Stochastic simulation of an aggregated model of the Italian economy: methodological and empirical aspects

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Stochastic Simulation of an Aggregated Model of the Italian Economy: Methodological and Empirical Aspects

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Preface

The IBM Scientific Center of Pisa, the University of Pisa and the University of Siena have been carrying on a research on Stochastic Simulation of Econometric Models.

The first results have been summarized at the 2nd Meeting on «Teoria dei Sistemi ed Economia», Udine, October 1975, in the paper «Simulazione Stocastica ed Analisi di un Modello Aggregato dell'Economia Italiana» (Stochastic Simulation and Analysis of an Aggregated Model of the Italian Economy).

The following Technical Reports present in detail the methodological aspects and the complete results of the research:

C. Bianchi, «Comparison of Alternative Estimates of an Aggregated Model of the Italian Economy», Technical Report CSP031/513-3541.

P. Corsi, «Eigenvalues and Multipliers of Alternative Estimates of an Aggregated Model of the Italian Economy», Technical Report CSP032/513-3542.

E. Cleur, «Spectral Analysis of an Aggregated Model of the Italian Economy», Technical Report CSP033/513-3543.

G. Calzolari, T.A. Ciriani, P. Corsi, «Generation and Testing of Pseudo-RandomNumbers with Assigned Statistical Properties to be Used in the Stochastic Similation of Econometric Models», Technical Report CSP034/513-3544.

C. Bianchi et alii, «Stochastic Simulation of an Aggregated Model of the Italian Economy: Methodological and Empirical Aspects», Technical Report CSP035/513-3545.

Contents

1. Introduction

The methods used during the identification stage of econometric models generally involve the use of linear structures in the parameters so that economic systems of the usual form may be represented by mathematical models of the following type:

$$\begin{cases} Ay_t + By_{t-1} + Cx_t + Mz_t = \epsilon_t \\ Z_t = Nf(y_t, y_{t-1}, x_t) \end{cases} \qquad (1)$$

in which A, B, C, M, N are coefficient matrices, y_t, y_{t-1}, x_t, endogenous, lagged endogenous and exogenous variable vectors respectively, z_t a vector of non-linear functions of these different types of variables, ϵ_t a vector of stochastic variables (disturbances) with zero mean and variance-covariance matrix Σ.

In econometrics, the term simulation means the solution of a type (1) mathematical model, in which the values of the current endogenous variables are the unknowns, once the estimated values of the parameters of the model and the actual or expected values of the exogenous variables have been assigned.

Simulation is said to be deterministic when each disturbance therein is equal to its expected value, that is zero.

Simulation is said to be stochastic when a randon disturbance taken from a multivariate distribution assigned a priori is attributed to each structural equation in the model.

There is obviously no guarantee that when a single shock is applied in a replication of stochastic simulation it gets any closer to the real value of the disturbance than when the shock applied is equal to the expected value of the disturbance in question.

At all events, the fundamental characteristic of stochastic simulation is that by executing a given number of replications, it is possible to start studying the empirical distributions of the solutions yielded by the type (1) class of models.

From the inferential aspect, there is immediate justification for adopting simulation methods for studying this class of models, when their statistical properties cannot be ascertained by direct deduction.

By analyzing the distributions of the solutions obtained by simulation, it is possible to ascertain whether or not the values observed relate to the population illustrated by a given model.

As regards the use of such models, via stochastic simulation tecniques it is possible to define confidence intervals for forecasts and for the results of alternative economic policy measures, thus reintegrating-from the decision aspect also - the probabilistic element which disappears, so to speak, in economic models after being determined during the estimation stage. [7].

In the case of non-linear models, simulation is the only practicable way of obtaining these distributions [6].

For linear models, useful information can be obtained by analytical means also [1], but it should be noted that analytical methods generally call for calculation of the eigenvalues of the characteristic equation of a system, and this may raise difficulties that cannot be resolved immediately in the more complex systems.

The order of the characteristic equation is defined by «mp», where «m» represents the number of equations and «p» the extent of the maximum lag relating to endogenous variables.

Where «m» = 30 (size of an average national economy model) and «p» = 3, the number of characteristic roots is about 90. The degree of accuracy

(1) See writings on this subject by E. Cleur [1] and P. Corsi [3] in which the model constituting the subject of the present research.is studied via the techniques of spectral analysis and dynamic analysis respectively.

required for calculation of the strategic roots may exceed the precision of the available algorithms (1).

In conclusion, for both non-linear and linear models the use of Montecarlo methods is justified for studying complete solutions of the systems used in economic applications.

Via analysis of a small model of the Italian economy, we aim in the article to demonstrate the use of Montecarlo methods in applications of this kind. We shall limit our considerations to the case of linear models only, laying emphasis on the characteristics of the method used and on its practical limitations, which are more obvious in linear models.

2. Some summary measures of the dicrepancy between deterministic solution and stochastic mean.

In the case of a linear model, (1) is simplified, in the sense that there is no «z» vector of non-linear functions. The structure is error-additive, not only in its structural form, but in the reduced and final forms also. This means that it is possible to separate the contribution of the deterministic part of the system from that of the stochastic part.

In all practical applications based on a limited number of replications, the discrepancy between the deterministic solution and the stochastic mean for a linear model depends entirely on this experimental factor.

As stated, for a linear model (1) may be rewritten as follows:

$$Ay_t + By_{t-1} + Cx_t = \epsilon_t \qquad (2)$$

If it is assumed as at the estimation stage that:

$$E(\epsilon_t) = 0 \qquad E(\epsilon_{t_i} \cdot \epsilon_{t_j}) = \begin{cases} \Sigma & t_i = t_j \\ 0 & t_i \neq t_j \end{cases} \qquad (3)$$

(1) See the writings on this subject by P. Howrey [5] and K. Mori [10], giving different results for calculation of the characteristic roots of an identical condensed version of the Wharton model. See also the difficulties met by P. Corsi [3] in calculating the spectrum of eigenvalues for the versions of this model, estimated using the single-equation and full-information maximum likelihood methods.

the solution is given by:

$$y_t = -A^{-1} \left\{ By_{t-1} + Cx_t \right\} + v_t \qquad (4)$$

where: $v_t = A^{-1} \epsilon_t$ (5)

$$\text{with:} \quad E(v_t) = 0 \qquad E(V_{t_i} \cdot V'_{t_j}) = \begin{cases} \Omega = A^{-1} \Sigma (A^{-1})' & t_i = t_j \\ 0 & t_i \neq t_j \end{cases} \qquad (6)$$

The experimental discrepancy between the stochastic mean and the deterministic solution for the model considered, with 20 (1) replications, is illustrated in tables 3 - 16 of section 3 hereunder. Brief analysis of the columns relating to the two types of solution reveals an average 2% discrepancy.

To illustrate the extent of the error committed by using a fairly limited number of replications (20 in our example), in table 1 we have compared certain indices for the deterministic solution and for the stochastic mean. For this, the stochastic solution was obtained by means of the McCarthy algorithm [9]. The following indices were used:

a) RMSE, root-mean-square error between observed values O_t and calculated values C_t

$$RMSE = \sqrt{\frac{\Sigma (C_t - O_t)^2}{\Sigma O_t^2}} \qquad \begin{array}{ll} RMSE = 0 & \text{if } C_t = O_t \\ RMSE = 1 & \text{if } \begin{cases} C_t = 0 \\ C_t = 2O_t \end{cases} \\ RMSE \to \infty & \text{if } C_t >> O_t \end{array}$$

(1) The number of replications was established bearing in mind the information given in [12, page 209] regarding the non-parametric tolerance interval and the values used in similar stochastic simulation experiments (see [2], [4], for example).

b) Theil inequality coefficients:

$$T_1 = \frac{\Sigma (c_t - o_t)^2}{\Sigma o_t^2} \qquad T_1 = 0 \quad \text{if } o_t = c_t; \qquad = 1 \quad \text{if } c_t = 0$$

$$T_2 = \frac{\Sigma (c_t - o_t)^2}{\Sigma (o_t - o_{t-1})^2} \qquad T_2 = 0 \quad \text{if } o_t = c_t; \qquad = 1 \quad \text{if } c_t = o_{t-1}$$

where:

c_t = annual percentage variation of calculated values (deterministic or mean stochastic) between periods t and t-1.

o_t = annual percentage variation of observed values between period t and t-1.

The value of each of these two statistics is zero when both the size and sign of the variables are accurately predicted.

T_1 has a value of 1 when no variation is predicted; T_2 has a value of 1 when one variation is predicted, equal to that of the previous year. Values higher than 1 are a clear indication of less accurate forecasting capacity than that of the previous formulae [13].

c) Regression coefficients of the model: $C_{st} = \alpha + \beta C_{dt} + v_t$ where C_{st} are the realizations of the stochastic simulation at different periods, and C_{dt} the results of the deterministic simulation. When there is no systematic correlation between the forecasting errors and the values predicted (efficient forecasting): $\beta = 1$.

If the forecast is both efficient and correct, it follows that: $\alpha = 0$.

Alongside the values of the coefficients are indicated the values generated by test F on the combined zero hypothesis ($\alpha = 0$, $\beta = 1$) and those of the separate «t» tests on the unbias and efficiency hypotheses. For all variables, these hypotheses are not rejected in 95% of cases.

The RMSE values are satisfactory for all variables in any case, as the highest value is .06226 (ILIT); on the other hand, the values of the Theil inequality coefficients tend to be high, even over 1 for KOCC (T1S) and CPN, RNLCF (T2S).

As already stated, the previous results provide indications as to the experimental discrepancy (due to the small number of replications) between the

Table 1

Summary measurements - McCarthy's Algorithm

	RMSE		T_1		T_2		Regression $C_{st} = \alpha + \beta C_{dt}$				
	Det.	Stoch.	Det.	Stoch.	Det.	Stoch.	α	β	F	t_α	t_β
CPN	.02215	.02456	.5346	.5883	1.0376	1.1418	-7.869	1.0014	389761.	-.0939	236.3
ILIT	.05873	.06226	.6440	.7355	.7172	.8190	-18.113	1.0091	59304.	-.5087	95.89
M	.05269	.06043	.6980	.8296	.5825	.6923	-28.421	1.0109	72413.	-.9427	143.69
WIT	.02241	.02538	.4084	.4881	.8689	1.0383	9.872	1.0005	402003.	.3157	327.76
KOCC	.02697	.02809	.9223	1.0291	.7065	.7883	-.569	1.0071	262799.	-.0937	15.07
PIT	.03020	.03115	.6105	.5967	.9358	.9148	16.301	0.9981	214579.	.3381	183.68
RNLCF	.01254	.01330	.3362	.3509	1.0027	1.0463	18.459	0.9999	1280128.	.3134	465.39

deterministic solution and the stochastic mean. In order to have information regarding the variance-covariance matrix of the results of stochastic simulation, comparison was drawn between the theoretical variance-covariance matrix of reduced form $A^{-1}\Sigma(A^{-1})'$ and the variance-covariance matrix calculated with 20 replications for each year of the sample period 1952-71, indicated in the table as $\hat{v}_t\hat{v}_t'$.

It will be seen from Table 2 that in the particular case of 20 replications, the percentage error on variances is 5% on average; in the experimental approximation of the convariances, however, the variability is greater.

Later in this study, we shall give closer attention to the problem of experimental error in relation to the number of replications, comparing the results obtained with the two error-generating algorithms used (Nagar and Mc-Carthy) (see section 3).

Table 2

Comparison between theoretical and calculated variance-covariance matrix.

$\hat{\Omega} = A^{-1}\hat{\Sigma}(A^{-1})' = 10^3 x$	191.13	-11.72	41.61	60.33	-.16	77.47	137.80
		39.48	28.42	19.51	.42	-20.17	-.66
			50.79	30.53	.31	-11.29	19.23
				52.49	.29	-3.19	49.31
					.006	-.35	-.054
						71.78	68.59
							117.90
$\hat{V}_t\hat{V}_t' = \Omega^* = 10^3 x$	182.15	-9.34	43.80	54.31	-.18	74.69	129.01
		37.78	26.78	21.43	.42	-19.77	1.66
			47.42	31.36	.28	-8.20	23.16
				49.76	.30	-5.39	44.37
					.006	-.36	-.051
						68.51	63.13
							107.51
$\frac{\hat{\Omega} - \hat{\Omega}^*}{\hat{\Omega}} =$	-4.7	-20.38	5.27	-9.98	15.82	-3.58	-6.38
		-4.92	-5.74	9.84	-.81	-2.00	-349.63
			-6.62	2.71	-10.40	-27.39	20.40
				-5.21	2.00	68.92	-9.99
					1.91	.688	-6.66
						-4.56	-7.97
							-8.81

3. Stochastic simulations over the sample period.

The results of the stochastic simulations carried out for the sample period (1952-1971) in relation to the model [11] - with coefficients estimated via the O.L.S. method, 20 replications, Nagar and McCarthy error-generating algorithms - are given in the following tables (Tables 3 - 16).

For the sake of clarity, the results obtained with the two algorithms used are shown in separate tables. In addition to the values observed, the deterministic solution and the stochastic mean, each table also gives the following values:

a) minimum and maximum value of the stochastic solution in the 20 replications;
b) standard deviation in the stochastic solution;
c) comparison between the annual percentage variation in the observed values, in those calculated deterministically and in the stochastic mean value.

The minimum and maximum values which define the stochastic simulation interval, provide an immediate indication as to whether the sample values belong to the population described by the model. This representation is indicated on the graph, by way of example, to show the solutions obtained with the McCarthy method, which is the one most written about (1).

Via the standard deviations in the stochastic solution it is naturally possible to define a confidence interval in relation to the solution of the model. In the case of linear models, this interval is a constant value for all endogenous variables and, as illustrated in the previous section, it can be defined a priori on the basis of linear transformation of the variance-covariance matrix of the structural errors. It may be seen from the standard deviation columns of tables 3 - 16 that there is marked experimental variability which is due to the small number of replications adopted. In this case, the most advisable solution is that of relating the standard deviation mean calculated for the sample period to each endogenous variable or, better still, estimating the variance on all sample data.

(1) See writings on the subject by E. R. Sowey [12], J.P Cooper, B.Fisher [2], G.R.Green [4], etc.

In non-linear models on the other hand, the variability of the standard deviation is intrinsic and it constitutes one of the aspects regarding which stochastic simulation is most useful.

Likewise, the third index, which relates to the annual percentage variations and is also significant in the analysis of turning points in the economy, is not of great importance in the stochastic simulation of linear models (in such cases, in fact, it is merely a measure of the experimental discrepancy between the deterministic solution and the stochastic mean, which we have already analyzed more thoroughly in relation to Table 1). The index becomes essential, however in the dynamic solution of non linear models.

On the basis of examination of the indices, there is little to add regarding the total verification of the model used. The structure involved has already been amply tested (1). The confidence intervals revealed by the stochastic simulation are too wide to be used in any application of economic policy. This is also a consequence of the very aggregative structure of the model and does not therefore make the experiments recorded any less valid as examples.

It is clear from the results that, if it is necessary to limit the number of replications for cost reasons, it is also necessary to exercise a certain care in using the results of stochastic simulation. In view of the importance of this problem, in the next section we show a number of results in terms of both mean and variance, in which the number of replications ranged from 20 to 300.

(1) See writings on the subject by E. Cleur [1] and P. Corsi [3].

OUTPUT FOR VARIABLE Y(1) FROM YEAR 1952 TO YEAR 1971 WITH 20 REPLICATIONS

	ACTUAL VALUE	DETERM. VALUE	MEAN STOC. VALUE	STANDARD DEVIAT.	MINIMUM	MAXIMUM	ACTUAL % CHANGE	DETERM. % CHANGE	MEAN STOC % CHANGE
1952	12017.40	11634.22	11612.20	294.00	10964.11	12192.93	0.0	0.0	0.0
1953	12664.60	12188.90	12416.72	341.12	11935.22	13152.20	5.39	4.77	6.93
1954	12895.00	12528.79	12406.59	417.78	11693.45	13263.78	1.82	2.79	-0.08
1955	13365.80	13445.03	13368.93	298.91	12782.93	13898.83	3.65	7.31	7.76
1956	14117.70	13859.36	13856.51	346.60	13346.94	14564.16	5.63	3.08	3.65
1957	14612.50	14718.54	14598.43	441.69	13673.16	15315.31	3.50	6.20	5.35
1958	15124.60	15105.08	15217.39	607.99	14096.82	16517.25	3.50	2.63	4.24
1959	15840.20	15806.39	16057.07	515.04	15239.32	17138.88	4.73	4.64	5.52
1960	16694.00	17212.83	17393.18	332.71	16697.04	17920.12	5.39	8.90	8.32
1961	17727.20	18232.17	18243.02	373.42	17664.91	18957.11	6.19	5.92	4.89
1962	18751.90	19215.82	19244.20	390.25	18370.99	20128.07	5.78	5.40	5.49
1963	20090.00	19901.48	19685.74	468.33	18900.43	20791.66	7.14	3.57	2.29
1964	20422.80	21022.27	20950.50	479.36	20068.56	22012.95	1.66	5.63	6.42
1965	21045.30	21767.98	21823.55	455.75	20802.37	22843.21	3.05	3.55	4.17
1966	22565.10	22533.97	22580.52	432.80	21713.74	23310.70	7.22	3.52	3.47
1967	24279.40	23564.32	23466.39	502.58	22645.86	24384.09	7.60	4.57	3.92
1968	25416.70	24916.39	24763.15	323.19	24133.19	25415.47	4.68	5.74	5.53
1969	26549.10	26819.14	26789.93	484.37	25801.91	27882.04	4.46	7.64	8.18
1970	28288.40	27544.03	27643.78	419.50	27066.80	28379.92	6.55	2.70	3.19
1971	28428.07	28866.67	29147.38	450.42	28289.56	30014.31	0.49	4.80	5.44

Table 3 : Results of stochastic simulation for the varizble CPN (Mc Carthy's algorithm)

OUTPUT FOR VARIABLE Y(2) FROM YEAR 1952 TO YEAR 1971 WITH 20 REPLICATIONS

	ACTUAL VALUE	DETERM. VALUE	MEAN STOC. VALUE	STANDARD DEVIAT.	MINIMUM	MAXIMUM	ACTUAL % CHANGE	DETERM. % CHANGE	MEAN STOC % CHANGE
1952	1869.70	1991.49	2055.52	206.45	1368.32	2354.78	0.0	0.0	0.0
1953	1966.50	2083.86	2117.85	256.29	1532.67	2672.88	5.18	4.64	3.03
1954	2084.80	2133.73	2074.39	195.73	1669.38	2329.69	6.02	2.39	-2.05
1955	2236.00	2214.50	2201.16	179.69	1836.99	2592.34	7.25	3.79	6.11
1956	2382.70	2384.57	2401.18	170.31	2038.80	2661.29	6.56	7.68	9.09
1957	2594.50	2498.88	2451.72	127.78	2167.51	2711.83	8.89	4.79	2.10
1958	2523.40	2690.16	2699.21	207.16	2326.52	3013.12	-2.74	7.65	10.09
1959	2706.30	2579.79	2563.26	159.84	2306.45	2816.69	7.25	-4.10	-5.04
1960	3134.60	2920.55	2927.88	180.34	2580.52	3186.85	15.83	13.21	14.23
1961	3647.30	3324.45	3320.84	203.61	2922.83	3746.98	16.36	13.83	13.42
1962	3888.40	3638.61	3669.11	176.35	3386.97	3979.54	6.61	9.45	10.49
1963	4171.00	3649.73	3670.59	231.95	3343.78	4256.69	7.27	0.31	0.04
1964	3538.00	3765.93	3793.51	195.48	3473.17	4282.81	-15.16	3.18	3.35
1965	3031.50	3172.05	3240.39	213.80	2950.17	3641.74	-14.34	-15.77	-14.58
1966	3217.80	3233.01	3200.91	165.84	2882.36	3483.66	6.15	1.92	-1.22
1967	3662.40	3833.78	3856.50	196.76	3476.34	4268.72	13.82	18.58	20.48
1968	4042.50	4241.30	4297.43	161.50	4049.67	4640.05	10.38	10.63	11.43
1969	4376.60	4599.34	4610.62	182.95	4118.59	4920.12	8.26	8.44	7.29
1970	4926.30	4872.81	4872.97	247.61	4403.99	5363.15	12.56	5.95	5.69
1971	5019.29	5186.78	5218.51	176.53	4876.08	5543.93	1.89	6.44	7.09

Table 4 · Results of stochastic simulation for the variable ILIT (McCarthy's algorithm)

OUTPUT FOR VARIABLE Y(3) FROM YEAR 1952 TO YEAR 1971 WITH 20 REPLICATIONS

	ACTUAL VALUE	DETERM. VALUE	MEAN STOC. VALUE	STANDARD DEVIAT.	MINIMUM	MAXIMUM	ACTUAL % CHANGE	DETERM. % CHANGE	MEAN STOC. % CHANGE
1952	2104.60	1961.79	1986.09	172.60	1445.00	2134.61	0.0	0.0	0.0
1953	2116.80	2067.41	2148.09	217.61	1796.03	2484.30	0.58	5.38	8.11
1954	2087.10	2070.36	1988.20	198.70	1583.87	2397.57	-1.40	0.14	-7.44
1955	2244.70	2315.63	2278.06	182.59	1951.92	2585.70	7.55	11.85	14.58
1956	2528.00	2396.26	2431.64	235.49	2075.41	2902.58	12.62	3.48	6.74
1957	2861.00	2632.15	2507.69	167.93	2142.76	2852.02	13.17	9.84	3.13
1958	2439.70	2710.14	2741.75	275.68	2235.90	3129.36	-14.73	2.96	9.33
1959	2567.50	2793.38	2831.34	160.40	2618.70	3158.00	5.24	3.07	3.27
1960	3526.00	3338.02	3376.19	169.09	3061.29	3708.11	37.33	19.50	19.24
1961	3785.00	3753.38	3769.58	257.90	3374.02	4370.68	7.35	12.44	11.65
1962	4135.30	4118.68	4193.45	221.29	3896.97	4728.62	9.25	9.73	11.24
1963	4744.00	4244.16	4224.05	207.55	3811.07	4638.50	14.72	3.05	0.73
1964	4255.30	4585.19	4612.14	231.82	4172.27	5049.85	-10.30	8.04	9.19
1965	4171.70	4493.20	4589.85	244.75	4251.03	5055.12	-1.96	-2.01	-0.48
1966	4739.30	4675.61	4653.85	228.00	4245.96	5072.72	13.61	3.95	1.39
1967	5285.00	5168.92	5099.00	227.38	4572.78	5487.91	11.51	10.67	9.57
1968	5434.80	5718.51	5737.11	191.40	5383.40	6179.89	2.83	10.63	12.51
1969	6338.10	6468.09	6467.88	218.64	6030.12	6948.92	16.62	13.11	12.74
1970	7083.20	6713.90	6723.61	267.78	6295.91	7371.01	11.71	3.80	3.95
1971	6997.11	7214.42	7373.76	224.15	6862.12	7725.21	-1.17	7.45	9.67

Table 5 - Results of stochastic simulation for the variable M (McCarthy's algorithm)

OUTPUT FOR VARIABLE Y(4) FROM YEAR 1952 TO YEAR 1971 WITH 20 REPLICATIONS

	ACTUAL VALUE	DETERM. VALUE	MEAN STOC. VALUE	STANDARD DEVIAT.	MINIMUM	MAXIMUM	ACTUAL % CHANGE	DETERM. % CHANGE	MEAN STOC. % CHANGE
1952	4551.20	4560.95	4577.42	264.68	3759.46	4859.62	0.0	0.0	0.0
1953	4935.80	4838.09	4931.31	162.92	4593.99	5245.17	8.45	6.10	7.73
1954	5362.60	5196.97	5148.12	238.56	4714.41	5642.57	8.65	7.42	4.40
1955	5678.70	5705.77	5725.70	168.27	5358.57	5978.42	5.89	9.79	11.22
1956	6005.10	6030.82	6025.37	175.82	5739.69	6373.56	5.75	5.70	5.23
1957	6420.20	6505.81	6460.83	219.71	6078.59	6939.25	6.91	7.88	7.23
1958	6718.10	6793.22	6853.27	233.54	6445.67	7288.16	4.64	4.42	6.07
1959	7221.60	7129.06	7164.59	189.49	6877.32	7726.20	7.49	4.94	4.54
1960	7929.00	8061.26	8122.55	262.86	7717.78	8609.21	9.80	13.08	13.37
1961	8639.80	8757.32	8760.41	285.74	8219.83	9362.91	8.96	8.63	7.85
1962	9451.70	9431.65	9500.26	221.06	9211.64	9946.87	9.40	7.70	8.45
1963	10554.00	9996.28	9871.94	274.51	9233.06	10473.09	11.66	5.99	3.91
1964	11055.80	10959.48	10935.68	210.82	10555.85	11289.73	4.75	9.64	10.78
1965	10979.00	11230.38	11284.30	227.43	10904.82	11722.77	-0.69	2.47	3.19
1966	11459.50	11536.21	11592.17	230.19	11243.41	11939.11	4.38	2.72	2.73
1967	12573.40	12355.65	12339.67	175.19	12108.08	12628.63	9.72	7.10	6.45
1968	13504.70	13530.22	13521.90	211.45	13102.68	13911.07	7.41	9.51	9.58
1969	14340.20	14702.86	14735.68	228.14	14368.05	15237.41	6.19	8.67	8.98
1970	16150.00	15734.20	15757.59	255.08	15313.84	16297.61	12.62	7.01	6.93
1971	17118.04	17585.06	17633.19	156.82	17197.00	17959.73	5.99	11.76	11.90

Table 6 - Results of stochastic simulation for the variable WIT (McCarthy's algorithm)

OUTPUT FOR VARIABLE Y(5) FROM YEAR 1952 TO YEAR 1971 WITH 20 REPLICATIONS

	ACTUAL VALUE	DETERM. VALUE	MEAN STOC. VALUE	STANDARD DEVIAT.	MINIMUM	MAXIMUM	ACTUAL % CHANGE	DETERM. % CHANGE	MEAN STO % CHANGE
1952	88.70	90.40	90.96	3.27	81.91	96.85	0.0	0.0	0.0
1953	88.90	90.61	91.05	2.77	85.50	96.87	0.23	0.23	0.10
1954	90.20	90.34	89.68	2.38	84.10	93.36	1.46	-0.30	-1.51
1955	90.90	90.30	90.36	2.21	85.66	96.03	0.78	-0.04	0.76
1956	90.40	90.54	90.47	2.10	86.70	94.40	-0.55	0.26	0.13
1957	90.20	90.42	89.97	1.94	86.33	95.09	-0.22	-0.12	-0.56
1958	87.80	90.64	90.96	2.44	86.96	94.75	-2.66	0.24	1.10
1959	89.80	89.38	88.81	1.99	85.14	92.63	2.28	-1.39	-2.37
1960	94.60	91.04	91.05	2.28	86.69	94.50	5.35	1.85	2.53
1961	94.60	91.99	92.10	2.52	88.55	97.19	0.0	1.04	1.16
1962	94.70	91.64	92.08	2.40	87.84	97.02	0.11	-0.38	-0.03
1963	95.60	89.73	89.63	2.98	86.67	99.28	0.95	-2.08	-2.65
1964	89.10	89.29	89.53	2.68	85.02	94.92	-6.80	-0.49	-0.11
1965	85.70	85.77	86.36	2.72	82.20	90.69	-3.82	-3.94	-3.54
1966	89.10	88.41	88.24	1.66	85.73	90.80	3.97	3.07	2.17
1967	91.10	92.78	93.11	2.38	88.45	97.41	2.24	4.95	5.51
1968	91.90	93.74	94.48	2.37	88.24	98.50	0.88	1.03	1.47
1969	91.70	93.49	93.71	2.70	86.33	98.36	-0.22	-0.26	-0.81
1970	94.30	93.15	92.72	3.09	85.70	98.69	2.84	-0.36	-1.06
1971	87.60	93.17	92.99	1.86	89.02	96.82	-7.10	0.02	0.29

Table 7 - Results of stochastic simulation for the variable KOCC (McCarthy's algorithm)

OUTPUT FOR VARIABLE Y(6) FROM YEAR 1952 TO YEAR 1971 WITH 20 REPLICATIONS

	ACTUAL VALUE	DETERM. VALUE	MEAN STOC. VALUE	STANDARD DEVIAT.	MINIMUM	MAXIMUM	ACTUAL % CHANGE	DETERM. % CHANGE	MEAN STO % CHANGE
1952	5135.00	5007.56	5007.10	243.30	4585.23	5578.95	0.0	0.0	0.0
1953	5261.60	5050.50	5138.36	223.91	4741.40	5629.08	2.47	0.86	2.62
1954	5395.50	5260.71	5210.17	238.84	4879.52	5622.02	2.54	4.16	1.40
1955	5820.20	5779.95	5708.12	227.86	5307.91	6219.37	7.87	9.87	9.56
1956	6098.10	5947.77	5931.57	248.88	5513.03	6323.53	4.77	2.90	3.91
1957	6473.80	6627.44	6629.62	267.76	6186.22	7011.68	6.16	11.43	11.77
1958	6635.60	6437.35	6467.08	322.94	5900.85	7111.93	2.50	-2.87	-2.45
1959	7295.90	7002.48	7163.12	308.75	6598.05	7948.55	9.95	8.78	10.76
1960	7918.30	8278.88	8367.11	224.26	8082.73	8878.79	8.53	18.23	16.81
1961	8373.00	8469.36	8457.28	212.01	7983.12	8836.07	5.74	2.30	1.08
1962	8608.80	8859.74	8775.21	243.59	8201.93	9451.79	2.82	4.61	3.76
1963	8644.00	8991.84	8941.37	297.64	8048.07	9422.50	0.41	1.49	1.89
1964	8728.80	9321.50	9274.27	275.25	8892.19	9940.47	0.98	3.67	3.72
1965	9368.70	9659.10	9632.45	257.58	8983.46	9987.54	7.33	3.62	3.86
1966	10258.10	10234.33	10209.51	206.81	9753.66	10552.86	9.49	5.96	5.99
1967	10851.50	10641.88	10652.54	276.99	10096.04	11076.12	5.78	3.98	4.34
1968	11910.30	11299.64	11192.21	221.90	10604.19	11518.54	9.76	6.18	5.07
1969	12659.20	12659.27	12608.73	273.93	12174.67	13184.69	6.29	12.03	12.66
1970	12831.50	12807.52	12874.30	250.50	12434.95	13281.53	1.36	1.17	2.11
1971	12038.20	11958.39	12063.33	351.35	11431.52	12625.70	-6.18	-6.63	-6.30

Table 8 - Results of stochastic simulation for the variable PIT (McCarthy's algorithm)

OUTPUT FOR VARIABLE Y(7) FROM YEAR 1952 TO YEAR 1971 WITH 20 REPLICATIONS

	ACTUAL VALUE	DETERM. VALUE	MEAN STOC. VALUE	STANDARD DEVIAT.	MINIMUM	MAXIMUM	ACTUAL % CHANGE	DETERM. % CHANGE	MEAN STOC. % CHANGE
1952	15042.00	14923.22	14940.13	276.78	14445.41	15665.95	0.0	0.0	0.0
1953	16101.00	15792.15	15973.28	248.84	15710.57	16716.06	7.04	5.82	6.92
1954	16629.00	16328.45	16229.08	375.03	15605.83	17056.62	3.28	3.40	1.60
1955	17728.00	17714.79	17662.92	200.33	17284.90	17994.28	6.61	8.49	8.84
1956	18511.00	18386.26	18364.64	283.37	17906.79	18773.14	4.42	3.79	3.97
1957	19485.00	19724.27	19681.46	304.67	19072.79	20156.63	5.26	7.28	7.17
1958	20442.00	20118.89	20408.66	444.04	19549.63	21488.42	4.91	3.01	3.69
1959	21822.00	21435.90	21632.08	382.62	21003.68	22458.81	6.75	5.50	5.99
1960	23122.00	23614.86	23764.37	274.79	23240.95	24172.41	5.96	10.16	9.86
1961	24944.00	25157.84	25148.88	290.43	24624.99	25783.42	7.88	6.53	5.83
1962	26436.00	26666.74	26650.86	241.21	26325.93	27148.82	5.98	6.00	5.97
1963	27800.00	27590.56	27415.27	373.89	26927.65	28351.25	5.16	3.46	2.87
1964	28625.00	29121.73	29050.57	356.90	28288.58	29799.16	2.97	5.55	5.96
1965	29674.00	30215.73	30242.99	351.86	29375.85	30948.26	3.66	3.76	4.10
1966	31431.00	31483.87	31515.08	345.44	30724.21	32138.61	5.92	4.20	4.21
1967	33551.00	33123.17	33117.90	340.90	32444.38	33706.49	6.74	5.21	5.09
1968	35709.00	35123.78	35008.07	268.45	34402.32	35513.83	6.43	6.04	5.71
1969	37757.00	38119.78	38102.06	371.01	37300.24	38791.77	5.74	8.53	8.84
1970	39594.00	39162.42	39252.64	318.38	38838.36	39839.14	4.87	2.74	3.02
1971	40241.00	40630.94	40784.03	391.39	40016.09	41532.32	1.63	3.75	3.90

Table 9 - Results of stochastic simulation for the variable RNLCF (McCarthy's algorithm)

OUTPUT FOR VARIABLE Y(1) FROM YEAR 1952 TO YEAR 1971 WITH 20 REPLICATIONS

	ACTUAL VALUE	DETERM. VALUE	MEAN STOC. VALUE	STANDARD DEVIAT.	MINIMUM	MAXIMUM	ACTUAL % CHANGE	DETERM. % CHANGE	MEAN STOC % CHANGE
1952	12017.40	11634.22	11560.67	412.11	10844.80	12188.23	0.0	0.0	0.0
1953	12664.60	12188.90	12278.19	410.74	11356.51	13134.87	5.39	4.77	6.21
1954	12895.00	12528.79	12475.51	459.70	11701.89	13698.23	1.82	2.79	1.61
1955	13365.80	13445.01	13573.32	281.58	13102.17	14022.29	3.65	7.31	8.80
1956	14117.70	13859.36	13771.65	384.19	13216.43	14744.94	5.63	3.08	1.46
1957	14612.50	14718.54	14725.19	492.54	14154.11	15947.76	3.50	6.20	6.92
1958	15124.60	15105.08	14934.01	366.31	14287.46	15914.58	3.50	2.63	1.42
1959	15840.20	15806.79	15723.09	386.31	14971.60	16262.78	4.73	4.64	5.28
1960	16694.00	17212.83	17334.21	351.63	16706.45	18044.95	5.39	8.90	10.25
1961	17727.20	18232.17	18330.71	417.07	17422.32	19208.31	6.19	5.92	5.75
1962	18751.90	19215.82	19194.58	512.33	18284.21	20353.98	5.78	5.40	5.80
1963	20090.00	19901.48	19966.20	446.36	19073.69	21021.63	7.14	3.57	2.95
1964	20422.80	21022.27	20915.23	387.65	19832.36	21547.73	1.66	5.63	4.75
1965	21045.30	21767.98	21749.01	359.53	21071.19	22332.48	3.05	3.55	3.99
1966	22565.10	22533.97	22722.65	382.56	21869.39	23526.00	7.22	3.52	4.48
1967	24279.40	23564.32	23503.94	388.59	22601.65	24158.26	7.60	4.57	3.44
1968	25416.70	24916.39	24958.48	351.70	24410.19	25667.28	4.68	5.74	6.19
1969	26549.10	26819.14	26787.93	421.30	25837.48	27501.40	4.46	7.64	7.33
1970	28288.40	27544.01	27618.31	408.84	26893.47	28421.42	6.55	2.70	3.10
1971	28428.07	28866.67	28861.43	449.61	28009.07	29640.08	0.49	4.80	4.50

Table 10 - Results of stochastic simulation for the variable CPN (Nagar's algorithm)

OUTPUT FOR VARIABLE Y(2) FROM YEAR 1952 TO YEAR 1971 WITH 20 REPLICATIONS

	ACTUAL VALUE	DETERM. VALUE	MEAN STOC. VALUE	STANDARD DEVIAT.	MINIMUM	MAXIMUM	ACTUAL % CHANGE	DETERM. % CHANGE	MEAN STOC % CHANGE
1952	1869.70	1991.49	2012.83	178.50	1725.16	2486.29	0.0	0.0	0.0
1953	1966.50	2083.86	2006.21	211.64	1673.91	2361.74	5.18	4.64	-0.33
1954	2084.80	2133.73	2113.48	210.26	1738.62	2458.15	6.02	2.39	5.35
1955	2236.00	2214.50	2142.58	232.61	1624.29	2469.70	7.25	3.79	1.38
1956	2382.70	2384.57	2330.62	141.78	1999.73	2692.39	6.56	7.68	8.78
1957	2594.50	2498.88	2447.47	237.01	2007.54	3018.88	8.89	4.79	5.01
1958	2523.40	2690.16	2574.94	189.91	2215.90	2912.55	-2.74	7.65	5.21
1959	2706.30	2579.79	2518.78	173.95	2176.03	2850.87	7.25	-4.10	-2.18
1960	3134.60	2920.55	2905.17	237.50	2328.28	3343.62	15.83	13.21	15.34
1961	3647.30	3324.45	3348.24	248.51	2944.75	3705.45	16.36	13.83	15.25
1962	3888.40	3638.61	3682.37	191.54	3438.14	4354.21	6.61	9.45	9.98
1963	4171.00	3649.73	3654.23	204.58	3246.78	3976.89	7.27	0.31	-0.76
1964	3538.80	3765.93	3832.13	190.10	3499.65	4192.54	-15.16	3.18	4.87
1965	3031.50	3172.05	3107.18	196.35	2727.59	3453.97	-14.34	-15.77	-18.92
1966	3217.80	3233.01	3216.32	190.49	2822.26	3486.93	6.15	1.92	3.51
1967	3662.40	3833.78	3909.30	136.82	3650.74	4240.95	13.82	18.58	21.55
1968	4042.50	4241.30	4277.77	224.74	3898.73	4726.53	10.38	10.63	9.43
1969	4376.60	4599.34	4562.15	211.00	4294.92	5082.39	8.26	8.44	6.65
1970	4326.30	4872.81	4819.63	184.14	4502.40	5183.01	12.56	5.95	5.64
1971	5019.29	5186.78	5240.40	184.67	4925.94	5683.15	1.89	6.44	8.73

Table 11 - Results of stochastic simulation for the variable ILIT (Nagar's algorithm)

OUTPUT FOR VARIABLE Y(3) FROM YEAR 1952 TO YEAR 1971 WITH 20 REPLICATIONS

	ACTUAL VALUE	DETERM. VALUE	MEAN STOC. VALUE	STANDARD DEVIAT.	MINIMUM	MAXIMUM	ACTUAL % CHANGE	DETERM. % CHANGE	MEAN STOC % CHANGE
1952	2104.60	1961.79	1929.14	125.59	1701.02	2205.83	0.0	0.0	0.0
1953	2116.80	2067.41	2006.69	195.08	1675.51	2313.58	0.58	5.38	4.02
1954	2087.10	2070.36	2075.43	223.77	1758.54	2553.51	-1.40	0.14	3.43
1955	2244.70	2315.63	2342.50	187.77	1885.50	2596.29	7.55	11.85	12.87
1956	2528.00	2396.26	2321.58	193.53	1992.07	2702.94	12.62	3.48	-0.89
1957	2861.00	2632.15	2598.40	231.52	2209.94	3100.01	13.17	9.84	11.92
1958	2439.70	2710.14	2539.50	202.29	1946.81	2826.36	-14.73	2.96	-2.27
1959	2567.50	2793.38	2783.58	172.81	2434.29	3230.30	5.24	3.07	9.61
1960	3526.00	3338.02	3354.26	173.11	2975.78	3649.09	37.33	19.50	20.50
1961	3785.00	3753.38	3799.48	282.05	3271.37	4352.93	7.35	12.44	13.27
1962	4135.30	4118.68	4131.89	216.69	3665.20	4413.99	9.25	9.73	8.75
1963	4744.00	4244.16	4289.97	217.35	3933.01	4555.52	14.72	3.05	3.83
1964	4255.30	4585.19	4626.07	125.91	4345.65	4852.14	-10.30	8.04	7.83
1965	4171.70	4493.20	4400.18	177.43	4095.56	4864.90	-1.96	-2.01	-4.88
1966	4739.30	4670.61	4722.99	212.65	4403.37	5027.01	13.61	3.95	7.34
1967	5285.00	5168.92	5202.62	244.53	4849.52	5676.51	11.51	10.67	10.16
1968	5434.80	5718.51	5706.42	230.72	5316.18	6238.85	2.83	10.63	9.68
1969	6338.10	6468.09	6440.60	271.49	5944.00	7039.41	16.62	13.11	12.87
1970	7080.20	6713.90	6684.77	250.23	6213.85	7126.24	11.71	3.80	3.79
1971	6997.11	7214.42	7226.68	252.28	6811.05	7631.41	-1.17	7.45	8.11

Table 12 - Results of stochastic simulation for the variable M (Nagar's algorithm)

OUTPUT FOR VARIABLE Y(4) FROM YEAR 1952 TO YEAR 1971 WITH 20 REPLICATIONS

	ACTUAL VALUE	DETERM. VALUE	MEAN STOC. VALUE	STANDARD DEVIAT.	MINIMUM	MAXIMUM	ACTUAL % CHANGE	DETERM. % CHANGE	MEAN STO % CHANGE
1952	4551.20	4560.05	4598.22	178.94	4219.51	4964.85	0.0	0.0	0.0
1953	4935.80	4838.09	4765.62	193.75	4380.63	5081.29	8.45	6.10	3.64
1954	5362.60	5196.97	5173.83	219.66	4785.33	5661.28	8.65	7.42	8.57
1955	5678.70	5705.77	5739.07	216.15	5237.22	6099.43	5.89	9.79	10.93
1956	6005.10	6030.82	5985.10	197.00	5702.74	6349.40	5.75	5.70	4.29
1957	6420.20	6505.81	6447.44	289.64	6013.14	7178.20	6.91	7.88	7.72
1958	6718.10	6793.22	6685.34	288.39	6138.05	7394.21	4.64	4.42	3.69
1959	7221.60	7129.06	7056.55	206.20	6774.51	7542.07	7.49	4.94	5.55
1960	7929.00	8061.26	8050.36	219.73	7574.25	8416.12	9.80	13.08	14.08
1961	8639.80	8757.32	8810.79	244.47	8306.80	9332.39	8.96	8.63	9.45
1962	9451.70	9431.65	9543.86	239.74	9006.21	9852.20	9.40	7.70	8.32
1963	10554.00	9996.28	9991.30	176.16	9598.93	10279.89	11.66	5.99	4.69
1964	11055.80	10959.48	10922.71	152.84	10643.18	11114.27	4.75	9.64	9.32
1965	10979.00	11230.38	11191.67	243.11	10628.47	11758.66	-0.69	2.47	2.46
1966	11459.50	11536.21	11583.49	166.69	11333.85	12000.14	4.38	2.72	3.50
1967	12573.40	12355.65	12350.86	241.55	11813.12	12781.15	9.72	7.10	6.62
1968	13504.70	13530.22	13529.74	219.87	13125.37	14062.19	7.41	9.51	9.54
1969	14340.20	14702.86	14670.98	281.41	14275.41	15191.76	6.19	8.67	8.44
1970	16150.00	15734.20	15744.19	247.45	15303.28	16178.26	12.62	7.01	7.32
1971	17118.04	17585.06	17602.30	183.21	17271.60	17880.17	5.99	11.76	11.80

Table 13 - Results of stochastic simulation for the variable WIT (Nagar's algorithm)

OUTPUT FOR VARIABLE Y(5) FROM YEAR 1952 TO YEAR 1971 WITH 20 REPLICATIONS

	ACTUAL VALUE	DETERM. VALUE	MEAN STOC. VALUE	STANDARD DEVIAT.	MINIMUM	MAXIMUM	ACTUAL % CHANGE	DETERM. % CHANGE	MEAN STOC % CHANGE
1952	88.70	90.40	90.97	1.96	87.30	94.86	0.0	0.0	0.0
1953	88.90	90.61	89.37	2.19	85.63	94.23	0.23	0.23	-1.76
1954	90.20	90.34	89.88	3.02	83.42	95.76	1.46	-0.30	0.58
1955	90.90	90.30	89.68	3.26	83.27	94.95	0.78	-0.04	-0.22
1956	90.40	90.54	90.11	1.71	86.99	93.81	-0.55	0.26	0.47
1957	90.20	90.42	89.68	2.57	85.07	95.18	-0.22	-0.12	-0.47
1958	87.80	90.64	89.52	2.34	85.45	92.56	-2.66	0.24	-0.18
1959	89.80	89.38	88.57	1.81	85.74	91.76	2.28	-1.39	-1.06
1960	94.60	91.04	90.56	3.08	81.52	95.34	5.35	1.85	2.25
1961	94.60	91.99	91.91	3.14	85.64	96.45	0.0	1.04	1.49
1962	94.70	91.64	92.22	1.87	89.79	97.74	0.11	-0.38	0.33
1963	95.60	89.73	89.75	2.42	83.67	94.39	0.95	-2.08	-2.67
1964	89.10	89.29	89.67	2.43	84.52	93.71	-6.80	-0.49	-0.09
1965	85.70	85.77	85.38	2.71	79.71	89.46	-3.82	-3.94	-4.79
1966	89.10	88.41	88.47	1.90	84.37	91.47	3.97	3.07	3.62
1967	91.10	92.70	93.19	1.97	89.81	97.15	2.24	4.95	5.35
1968	91.90	93.74	93.60	2.74	89.26	99.54	0.88	1.03	0.43
1969	91.70	93.49	93.19	3.01	88.35	100.34	-0.22	-0.26	-0.44
1970	94.30	93.15	92.69	2.40	88.80	97.03	2.84	-0.36	-0.54
1971	87.60	93.17	93.69	2.22	88.85	98.77	-7.10	0.02	1.09

Table 14 - Results of stochastic simulation for the variable KOCC (Nagar's algorithm)

OUTPUT FOR VARIABLE Y(6) FROM YEAR 1952 TO YEAR 1971 WITH 20 REPLICATIONS

	ACTUAL VALUE	DETERM. VALUE	MEAN STOC. VALUE	STANDARD DEVIAT.	MINIMUM	MAXIMUM	ACTUAL % CHANGE	DETERM. % CHANGE	MEAN STOC % CHANGE
1952	5135.00	5007.56	4949.83	217.63	4599.64	5301.60	0.0	0.0	0.0
1953	5261.60	5050.50	5195.30	292.97	4758.88	5793.03	2.47	0.86	4.96
1954	5395.50	5260.71	5205.24	347.96	4515.31	5791.67	2.54	4.16	0.19
1955	5820.20	5779.95	5776.13	343.49	5180.09	6557.82	7.87	9.87	10.97
1956	6098.10	5947.77	5926.50	193.19	5541.96	6433.15	4.77	2.90	2.60
1957	6473.80	6627.44	6674.81	282.18	6263.84	7361.72	6.16	11.43	12.63
1958	6635.60	6437.35	6429.60	259.32	5975.76	6923.56	2.50	-2.87	-3.67
1959	7295.90	7002.48	6940.48	191.28	6531.01	7246.72	9.95	8.78	7.95
1960	7918.30	8278.88	8379.55	257.30	7912.87	8832.40	8.53	18.23	20.73
1961	8373.00	8469.76	8492.07	292.44	8083.83	9058.80	5.74	2.30	1.34
1962	8608.80	8859.74	8956.78	274.56	8431.59	9482.46	2.82	4.61	5.47
1963	8644.00	8991.86	9020.22	314.64	8501.05	9834.17	0.41	1.49	0.71
1964	8728.80	9321.58	9276.63	296.33	8646.83	9906.39	0.98	3.67	2.84
1965	9368.70	9659.10	9706.99	205.44	9416.37	10217.41	7.33	3.62	4.64
1966	10258.10	10234.33	10306.59	249.99	9838.65	10897.66	9.49	5.96	6.18
1967	10851.50	10641.88	10628.09	230.60	10029.15	11001.59	5.78	3.98	3.12
1968	11910.30	11299.64	11390.72	226.21	10813.59	11774.15	9.76	6.18	7.18
1969	12659.20	12659.27	12650.25	262.57	12111.76	13168.46	6.29	12.03	11.06
1970	12831.50	12807.52	12847.73	266.19	12212.31	13186.20	1.36	1.17	1.56
1971	12038.20	11958.39	11977.25	256.25	11506.73	12510.48	-6.18	-6.63	-6.78

Table 15 - Results of stochastic simulation for the variable PIT (Nagar's algorithm)

OUTPUT FOR VARIABLE Y(7) FROM YEAR 1952 TO YEAR 1971 WITH 20 REPLICATIONS

	ACTUAL VALUE	DETERM. VALUE	MEAN STOC. VALUE	STANDARD DEVIAT.	MINIMUM	MAXIMUM	ACTUAL % CHANGE	DETERM. % CHANGE	MEAN STOC % CHANGE
1952	15342.00	14923.22	14903.66	328.84	14265.45	15391.78	0.0	0.0	0.0
1953	16101.00	15792.15	15864.51	347.62	15256.38	16753.78	7.04	5.82	6.45
1954	16629.00	16328.45	16249.85	381.30	15534.61	17217.37	3.28	3.40	2.43
1955	17728.00	17714.79	17744.31	252.93	17414.53	18606.93	6.61	8.49	9.20
1956	18511.00	18386.26	18319.30	292.52	17683.83	18893.91	4.42	3.79	3.24
1957	19485.00	19724.27	19713.26	409.83	19062.94	20748.43	5.26	7.28	7.61
1958	20442.00	20318.89	20203.24	262.56	19810.18	20977.44	4.91	3.01	2.49
1959	21822.00	21435.90	21301.40	334.14	20738.26	21864.65	6.75	5.50	5.44
1960	23122.00	23614.86	23704.62	292.15	23195.95	24330.48	5.96	10.16	11.28
1961	24944.00	25157.84	25234.07	321.70	24690.41	26045.09	7.88	6.53	6.45
1962	26436.00	26666.74	26876.06	389.66	26158.44	27710.09	5.98	6.00	6.51
1963	27800.00	27590.06	27613.47	335.63	27014.77	28488.77	5.16	3.46	2.74
1964	28625.00	29121.70	29039.99	295.25	28255.86	29640.46	2.97	5.55	5.17
1965	29674.00	30215.73	30224.90	276.03	29763.86	30607.37	3.66	3.76	4.08
1966	31431.00	31483.87	31603.48	263.79	31160.96	32272.79	5.92	4.20	4.56
1967	33551.00	33123.17	33104.63	280.05	32531.44	33489.32	6.74	5.21	4.75
1968	35709.00	35123.78	35214.44	264.80	34669.42	35672.20	6.43	6.04	6.37
1969	37757.00	38119.78	38078.88	326.02	37496.11	38925.44	5.74	8.53	8.13
1970	39594.00	39162.42	39212.67	359.21	38494.56	39855.84	4.87	2.74	2.98
1971	40241.00	40630.94	40667.06	340.96	39902.60	41385.63	1.63	3.75	3.71

Table 16 - Results of stochastic simulation for the variable RNLCF (Nagar's algorithm)

4. Analysis of the results of stochastic simulation with variation in the number of replications

We present in this section some of the preliminary results obtained, via the Nagar and McCarthy algorithms, with variation in the number of replications from a minimum of 20 to a maximum of 300. For the sake of simplicity and compactness, the results are presented as an RMSE comparison (table 17) and as a comparison between the Theil inequality coefficients of the deterministic solution and the same indices of the stochastic mean solution (tables 18 - 19).

As regards the RMSE, when the number of replications is low, the results obtained with the Nagar algoritm are nearer to the RMSE of the deterministic solution: The best result is obtained with 200 replications; the McCarthy algorithm prevails slightly when the number of replications reaches 300 (though the differences between the results with the two algorithms are very small).

The same thing applies in the case of the Theil inequality coefficients. It seems we can therefore conclude that, when the number of replications used is not very high (as is generally the case, for reasons of time and cost), the numbers generated on the basis of the Nagar algorithm prevail slightly and this algorithm is also the quickest in this case.

In fact, even though the transformation matrix has to be calculated for the Nagar algorithm, it only has to be calculated once, and for each replication the vector of the independent normal random numbers (which serves as starting point) contains only 5 elements, (the number of structural equations in the model). For the McCarthy algorithm, on the other hand, the vector used contains 20 elements (the same as the number of years in the sample period).

The McCarthy algorithm has the advantage of being much more simple from the computational aspect, and it is the only one that can be used when the number of structural equation in the model exceeds the number of years in the sample period.

As has already been implied, it must be noted that in the operations car-

Table 17

RMSE values for the deterministic solution and for the stochastic mean with variation in the number of replications

	Determinist.	20	50	100	200	300
		McCarthy				
CPN	1.037	1.142	1.183	1.053	.986	1.041
ILIT	.717	.819	.792	.783	.748	.741
M	.582	.692	.601	.588	.599	.587
WIT	.869	1.038	.925	.889	.863	.869
KOCC	.706	.788	.783	.719	.736	.712
PIT	.936	.915	1.027	.977	.921	.934
RNLCF	1.003	1.046	1.096	1.045	.976	1.010
		Nagar				
CPN	1.037	1.172	1.054	.982	1.030	1.023
ILIT	.717	.764	.789	.723	.739	.749
M	.582	.558	.615	.572	.587	.596
WIT	.869	.959	.963	.850	.874	.869
KOCC	.706	.761	.747	.705	.715	.721
PIT	.936	1.069	.878	.931	.913	.961
RNLCF	1.003	1.126	.984	.998	.986	1.000

Table 18

T1 values for the deterministic solution and for the stochastic mean, with variation in the number of replications.

	Determinist.	20	50	100	200	300
			McCarthy			
CPN	.535	.588	.609	.543	.508	.536
ILIT	.644	.735	.711	.703	.672	.665
M	.698	.829	.720	.705	.718	.705
WIT	.408	.488	.435	.418	.406	.408
KOCC	.922	1.029	1.022	.938	.961	.929
PIT	.610	.597	.670	.637	.601	.609
RNLCF	.336	.351	.368	.350	.327	.339
			Nagar			
CPN	.535	.603	.543	.506	.531	.527
ILIT	.644	.686	.709	.649	.664	.673
M	.698	.669	.737	.686	.703	.714
WIT	.408	.451	.453	.400	.411	.408
KOCC	.922	.994	.976	.920	.933	.941
PIT	.610	.697	.573	.607	.596	.627
RNLCF	.336	.377	.330	.335	.331	.335

Table 19

T2 values for the deterministic solution and for the stochastic mean with variation in the number of replications.

	Determinist.	20	50	100	200	300
			McCarthy			
CPN	.0221	.0245	.0234	.0223	.0215	.0218
ILIT	.0587	.0623	.0607	.0599	.0608	.0594
M	.0526	.0604	.0554	.0533	.0531	.0530
WIT	.0224	.0254	.0235	.0225	.0225	.0222
KOCC	.0269	.0281	.0283	.0273	.0280	.0268
PIT	.0302	.0301	.0317	.0312	.0297	.0301
RNLCF	.0125	.0133	.0130	.0128	.0122	.0124
			Nagar			
CPN	.0221	.0233	.0219	.0216	.0221	.0217
ILIT	.0587	.0609	.0572	.0593	.0587	.0594
M	.0526	.0508	.0551	.0515	.0525	.0533
WIT	.0224	.0230	.0236	.0220	.0227	.0226
KOCC	.0269	.0276	.0266	.0269	.0268	.0272
PIT	.0302	.0313	.0286	.0295	.0295	.0299
RNLCF	.0125	.0133	.0121	.0124	.0125	.0123

ried out starting from the same seed [(1)] for the generation of the random numbers, the pseudo-random numbers used are not exactly the same, because of the difference between the two algorithms.

From analysis of tables 17 - 18 - 19 we may conclude that the number of replications that has to be carried out before the experimental error becomes negligible is in fact too high, from the practical application aspect. It should be noted, however, that to ensure the correct statistical use of the model, it is sufficient for the experimental error to be less than the RMSE of the deterministic solution and, clearly, this result is also achieved with the 20 replications selected by way of example.

Similar results are obtained in terms of variances too. In this connection, the following experiment was performed regarding the value of the standard deviation of the stochastic solution, for the y1 (CPN) variable for 1953, generating the errors by means of the McCarthy algorithm (see Table 3, line 1, Standard Deviation column).

Tests were carried out with 20, 50, 100, 200, and 300 replications with random numbers calculated on the basis of 4 different initial values. The asymptotic value for the consumption variance, calculated on the basis of $A^{-1}\Sigma(A^{-1})'$ is 437 (see Table 2).

In this case also, about 300 replications are necessary to render the experimental error negligible. Analysis of Table 20 (which contains these results) shows the influence of the initial value for generating the uniform random numbers on the dispersion of the standard deviation of the CPN variable.

Whith 20 replications, for example, the dispersion due to the initial values (294 - 395 - 453 - 519) is almost equal to that obtained with the first initial value (294 - 341 - 417 - ... - 450) with 20 replications over the entire sample period (see Table 3, column 4).

(1) This term signifies the starting value used to generate uniform random numbers.

Table 20

Standard deviation values of CPN variable, with variation of the initial value and of the number of replications.

	Replications				
	20	50	100	200	300
Seed 1	294	367	363	429	440
Seed 2	519	502	475	481	455
Seed 3	395	369	432	439	420
Seed 4	453	454	400	426	408

5. Conclusions

By the analysis of the results, we can conclude that, when a small number of replications is performed, the experimental discrepancy between deterministic solution and stochastic mean is not very little. Moreover, it is possible to see that the number of replications necessary to make this experimental error negligible is too high, from the practical application aspect.

Nevertheless, the correct statistical use of the model is ensured even with the 20 replications performed by way of example.

As far as a comparison between the two error-generating algorithms is concerned, we can say that there is no clear cut prevalence of any of them, even if the Nagar's algorithm seems to be more convenient for a small number of replications.

To sum up, even if the adopted model is linear, it is possible to get useful information on the characteristics of stochastic simulation and on its practical limitations.

Appendix: Solution Algorithm

The Gauss-Seidel method was used for solution of the model, for the following main reasons:

a) it is easy to put on to a computer
b) it can be used for the solution of linear and non-linear models, without distinction
c) it can be used without any difficulty for taking into account particular problems such as: saturation, discontinuity, changes in model parameters, etc.

Given a system of linear or non-linear equations, (leaving aside the disturbance element):

$$y_{it} = f_i\,(y_{1t}, \ldots, y_{nt}, y_{1t-1}, y_{nt-p}, x_{1t}, \ldots x_{qt-q}) \qquad i = 1, \ldots, n$$

the Gauss algorithm used for the numerical solution of this system is (passing from iteration r - 1 to iteration r):

$$f_i\,(y_{1t}^{(r-1)} \ldots y_{it}^{(r-1)} \ldots, y_{nt}^{(r-1)}, y_{1t-1}, y_{nt-p}, x_{1t}, \ldots x_{qt-q}) \qquad i = 1 \ldots, n$$

Seidel's modification consists in making use - in each iteration - of data already calculated during same iteration and, naturally, during the previous one:

$$y_{it}^{(r)} = f_i\,(y_{1t}^{(r)}, \ldots y_{i-1t}^{(r)}\; y_{it}^{(r-1)}, \ldots y_{nt}^{(r-1)}, \ldots y_{1t-1}, y_{nt-q}, x_{1t}, x_{qt-q})$$

$$i = 1 \ldots, n$$

The iterative process comes to an end when with ϵ assigned:

$$\left| \frac{y_{i,t}^{(r)} - y_{i,t}^{(r-1)}}{y_{i,t}^{(r-1)}} \right| < \epsilon \qquad i = 1, \ldots\ldots, n$$

In our model, assuming that $\epsilon = 10^{-5}$, convergence is generally reached with less than 15 iterations.

Two factors affect convergence in this algorithm; the first is the normalization procedure (that is the choice of the variable rendered explicit for each equation in the model), and the other is the ordering of the equations (but only if the Seidel modification method is used).

As regards economic models in particular, it should be noted that convergence is facilitated if the equations are normalized and ordered so as to reflect the causal flow of economic events, which is implicit in the model. [8]

References

[1] E., Cleur, «Spectral analysis of an aggregated model of the Italian economy» *IBM Italia Technical Report* CSP 033/513-3543, 1976.

[2] J P., Cooper, S., Fischer «Stochastic simulation of monetary rules in two macroeconometric models», *J.A.S.A., 1973.*

[3] P., Corsi, «Eigenvalues and multipliers of alternative estimates of an aggregated model of the Italian economy», *IBM Italia, Technical Report* CSP 032/513-3542. 1976

[4] G.R., Green, M., Lienberg, A.A., Hirsch «Short and long term simulations with the OBE econometric model», in *Econometric models of cyclical behaviour,* edited by B.G., Hickman, Studies in Income & Wealth, n.36, National Bureau for Economic Research, 1972.

[5] E.P., Howrey «Dynamic properties of a condensed version of the Wharton model», in *Econometric models of cyclical behaviour,* edited by B.G., Hickman, Studies in Income & Wealth, n. 36, National Bureau for Economic Research, 1972.

[6] E.P., Howrey, «Stabilization policy in linear stochastic systems», *Rev. of Econ. and Stat.,* 1967.

[7] L.R., Klein, «Dynamic analysis of economic systems», *Int-Journ. Math. Educ. Sci. Tech.,* 1973.

[8] L.R. Klein, M.K. Evans, M. Hartley *Econometric Gaming:a Computer kit for macroeconomic analysis,* Mac Millan Co, 1969.

[9] M. McCarthy, «Some notes on the generation of pseudo-structural errors for use in stochastic simulation studies», in *Econometric models of cyclical behaviour,* edited by B.G. Hickman, Studies in

Income & Wealth n. 36, National Bureau for Economic Research, 1972.

[10] K. Mori, «Generalized eigenvalue problem of an econometric model», mimeographed, *paper delivered at the Second World Congress of the Econometric Society,* Cambridge, England, 1970.

[11] B. Sitzia, M. Tivegna, «Un modello aggregato dell'economia italiana», *Contributi alla Ricerca Economica,* Banca d'Italia, 1975.

[12] E.R. Sowey «Stochastic simulation of macroeconometric models: methodology and interpretation», in *Econometric studies of macro and monetary relations,* edited by A.A. Powell, R.A. Williams, North Holland, 1973.

[13] H. Theil *«Economic forecasts and policy»,* North Holland, 1970.