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the Schumpeterian Cobb-Douglas
Production Function

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Cobb-Douglas Production Function

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Abstract

In this paper, we institute the role of entrepreneurs in technical progress and the mechanism of tools multiplication into the Cobb Douglas Production Function. After the advancements, the technology component in the function has technical meaning and is potentially observable. Unlimited technical progress becomes possible and automatic under sticky competitive markets. The coexistence of sustained growth, decline and stagnation across countries and time becomes obvious and the target of public policies for achieving sustained growth is also clear and precise.

Key words: Entrepreneur, Sticky Competition, Cobb-Douglas Production Function, Endogenous Growth, Technical Progress, Tools Variety.

JEL Classification: O4, D2.
1. Introduction

‘My own favourite how-to-do-it injunctions are: (i) keep it simple; (ii) get it right; and (iii) make it plausible. (By getting it right, I mean finding a clear, intuitive formulation, not merely avoiding algebraic errors.’
<Robert Solow, 2007>

The Cobb-Douglas Production Function (CDOPF) has been chosen to investigate numerous economic problems for decades. It derives merits from its simplicity, easy to handle, transparent and encompassing in capturing the key elements determining aggregate output and growth. However, the production function is not developed on the basis of any knowledge of engineering, technology, or management of the production process. The technical element is exogenous, not independently measured and is considered only as the residual in the growth accounting exercise. Moreover, the diminishing marginal product in the function generates the gloomy prediction that every economy will finally stop growing and more importantly, the prediction is counterfactual. Despite all these criticisms, it is still the most popular functional form adopted in numerous economic investigations that have generated many useful theoretical and empirical results. Because of the remarkable merits, it is highly desirable to retain the basic structure of CDOPF along with the merits while eliminate the above-mentioned shortcomings.

In our model, the insights of Schumpeter (1934) on entrepreneurs and innovations, the framework constructed by Shell (1973) on competition through innovations and the role of input/product variety proposed in Dixit and Stiglitz (1977) are instituted into the CDOPF. With the advancements, technology and its progress in the CDOPF are no longer a black box and being exogenous. They are embedded in the combinations of tools, machines and materials adopted by profit-maximizing entrepreneurs in production. In a sticky competitive environment, the potential short-run innovative rent generated from the lower production cost than the market provides incentive to entrepreneurs for conducting innovative activities. In the longer-run, the advanced production method is imitated by competitors. The spillovers of advancements and innovative activities to the whole economy
raise the marginal product of investment and therefore induce more capital accumulation. Sustained increase in output is therefore automatic under the market mechanism. The coexistence of sustained growth, decline and stagnation across countries and time becomes obvious and the target of public policies for achieving sustained growth is also clear and precise.

The divergence of our model from the endogenous growth theories such as Romer (1990) is that sustained growth is driven by profit-maximizing entrepreneurs under a price taking sticky competitive market structure. The framework is more straight-forward than the monopolistic R&D framework. The implications from the model are also richer and have stronger explanatory power. The growth determining factors proposed by endogenous growth models such as R&D activities, stock of knowledge, human capital and government supports become neither necessary nor sufficient to sustained growth. We instill technical meaning into the technology component in the CDOPF and explain the automatic technical progress directly from the profit motive of entrepreneurs in their production process. This Schumpeterian Cobb Douglas Production Function (SCDOPF) is the direct extension of the CDOPF. It is highly transparent and operational which makes many important issues on economic growth and development have obvious and straight-forward answers. The voluminous literature questing for sustained growth is gauged into the level and effectiveness of entrepreneurial activities that drive the growth of tools variety.

The rest of the paper is organized as follows. Section 2 derives the SCDOPF by incorporating the role of entrepreneurs, tools variety, competition through innovations and the mechanism of tools multiplication into the CDOPF. It is followed by the discussion about its direct implications on some important issues of economic development and growth. The final section concludes the paper.

2. Entrepreneurs, Tools Variaty, Sticky Competition and SCDOPT

*Economic leadership in particular must be distinguished from “invention.” As long as they are not carried into
practice, inventions are economically irrelevant. And to carry any improvement into effect is a task entirely different from the inventing of it, and a task, moreover, requiring entirely different kinds of aptitudes. Although entrepreneurs of course may be inventors just as they may be capitalists, they are inventors not by nature of their function but by coincidence and vice versa … It is, therefore, not advisable, and it may be downright misleading, to stress the element of invention as much as many writers do.’  < Schumpeter, 1934 >

A theory of economic growth is a dynamic theory of production. It investigates how the input-output relationship changes over time. A major focus of researches on economic growth is therefore searching for a production function that can capture the fundamental inputs in production as well as how and why they might change over time. In recent decades, the endogenous growth theory can be considered as the effort to establish a production function that can explain sustained economic growth. The simplest theoretical model of endogenous growth is the AK model of Rebelo (1991) where he postulated that output is proportional to a broad concept of capital ($K$) that could include stock of knowledge, technology, and organizational technique. Capital can therefore be built up over time without limit that makes sustained growth possible. Romer (1986 and 1990) attempts to introduce the possibility of sustained growth by invoking knowledge or human capital as an additional factor of production which does not subject to the law of diminishing productivity. The specification in Romer (1990) emphasizes the importance of R&D while Romer (1986) shows that specialization and scale are important determinants of the rate of growth. As a result of the changes in assumption, human capital becomes the determining factor of sustained growth in his later contribution instead of country or population size in Romer (1986). The contributions initiate various attempts to endogenize technical progress and to find other possible causes of sustained economic growth such as product quality improving innovations as modeled in Grossman and Helpman (1991a) and Aghion and Howitt (1992).

However, there is no shortage of negative comments on the endogenous growth models. For instance, as observed in Jones (1995), the endogenous growth literature has turned to a class of models in which growth is driven by technological change that in turn
determined by R&D efforts of profit-maximizing agents in a monopolistic R&D sector. Introducing human capital and the R&D sector as the drivers of technical progress has made the implications from the models highly sensitive to their specific assumptions. In addition, the models fail to generate supportive empirical literature and the implications can easily be refuted by casual observations. After a thorough review on the endogenous growth models, Pack (1994) concluded that: ‘the long-term imprint of any growth theory must ultimately depend on the extent to which it generates a productive empirical literature. In this task, endogenous growth theory has led to little tested empirical knowledge.’ Obviously, there is a lot of room to advance in the endogenous growth literature by making the implications less restricted by the assumptions and parameters in the models while at the same time, having stronger explanatory power on the real world phenomena related to the causes and mechanism of sustained economic growth.  

2.1 Innovation, Entrepreneur and Tools Variety

"Anything that we have to learn to do we learn by the actual doing of it: people become builders by building and instrumentalists by playing instruments.”  <Aristotle. Cited in: Dasgupta and Stiglitz (1988)>

Schumpeter (1934) emphatically distinguishes innovations from inventions. An invention is an idea, a sketch or model for a new or improved device, product, process or system. Such inventions do not necessarily lead to technical innovations. In fact, the majority do not. An innovation in economic sense is accomplished only with the first commercial transaction involving the new production method, device, product is completed. The chain of events from invention to economic application is often long and risky. Schumpeter always stresses the crucial role of entrepreneurs in this complex innovative process. As stated in Schumpeter (1934), the carrying out new combinations can no more be a vocation than the making and execution of strategic decisions. The entrepreneur’s essential function must always appear mixed up with other kinds of activity. However, everyone is an entrepreneur only when he actually carries out new combinations. In this section, we extend the CDOPF
by incorporating the insights of Schumpeter (1934) relating to the role of tools, the nature of innovations, the function of entrepreneurs and competition through innovations in economic development process. The most common type of innovations is the introduction of new tools variety that can raise the productivity of the firm. Real-world examples include steam engine, train, and computer that generate new mix of tools variety for mining, transportation, education, manufacturing and finance. They substantially lower the costs for satisfying certain needs and wants. Of course, innovations need not be as revolutionary as the examples. Most innovations are just incremental improvements based on old production methods. In the production process, a routine job of entrepreneurs is to select an optimal combination of tools to minimize/maximize cost/profit. In the competitive environment with imitation lag, entrepreneurs are supported and induced to innovate by innovative quasi-rent and/or are pressed to innovate for survival.

There are two fundamental inputs, labor and a set of tools that generate capital services in the model. Behind the production scene is entrepreneur who is responsible for decision-making, risk-taking and most importantly, conducting innovative activities. Innovation is defined as introducing a new mix of tools in the production process that can raise the productivity of capital service. The output level \( Y_j \) depends on the amount of capital service \( K_j \) and labor \( L_j \) employed by representative firm \( j \), such that:

\[
Y_j = K_j^\alpha L_j^{\beta} \quad \text{with} \quad \alpha + \beta = 1 \quad \text{and} \quad 0 < \alpha < 1 .
\] (E1)

In order to introduce the contribution of innovations in the production process, we follow the product/input variety literature attributed to Dixit and Stiglitz (1977). A combination of tools is aggregated by the CES function that gives a positive value to an increase in tools variety in generating the capital service, such that:

\[1\] Man-made tools for enhancing production include simple tools, machines and materials. Each variety provides differentiated services that are complimentary to each other in production. They are combined to generate 'capital service' for specific production to meet certain needs and wants.
\[ K_j = (\sum_i X_{ij}^\theta)^{1/\theta}, \]  
(E2)

with \(0 < \theta < 1\) and, \(i = 1\) to \(v_j\).\(^2\)

The parameter \(\theta\) is greater than zero and less than one that governs the elasticity of substitution between the tools. A higher value of \(\theta\) indicates that the tools variety \((X_{ij})\) can be more easily substituted for each other in the production of capital services while a lower values of \(\theta\) correspond to greater differentiation among the set of tools. The capital service function has identical structure as in Dixit and Stiglitz (1977) that are followed closely by the others. Similar functional form is then adopted in the works of Ethier (1982) for introducing the gains caused by increase in input variety to study the implications on international trade. The conventional Cobb-Douglas production function treats all man-made tools for enhancing production are perfect substitutes that is equivalent to considering \(\theta\) equals one.\(^3\)

The measurement unit of each tool is normalized so that the unit price of each tool equals to \(r\). For all output level, a profit-maximizing entrepreneur in firm \(j\) choose an optimal level of each tool, \(X_{ij}^*\), \(i = 1, \ldots, h, \ldots v_j\), to maximize the value of capital service \(K_j = (\sum_i X_{ij}^\theta)^{1/\theta}\) subject to a given \(C_j^k\) allocated for capital service with \(C_j^k = \sum_i rX_{ij}\).

The first-order conditions and the symmetry imply that for all \(i\), \(X_{ij}^* = X_{hj}^* = X_j^*(r, C_j^k, V_j)\).

\(^2\) Obviously, the labor service (L) can be treated in an identical manner such that labor service depends on a combination of workers with different skills and human capital. However, the sacrificed technical detail allows us to simply and compactly formalize important ideas about the role of tools variety in the production function.

\(^3\) The source of sustained growth based on CDOPF is driven by exogenous growth in technology. In Romer (1990), capital service is the outcome of the aggregation of tools that are additively separable. Technology/tools variety is generated by a profit-maximizing monopolistic R&D sector with inputs including existing stock of knowledge and human capital. The conclusions of the model such as the rate of technological change is sensitive to the rate of interest, a larger total stock of human capital will experience higher sustained growth and free international trade can speed up growth are mainly driven by the assumptions.
with \( \frac{\partial X_j^*}{\partial r} < 0 \), \( \frac{\partial X_j^*}{\partial C_j^k} > 0 \), \( \frac{\partial X_j^*}{\partial V_j} < 0 \); where \( V_j \) is the number of tools variety exogenously given to firm \( j \). Therefore, for all output level,

\[
K_j^* = \left[ \sum_i X_i^* \right]^{1/\theta} = V_j^{1/\theta} X_j^*, \tag{E3}
\]

(E3) indicates that the Dixit-Stiglitz capital service function can be decomposed into the variety component, \( V_j \) and the tools component \( X_j \). Capital service now has an exact definition. It is an aggregation of the tools adopted by the cost-minimizing entrepreneurs in production. Moreover, the higher the \( V_j \), the higher the productivity of \( K_j^* \) and the lower the average cost of capital service \( (C_j^k / K_j^*) \) for all \( C_j^k \). Substituting \( K_j^* \) into (E1), we have:

\[
Y_j = V_j^a X_j^a L_j^\beta = A_j X_j^a L_j^\beta; \quad \text{where } A_j \text{ equals } V_j^a. \tag{E4}
\]

The formulation results in \( A_j \) equals \( V_j^{a/\theta} \) in the CDOPF. The ‘\( A_j \)’ in the CDOPF has observable and transparent definition which is the level of tools variety adopted by the profit-maximizing entrepreneurs in production. The higher the \( V_j \), the higher the productivity of capital service and the lower the average cost of production.

### 2.2 Creative Destruction in the Price-taking Sticky Competitive Market Structure

“Development in our sense is then defined by the carrying out of new combinations….new good….new quality of a good….new method of production….new way of handling a commodity….new market….new sources of supply of raw materials….new organization of any industry….new combinations should be carried out by the

\[
\frac{\partial K^*}{\partial V_j} = \frac{1}{rV_j / C_j^k} \left[ \frac{1}{V_j^a} + V_j^a \left( \frac{0 - r / C_j^k}{(rV_j / C_j^k)^2} \right) \right] = V_j^{a-2} \frac{1}{r / C_j^k} (\frac{1}{\theta} - 1) > 0. \quad \text{An increase in } V_j \text{ has two effects on the value of capital service. The first component indicates its direct positive effect while the second component indicates its negative displacement effect that lowers the level of each } X_j^a. \quad \text{Given the cost/resource constraint, employing additional tools variety has to reduce the quantity of other tools variety. However, the direct effect always dominant the indirect effect when } \theta \text{ is less than one. For simplicity, we drop the star in denoting } X_j^* \text{ in the production function. This has no effect on our conclusion as the positive direct effect always dominates.}
same people who control the productive or commercial process which is to be displaced by the new...new combinations are, as a rule, embodied, as it were, in new firms which generally do not arise out of the old ones but start producing beside them.”  <Schumpeter (1934). Italics mine.>

Schumpeter (1934) described a competitive mechanism that spurs innovation during a creative destruction process among firms. Innovative effort is motivated by the resulting source of profit that exceeds the normal level. He also described how competitors’ imitation erodes the profit and forces the profit-maximizing firm to advance further if the innovative quasi-rent (IR) is not to dry up. The unmatched production and growth performances of free enterprise economies are mainly due to the competition for the IR that constantly reduces cost or increases production. Shell (1973) formulates a model that describes the process under the price-taking sticky competitive market structure. Under the framework, the level of technology may differ among many small firms. The reasons can be due to the high transmission costs of technology between firms. Firms with advanced technologies have incentives for not revealing their technologies, and employ secrecy to achieve this end. Patents can also give some limited legal protection to the leading firms and therefore increase the level of stickiness. Other potential sources are also described in Gerschenkron (1968): ‘Very few modern industrial entrepreneurs were truly great innovators, in the sense of being the first to apply a revolutionary, unprecedented technique. Most of them were imitators, a part of what Schumpeter called the “secondary wave,” which spread a new signal innovation over broad segments of industrial economy. But, as everyone who has ever worked inside a modern enterprise knows, the distinction between the innovator and imitator is a very uncertain one. Every imitation requires a great deal of energy to overcome inertia, to abandon the accustomed way of doing things. It raised a million technical and economic problems that must be solved. And they will not be solved unless there are alert minds to welcome the new and to see the solutions and strong wills to carry the tasks to successful termination.’ The key idea of the creative destruction process under the sticky
competitive environment specified in Shell (1973) can be captured in Figure 1.

**Figure 1:**
Price-taking Sticky Competitive Market and Competition through Innovation

Note: AC$_{t0}$ has three sections: P$_e$a is the AC of the advanced firms, ac belongs to the catching-up firms and cs is the AC of the lagged firms. AC$_{t1}$ of P$_e$es shows the effect on the AC when more firms caught-up while AC$_{te}$ of P$_e$es corresponds to the long-run equilibrium where all firms produce with the same technique in the market given the demand curve D.

Initially at $t_0$, the economy has three types of firm and the respective average cost of production (AC): the leading firms produce with highest $V$ and have the lowest AC that equals to P$_e$; the intermediate firms produce with less $V$ with AC ranging from a to c while the lagged firms with AC of P$_0$ have the lowest $V$. Since it is always possible to produce the final good using well-known traditional technology, there is unlimited supply at P$_0$. The AC$_{t0}$ of P$_e$acs is therefore the supply curve of the product at $t_0$. Given the market demand curve D, the market price equals P$_0$ and the output produced is Q$_0$. The leading and intermediate firms are earning the total innovative quasi-rent (IR) amounting to P$_0$caP$_e$ while at the same equilibrium price, the potential IR to be captured amounts to ac e$_0$f.

The existence of IR implies that e$_0$ will not be in equilibrium. Besides the
leading firms will expand their output by setting up new small production units, all other firms will try to capture the IR by imitating the production method of the leading firms such that the supply curve moves to $AC_{t1}$ with the equilibrium $e_1$ and finally $AC_{te}$ with the long-run equilibrium $e$ when more and more lagged firms catch up. In the process, all stagnant lagged firms with $AC$ higher than $P_e$ are driven out from the market when the market price decreases as in the $e_1$ and $e$.\(^5\) In the long run, all firms adopt the same advanced technology when the technology is a common knowledge, available to and adopted by all entrepreneurs. All survival firms produce with the same $AC$ before some firms find a better method of production that initiates the catch-up process again. In this sticky competitive environment, innovative activity is a routine activity of the entrepreneurs in all firms targeting for IR and/or for survival.

In the long-run equilibrium, the aggregate output is obtained by summing up the output of the $n$ identical firms in the economy:

$$GDP = Y = nY_j = nV^\alpha X_j^\alpha L_j^\beta = \bar{V}^\alpha X^\alpha L^\beta,$$

where $X$ and $L$ equals to $nX_j$ and $nL_j$ respectively. (E5) suggests that when resources are under-utilized in an economy, increase in the number of firms with the same technology can generate short-term growth when $n$ increases. However, when resources are fully utilized, expansion of $V$ is the only way to unlimited growth.

\(^5\) Notice that the exit of stagnant firms does not imply the obsolescence of old production method and tools. In most cases, the main characteristics of many simple tools/products are embodied in the advanced tools/products that can better satisfy certain needs or wants. The original simple tools/products are embodied rather than obsolete. The creative destruction process is therefore modeled as the expansion of leading firms and the exit of stagnant lagged firms instead of being treated as the obsolescence of ‘inferior products’ as in Aghion and Howitt (1992), for instance. Moreover, old technology is always observed to be adopted if it is profitable. New methods that are cost effective in some countries/environment may not be so in other places. Due to the complementarities among inputs, ‘better tools’ are very difficult to define. For examples, car may be superior or inferior to horse for satisfying the travelling needs which depends on the condition of roads; candle lighting is more effective than light-bulbs if electricity is not available or very expensive.
While IR motivates and supports each firm to innovate during production, this framework is the direct extension of the CDOPF and preserves the vastly adopted price-taking competitive market structure that is much more general, transparent, less restrictive and easier to handle than the product/input variety growth models based on monopolistic R&D sector. However, when \( \alpha/\theta \) is less than one, introducing the tools variety into the production function may not be able to escape the dismal conclusion that economic growth will finally stop. Tools variety may remain subject to the law of diminishing productivity and the economy will finally approach a steady state of zero growth even \( V \) increases over time.\(^6\)

2.3 The Unlimited Possibilities of Innovations

In reviewing the evolution process of production technology, innovations/inventions are complimentary to each other. In the words of Rosenberg (1976) ‘for technological change as in other aspects of human ingenuity, one thing often leads to another – not in a strictly deterministic sense, but in the more modest sense that doing some things successfully creates a capacity for doing other things … experience in the production of firearms made it a relatively simple matter to produce sewing machines, just as the skills acquired in producing sewing machines and bicycles greatly facilitated the production of the automobile.’ (p. 30) The observation suggests that a ‘new’ tool/machine/material is not really new in most cases. It embodies all or most of the characteristics/components of the old tools/machines/materials although it is modified or upgraded by new way of combination and/or by new devices/materials. As a result, the likelihood of devising a new tool depends on the number of existing tools variety. This idea is exemplified by the evolution of drills, watch, automobile and various machines. It becomes increasingly obvious that tools variety

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\(^6\) Sustained growth is feasible if \( \alpha/\theta \) is greater than or equal to one. However, this will make the model fall into the ‘linearity critique’ on endogenous growth models. That is, the possibility of sustained growth depends on the magnitude of the parameters that is purely arbitrary without behavioral foundations. (Jones, 2005)
multiplies in the sense that the larger the stock of tools/machines/materials, the larger will be the increase in tools variety over time. Moreover, the potential number of tools variety is unlimited.

This mechanism of tools multiplication find corollary in other discipline like biology. In biology, the growth of organism is defined by the multiplication of cells. Bacterial growth is the division of one bacterium into two identical daughter cells during a process called binary fission. The two daughter cells may not both survive. However, if on average, the number of survival exceeds unity, the bacterial population will undergo exponential growth when the environment is favorable.

Similar corollary can be found in the ancient Chinese Tao philosophy. The Tao philosophers suggest that the complexity of nature is generated by the multiplication and combination of Yin and Yan. The ‘singularity’ divides into Yin and Yan. The existing Yin and Yan further combine with either Yin or Yan which generates the four ‘phenomena’. The process continues and the variety/complexity of natural phenomena multiplies with the base of 2 following a sequence of: $2^0$, $2^1$, $2^2$, $2^3$ and so on. The number of multiplication therefore defines the number of the variety or the level of complexity/development in the nature.

Romer (2007) suggests that similar to the exploratory synthesis in physical chemistry, the possibilities of discovering new recipe/mixture of materials do not merely add up, they multiply. He proposes that the source of economic growth is driven by the combinatorial explosion of ideas that potentially has no limit and is not subject to the law of diminishing productivity.

The commonality among different disciplines and insights suggests that tools variety does evolve through multiplication and limited only by human imagination. We therefore assume that tools variety evolves through multiplication, such that:

$$V = e^\gamma,$$  

(E6)
where \( \gamma \geq 0 \) is a variable indicating the number of multiplication. The higher the \( \gamma \), the higher the level of development/technology.\(^7\) More importantly, unlike the inputs like tools and labor, tools variety is not limited by the non-producible land. It implies that \( \gamma \) or \( V \) has no limit by its nature.

Substituting (E6) into equation (E5), we have:

\[
Y = e^{\gamma(\alpha/\theta)} X^\alpha L^\beta = e^{a} X^\alpha L^\beta = AX^\alpha L^\beta ,
\]

with \( \gamma \geq 0 \); \( a = \gamma(\alpha/\theta) \geq 0 \), and \( A = e^a \).

Total differentiation implies:

\[
\frac{dY}{Y} = da + \frac{dX}{X} + \frac{dL}{L} = \frac{\alpha}{\theta} d\gamma + a \frac{dX}{X} + \beta \frac{dL}{L} = \frac{\alpha}{\theta} \frac{dV}{V} + \alpha \frac{dX}{X} + \beta \frac{dL}{L} ,
\]

with \((\alpha/\theta) \geq 1\).\(^8\)

After the role of entrepreneurs and the combinatorial nature of tools variety are instituted into the CDOPF, we have a clear definition of technology and its evolution mechanism while sustained growth becomes possible and automatic.\(^9\) At the same time, we retain the simplicity and all the other merits of the CDOPF. As commonly observed across countries and time, (E8) suggests that an economy can experience increasing sustained growth when \( d\gamma \)

\(^7\) Choosing ‘e’ as the base of multiplication is for convenience only. It can be any number larger than unity. Exponential gets its name from the number of times the number is multiplied by itself. Linear growth is incremental and gradual while exponential growth is drastic and dramatic.

\(^8\) As shown in footnote 4, \( \frac{\partial K_j^*}{\partial V^j} = V_j^{1/\theta - 2} \frac{1}{r/C_j^k} \left( \frac{1}{\theta} - 1 \right) > 0 \). It indicates that the marginal productivity of a new tools variety to capital service is maximized when \( \theta \) approaches zero. Since the possibilities of innovation are unlimited, other things being equal, entrepreneurs tend to adopt/invent new tools variety with the lowest \( \theta \), that is, tools of higher differentiation to the existing tools, for maximizing the IR. Therefore, under proper environment, the ratio \((\alpha/\theta)\) is likely to be greater than one on average. Escaping the law of diminishing productivity becomes possible, automatic and supported by micro-foundations.

\(^9\) Although escaping the law of diminishing productivity becomes possible and automatic, economic growth will not become sustaining or explosive automatically. It is confined by the growth rate of tools variety that is determined by many vary institutional factors documented by a long list of theoretical and empirical papers on long-run growth and development. (for instance, the paper series contributed by Mo, various years)
increases over time. On the other hand, an economy can be stagnant when \( d\gamma \) equals zero, or can be declining in growth rate when \( d\gamma \) is decreasing. As indicated in (E7), if \( \gamma \) remains unchanged and is exogenous, the production function becomes the CDOPF with identical growth implications in related literature.

3. Direct Answers and Implications on Major Growth Issues


Building upon the contributions from Cobb-Douglas (1928), Schumpeter (1934), Shell (1973) and the product/input variety literatures attributed to Dixit-Stiglitz (1977), our model provides a clear and precise target for all decision-makers who care about growth. Moreover, the model has some obvious, straight-forward answers for many important issues related to economic development and growth. We list some of them as follows:

1. The differences in the level and growth rate of tools variety account for most of the differences in the level and growth rate of GDP across time and countries. Poor countries remain poor because they have low rate of tools multiplication adopted in the production process due to their bad motivation system that discourages entrepreneurs to innovate and/or apply the global stock of tools variety for productive uses. The model suggests that nothing, sustained growth, decline or stagnation is inevitable. The rate of tools multiplication across countries explains the structure of the long-run world income distribution over time.

2. In the long-run equilibrium, the innovations by individual firm \( j \) will be imitated by all other production units in the economy, such that: \( Y = GDP = nY_j = nV_j^{\alpha/\theta}X_j^\alpha L_j^\beta = V^{\alpha/\theta}X^\alpha L^\beta \). Moreover, the inter-firm innovation spillover effects among entrepreneurs through learning from watching, interactions and imitation
imply that a firm \( j \)'s innovation ability depends on the number of firms in its neighborhood such that: \( dV_j = g(n, Z) \), with \( g_n > 0 \) and \( Z \) is a vector of other factors like public policies and infrastructures. At the same time, higher \( V_j \) raises the imitation opportunities and therefore the potential IR to existing and new firms which will attract expansion of new ventures, such that \( dn = dn(dV_j) \) with \( dn' > 0 \). Therefore, in the development process with expanding \( V_j \) among firms, \( V \) and \( n \) tends to reinforce each other such that:

\[
\frac{dY}{Y} = \frac{dn(dV)}{n} + \frac{\alpha}{\theta} \frac{dV(n,Z)}{V} + \frac{\alpha}{\alpha} \frac{dX(dV)}{X} + \frac{\beta}{L} \frac{dL}{L}.
\]  

(E9)

The interdependent nature of the above growth variables implies that productivity, intensity of entrepreneurial activities, expansion of tools variety, number of firms and investments tend to move together in the growth process across countries and time.\(^{10}\)

3. Similarly, the mechanism in (E9) also induces within country concentration. The higher level and growth rate of specific tools variety in certain places will therefore induce related firms to flow in to capture the benefits generated by the innovation spillovers and interaction opportunities between entrepreneurs. Industries and firms that can be benefited from the innovative activities of each other will therefore form natural clusters.

4. Factors of production flow to the same places. Economies with motivation structure and environment favorable to entrepreneurial activities will enjoy higher multiplication rate of tools variety and higher potential IR. Entrepreneurs, physical and financial capital will flow to the economies with favorable ‘economic substrates’ and vice versa.

5. The enormous increase in standards of living during the last few centuries and recent decades can be attributed to the explosion of tools variety generated by the discoveries and applications of some general technology, along with important breakthroughs in transportation and information technology. They initiate a globalization process that

\(^{10}\) Higher \( dV \) raises \( dX \). The marginal product of \( X \) equals \( \alpha V^{a/\theta} X^{a-1} L^\beta \). Given the same rental price of tools, higher \( dV \) implies higher profit-maximizing \( dX \), other things being equal.
results in rapid expansion of market size and therefore a drastic increase in the demand curve in Figure 1 to each firm in the world. The enormous increase in the potential IR $c_a e_n$ incentivizes the proliferation, imitation, application and invention of new tools in the national as well as in the firm level. In notation, $dn/n$, $dV/V$ and $dX/X$ in (E9) are positively driven by the potential IR that expands rapidly to all firms in the globalization process.

6. In the model, entrepreneurs are assumed to conduct innovations in their regular production process and no extra market resources are expended in the process apart from their implicit opportunity costs. Market-driven innovative activities and technical progress are always Pareto-improving.

7. The extensive spillover effects of innovations and the multiplicative mechanism of tools variety imply that the external effects of individual entrepreneurial activities to the whole society tend to be extensive and cumulative over time. Decentralized innovative activities tend to be sub-optimal and therefore market-driven innovative/entrepreneurial activities should be encouraged and subsidized by public policies.

8. Since tools variety in the production process is the key to technical progress, entrepreneurial activities are essential to sustained economic growth which apply and/or transform new ideas, new knowledge, new invention to new mix of tools variety for producing final goods and services. All other factors emphasized in the endogenous growth literature like R&D, education, knowledge, inventions and public policies become neither necessary nor sufficient. They should only be considered as supplementary to entrepreneurial activities. The voluminous literature questing for sustained growth is

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11 Probably it is not uncommon that most entrepreneurs can satisfy their creative instinct in their innovative activities and find them enjoyable.

12 In (E5), $Y = GDP = n Y_j = n V_j^{\alpha/\theta} X_j^{\theta} L_j^\beta$ in the long-run equilibrium. In equilibrium, $\frac{\partial Y}{\partial V_j} = n \frac{\alpha}{\theta} V_j^{\alpha/\theta - 1} X_j^\alpha L_j^\beta$. The scale of spillover effect generated by the innovation of firm $j$ is proportional to the number of firms in its neighborhood.
gauged into entrepreneurial activities that drive the growth rate of tools variety.

4. Conclusions

The development process and technological improvements since the industrial revolution are characterized by substantial increases in entrepreneurial activities and the varieties of tools/machines/materials in production. However, the CDOPF does not include the role of entrepreneurs and representations of the advances in plant and machinery that make modern industrial economies possible. In this paper, we provide a simple integration of different ideas by incorporating entrepreneurial activities and the variety of tools/machines/materials and its evolution mechanism into the production function largely based on the insights of Schumpeter (1934). This resolves many important puzzles in the studies of economic development and growth.

The divergence of our model from the recent endogenous growth theories such as Romer (1990) is that sustained growth is driven by profit-maximizing entrepreneurs under a price taking sticky competitive market structure. The framework is more straight-forward than the monopolistic R&D framework. The implications from the model are also richer, have stronger explanatory power and less restrictive by specific assumptions. In our model, it is not population or ideas or human capital or number of researchers determines the rate of sustained growth but the entrepreneurial activities that drive the rate of tools multiplication. Escaping the law of diminishing productivity becomes possible, automatic and supported by micro-foundations. Investigations on the causes and phenomena about economic growth and development become much easier and straight-forward. Our contributions have made the mechanism of endogenous growth much simpler, intuitive and closer to the theoretical and empirical models based on CDOPF while

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13 Mo (2010b) applies the ideas related to SCDOPF for analyzing trade liberalization policy for sustained economic growth in developing economies. Combining the insights from institutional literatures with the SCDOPF, the factors, channels, mechanism and the key to sustained economic growth becomes clear, straight-forward and precise as demonstrated in Mo (2010c).
at the same time, having stronger explanatory power on the real world observations. The shortcomings of endogenous growth models as criticized by Jones (1995) and Pack (1994), among others, are eliminated.
References


Green, John (1964) *Aggregation in Economic Analysis*, Princeton.


(2010c) “Institutions, Entrepreneurship and Channels to Sustained Economic Growth,” School of Business, Hong Kong Baptist University.


