On Elements of Axiomatizing Eventology

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10. December 2009
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Received 10.12.2009, received in revised form 10.03.2010, accepted 20.04.2010

Elements of eventologic axiomatics are offered. Eventology [1], a new direction of probability theory and philosophy, offers the original many-event-based approach to the description of many-agent being and co-being, entering human agents, together with his/her beliefs, directly in the frameworks of scientific research in the form of eventologic distribution of his/her own events. This allows us, by putting together probabilistic and many-event-based representation of information and philosophical concept of event as co-being [2], to offer an axiomatizing eventology which expands Kolmogorov's axiomatic of probability theory [3] and axiomatizes an overlapping sciences mathematical eventolanguage for the description of many-agent being and co-being.

Keywords: event, co-being, probability, Kolmogorov’s axiomatics, eventology, axiomatizing eventology, eventologic axiomatics, universal elementary event, universal event, universal measurable space, universal probability space, eventologic space, name of event, set of names of events, sufficiency of eventologic space, simpliciality of eventologic space.

The contemporary eventology [1] is a scientific theory with mathematical tools based on Kolmogorov’s axiomatics of probability theory and with philosophical features based on the following ideas about event and probability:

• “event is always co-being” (Bakhtin, 1920, [2]);
• “event is a set of alternative outcomes and it comes if one of outcomes comes” (Kolmogorov, 1933, [3]);
• “matter is simply a convenient way of connecting of events together” (Russell, 1946, [4]);
• “mind arises there and then, where and when there is an ability of probabilistic choosing” (Lefebvre, 2003, [5]);
• “mind is an ability of co-being, i.e., an ability of probabilistic connecting of events together into a set and of probabilistic choosing an event among the set of events” (Vorobyev, 2001, [1]).

For last years eventology managed, having connected probabilistic and many-events-based representations of matter [4] and mind [1] with philosophical interpretation of event as co-being [2], to be achieved the level of theoretic development such that an axiomatic definition of eventology has appeared possible.

Recently the variant of expansion of Kolmogorov’s axiomatics of probability theory, the *Natural Axiomatic System* (Xiong, 2003, [6]) has been published. This axiomatic system plunges all probability spaces of various special probability models into the common causal space, a mathematical model of casual universe.

We assume to develop the **Eventologic Axiomatic System (EAS)** which absorbs Kolmogorov’s Axiomatic System (KAS) [3] and the **Natural Axiomatic System (NAS)** [6], adds new undefinable concepts and relations, and new postulates, and formalizes the basic eventologic ideas about event and probabilities listed above.

Here the elements of eventologic axiomatics are offered only. We begin with a new statement of Kolmogorov’s axiomatic system, we add to it new undefinable eventologic concepts and relations, and then we add two eventologic axioms. It has been more recently opened that these two new axioms were silently assumed true in our proofs of theorems about the maximal properties of an entropy of eventologic distributions [7, 8]. Still these

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1. Axiomatic definition is one of the supreme forms of scientific definition; definition of the scientific theory through the set of initial undefined notions and relations and the set of axioms (postulates) which limit an area of its possible interpretations. For an axiomatic definition of the scientific theory the high level of knowledge of researched area, which allows the theory to reduce studied objects and their relations to rather simple objects and relations, is required.
eventologic axioms are not enough for formalization of all basic eventologic ideas about event and probability. Our following researches will be devoted to end of construction of the EAS.

1. Kolmogorov’s axiomatics

Kolmogorov’s axiomatics of probability theory is used by us without changes as fundamental part of the EAS. First we shall give statement of KAS in our new form which is not changing Kolmogorov’s contents however. Thus to distinguish Kolmogorov’s concepts and denotations from its analogues introduced in the EAS, we shall add to each initial Kolmogorov’s denotation or definition a sign of “stroke”.

1. Undefined concepts and relations

In Kolmogorov’s axiomatics the following concepts are undefined:

(a’) set of random experiments E and random experiment $\varepsilon \in E$;
(b’) outcomes, or elementary events, of random experiment $\varepsilon$ are denoted by $\omega_\varepsilon'$.

2. Definitions of initial notions and relations

Notions of

(1’ elementary event,
(2’) space of elementary events, or a certain event,
(3’) event,
(4’) algebra of events,
(5’) probability of event,

and a relation

(6’) “to come, to happen, to occur”,

belong to initial notions and relations of Kolmogorov’s axiomatics. We define these notions and relations in the own way as follows.

Definition 1’ (elementary event). Outcomes $\omega_\varepsilon'$ of the random experiment $\varepsilon$ are called elementary events.

Definition 2’ (space of elementary events, or a certain event). The set $\Omega_\varepsilon'$ of all elementary events $\omega_\varepsilon'$ is called a space of elementary events, or a certain event.

Definition 3’ (event and a set of events). A set $\mathcal{F}_\varepsilon'$ of subsets of $\Omega_\varepsilon'$ is called a set of events, and each element $x_\varepsilon' \in \mathcal{F}_\varepsilon'$ is called a random event, or simply event $x_\varepsilon' \subseteq \Omega_\varepsilon'$.

Definition 4’ (algebra of events). A set of events is called an algebra of events if it contains the certain event $\Omega_\varepsilon'$ and if it is closed relatively set theoretic operations with a finite set of events.

Definition 5’ (probability of event). The function $P_\varepsilon'$ on the $\mathcal{F}_\varepsilon'$ is called the probability, and its value $P_\varepsilon'(x_\varepsilon')$ is called the probability of event $x_\varepsilon' \in \mathcal{F}_\varepsilon'$.

Definition 6’ (“to come, to happen, to occur”). We say that

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According to Kolmogorov [3].
a) the elementary event $\omega'_\varepsilon \in \Omega'_\varepsilon$ “comes, happens, occurs” if the random experiment $\varepsilon$ comes to the end with outcome $\omega'_\varepsilon$;

b) the event $x'_\varepsilon \in F'_\varepsilon$ “comes, happens, occurs” if the elementary event $\omega'_\varepsilon$ “comes, happens, occurs” such that $\omega'_\varepsilon \in x'_\varepsilon$.

3. Kolmogorov’s axioms

1. Finite probability spaces

We formulate first four Kolmogorov’s axioms as follows.

Axiom I’ (algebraicability of a set of events). The set of events $F'_\varepsilon$ is an algebra of events.

Axiom II’ (reality and non-negativity of probabilities). The function $P'_\varepsilon$ is real and non-negative on $F'_\varepsilon$: the probability of any event $x'_\varepsilon \in F'_\varepsilon$ is always the real non-negative number $P'_\varepsilon(x'_\varepsilon) \geq 0$.

Axiom III’ (normalization of probability). The probability of the certain event $\Omega'_\varepsilon \in F'_\varepsilon$ is equal to unit: $P'_\varepsilon(\Omega'_\varepsilon) = 1$.

Axiom IV’ (additivity of probability). $P'_\varepsilon(x'_\varepsilon + y'_\varepsilon) = P'_\varepsilon(x'_\varepsilon) + P'_\varepsilon(y'_\varepsilon)$ if $x'_\varepsilon \cap y'_\varepsilon = \emptyset$, i.e. if events $x'_\varepsilon$ and $y'_\varepsilon$ from $F'_\varepsilon$ are not intersected.

Definition A (probability space). The set of objects $(\Omega'_\varepsilon, F'_\varepsilon, P'_\varepsilon)$ satisfying to axioms I’ - IV’ is called a probability space.

2. Infinite probability spaces

The following Kolmogorov axiom allows to define the expanded concepts of event and probability space.

Axiom V’ (continuity of probability). For any decreasing sequence of events $x'_{\varepsilon 1} \supseteq x'_{\varepsilon 2} \supseteq \ldots \supseteq x'_{\varepsilon n} \supseteq \ldots$ from $F'_\varepsilon$ such that $\bigcap_n x'_{\varepsilon n} = \emptyset$ the limit of probabilities of these events is equal to zero: $\lim_n P'_\varepsilon(x'_{\varepsilon n}) = 0$.

Remark (continuity of events). A continuity of probability (Axiom V’) follows from a continuity of events (Axiom V)\textsuperscript{\textcopyright}.

Definition B (probability space in expanded sense). The set of objects $(\Omega'_\varepsilon, F'_\varepsilon, P'_\varepsilon)$ satisfying to axioms I’ - IV’ is called a probability space in expanded sense.

It is known that there is the least sigma-algebra of events\textsuperscript{**} $F'_{\varepsilon \sigma} = \sigma(F'_{\varepsilon})$ containing $F'_\varepsilon$. Moreover, the following theorem is true.

Theorem (\(\sigma\)-continuation of probability). The non-negative additive function $P'_\varepsilon$ defined on measurable space of events $(\Omega'_\varepsilon, F'_\varepsilon)$ can be uniquely continued with invariance of all properties (non-negativity, normalization

\textsuperscript{\textcopyright}According to Kolmogorov: a field of probabilities.

\textsuperscript{\textcopyright}Axiom V(a continuity of event) still should be formulated.

\textsuperscript{\textcopyright}According to Kolmogorov: a field of probabilities in expanded sense.

\textsuperscript{**}A set of events is called a sigma-algebra of events if it contains the ceratin event and if it is closed relatively set theoretic operations with a countable set of events.
and additivity) on all sets from $\mathcal{F}_{\sigma}^\prime$ to receive the countable-additive function $P_{\sigma}^\prime$.

Thus, each probability space $(\Omega_1^\prime, \mathcal{F}_1^\prime, P_1^\prime)$ can mathematically correctly is $\sigma$-continued up to the infinite probability space $(\Omega_\infty^\prime, \mathcal{F}_\infty^\prime, P_\infty^\prime)$ which in modern probability theory is called probability space simply.

At the same time elements of sigma-algebra $\mathcal{F}_{\sigma}^\prime$ of infinite probability space can be considered only as “ideal events”†† (“$\sigma$-events”). Nothing in the real world can be compared empirically to “ideal events”. However, if the reasoning uses probabilities of “ideal events” and leads to the definition of probability of “real event” (i.e. events from $\mathcal{F}_\sigma^\prime$) this definition automatically will be consistent from the empirical point of view obviously [3].

2. Elements of axiomatics of eventology

1. Undefined notions and relations

In the EAS the following concepts are undefined:

(a) World (physical and mental), Being (physical and mental);

(b) instant states of World, instant outcomes of Being, are denoted by $\omega$.

2. Definitions of initial notions relations

1. Analogues of Kolmogorov’s initial notions and relations

Together with all Kolmogorov’s initial concepts and relations the following ones concern to initial concepts and relations of the eventologic axiomatics.

(1) universal elementary event,

(2) space of universal elementary events, or a universal certain event,

(3) universal event,

(4) algebra of universal events,

(5) probability of universal event,

and a relation

(6) “to come, to happen, to occur”,

which are defined almost similarly as in Kolmogorov’s axiomatics.

Definition 1 (universal elementary event). Instant states $\omega$ of World, or instant outcomes $\omega$ of Being, are called universal elementary events.

Definition 2 (space of universal elementary events, or an universal certain event). The set $\Omega$ of all elementary events $\omega$ is called a space of universal elementary events, or a universal certain event.

Definition 3 (universal event and a set of universal events). A set $\mathcal{F}$ of subsets of $\Omega$ is called a set of universal events, and each element $x \in \mathcal{F}$ is called an universal event $x \subseteq \Omega$.

Definition 4 (algebra of universal events). A set of universal events is called an algebra of universal events if it contains the universal ceratin event $\Omega$ and if it is closed relatively set theoretic operations with a

††Kolmogorov’s term [3].
finite set of universal events.

**Definition 5 (probability of universal event).** The function $P$ on the $\mathcal{F}$ is called the *probability*, and its value $P(x)$ is called the *probability of universal event $x \in \mathcal{F}$.*

**Definition 6 (“to come, to happen, to occur”).** We say that
a) the universal elementary event $\omega \in \Omega$ “comes, happens, occurs” if World appears in the instant state $\omega$, or $\omega$ serves as the instant outcome of Being;
b) the universal event $x \in \mathcal{F}$ “comes, happens, occurs” if the universal elementary event $\omega$ “comes, happens, occurs” such that $\omega \in x$.

2. Analogues of Kolmogorov’s axioms

We formulate analogues of Kolmogorov’s axioms as follows.

**Axiom I (algebraicability of a set of universal events).** The set of universal events $\mathcal{F}$ is an algebra of universal events.

**Axiom II (reality and non-negativity of probabilities).** The function $P$ is real and non-negative on $\mathcal{F}$: the probability of any universal event $x \in \mathcal{F}$ is always the real non-negative number $P(x) \geq 0$.

**Axiom III (normalization of probability).** The probability of the universal certain event $\Omega \in \mathcal{F}$ is equal to unit: $P(\Omega) = 1$.

**Axiom IV (additivity of probability).** $P(x + y) = P(x) + P(y)$ if $x \cap y = \emptyset$, i.e. if events $x$ and $y$ from $\mathcal{F}$ are not intersected.

**Definition A (universal probability space).** The set of objects $(\Omega, \mathcal{F}, P)$ satisfying to axioms I’ – IV’ is called an *universal probability space*.

3. Eventologic initial concepts and relations

The eventologic axiomatics adds new notions to Kolmogorov’s initial notions and relations:

(7) *name of universal event* and a *set of names of universal events*,

and a new relation:

(8) “to have a name” for universal event,

which are defined as follows.

**Definition 7 (name of universal event and a set of names of universal events).** The partition $\mathfrak{N}$ of the set of universal events $\mathcal{F}$ is called the *set of names*, and each element $x \in \mathfrak{N}$ is called the *name of universal events $x \in x$*.

**Definition 8 (relation “to have a name” for universal event).** We say that the universal event $x \in \mathcal{F}$ “has the name” $x \in \mathfrak{N}$ if $x \in x$. 

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Definition 9 (empty name of universal event). The name $x_\emptyset \in \mathcal{R}$ is called the empty name and consists of universal events $x \in x_\emptyset$ which “have an empty name”; we also say that universal events $x \in x_\emptyset$ “have no name”.

3. First two eventologic axioms: a sufficiency and a simpliciality of eventologic space

It has appeared that for a construction of the eventologic theory and for a proof of eventologic statements additional new axioms are required. Here two new eventologic axioms are formulated. These axiom were required to prove theorems of a maximum of an entropy of multiplicative-truncated and general Gibbsean eventologic distributions [7, 8]. It was found out that aprioristic assumptions of some probability space properties are required for the proof of these theorems. These properties always silently were assumed by us, but not formulated in an explicit form.

The given assumptions have an all-eventologic character, play a fundamental role for development of eventology as a whole, and deserve to be allocated as two special eventologic axioms about a sufficiency and a simpliciality of eventologic space.

Axiom VI (sufficiency of universal probability space). The universal probability space $(\Omega, \mathcal{F}, P)$ is sufficient to define Kolmogorov’s probability space $(\Omega_\varepsilon, \mathcal{F}_\varepsilon, P_\varepsilon)$ for any Kolmogorov’s random experiment $\varepsilon \in \mathcal{E}$.

Remark. The expression “is sufficient to define” has the following sense: 1) to each outcome $\omega'_\varepsilon \in \Omega'_\varepsilon$ of the experiment $\varepsilon \in \mathcal{E}$ there corresponds the universal event $\omega_\varepsilon \in \mathcal{F}$, 2) to the set of all outcomes of the experiment $\varepsilon \in \mathcal{E}$ there corresponds $\mathcal{F}$-measurable partition of the space of universal elementary events:

$$\Omega = \sum_{\omega_\varepsilon : \omega'_\varepsilon \in \Omega'_\varepsilon} \omega_\varepsilon,$$

3) to each Kolmogorov’s event $x'_\varepsilon \in \mathcal{F}'_\varepsilon$ there corresponds the universal event

$$x_\varepsilon = \sum_{\omega_\varepsilon : \omega'_\varepsilon \in x'_\varepsilon} \omega_\varepsilon,$$

and 4) the set of probabilities $P(\omega_\varepsilon)$ of all universal events $\omega_\varepsilon$ such that $\omega'_\varepsilon \in \Omega'_\varepsilon$ defines Kolmogorov’s probability measure $P'_\varepsilon$ on the set of outcomes of experiment $\varepsilon$ by formula: $P'_\varepsilon(\omega'_\varepsilon) = P(\omega_\varepsilon)$ and, hence, on Kolmogorov’s algebra of events $\mathcal{F}'_\varepsilon$.

Definition 10 (probability simplex). Let $X \subseteq \mathcal{R} - \{x_\emptyset\}$ be a finite set of nonempty names of universal events. The $2^{|X|}$-vertex simplex is called the probability simplex generated by $X$ and is denoted by

$$S_{2^{|X|}}(X) = \left\{ p : p(X) \geq 0, \sum_{X \subseteq \mathcal{X}} p(X) = 1 \right\}$$

if it is defined as a set of points $p = \{p(X), X \subseteq X\}$ from $2^{|X|}$-dimensional real space such that its components are numbered by subsets $X \subseteq \mathcal{X}$ and satisfy to restrictions of local normalization:

$$0 \leq p(X) \leq 1, \quad X \subseteq \mathcal{X},$$

and to the restriction of global normalization:

$$\sum_{X \subseteq \mathcal{X}} p(X) = 1.$$
Definition 11 (eventologic distribution). Let $X \subseteq F$ be a finite set of the universal events with names from the finite set of nonempty names $X \subseteq \mathbb{N} - \{\emptyset\}$ of the same power: $|X| = |X|$. The set of probabilities $\{p(X), X \subseteq X\}$, where $p(X) = P(\text{ter}(X)), X \subseteq X$ are the probabilities of universal terrace-events
\[
\text{ter}(X) = \bigcap_{x \in X} x \bigcap_{x' \in X'} x' \in F,
\]
all together forming, as is known, the $\mathcal{F}$-measurable partition of the space of universal elementary events: $\Omega = \sum_{X \subseteq X} \text{ter}(X)$, is called the eventologic distribution (E-distribution) of the set of universal events $X \subseteq F$.

Definition 12 (event-probability (eventologic) simplex). Let $(\Omega, \mathcal{F}, P)$ be the universal probability space, $X \subseteq \mathbb{N} - \{\emptyset\}$ be the finite set of nonempty names of universal events, and $X_p \subseteq F$ be any finite set of the universal events with names from the finite set of nonempty names $X \subseteq \mathbb{N} - \{\emptyset\}$ of the same power: $|X| = |X_p|$ and with the E-distribution $p \in S_{2|X|}$. The totality of sets of universal events $X_p$ with E-distributions $p$, which fill all probability simplex $S_{2|X|}(X)$, is called the event-probability (eventologic) simplex generated by $X$ and is denoted by
\[
S_{2X} = \{X_p, p \in S_{2|X|}\}.
\]

Definition 13 (simplicial universal probability space). The universal probability space with the algebra of universal events, which contains all sets of universal events from the event-probability simplex $S_{2X}$ generated by $X$ for each finite set of nonempty names $X \subseteq \mathbb{N} - \{\emptyset\}$, is called the simplicial universal probability space concerning set of names $\mathbb{N}$ and probabilities $P$, or $(\mathbb{N}, P)$-simplicial universal probability space.

Axiom VII (simpliciality of the universal probability space). The universal probability space $(\Omega, \mathcal{F}, P)$ is the simplicial universal probability space concerning finite set of names $\mathbb{N}$ and probabilities $P$.

Definition 14 (eventologic space). The universal probability space $(\Omega, \mathcal{F}, P)$ is called the universal probability space with the set of names $\mathbb{N}$, or $\mathbb{N}$-eventologic space, or eventologic space (E-space), and is denoted by $(\Omega, \mathcal{F}, \mathbb{N}, P)$ if it satisfies to axioms VI and VII.

3. Discussion

1. About laconic eventologic terminology

The offered elements of the EAS contain KAS of probability theory.

In this first text on axiomatizing eventology for the sake of invariance of Kolmogorov’s terminology we used new terminology which is designed from former Kolmogorov’s one by addition of the term “general” for a denotation of eventologic concepts. For example, Kolmogorov’s “event” corresponds to the “general event” from the EAS, Kolmogorov’s “probability space” corresponds to the “universal probability space” from the EAS, etc.

Further we are going to pass to the more brief eventologic terminology where terms from KAS are called “Kolmogorov’s terms”. This will allow us to do without addition of the term “universal”. For example, “events” from KAS will be called “Kolmogorov’s events”, and “general events” from the EAS will be called “events”, etc. This will make eventologic texts more laconic and, hardly, can lead to misunderstanding.

*In the text finite eventologic spaces are considered only.*
2. Eventology is a science about event probability simplecies

The eventology is a science about events and their probabilities, or more precisely about event-probability simplecies where “eventologic” always means “event-probability”. Therefore we can tell that this science offers in axioms VI and VII the event-probability-simplex analogy of Kolmogorov’s probability space, the eventologic space (E-space).

![Figure 1: The simplicial manifold (complex) in three-dimensional space.](image)

Of course it is necessary to distinguish a probability simplex and an event-probability simplex. If the first one is a set of $2^{|X|}$-dimensional points (with the components specifically numbered by subsets of events) such that each point is interpreted as a set of probabilities of events-terraces, then the second one is a totality of sets of events with E-distributions filling the probability simplex completely.

Let’s notice, that if an E-distribution is understood not only as a set of probabilities $p$, but also as a set of corresponding terrace-events generated by $X_p$ then we can speak that an event-probability (eventologic) simplex is a simplicial$^1$ totality of E-distributions generated by a fixed finite set of nonempty names of events. Thus for any $X \subseteq \mathcal{G} - \{\emptyset\}$ in the E-space $(\Omega, \mathcal{F}, \mathcal{G}, \mathcal{P})$ there is the simplicial set of E-distributions generated by $X$.

References


$^1$Simplicial geometry is a section of geometry studying simplicial manifolds (fig. 1).
О началах аксиоматизации эвентологии

Олег Юрьевич Воробьев

Предлагаются начала эвентологической аксиоматики. Эвентология [Воробьев, 2001], новое направление теории вероятностей и философии, предлагает оригинальный многособытийный подход к описанию многоагентного бытия и со-бытия, вводя человеческих агентов непосредственно в рамки научного исследования в виде эвентологических распределений множеств их собственных событий. Соединение вероятностного и многособытийного представления материи [Рассел, 1946] и разума [Воробьев, 2001], а также философского понятия события как со-бытия [Бахтин, 1920] предполагает такую аксиоматизацию эвентологии, которая, расширяя кольмогоровскую аксиоматику теории вероятностей [Колмогоров, 1933], формулирует аксиомы перекрывающего науки математического эвентоязыка для описания многоагентного бытия и со-бытия.

Ключевые слова: событие; со-бытие; всеобщее событие; вероятность; кольмогоровская аксиоматика; эвентология; аксиоматика эвентологии; всеобщее вероятностное пространство; эвентологическое пространство; имя события; множество имен событий; достаточность эвентологического пространства; симплициальность эвентологического пространства.