(\pi_t^{\bullet}\right) d\pi_t^{\bullet} \quad , \quad \bullet = e, p \tag{4}$$

$$c_t^{\bullet} = \Pr\left(\pi_t^{\bullet} < -l_t^{\bullet}\right) = \int_{-\infty}^{-l_t^{\bullet}} f_t\left(\pi_t^{\bullet}\right) d\pi_t^{\bullet} \quad , \quad \bullet = e, p \tag{5}$$

where: $f_t\left(\pi_t^\bullet\right)$ denotes density function of perceived ($\bullet = p$) or expected ($\bullet = e$) inflation; $a_{1t}^\bullet$, $a_{2t}^\bullet$, $a_{3t}^\bullet$, $b_t^\bullet$ and $c_t^\bullet$ denote percentages of respondents selecting subsequent survey responses, $sf_t^\bullet$ stands for the scaling factor, while $s_t^\bullet$ and $l_t^\bullet$ determine the width of sensitivity intervals surrounding zero and the scaling factor respectively. Solving the system, parameters of the density function and both sensitivity intervals are calculated.

3. Quantification of survey responses when the scaling factor is non-positive

Methods described above assume that the scaling factor is positive and unique for the surveyed population. However, there are episodes, when the scaling factor becomes non-positive. In such circumstances it is difficult to interpret survey responses using standard assumptions of probability methods.

To make the probability method work when the scaling factor is non-positive, we propose the following additional assumptions. Firstly, we assume that in such cases, implied perceived or expected inflation in the group of respondents selecting three initial responses is driven by a positive benchmark value, $bv_t^\bullet$. Secondly, it is assumed that when the scaling factor is non-positive, implied price changes perceived or expected by respondents declaring a reduction of the price level, is equal to a certain value $\pi_t^{c\bullet}$ on average.

It leads to the following set of equations:

$$a_{1t}^\bullet = \Pr\left(\pi_t^\bullet > sf_{+t}^\bullet + s_t^\bullet\right) = \int_{bv_t^\bullet + s_t^\bullet}^{\infty} f_t\left(\pi_t^\bullet\right) d\pi_t^\bullet \quad , \quad \bullet = e, p \tag{6}$$

$$a_{2t}^\bullet = \Pr\left(sf_{+t}^\bullet - s_t^\bullet < \pi_t^\bullet < sf_{+t}^\bullet + s_t^\bullet\right) = \int_{bv_t^\bullet - s_t^\bullet}^{bv_t^\bullet + s_t^\bullet} f_t\left(\pi_t^\bullet\right) d\pi_t^\bullet \quad , \quad \bullet = e, p \tag{7}$$

$$a_{3t}^\bullet = \Pr\left(l_t^\bullet < \pi_t^\bullet < sf_{+t}^\bullet - s_t^\bullet\right) = \int_{l_t^\bullet}^{bv_t^\bullet - s_t^\bullet} f_t\left(\pi_t^\bullet\right) d\pi_t^\bullet \quad , \quad \bullet = e, p \tag{8}$$

$$b_t^\bullet = \Pr\left(-l_t^\bullet < \pi_t^\bullet < l_t^\bullet\right) = \int_{-l_t^\bullet}^{l_t^\bullet} f_t\left(\pi_t^\bullet\right) d\pi_t^\bullet \quad , \quad \bullet = e, p \tag{9}$$

$$c_t^\bullet = \Pr\left(\pi_t^\bullet < -l_t^\bullet\right) = \int_{-\infty}^{-l_t^\bullet} f_t\left(\pi_t^\bullet\right) d\pi_t^\bullet \quad , \quad \bullet = e, p \tag{10}$$

$$\begin{aligned} &\text{for } sf_t^\bullet \le 0: \quad E\left(\pi_t^\bullet \mid \pi_t^\bullet < -l_t^\bullet\right) = \pi_t^{c\bullet} \quad , \quad \bullet = e, p \\ &\text{for } sf_t^\bullet > 0: \quad bv_t^\bullet = sf_t^\bullet \quad , \quad \bullet = e, p \end{aligned} \tag{11}$$

Solution of above equations – dependant on the type of the distribution applied and the parameter $\pi_{ct}^\bullet$ – gives both sensitivity intervals, parameters of the distribution as well as the benchmark value.

To determine $\pi_t^{c\bullet}$, i.e. the average perceived or expected inflation in the group of respondents declaring fall in prices, we propose the following approach. In the first step, using observations characterized by a positive scaling factor, we derive $\pi_t^{c\bullet}$ based on the outcomes from the probability quantification method, i.e.:

$$\text{for } sf_t^{\bullet}>0: \quad \pi_t^{c\bullet} = E\left(\pi_t^{\bullet} \,|\, \pi_t^{\bullet} < -l_t^{\bullet}\right) \quad , \quad \bullet = e, p \tag{12}$$

In the second step we estimate a behavioral model for $\pi_{ct}^{\bullet}$, e.g. regressing it on its lagged values and the scaling factor:

$$\text{for } sf_t^{\bullet}>0: \quad \pi_t^{c\bullet} = \beta_0 + \beta_1\pi_{t-1}^{c\bullet} + \beta_2 sf_t^{\bullet} + \varepsilon_t^{\bullet} \quad , \quad \bullet = e, p \tag{13}$$

Using estimates of the coefficients *β0*, *β1* and *β2*, we derive the average perceived or expected inflation in the analysed subgroup of respondents for the periods of a non-positive scaling factor, assuming that when it is becoming increasingly negative, $\pi_t^{c\bullet}$ goes down:

$$\text{for } sf_t^{\bullet} \le 0: \quad \pi_t^{c\bullet} = \widehat{\beta}_0 + \widehat{\beta}_1\pi_{t-1}^{c\bullet} - \widehat{\beta}_2 sf_t^{\bullet} \quad , \quad \bullet = e, p \tag{14}$$

Normal distribution

Normality of the distribution of perceived or expected inflation is a common assumption in probability methods.[2] Under such distribution, equations (6)-(10) can be expressed in the following way, using density ($f_t^{\bullet}$) and cumulative standard normal distributions ($F_t^{\bullet}$):

$$a_{1t}^{\bullet} = 1 - F_t^{\bullet}\left(\frac{bv_t^{\bullet} + s_t^{\bullet} - \overline{\pi_t^{\bullet}}}{\sigma_t^{\bullet}}\right) \quad , \quad \bullet = e, p \tag{15}$$

$$a_{2t}^{\bullet} = F_t^{\bullet}\left(\frac{bv_t^{\bullet} + s_t^{\bullet} - \overline{\pi_t^{\bullet}}}{\sigma_t^{\bullet}}\right) - F_t^{\bullet}\left(\frac{bv_t^{\bullet} - s_t^{\bullet} - \overline{\pi_t^{\bullet}}}{\sigma_t^{\bullet}}\right) \quad , \quad \bullet = e, p \tag{16}$$

$$a_{3t}^{\bullet} = F_t^{\bullet}\left(\frac{bv_t^{\bullet} - s_t^{\bullet} - \overline{\pi_t^{\bullet}}}{\sigma_t^{\bullet}}\right) - F_t^{\bullet}\left(\frac{l_t^{\bullet} - \overline{\pi_t^{\bullet}}}{\sigma_t^{\bullet}}\right) \quad , \quad \bullet = e, p \tag{17}$$

$$b_t^{\bullet} = F_t^{\bullet}\left(\frac{l_t^{\bullet} - \overline{\pi_t^{\bullet}}}{\sigma_t^{\bullet}}\right) - F_t^{\bullet}\left(\frac{-l_t^{\bullet} - \overline{\pi_t^{\bullet}}}{\sigma_t^{\bullet}}\right) \quad , \quad \bullet = e, p \tag{18}$$

$$c_t^{\bullet} = F_t^{\bullet}\left(\frac{-l_t^{\bullet} - \overline{\pi_t^{\bullet}}}{\sigma_t^{\bullet}}\right) \quad , \quad \bullet = e, p \tag{19}$$

The solution of equations (15)-(19) leads to the following expressions defining the mean of the distribution ($\overline{\pi_t^{\bullet}}$), its standard deviation (σ_t) and both sensitivity intervals:

$$\overline{\pi_t^{\bullet}} = \frac{bv_t^{\bullet} \cdot \left(G_t^{\bullet} + H_t^{\bullet}\right)}{G_t^{\bullet} + H_t^{\bullet} - \left(E_t^{\bullet} + F_t^{\bullet}\right)} \quad , \quad \bullet = e, p \tag{20}$$

$$\sigma_t = \frac{-2 \cdot bv_t^{\bullet}}{G_t^{\bullet} + H_t^{\bullet} - \left(E_t^{\bullet} + F_t^{\bullet}\right)} \quad , \quad \bullet = e, p \tag{21}$$

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[2] Except theoretical justifications of using this type of distribution (e.g. Carlson 1975), many empirical studies support this choice (e.g. Mitchell 2002).

$$s_t = \frac{bv_t^{\bullet} \cdot \left(F_t - E_t\right)}{G_t^{\bullet} + H_t^{\bullet} - \left(E_t^{\bullet} + F_t^{\bullet}\right)} \quad , \quad \bullet = e, p \tag{22}$$

$$l_t = \frac{bv_t^{\bullet} \cdot \left(H_t - G_t\right)}{G_t^{\bullet} + H_t^{\bullet} - \left(E_t^{\bullet} + F_t^{\bullet}\right)} \quad , \quad \bullet = e, p \tag{23}$$

where: $E_t^{\bullet} = F_t^{\bullet^{-1}}\left(1 - a_{1t}^{\bullet}\right)$, $F_t^{\bullet} = F_t^{\bullet^{-1}}\left(1 - a_{1t}^{\bullet} - a_{2t}^{\bullet}\right)$, $G_t^{\bullet} = F_t^{\bullet^{-1}}\left(1 - a_{1t}^{\bullet} - a_{2t}^{\bullet} - a_{3t}^{\bullet}\right)$, $H_t^{\bullet} = F_t^{\bullet^{-1}}\left(c_t^{\bullet}\right)$.

The benchmark value $bv_t^{\bullet}$ used in the above equations equals the scaling factor if it is positive:

$$\text{for } sf_t^{\bullet} > 0: \quad bv_t^{\bullet} = sf_t^{\bullet}, \quad \bullet = e, p \tag{24}$$

Otherwise the benchmark value is calculated using the condition (11), i.e. that perceived or expected inflation in the group of respondents declaring fall in prices equals $\pi_t^{c\bullet}$, as estimated from the equation (14). To express this condition we use the formula of the mean of the truncated distribution (Greene 2003, p. 759):

$$\text{for } sf_t^{\bullet} \leq 0: \quad E\left(\pi_t^{\bullet} \mid \pi_t^{\bullet} < -l_t^{\bullet}\right) = \overline{\pi_t^{\bullet}} + \sigma_t^{\bullet} \frac{-f_t^{\bullet}\left(\dfrac{-l_t^{\bullet} - \overline{\pi_t^{\bullet}}}{\sigma_t^{\bullet}}\right)}{F_t^{\bullet}\left(\dfrac{-l_t^{\bullet} - \overline{\pi_t^{\bullet}}}{\sigma_t^{\bullet}}\right)} = \pi_t^{c\bullet} \quad , \quad \bullet = e, p \tag{25}$$

Replacing parameters of the normal distribution with their definitions (20) and (21), the benchmark value is derived in the following way:

$$\text{for } sf_t^{\bullet} \leq 0: \quad bv_t^{\bullet} = \pi_t^{c\bullet} \cdot \frac{G_t^{\bullet} + H_t^{\bullet} - \left(E_t^{\bullet} + F_t^{\bullet}\right)}{G_t^{\bullet} + H_t^{\bullet} + \dfrac{4}{c_t^{\bullet}} e^{-\frac{1}{2}H_t^{\bullet 2}}} \quad , \quad \bullet = e, p \tag{26}$$

Uniform distribution

Under the uniform distribution defined over the interval $\left(\overline{\pi_t^{\bullet}} - q_t^{\bullet} ; \overline{\pi_t^{\bullet}} + q_t^{\bullet}\right)$, equations (6)-(11) take the following form:

$$a_{1t}^{\bullet} = \frac{1}{2q_t^{\bullet}} \cdot \left(\overline{\pi_t^{\bullet}} + q_t^{\bullet} - bv_t^{\bullet} - s_t^{\bullet}\right) \quad , \quad \bullet = e, p \tag{27}$$

$$a_{2t}^{\bullet} = \frac{s_t^{\bullet}}{q_t^{\bullet}} \quad , \quad \bullet = e, p \tag{28}$$

$$a_{3t}^{\bullet} = \frac{1}{2q_t^{\bullet}} \cdot \left(bv_t^{\bullet} - s_t^{\bullet} - l_t^{\bullet}\right) \quad , \quad \bullet = e, p \tag{29}$$

$$b_t^{\bullet} = \frac{l_t^{\bullet}}{q_t^{\bullet}} \quad , \quad \bullet = e, p \tag{30}$$

$$c_t^{\bullet} = \frac{1}{2q_t^{\bullet}} \cdot \left(-l_t^{\bullet} - \overline{\pi_t^{\bullet}} - q_t^{\bullet}\right) \quad , \quad \bullet = e, p \tag{31}$$

$$\text{for } sf_t^\bullet \le 0: \quad \frac{-l_t^\bullet + \overline{\pi_t^\bullet} - q_t^\bullet}{2} = \pi_t^{c\bullet} \quad , \quad \bullet = e, p$$
$$\text{for } sf_t^\bullet > 0: \quad bv_t^\bullet = sf_t^\bullet, \quad \bullet = e, p \tag{32}$$

The solution is as follows:

$$\overline{\pi_t^\bullet} = \frac{bv_t^\bullet \cdot \left(1 - b_t^\bullet - 2c_t^\bullet\right)}{2a_{3t}^\bullet + a_{2t}^\bullet + b_t^\bullet} \quad , \quad \bullet = e, p \tag{33}$$

$$q_t^\bullet = \frac{bv_t^\bullet}{2a_{3t}^\bullet + a_{2t}^\bullet + b_t^\bullet} \quad , \quad \bullet = e, p \tag{34}$$

$$s_t^\bullet = \frac{a_{2t}^\bullet \cdot bv_t^\bullet}{2a_{3t}^\bullet + a_{2t}^\bullet + b_t^\bullet} \quad , \quad \bullet = e, p \tag{35}$$

$$l_t^\bullet = \frac{b_t^\bullet \cdot bv_t^\bullet}{2a_{3t}^\bullet + a_{2t}^\bullet + b_t^\bullet} \quad , \quad \bullet = e, p \tag{36}$$

$$\text{for } sf_t^\bullet \le 0: \quad bv_t^\bullet = \pi_t^{c\bullet} \cdot \frac{2a_{3t}^\bullet + a_{2t}^\bullet + b_t^\bullet}{-b_t^\bullet - c_t^\bullet} \quad , \quad \bullet = e, p$$
$$\text{for } sf_t^\bullet > 0: \quad bv_t^\bullet = sf_t^\bullet, \quad \bullet = e, p \tag{37}$$

4. Results

To empirically illustrate suggested adjustments in the probability method, we quantify consumer inflation expectations in the euro area based on qualitative survey data from the European Commission Consumer Survey covering years 1985-2010.[3] Except quantifying expectations for the euro area, similar measures are presented for Ireland – the economy, in which the recent financial crisis led to the highest negative inflation among euro zone economies.

Three measures of inflation expectations are derived. The first one is calculated assuming normal distribution of expected inflation and official current inflation as a measure of perceived inflation. The second measure replaces the assumption of normal distribution with the assumption of uniform distribution. The third measure assumes normality of expected inflation, but it uses survey-based measure as a proxy for perceived inflation.[4]

For the periods, in which scaling factors were non-positive, the average perceived or expected inflation in the group of respondents declaring fall in prices is calculated on the basis of estimation results of equation (13), presented in Table 1. Having estimated the value of the parameter $\pi_t^{c\bullet}$, the benchmark value $bv_t^\bullet$, used by the respondents declaring increase in prices, is derived with the parameters of the distribution of perceived or expected inflation.

Figure 1 presents survey data on consumer inflation perception and expectations and the results of quantification of perceived and expected inflation. It can be observed that the magnitude of the fall in the price level in Ireland – that reached as much as -6.5% in November and December 2009 – affected consumer opinions of perceived price changes considerably, with the fraction of

[3] See: http://ec.europa.eu/economy_finance/db_indicators/surveys/index_en.htm.
[4] Similar measures were used in a number of studies concerning consumer inflation expectations in the EU economies (e.g. Berk 1999, Forsells and Kenny 2004, Łyziak 2010).

respondents claiming that in the course of last 12 months prices have fallen increasing from negligible levels to 57% in January 2010.[5] The magnitude of the fall in prices in the euro area was relatively small and short-lived, therefore survey opinions on current price changes were affected less. Both in the euro area as a whole and in Ireland the distribution of responses to the survey question on future inflation was affected less than the survey opinions on past inflation.

As far as the results obtained from quantification methods are concerned, it should be noted that both inflation perception and expectations in the euro area were non-negative despite negative HICP inflation rates in the recent period. However, there was a relatively long episode of non-positive inflation perception in Ireland, although the maximum perceived price fall (-1.1%) was almost six times weaker than the fall in prices measured with official statistics. Even if we find evidence of non-positive perceived inflation in Ireland, inflation expectations – due to the fact that survey data on inflation expectations were affected to a lower extent[6] – were non-negative with some exceptions in May and July 2009, when both measures using current official inflation as a proxy for perceived inflation became slightly negative.

We can therefore conclude that both in the euro area as a whole and in Ireland, characterized by significantly negative inflation rates, the financial crisis and episodes of non-positive inflation did not create consumer deflationary expectations, although the level of inflation expectations was significantly reduced.

[5] For the period May 2008-April 2009 survey data from the European Commission Consumer Survey is not available for Ireland.

[6] The fractions of respondents expecting prices to increase was not lower than 27% during the episode of negative inflation perception with the fraction of respondents expecting prices to remain approximately the same not lower than 37%.

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Figures and Tables

Table 1. Estimating average perceived/expected inflation for respondents declaring fall in prices

	β_0	β_1	β_2	adj. R^2
Inflation perception – EMU				
normal distr.	-0.000 (.000)	0.672 (.049)	-0.145 (.024)	0.99
Inflation expectations – EMU				
scaled with current HICP inflation, normal distr.	-0.002 (.000)	0.237 (.046)	-0.362 (.028)	0.98
scaled with current HICP inflation, uniform distr.	-	0.883 (.039)	-0.055 (.018)	0.98
scaled with perceived inflation, normal distr.	-0.000 (.000)	0.682 (.072)	-0.006 (.001)	0.83
Inflation perception – Ireland				
normal distr.	-	0.993 (.005)	-	0.98
Inflation expectations – Ireland				
scaled with current HICP inflation, normal distr.	-	0.966 (.011)	-0.014 (.005)	0.94
scaled with current HICP inflation, uniform distr.	-0.000 (.000)	0.887 (.033)	-0.001 (.000)	0.80
scaled with perceived inflation, normal distr.	-	0.976 (.007)	-0.007 (.003)	0.99

Figure 1. Survey data and quantification results – EMU and Ireland