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**Conditional R&D Subsidies**

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### **Abstract**

*This paper introduces a new type of R&D subsidy, which is conditional on the success of the R&D project. In a three-stage model, the government chooses a subsidy(ies) in the first stage; in the second stage, a monopolist chooses R&D effort which determines the size or the probability of success of the R&D project; in the last stage, the firm chooses its output. It is found that conditional subsidies can yield the same level of innovation and welfare as unconditional subsidies. However, when the probability of success is sufficiently low (be it endogenous or exogenous), conditional subsidies yield suboptimal levels of innovation and welfare. When the firm chooses the probability of success, conditional subsidies can have the advantage of a lower expected cost of the subsidy to the government. I consider the simultaneous use of conditional and unconditional subsidies, and show that different combinations of the two can lead to the same levels of innovation and welfare as unconditional subsidies alone. Finally, reverse conditional subsidies, which the firm gets only if the project fails, are considered. It is found that they yield the same level of innovation as unconditional subsidies, except when the probability of success is sufficiently high. Comparing conditional subsidies with reverse conditional subsidies, conditional subsidies yield higher (lower) welfare when the probability of success is high (low).*

**Keywords:** R&D subsidies, Innovation, R&D policy, Innovation policy

**JEL codes:** L12, L52, O31, O38

### **Sommaire**

*Cet article présente un nouveau type de subvention à la R&D, qui est fonction du succès du projet de recherche. Dans un modèle à trois étapes, le gouvernement choisit une/des subvention(s) à la première étape; à la deuxième étape, un monopoleur choisit la taille du projet ou la probabilité de succès; à la dernière étape, la firme choisit son output. Les subventions conditionnelles peuvent générer les mêmes niveaux d'innovation et de bien-être que les subventions non-conditionnelles. Cependant, lorsque la probabilité de succès est suffisamment faible (qu'elle soit endogène ou exogène), les subventions conditionnelles produisent des niveaux d'innovation et de bien-être sub-optimaux. Lorsque la firme choisit la probabilité de succès, les subventions conditionnelles peuvent avoir l'avantage d'un coût attendu inférieur pour le gouvernement. On considère l'utilisation simultanée des subventions conditionnelles et non-conditionnelles, et l'on montre que plusieurs combinaisons des deux peuvent produire les mêmes niveaux d'innovation et de bien-être que les subventions non-conditionnelles. Finalement, des subventions conditionnelles que la firme obtient seulement si le projet échoue, sont considérées. Ces subventions génèrent le même niveau d'innovation que les subventions non-conditionnelles, sauf lorsque la probabilité de succès est suffisamment élevée. Comparant les subventions conditionnelles reliées au succès et celles reliées à l'échec, on trouve que celles reliées au succès génèrent un niveau de bien-être supérieur (inférieur) lorsque la probabilité de succès est élevée (faible).*

**Mots clés:** Subventions à la R&D, Innovation, Politique de R&D, Politique d'innovation

**Codes JEL:** L12, L52, O31, O38

## 1. Introduction<sup>1</sup>

R&D subsidies are used by governments to encourage firms to increase their investments in R&D. R&D subsidies are the fastest growing and second largest type of government aid to industry in developed countries (Nevo, 1998). R&D subsidies are the subject of a large economic literature. Examples are Petrakis and Poyago-Theotoky (2002), Busom (2000), Klette et al. (2000), Ekholm and Torstensson (1997), and Davidson and Segerstrom (1998). While a variety of forms of R&D subsidies have been studied, a common feature of those subsidies is that they are unconditional, that is, their provision or their level is not related to the technical or commercial success of the research project.

Yet, there may be benefits to relating the provision of a R&D subsidy, or its level, to the success/failure of a R&D project, or to the level of risk associated with the project. Such a scheme, by rewarding success, may induce firms to perform better research, to increase the chances of success of the project and/or the quality of the outcome. Moreover, with firms having different research capabilities, R&D subsidies linked to success may select the best firms, and eliminate those performing the least successful research. Making financing conditional on success can reduce adverse selection and moral hazard (Aghion and Howitt, 1998). With limited government funds available for subsidies, it may make sense to allocate those budgets to the best R&D projects and to the best performers.<sup>2</sup>

Empirically, there is evidence that some countries incorporate some form of conditionality into their support for R&D. Tanayama et al. (2004) find that in Finland, R&D subsidies are increasing in technical challenge; this can be interpreted as a policy whereby the government allows higher R&D subsidies to more risky or difficult projects.<sup>3</sup> Fölster (1991) analyzes the different types of R&D subsidies in use. One type of R&D support is a loan that is repaid to the government only if the R&D project is successful. This is equivalent to a R&D subsidy based on failure; this “reverse” subsidy will also be considered in the current paper. Fölster also identifies royalty grants,

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<sup>1</sup>I would like to thank the participants in the EARIE conference (Porto) for useful comments.

<sup>2</sup>There is also the possibility that even failed R&D projects are useful in that the human capital built from such projects is transferred to other firms through employee mobility. Although Møen (2004) finds little support for this hypothesis using Norwegian data.

<sup>3</sup>In Finland, during the evaluation of subsidy provision, R&D projects are classified based on several criteria. One of these criteria is the risk associated with the R&D project; the projects are ranked from 0 to 5, with 0 representing the “no risk” projects, and 5 representing the most challenging projects. Rejected projects were on average safer than all applications (Tanayama et al., 2004).

which are paid to the government based on the sales from the invention; hence, the higher the (commercial) success of the project, the lower is the real subsidy rate. Or, the government, in exchange for the subsidy, receives a stock option which can be exercised if the stock value rises significantly, i.e., if the project is commercially successful. Moreover, based on a sample of Swedish firms, Fölster finds that “selective” subsidies (such as royalty grants, conditional loans and stock options) induce more additional private R&D than more traditional subsidies, such as tax incentives, project grants and project loans. In France, the *Agence nationale de valorisation de la recherche* provides innovation support which is reimbursed if the project succeeds (Guellec, 2001), which amounts to a subsidy based on failure. *Technology Partnerships Canada*, a program aimed at providing seed money to promote innovation in automotive, aerospace, defense and high-tech companies, incorporates repayment of royalties based on commercial success. Until 1996, Australian firms benefited from a higher deduction rate for R&D expenditures when the investments were fully at risk (Department of Finance Canada, 1997).

Given that some form of conditionality in the support of R&D is being used by several countries,<sup>4</sup> there is a gap in the economic literature, which is the study of the properties and consequences of such conditional subsidies. To fill this gap in our understanding of the impact of R&D policies, this paper proposes a R&D subsidy which is conditional upon the success of the project: if the R&D project succeeds, the firm obtains the subsidy; if the R&D project fails, the firm has to cover all the expenses itself.<sup>5</sup> Moreover, the simultaneous use of conditional and unconditional R&D subsidies is considered. Finally, I consider the use of reverse conditional R&D subsidies, where the firm has to pay the subsidy back to the government (or, equivalently, does not get the subsidy) in case of success of the project. The logic behind such a subsidy is that if the project fails, the firm may not have the means to pay the subsidy back. If the project succeeds, however, the firm benefits commercially, and has the financial ability of paying back the subsidy. In this case the subsidy is like a conditional loan.

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<sup>4</sup>The tax system may contain hidden conditionalities. For instance, the corporate income tax coupled with a (unconditional) tax incentive for R&D incorporates an implicit conditional subsidy (which the firm keeps only if the project fails), since greater technological and commercial success imply a higher repayment rate.

<sup>5</sup>Mansfield et al. (1977) identify three types of success probabilities: technical completion (0.57), commercialization given technical completion (0.65), and financial success given commercialization (0.74). The numbers in brackets represent those probabilities for 16 firms operating in the chemical, pharmaceutical, electronics, and petroleum industries. These numbers show that the probability of failure is significant.

To consider these issues, the paper adopts a three-stage framework. The government chooses a subsidy(ies) in the first stage; in the second stage, a monopolist chooses R&D effort which determines the size of the R&D project or the probability of success of the project; in the last stage, the firm chooses its output. It is found that conditional subsidies can yield the same level of innovation and welfare as unconditional subsidies. However, when the probability of success is sufficiently low (be it endogenous or exogenous), conditional subsidies yield suboptimal levels of innovation and welfare. When the firm chooses the probability of success, conditional and unconditional subsidies yield the same levels of innovation and welfare, but conditional subsidies can have the advantage of a lower expected cost of the subsidy to the government. I consider the simultaneous use of conditional and unconditional subsidies, and show that different combinations of the two can lead to the same levels of innovation and welfare as unconditional subsidies alone. Finally, reverse conditional subsidies yield optimal levels of innovation and welfare, except when the probability of success is sufficiently high. Comparing conditional subsidies with reverse conditional subsidies, conditional subsidies yield higher (lower) welfare when the probability of success is high (low). In all, eight different models are considered, allowing for different policy instruments (conditional and/or unconditional subsidies) and different choices of innovation effort by the firm (innovation size versus probability of success); more specifically, for each choice variable by the firm, four combinations of policy instruments are considered: unconditional subsidies only, conditional subsidies only, conditional and unconditional subsidies together, and reverse conditional subsidies only.

There is a small literature on R&D uncertainty. Poyago-Theotoky (1998) analyses the role of a public firm in a mixed duopoly under uncertainty. Miyagiwa and Ohno (2002) analyze cooperative R&D in an oligopoly with spillovers and uncertainty. Lambertini (2004) studies the impact of demand uncertainty on R&D. However, the interaction between uncertainty and R&D subsidization has not been analyzed.<sup>6</sup>

Existing modes of rewarding innovation themselves contain some form of conditionality.

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<sup>6</sup>Tassey (1996) discusses the use of tax incentives vs. direct funding to deal with different types of technological and commercial risks, but does not discuss conditional funding.

Patents are granted only to a successful innovation, and hence constitute a conditional reward.<sup>7</sup> Lakdawalla and Sood (2004) distinguish between patents and prizes, which constitute a conditional reward, and research subsidies, which are unconditional. They show that inventions that replace existing products are more likely to benefit from conditional rewards, while new products that enhance existing products are better served by research subsidies. However, they do not consider conditional subsidies. Moreover, at the time of launching a new product, it may be difficult in practice to determine whether a new invention will replace an existing one, or coexist with existing products/processes and enhance their value. This is something that is revealed well into the life-cycle of a new product/technology, long after the innovator has been (legally) rewarded and/or research has been subsidized.

The modelling of innovation in this paper is close to Matsumura (2003). Matsumura studies cost-reducing R&D under uncertainty. Innovation efforts can either reduce production costs, or increase the probability of success. Matsumura (2003) also finds that the optimal subsidy is increasing in the riskiness of the project.<sup>8</sup>

Kiahara and Matsumura (2006) consider subsidies based on realized cost differences; this makes the subsidy conditional on the relative R&D performances of firms. In a duopoly where firms choose innovation size and the probability of success, and show that they induce efficient R&D levels when demand is linear. The application of their method becomes more difficult when there are no cost differences between firms, when reliable information about costs is difficult to obtain, or when there is only one firm operating in the relevant market or using the relevant technology.

The paper is organized as follows. The following section presents the basic model. Section 3 focuses on the case where the firm chooses the size of the project. Section 4 analyses the case where the probability of success is endogenous, holding the size of the innovation fixed. Section 5 studies reverse conditional subsidies, which are granted only in case of failure of the project; this type of subsidy is analyzed with both endogenous and exogenous probabilities of success. The last section concludes.

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<sup>7</sup>In a duopoly setting with spillovers, Helm and Schöttner (2004) consider an output subsidy given to the firm that develops the best technology. They find that such a subsidy can induce under or over-investment in R&D relative to a general output subsidy.

<sup>8</sup>He goes even further, suggesting that risky investments should be subsidized, while safer investments should be taxed. This is because in his model, when firms choose the probability of success, they invest too much in safe projects.

## 2. The model

A monopolist produces an output  $y$  using a constant returns to scale technology. The firm can invest in process R&D to reduce its production costs. There is uncertainty, in that the R&D project may succeed or fail. Two environments are studied. In the first environment, the probability of success is given, and the firm can affect only the size of the cost reduction. In the second environment, the size of the R&D project is given, and the resources invested by the firm contribute to increasing the probability of success.

Let  $y$  denote output,  $p(y)=A-y$  the inverse demand function,  $\alpha$  the initial production cost,  $x$  R&D output,  $\theta \in (0,1)$  the probability of success,  $\gamma x^2$  the cost of R&D, and  $\lambda \theta^2$  the cost of inducing a probability of success  $\theta$ . The parameters  $\gamma$  and  $\lambda$  capture the efficiency of R&D.<sup>9</sup> At the time of producing all uncertainties are resolved, and the firm knows its cost of production. In the *good* outcome, the project succeeds, the firm enjoys the cost reduction  $x$ , hence the marginal production cost is given by

$$c_g(x) = \alpha - x \quad (1)$$

In the *bad* outcome, R&D does not result in any cost reduction, and the firm produces using the initial technology, with marginal cost  $\alpha$ :

$$c_b = \alpha \quad (2)$$

The convexity of innovation costs and the linearity of their benefits ensure that the profit function is concave in innovation effort ( $x$  or  $\theta$ , depending on the model). Moreover,  $A > \alpha > x$ .<sup>10</sup>

Let  $s_u \in [0,1]$  denote the unconditional R&D subsidy, and  $s_c \in [0,1]$  denote the conditional subsidy. When both subsidies are used, it must be that  $s_u + s_c \leq 1$ : the total amount paid to the firm cannot exceed 100% of total R&D expenditures.

Decisions are taken in three stages. In the first stage, the government chooses the level of the subsidy(ies) to maximize expected social welfare, defined as the sum of producer and consumer surplus, minus the cost of the subsidy (all in expectations). The government can commit to these subsidies. In the second stage, the firm chooses the level of resources to devote to innovation (to affect  $x$  or  $\theta$ ) to maximize its expected profits. In the third stage, the firm chooses its output to

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<sup>9</sup>Because  $\theta \in (0,1)$ , the parameter  $\lambda$  also represents an upper bound on R&D investment when  $\theta$  is endogenous.

<sup>10</sup>When  $x$  is endogenous, this means that its arguments are such that  $x^* < \alpha$ .



maximize its profits. This order of decision making is reasonable in that government policy is set for long periods of time. The firm invests in technology taking government policy as given, and at the time of production, the technology is fixed in the short run.

At the production stage, the firm knows whether the project has succeeded or not, and hence uncertainty is resolved. Therefore, the choice of output is contingent on the success or failure of the project. Let  $y_g$  represent the level of output chosen when the outcome is *good*, and  $y_b$  represent the level of output when the outcome is *bad*. If the project succeeds, the profit-maximizing level of output is given by:

$$y_g = \frac{A - \alpha + x}{2} \quad (3)$$

If the project fails, the firm produces

$$y_b = \frac{A - \alpha}{2} \quad (4)$$

These levels of output are common to all the models that will be considered below. Therefore, for each model, it will suffice to consider the R&D and subsidy stages.

### 3. Exogenous probability of success

We first consider the case where the firm chooses  $x$ , with  $\theta$  exogenous. Three cases are considered: unconditional subsidies only, conditional subsidies only, and the simultaneous use of conditional and unconditional subsidies.

#### 3.1 Model 1: Exogenous probability of success with unconditional R&D subsidies

As a benchmark, consider the basic model where the firm chooses the size of the cost reduction,  $x$ , the government chooses  $s_u$ , and  $\theta$  is exogenous. The expected profits of the firm are given by:<sup>11</sup>

$$\pi(x, s_u) = \theta(p(y_g(x)) - c_g(x))y_g(x) + (1 - \theta)(p(y_b) - c_b)y_b - (1 - s_u)\gamma x^2 \quad (5)$$

In the second stage<sup>12</sup> the firm chooses the level of R&D to maximize (5):

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<sup>11</sup>The expectation sign is dropped for notational convenience.

<sup>12</sup>All second-order conditions for stages 1 and 2 are given in the Appendix.

$$x(s_u) = \frac{(A - \alpha)\theta}{4\gamma(1 - s_u) - \theta} \quad (6)$$

Expected social welfare is given by:

$$W(s_u) = \theta CS_g(s_u) + (1-\theta) CS_b + \pi(s_u) - s_u \gamma x^2(s_u) \quad (7)$$

where  $CS = y^2/2$  denotes consumer surplus, and can take the value  $CS_g$  (with  $y=y_g$ ) in the *good* outcome and  $CS_b$  (with  $y=y_b$ ) in the *bad* outcome. Note that  $CS_b$  does not depend on  $s_u$ , because  $y_b$  does not depend on  $x$ , and hence does not depend on  $s_u$ . Moreover,  $\pi(s_u)$  represents expected profits, as given by (5). Maximizing (7) w.r.t.  $s_u$  yields:

$$\frac{\partial W}{\partial s_u} = \frac{4(A - \alpha)^2 (1 - 3s_u) \gamma^2 \theta^2}{(4(1 - s_u) \gamma - \theta)^3} = 0 \quad (8)$$

which yields

$$s_u = \frac{1}{3} \quad (9)$$

The optimal unconditional subsidy is to subsidize the third of the costs of R&D. The subsidy is positive, given that the firm invests too little in R&D, because it does not capture the benefits to consumers from the innovation. Facing this subsidy, the firm chooses the socially optimal<sup>13</sup> level of  $x$ . More importantly, the optimal subsidy in this case is independent of all model parameters, and especially of the probability of success,  $\theta$ . This result will be important for the comparison with the results to be derived later.

**Proposition 1.** *With an exogenous probability of success and unconditional R&D subsidies only, the optimal R&D subsidy is constant:  $s_u=1/3$ , and induces the socially optimal level of innovation.*

The result of Proposition 1 is actually well known and straightforward to derive,<sup>14</sup> but is presented as a benchmark case to better situate the results that follow. Table 1 summarizes the results of this model, as well as the models to follow, to facilitate the comparisons between the different settings.

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<sup>13</sup>Social optimality here is in a second-best sense, where the output distortion due to monopoly power is beyond the control of the government.

<sup>14</sup>It also holds under more general investment cost functions.

[Table 1 here]

### 3.2 Model 2: Exogenous probability of success with conditional R&D subsidies

Now assume that the government grants only a conditional (on success) subsidy. The expected profits of the firm become:

$$\pi(x, s_c) = \theta(p(y_g(x)) - c_g(x))y_g(x) + (1-\theta)(p(y_b) - c_b)y_b - (1-\theta s_c)\gamma x^2 \quad (10)$$

Note that the firm gets the subsidy with probability  $\theta$ . In the second stage the firm chooses R&D to maximize (10). This yields:

$$x(s_c) = \frac{(A - \alpha)\theta}{4\gamma(1 - \theta s_c) - \theta} \quad (11)$$

Expected social welfare is given by:

$$W(s_c) = \theta CS_g(s_c) + (1-\theta) CS_b + \pi(s_c) - \theta s_c \gamma x^2(s_c) \quad (12)$$

The foc for the choice of  $s_c$  is:

$$\frac{\partial W}{\partial s_c} = \frac{4(A - \alpha)^2(1 - 3\theta s_c)\gamma^2\theta^3}{(4(1 - \theta s_c)\gamma - \theta)^3} \geq 0 \quad (13)$$

which yields

$$s_c = \min\left\{\frac{1}{3\theta}, 1\right\} \quad (14)$$

The optimal subsidy is declining in the probability of success. This means that riskier projects should receive higher subsidies. This result may sound counterintuitive, but it is in line with the objective of maximizing expected social welfare. The government aims at inducing the firm to choose the socially optimal level of  $x$ . From (11) we see that the firm's choice of R&D depends on the subsidy only through the term  $\theta s_c$ , which represents the expected subsidy for the firm. There is a unique level of the expected subsidy which induces the optimal choice of  $x$  by the firm. To maintain  $\theta s_c$  at that optimal level, the government has to move  $s_c$  in the opposite direction of the change in  $\theta$ . That is, the government has to give higher subsidies to riskier projects, and lower subsidies to safer projects. This higher subsidy is not necessarily more costly for the government, because the probability that the subsidy is paid is low when  $\theta$  is low.

The reason why the solution takes the form of a *min* function is that if  $\theta < 1/3$ , the formula  $1/3\theta$  would yield  $s_c > 1$ , which is inadmissible. Hence for very risky projects, the government covers

100% of the expenses in the case of success, but even this may be suboptimal. Social welfare would increase if the government could cover more than 100% of R&D costs, because the higher subsidy would increase R&D.

Taking the case  $\theta \geq 1/3$ , we can write the optimal conditional subsidy as  $\theta s_c = 1/3$ . This form makes it directly comparable with the optimal  $s_u$  given by (9). We see that in both cases the (optimal) expected subsidy, which we denote  $E_s$ , is the same,  $E_s = 1/3$ . Therefore the objective of the government in setting the conditional subsidy is to make the expected subsidy the same for the firm as in the case of the unconditional subsidy. The firm is indifferent between the two policy instruments, because it is risk neutral. For  $\theta < 1/3$ , however, this is not feasible, and the expected subsidy (and therefore R&D, profits, consumer surplus, and welfare) is lower with the conditional subsidy.

Therefore, comparing R&D levels between the two models, they will be identical for  $\theta \geq 1/3$ , but the unconditional subsidy will yield higher levels of R&D for  $\theta < 1/3$ . This suggests that from a welfare point of view, the two policy instruments are equivalent for relatively safe R&D projects, but the unconditional subsidy is strictly superior for very risky R&D projects.

The optimal  $s_c$  depends only on  $\theta$ . Moreover, it is at least as large (it takes its minimal value of  $1/3$  when  $\theta \rightarrow 1$ ) as the optimal  $s_u$  derived in section 3.1, to compensate for the risk associated with conditionality.

**Proposition 2.** *With an exogenous probability of success and conditional R&D subsidies only, the optimal R&D subsidy is  $s_c = \min\{1/3\theta, 1\}$ , and declines with the probability of success. It induces the socially optimal level of innovation iff  $\theta \geq 1/3$ .*

### 3.3 Model 3: Exogenous probability of success with conditional and unconditional R&D subsidies

Consider now the more general case where the government can simultaneously use conditional and unconditional subsidies. The expected profit of the firm becomes:

$$\pi(x, s_u, s_c) = \theta(p(y_g(x)) - c_g(x))y_g(x) + (1-\theta)(p(y_b) - c_b)y_b - (1-s_u - \theta s_c)\gamma x^2 \quad (15)$$

The firm gets  $s_u$  in all cases, and gets in addition  $s_c$  if the project succeeds. The expected profit-

maximizing level of R&D is given by

$$x(s_c, s_u) = \frac{(A - \alpha)\theta}{4\gamma(1 - s_u - \theta s_c) - \theta} \quad (16)$$

Expected social welfare is given by:

$$W(s_u, s_c) = \theta CS_g(s_u, s_c) + (1 - \theta) CS_b + \pi(s_u, s_c) - (s_u + \theta s_c)\gamma x^2(s_u, s_c) \quad (17)$$

Maximizing  $W$  w.r.t.  $s_u$  and  $s_c$  yields the following focs:

$$\frac{\partial W}{\partial s_u} = \frac{4(A - \alpha)^2(1 - 3(s_u + \theta s_c))\gamma^2 \theta^2}{(4(1 - s_u - \theta s_c)\gamma - \theta)^3} = 0 \quad (18)$$

$$\frac{\partial W}{\partial s_c} = \frac{4(A - \alpha)^2(1 - 3(s_u + \theta s_c))\gamma^2 \theta^3}{(4(1 - s_u - \theta s_c)\gamma - \theta)^3} = 0$$

Solving for  $s_u$  and  $s_c$  yields

$$s_u + \theta s_c = \frac{1}{3} \quad (19)$$

The solution is such that  $E_s=1/3$ , the same expected subsidy as in the two previous models. There are different combinations of  $s_u$  and  $s_c$  which can lead to the optimal outcome. Notice the substitutability of  $s_u$  and  $s_c$ : an increase in one must necessarily lead to a decrease in the other.

Because the expected subsidy is the same as in the model with  $s_u$  only, all other variables (R&D, welfare, etc.) will have the same levels. Both the firm and the government are indifferent between the different combinations leading to the optimal expected subsidy.

However, expected welfare here is higher than in the model with  $s_c$  only. Remember that in the latter case, when  $\theta < 1/3$ , it is not possible to set the subsidy high enough to achieve  $E_s=1/3$ ; this results in lower R&D and lower welfare. Therefore, using both  $s_u$  and  $s_c$  is superior (when  $\theta$  is low) to using  $s_c$  only.

To illustrate how different combinations of conditional and unconditional subsidies can be used to achieve the optimum, we consider three combinations. The first combination is such that  $s_c$  is set as high as possible, and consequently  $s_u$  is set as low as possible. This combination is useful when the government wants to rely as much as possible on conditional subsidies. The method to achieve this combination depends on  $\theta$ . If  $\theta \geq 1/3$ , then we would simply set  $s_u=0$  and  $s_c=1/3\theta$ . If,

however,  $\theta < 1/3$ ,  $s_c$  alone is not sufficient to attain the optimal expected subsidy; in this case we have to set  $s_u = 1 - s_c$  and substitute this equality into (19). This yields

$$s_u = \frac{1 - 3\theta}{3(1 - \theta)}, \quad s_c = \frac{2}{3(1 - \theta)} \quad (20)$$

The second combination is such that  $s_u$  is set as high as possible, while  $s_c$  is set as low as possible. This combination is relevant when the government wants to rely mainly on unconditional subsidies. It is easy to see from (19) that it is always possible to set  $s_c = 0$  and  $s_u = 1/3$ , achieving the optimal expected subsidy and insulating the firm from uncertainty with respect to the subsidy to be received. Finally, the third combination is such that  $s_u = s_c = s$ : the government wants to set a unique level for the subsidy, with the firm getting  $s$  before the project is undertaken, and getting another time  $s$  if the project succeeds. Substituting this equality into (19) yields  $s = 1/3(1 + \theta)$ .

Figure 1 shows the three types of combinations. The first combination is represented by two curves,  $s_u \text{ min}$  and  $s_c \text{ max}$ . When  $\theta < 1/3$ ,  $s_c$  is increasing in  $\theta$ , while  $s_u$  is decreasing in  $\theta$ ; when  $\theta \geq 1/3$ ,  $s_u = 0$  and  $s_c$  is decreasing in  $\theta$ . The second combination is simply the horizontal line  $s_u = 1/3$ . The last combination is represented by the declining line.

[Figure 1 here]

This analysis shows the advantage of using both  $s_u$  and  $s_c$  over using  $s_c$  only. When  $\theta \geq 1/3$ , the two approaches yield the same level of expected welfare. When, however,  $\theta < 1/3$ , expected welfare is higher when both  $s_u$  and  $s_c$  are used, because in that case it is possible to reach the optimal expected subsidy only by using a strictly positive  $s_u$ .

The fact that the optimal subsidy sometimes declines with  $\theta$  does not change the expected relationship between  $x$  and  $\theta$ . It is easy to verify that  $x$  always increases with  $\theta$  when  $s_u$  and  $s_c$  are chosen optimally.

From a policy point of view, conditional and unconditional R&D subsidies are substitutes: they should be set such that their weighted sum is equal to  $1/3$ ; this implies that increasing one must be paralleled by a decrease in the other.

**Proposition 3.** *With an exogenous probability of success and both conditional and unconditional R&D subsidies, the locus of optimal R&D subsidies is such that  $s_u + \theta s_c = 1/3$ . This implies that the*

expected subsidy is constant, and that a higher probability of success will be associated with a lower value of  $s_u$  and/or  $s_c$ . This locus induces the socially optimal level of innovation.

#### 4. Endogenous probability of success

We now change the focus of the model, by letting the firm choose the probability of success of the project, rather than the extent of cost reduction. By devoting more resources to innovation, the firm can increase the probability of success of a project leading to a cost reduction of a fixed exogenous size,  $x$ . Let  $\lambda\theta^2$  represent the cost of inducing a probability of success  $\theta$ . Now the subsidy is aimed at covering part of the costs of increasing the probability of success (because  $x$  is exogenous, its cost is irrelevant).

Note that in all three models considered until now, the functional relationship between R&D investment and the expected subsidy is the same (equations 6, 11, and 16). This is because when the probability of success is exogenous, all what matters for R&D investment is the expected subsidy. As we will see, this will no longer be true when the probability of success is endogenous, because in that case the choice of  $\theta$  will affect both the expected subsidy and the expected benefit of R&D.

##### 4.1 Model 4: Endogenous probability of success with unconditional R&D subsidies

Consider first the case where the government uses only an unconditional subsidy. The expected profit of the firm is:

$$\pi(\theta, s_u) = \theta(p(y_g) - c_g)y_g + (1 - \theta)(p(y_b) - c_b)y_b - (1 - s_u)\lambda\theta^2 \quad (21)$$

In the second stage the firm chooses  $\theta$  to maximize its expected profits, which yields

$$\theta(s_u) = \frac{x(2(A - \alpha) + x)}{8(1 - s_u)\lambda} \quad (22)$$

Expected social welfare is given by:

$$W(s_u) = \theta(s_u)CS_g + (1 - \theta(s_u))CS_b + \pi(s_u) - s_u\lambda\theta^2(s_u) \quad (23)$$

In the first stage, the government chooses  $s_u$  to maximize (23). This yields the following first order condition:

$$\frac{\partial W}{\partial s_u} = \frac{(1 - 3s_u)[x(2(A - \alpha) + x)]^2 x^2}{64(1 - s_u)^3 \lambda} = 0 \quad (24)$$

Solving for the optimal unconditional subsidy we obtain

$$s_u = \frac{1}{3} \quad (25)$$

Note that this model is symmetric with respect to the model in section 3.1, where the firm chooses  $x$  and the government chooses  $s_u$ , with the roles of  $x$  and  $\theta$  interchanged. Therefore, the optimal  $s_u$  is the same.<sup>15</sup>

**Proposition 4.** *With an endogenous probability of success and unconditional R&D subsidies only, the optimal R&D subsidy is  $s_u=1/3$ . It induces the socially optimal level of innovation.*

#### 4.2 Model 5: Endogenous probability of success with conditional R&D subsidies

We now alter the previous model by letting the government use only conditional subsidies. In contrast to the previous model, the symmetry with the model where the firm chooses  $x$  and the government chooses  $s_c$  breaks down, because now the firm can affect not only the expected cost reduction, but also the probability of obtaining the subsidy.

The expected profit of the firm is given by:

$$\pi(\theta, s_c) = \theta(p(y_g) - c_g)y_g + (1-\theta)(p(y_b) - c_b)y_b - (1-\theta s_c)\lambda\theta^2 \quad (26)$$

In the second stage the firm chooses  $\theta$  to maximize (26). The solution to this problem has two roots, only one of which lies strictly between 0 and 1:

$$\theta(s_c) = \frac{8\lambda - \sqrt{64\lambda^2 - 48\lambda x(2(A - \alpha) + x)s_c}}{24\lambda s_c} \quad (27)$$

Expected social welfare is given by

$$W(s_c) = \theta(s_c)CS_g + (1-\theta(s_c))CS_b + \pi(s_c) - \theta(s_c)s_c\lambda\theta^2(s_c) \quad (28)$$

Maximizing (28) w.r.t.  $s_c$  and solving for  $s_c$ , we obtain

$$s_c = \min\left\{\frac{32\lambda}{27x(2(A - \alpha) + x)}, 1\right\} \quad (29)$$

It is obvious that  $s_c$  increases with  $\lambda$  and  $\alpha$ , and decreases with  $A$  and  $x$ . These counterintuitive results mean that the optimal conditional subsidy is higher when the size of the innovation is lower,

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<sup>15</sup>Given this solution for  $s_u$ , and given the solution for  $\theta$  from (23), it must be that  $\lambda > 3(x(2A - \alpha) + x)/16$  to get  $\theta < 1$ . This constraint applies to models 5, 6, and 8 below too.



innovation is more costly, demand is lower and production costs are higher. These are all factors which contribute to reducing the private and social value of the innovation.

To understand this result, we look at how the probability of success chosen by the firm varies with the model's parameters. Substituting the optimal conditional subsidy into  $\theta(s_c)$  from (29) yields the equilibrium choice of  $\theta$ :

$$\theta^* = \frac{3x(2(A - \alpha) + x)}{16\lambda} \quad (30)$$

As expected,  $\theta$  increases with  $A$  and  $x$  and declines with  $\lambda$  and  $\alpha$ . Therefore, the model parameters affect  $\theta$  and  $s_c$  in opposite directions. The effects of the parameters on  $\theta$  are intuitive. To understand why the government reacts in the opposite direction, it is useful to calculate the expected subsidy,  $E_s = \theta^* s_c$ :

$$\begin{aligned} \theta^* s_c &= \frac{3x(2(A - \alpha) + x)}{16\lambda} \frac{32\lambda}{27x(2(A - \alpha) + x)} \\ &= \frac{2}{9} \end{aligned} \quad (31)$$

Note that (29) can be rewritten as

$$s_c = \min\left\{\frac{2}{9\theta}, 1\right\} \quad (32)$$

The optimal expected subsidy is constant. Therefore, as a change in the environment pushes  $\theta$  in one direction, the government reacts by moving  $s_c$  in the other direction, so as to maintain the expected subsidy constant. In choosing  $s_c$ , the government takes into account that the choice of  $\theta$  by the firm depends on  $s_c$ . Therefore, while the level of the conditional subsidy and the endogenous probability of success depend on the model parameters, the expected subsidy does not.

In fact, the expected subsidy is lower in this model than when the firm chooses  $x$  (section 3.2) and when the firm chooses  $\theta$  while the government uses only an unconditional subsidy (section 4.1); in both cases the optimal expected subsidy was  $1/3$ . This result has to do with the constant optimal expected subsidy: the government uses the conditional subsidy to maintain the expected subsidy at its optimal level.

When only  $s_c$  is used, there is a risk that the government may not be able to implement the optimal subsidy, because this would require covering more than 100% of the expenses on R&D in

case of success. From (29) it is clear that this is more likely to happen when  $\lambda$  and  $\alpha$  are high, and  $A$  and  $x$  are low. Of particular interest is the effect of  $x$ : conditional subsidies only are less likely to induce the socially optimal level of innovation when the size of the R&D project is small, because a small  $x$  induces a small  $\theta$ , in which case  $s_c$  cannot be set high enough to induce the optimal  $\theta$ . From (32), we see that this problem occurs when  $\theta < 2/9$ . Therefore, if the model parameters induce the firm to choose  $\theta < 2/9$ , the government will pay 100% of R&D expenditures in case of success, but the choice of  $\theta$  by the firm will be suboptimal.

**Proposition 5.** *With an endogenous probability of success and when the government uses only conditional subsidies, the optimal conditional subsidy declines with the size of the innovation and demand, and increases with the cost of innovation and production costs. The optimal expected subsidy is  $E_s = 2/9$ . The socially optimal level of innovation is induced iff  $\theta \geq 2/9$ .*

### 4.3 Comparison between conditional and unconditional subsidies with $\theta$ endogenous

It is useful to compare the performance of the last two models. Comparing  $s_c$  and  $s_u$ , we note that factors which increase  $s_c$  will increase the likelihood that  $s_c > s_u$ . Hence, a lower innovation size, a smaller market, a higher innovation cost, and a higher production cost all increase the chances that  $s_c > s_u$ . It is when the conditions are particularly “unfavorable” that the conditional subsidy is likely to be higher than the unconditional subsidy. This is in contrast to the models where the firm chooses  $x$  (sections 3.1 and 3.2), where the conditional subsidy is always higher than the unconditional subsidy.

We now compare the choices of  $\theta$  between the two models. Substituting the optimal  $s_u$  into (22) yields the equilibrium level of  $\theta$  with unconditional subsidies:

$$\theta^* = \frac{3x(2(A - \alpha) + x)}{16\lambda} \quad (33)$$

But this is equal to the equilibrium value of  $\theta$  with conditional subsidies when  $s_c$  is chosen optimally, given by (29). Hence, the optimal subsidies are such that they induce the same choice of the probability of success by the firm. This is not surprising: social welfare is maximized when the firm chooses the socially optimal level of  $\theta$ . Different types of subsidies should lead to the same choice of  $\theta$ .

The equality between  $\theta s$  in the two models implies that they yield identical levels of production costs, output and expected welfare. Given that the  $\theta s$  are the same, it follows that expected production costs and output are the same. The only difference between the two models is in terms of the expected cost of the subsidy, which is higher with unconditional subsidies, because in this case  $E_s=1/3$ , whereas it is  $2/9$  with conditional subsidies. But given that the subsidy is simply a transfer from the government to the firm, its level vanishes from the welfare function. In the presence of a positive shadow cost of public funds, however, conditional subsidies would yield a clear advantage over unconditional subsidies.

Hence a social planner having to choose between conditional and unconditional subsidies strictly prefers the latter when the probability of success is exogenous (and the firm chooses the size of the innovation), but is indifferent between the two when the probability of success is endogenous (and the size of the innovation is fixed).

#### 4.4 Model 6: Endogenous probability of success with conditional and unconditional R&D subsidies

Consider now the more general model where the firm chooses  $\theta$  and the government uses  $s_u$  and  $s_c$ . Expected profits are

$$\pi(\theta, s_u, s_c) = \theta(p(y_g) - c_g)y_g + (1 - \theta)(p(y_b) - c_b)y_b - (1 - s_u - \theta s_c)\lambda\theta^2 \quad (34)$$

The output stage is as above. Maximizing expected profits w.r.t.  $\theta$  yields

$$\theta(s_u, s_c) = \frac{8\lambda(1 - s_u) - \sqrt{(8\lambda(1 - s_u))^2 - 48\lambda s_c x(2(A - R) + x)}}{24\lambda s_c}$$

Expected social welfare is

$$W(s_u, s_c) = \theta(s_u, s_c)CS_g + (1 - \theta(s_u, s_c))CS_b + \pi(s_u, s_c) - (s_u + \theta s_c)\lambda\theta^2(s_u, s_c) \quad (36)$$

Maximizing social welfare w.r.t.  $s_u$  and  $s_c$  yields

$$s_u + \frac{9x(2(A - \alpha) + x)}{32\lambda} s_c = \frac{1}{3} \quad (37)$$

As in the model with exogenous  $\theta$  and conditional and unconditional subsidies (section 3.3), there are different combinations which can be used to attain the optimal subsidy. To analyze this result,

we substitute the optimal subsidies into  $\theta(s_u, s_c)$  to obtain the equilibrium probability of success. Given that all combinations satisfying (37) are socially optimal, any combination can be used for this substitution, and will lead to the same value of  $\theta$ . We choose  $s_u=0$  and solve for  $s_c$  from (37), and substitute this value of  $s_c$  into (35) to obtain

$$\theta^* = \frac{3x(2(A - \alpha) + x)}{16\lambda} \quad (38)$$

which is equal to the equilibrium value of  $\theta$  in the previous two models. Therefore, in all three models with an endogenous probability of success, the goal of the government is to induce this socially optimal choice of  $\theta$ . In each case, it uses the available instrument(s) to achieve this objective.

Note that (37) can be rewritten as

$$s_u + \frac{3}{2}\theta^* s_c = \frac{1}{3} \quad (39)$$

where  $\theta^*$  is given by (38). This way of writing the locus of optimal subsidies shows the inverse link between R&D subsidies and the probability of success: factors which increase  $\theta^*$  induce the government to reduce  $s_c$  and/or  $s_u$ .

Note that the coefficient of  $s_c$  in (39) is larger than  $\theta$ , hence the r.h.s. of (39) cannot be interpreted as an expected subsidy. To compare the expected subsidy in this model with the previous models, we compute the expected subsidy in two extreme cases, when  $s_c=0$  and when  $s_u=0$ . When  $s_c=0$ , it is obvious from (37) that  $E_s=1/3$ , the same value as in the model with unconditional subsidies only. When  $s_u=0$ , the expected subsidy is

$$\begin{aligned} E_s &= \theta^* s_c \\ &= \frac{3x(2(A - \alpha) + x)}{16\lambda} \frac{32\lambda}{27x(2(A - \alpha) + x)} \\ &= \frac{2}{9} \end{aligned} \quad (40)$$

Therefore, when only  $s_c$  is used, the expected subsidy is as in the previous model, where the use of  $s_u$  was not an option. Hence  $E_s \in [2/9, 1/3]$ . The lower bound obtains when  $s_u=0$ . As  $s_u$  is allowed to increase,  $s_c$  declines, and  $E_s$  increases, until it reaches its maximum value of  $1/3$  when  $s_c=0$  and  $s_u=1/3$ .

The goal of the government is to induce the optimal  $\theta$ . The expected subsidy is allowed to vary to achieve this objective. As the subsidy is simply a transfer from the government to the firm, what really matters for social welfare is the choice of the probability of success by the firm, irrespective of how it is financed.

This illustrates one difference with the model where  $\theta$  was exogenous, where it was found that both the expected subsidy and the choice of  $x$  were the same across all three models (except when  $\theta$  was too low and only  $s_c$  could be used). The real objective of the government there was to induce the optimal  $x$ ; the fixed expected subsidy was the appropriate instrument to achieve this objective, because  $\theta$  was exogenous. However, with an endogenous  $\theta$ , the expected subsidy may need to vary to achieve the optimal  $\theta$ .

When both  $s_u, s_c > 0$ , changes in the parameters will force the government to adjust the subsidies. From (37), we see that increases in demand or in the size of the innovation will force the government to reduce  $s_u$  and/or  $s_c$ , while increases in production costs or in the cost of the innovation will induce the government to increase  $s_u$  and/or  $s_c$ . Again, the government moves the subsidies in a direction which is opposite to the direction in which the firm moves  $\theta$ .

**Proposition 6.** *With an endogenous probability of success and when the government can use both conditional and unconditional subsidies, the government chooses a mixture of  $s_u$  and  $s_c$  based on (37) to induce the firm to choose the optimal  $\theta$  given by (38). The optimal subsidies decline with demand and the size of the innovation, and increase with production costs and innovation costs. The expected subsidy  $E_s \in [2/9, 1/3]$ . The socially optimal level of innovation is induced.*

## 5. Reverse conditional R&D subsidies

In some cases the firm gets the subsidy only if the project fails. For instance, if the project succeeds and the firm generates revenues from the commercialization of its products, the firm has to pay the government part or all of the subsidy it received. We consider this type of subsidy when the firm chooses  $x$ , and then when it chooses  $\theta$ .

### 5.1 Model 7: Exogenous probability of success with reverse conditional R&D subsidies

Consider the model of section 3.2, where the firm chooses  $x$  and  $\theta$  is exogenous. Only now,

with a reverse conditional subsidy, the firm gets the subsidy only if the project fails, which occurs with probability  $1-\theta$ . Let  $s_c^R$  denote this reverse conditional subsidy. The expected profits of the firm are:

$$\pi(x, s_c^R) = \theta(p(y_g(x)) - c_g(x))y_g(x) + (1-\theta)(p(y_b) - c_b)y_b - (1-(1-\theta)s_c^R)\gamma x^2 \quad (41)$$

The choice of output is unchanged. In the second stage the firm chooses R&D, which yields

$$x(s_c^R) = \frac{(A - \alpha)\theta}{4\gamma(1 - (1 - \theta)s_c^R) - \theta} \quad (42)$$

Expected social welfare is given by:

$$W(s_c^R) = \theta CS_g + (1-\theta) CS_b + \pi(s_c^R) - (1-\theta)s_c^R \gamma x^2(s_c^R) \quad (43)$$

The foc for the choice of  $s_c^R$  is:

$$\frac{\partial W}{\partial s_c^R} = \frac{4(A - \alpha)^2 \gamma^2 (1 - 3\theta s_c^R (1 - \theta)) \theta^2 (1 - \theta)}{(4(1 - s_c^R (1 - \theta)) \gamma - \theta)^3} \geq 0 \quad (44)$$

Solving for  $s_c$  yields

$$s_c^R = \min\left\{\frac{1}{3(1 - \theta)}, 1\right\} \quad (45)$$

Notice that now the optimal conditional subsidy is increasing in  $\theta$ . Hence safer R&D projects should receive higher subsidies. Remember that the goal of the government is to induce the optimal choice of  $x$ . When  $\theta$  is low, the probability that the firm will have to repay the subsidy is low; hence a small subsidy is sufficient to induce the firm to choose the optimal  $x$ . When  $\theta$  is high, however, there are good chances that the subsidy will have to be repaid. To compensate for that, the subsidy has to be large enough. Hence the incentive effects of the subsidy when it is based on failure are opposite to those obtained when the subsidy was conditional on success.

From (45) we see that when  $\theta \geq 2/3$  the government sets  $s_c^R=1$ . When  $\theta$  is very high, the government would like to set the subsidy at a very high level to compensate for the fact that the firm will very likely have to pay it back; in fact the government would like to set  $s_c > 1$ . But as this is not allowed, the government settles for  $s_c^R=1$ . This means that for this range of  $\theta$ , the subsidy is suboptimal, and induces a suboptimal R&D effort. Again, because  $\theta$  increases with  $x$ , this effect is more likely to obtain when the innovation is important.

When  $s_c^R$  takes its optimal value,  $E_s = (1-\theta)s_c^R = (1-\theta)/[3(1-\theta)] = 1/3$ . This is the same expected subsidy obtained in the model with the conditional subsidy based on success (model 2). The same

expected subsidy is required to induce the optimal R&D investment.

To compare  $s_c$  with  $s_c^R$ , figures 2 and 3 illustrate the equilibrium subsidies and R&D outputs under the two subsidy schemes.<sup>16</sup> The two subsidies are equal when  $\theta=0.5$ . For lower values of  $\theta$ , the subsidy based on success is higher, while the opposite is true for higher values of  $\theta$ . As for R&D, the two subsidy schemes yield exactly the same R&D effort when  $\theta \in [1/3, 2/3]$ . When  $\theta < 1/3$ , the subsidy based on failure yields a slightly higher R&D effort. This is due to the fact that in this range the subsidy based on success, even though it is higher, is suboptimal. When  $\theta > 2/3$ , the subsidy based on success yields significantly higher R&D effort. This is because in this range the subsidy based on failure is suboptimal. While figure 3 may suggest that in this range R&D is constant when the subsidy is based on failure, in fact it is slightly increasing. In this range R&D is very flat because the subsidy is stagnating while  $\theta$  is increasing, which results in a reduction in the expected subsidy; this dampens the incentives for R&D considerably.

[Figures 2 and 3 here]

As figure 3 shows, the gap in R&D due to the suboptimality of the subsidy based on success when  $\theta$  is low is marginal, while the gap in R&D due to the suboptimality of the subsidy based on failure when  $\theta$  is high is significant. This suggests that it is less costly to have a suboptimal R&D subsidy when the probability of success is low, in which case R&D is low anyway, than when the probability of success is high, in which case R&D is higher and is more socially valuable.

Comparing expected profits between the two conditional subsidy schemes, it is easy to see that the firm loses from the suboptimality of the subsidy. Consider the range  $\theta \in [0, 1/3]$ . Let  $\pi^R$  represent profits with the reverse subsidy, and  $\pi^{suc}$  represent profits with the subsidy conditional on success. In this range,  $s_c=1$ , while  $s_c^R=1/3(1-\theta)$ . Substituting into the profit functions and taking the difference yields

$$\pi^R - \pi^{suc} = - \frac{(A - \alpha)^2 \gamma \theta^2 (1 - 3\theta)}{(8\gamma - 3\theta^2)(4\gamma(1 - \theta) - \theta)} > 0 \quad \text{for } \theta \in (0, \frac{1}{3}) \quad (46)$$

In this range the subsidy based on success is suboptimal, while the subsidy based on failure is optimal, hence expected profits are higher with the latter subsidy.

Conversely, in the range  $\theta \in [1/3, 2/3]$ ,  $s_c=1/3\theta$ , while  $s_c^R=1$ . Substituting into the profit

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<sup>16</sup>Figure 3 is based on the numerical parametrization  $A=1000$ ,  $\alpha=50$ ,  $\gamma=60$ .

functions and taking the difference yields

$$\pi^R - \pi^{suc} = -\frac{4(A - \alpha)^2 \gamma^2 (2 - 3\theta)^2}{3(8\gamma - 3\theta^2)(4\gamma - \theta)^2} < 0 \quad \text{for } \theta \in (\frac{2}{3}, 1] \quad (47)$$

In this range the subsidy based on success is optimal, while the subsidy based on failure is suboptimal, hence expected profits are higher with the subsidy based on success.

The welfare comparison goes in the same direction as the comparison of R&D and profits. Hence, for risky R&D projects, a subsidy which is paid to the firm only in the case of failure is preferred. For safe R&D projects, a subsidy paid in the case of success is superior. For intermediate levels of risk, the two subsidy schemes are equivalent. It is important to remember that here we compare only the two conditional subsidy schemes, and do not take into account other schemes which make use of unconditional subsidies (exclusively, or in conjunction with conditional subsidies), which easily resolve this suboptimality problem.

**Proposition 7.** *When the probability of success is exogenous and the government uses a reverse subsidy scheme such that the subsidy is paid only when the R&D project fails, the optimal subsidy is given by (45) and increases with the probability of success. The socially optimal level of innovation is induced iff  $\theta \leq 1/3$ .*

**Proposition 8.** *With an exogenous probability of success, comparing only conditional subsidies, the subsidy based on failure generates more R&D and welfare in the range  $\theta \in [0, 1/3]$ , and generates less R&D and welfare in the range  $\theta \in [2/3, 1]$ . The two subsidy schemes are equivalent for intermediate levels of risk.*

## 5.2 Model 8: Endogenous probability of success with reverse conditional R&D subsidies

It is useful to consider the use of reverse subsidies when the firm chooses the probability of success. On the one hand, this will allow us to compare regular conditional subsidies (from section 4.2) with reverse conditional subsidies with  $\theta$  endogenous. On the other hand, this will establish whether the comparison between regular conditional subsidies and reverse conditional subsidies performed above is robust to the choice variable of the firm.

The subsidy is as in the last section, except that the firm chooses  $\theta$ , and  $x$  is exogenous. The



expected profits of the firm are:

$$\pi(\theta, s_c^R) = \theta(p(y_g) - c_g)y_g + (1 - \theta)(p(y_b) - c_b)y_b - (1 - (1 - \theta)s_c^R)\lambda\theta^2 \quad (48)$$

Optimal output is as above. The expected profit-maximizing choice of  $\theta$  is

$$\theta(s_c^R) = \frac{\sqrt{12\lambda x(2(A - \alpha) + x)s_c^R + (2\lambda(1 - s_c^R))^2 - 4\lambda(1 - s_c^R)}}{12\lambda s_c^R} \quad (49)$$

Expected welfare is given by:

$$W(s_c^R) = \theta(s_c^R)CS_g + (1 - \theta(s_c^R))CS_b + \pi(s_c^R) - \theta(s_c^R)s_c^R\lambda\theta^2(s_c^R) \quad (50)$$

Maximizing (50) w.r.t.  $s_c^R$  yields

$$s_c^R = \min\left\{\frac{32\lambda}{3(32\lambda - 9x(2(A - \alpha) + x))}, 1\right\} \quad (51)$$

The reverse conditional subsidy increases with  $A$  and  $x$ , and decreases with  $\alpha$  and  $\lambda$ . This is in contrast to model 5, where the conditional subsidy varied in opposite directions with the model parameters. The reason for that differential lies in the effect of the change in firm behavior on the expected subsidy. In both models there is an optimal expected subsidy, which is required to induce the firm to choose the optimal  $\theta$ . Consider, for example, the effect of an increase in  $x$  on  $\theta$  and on the conditional subsidy. In both models (5 and 8), an increase in  $x$  increases the socially optimal level of  $\theta$ , and also the  $\theta$  chosen by the firm. However, remember that the firm will choose the socially optimal  $\theta$  iff it receives the optimal expected subsidy. In model 5, the increase in  $\theta$  by the firm increases the expected subsidy, which is given by  $\theta s_c$ . Therefore, to maintain the expected subsidy at its optimal level, the social planner moves  $s_c$  in the opposite direction. In model 8, however, when the firm increases  $\theta$  in response to an increase in  $x$ , it reduces the probability of obtaining the subsidy, because the subsidy is obtained only if the project fails. Therefore  $E_s = (1 - \theta)s_c^R$  is reduced; to maintain the optimal level of  $\theta$ , the government has to increase  $E_s$ , and the only way to do this is to increase  $s_c^R$ . The effects of changes in  $A$ ,  $\alpha$ , and  $\lambda$  can be understood in the same manner.

Given the relative analytical complexity of (49) and (51), the behavior of innovation and subsidies in this model is best analyzed using numerical simulations. This is done in table 2. The left part of table 2 illustrates the optimal reverse conditional subsidy, the choice of  $\theta$  by the firm, the probability of obtaining the subsidy  $(1 - \theta)$ , the expected subsidy, and total welfare for different parameter configurations. Three sets of benchmark configurations are considered, and for each

benchmark configuration each parameter is varied.

[Table 2 here]

First note that  $\theta$  increases with  $x$  and  $A$ , and declines with  $\alpha$  and  $\lambda$ . This is intuitive, and in accordance with the comparative statics derived above for both  $x$  and  $\theta$ .

One difference between models 5 and 8 is that in model 5, at an interior solution, the optimal expected subsidy is fixed at  $E_s=2/9$ . In model 8, however, even at an interior solution, the optimal expected subsidy is not constant, but varies with the model parameters. Moreover, the optimal expected subsidy at an interior solution varies in the same direction as  $\theta$  and  $s_c$ : it increases with  $A$  and  $x$ , and declines with  $\alpha$  and  $\lambda$ .

Consider now the three configurations in the left part of table 2. The main difference between these configurations, beyond the specific numerical values used, is that in the first and third configurations, the solution is interior, and the subsidy, expected subsidy, and  $\theta$  are all socially optimal. In the second configuration, however, welfare is increasing in  $s_c^R$  at  $s_c^R=1$ , hence the optimal subsidy is  $s_c^R=1$ . In this case the subsidy is suboptimal, and so is the choice of  $\theta$  by the firm. Note that the second configuration obtains for  $\lambda=3000$ , which is lower than the value of  $\lambda$  used for the first and third configurations. This is consistent with the comparative statics described above, where the optimal  $s_c^R$  decreases with  $\lambda$ . As  $\lambda$  becomes very low,  $s_c^R$  reaches its maximum of 1, and the subsidy becomes suboptimal. Because  $x$  has the opposite effect of  $\lambda$  on  $\theta$  and  $s_c^R$ , it can also be said that the reverse subsidy is more likely to be suboptimal when  $x$  is high.

One difference between the interior and corner solutions is that in the first and third configurations (interior solutions),  $E_s$  increases following any change which increases  $s_c^R$ . This means that the increase in  $s_c^R$  is larger than the decrease in  $1-\theta$ , which increases  $E_s$ . In the second configuration, however, we have a corner solution, and  $E_s$  declines when  $x$  or  $A$  increase, for instance. This is because  $s_c^R$  is already set at its maximum value; hence any change which increases  $\theta$  reduces the expected subsidy.

It is also obvious from the table that welfare moves in the same direction as  $s_c^R$ , the expected subsidy (at an interior solution), and  $\theta$ . Therefore, a change in the environment which increases (decreases) the optimal  $s_c^R$  will increase (decrease) welfare.

### 5.3 Comparison between conditional and reverse conditional subsidies when the probability of success is endogenous

One relevant question is whether conditional subsidies awarded in the case of success (model 5) are preferred to conditional subsidies awarded in the case of failure (model 8). This comparison was already established between models 2 and 7, where the choice variable of the firm was  $x$ . Here we wish to answer this question in the context of  $\theta$  as a choice variable.

The right side of table 2 illustrates the optimal subsidy, the choice of  $\theta$  by the firm, the expected subsidy, as well as welfare from model 5, where the regular conditional subsidy was used. These values come from the solutions given by (29) and (30). The same numerical parametrization is used as for model 8 to facilitate the comparison.

The three configurations allow us to establish a complete comparison between the two models, because each model can have either an interior or a corner solution for the choice of subsidy by the government. In configuration 1, model 8 is at an interior solution, while model 5 is at a corner solution. In the second configuration, model 5 is at an interior solution, while model 8 is at a corner solution. Finally, in the third configuration, both models are at interior solutions. The structure and the results of model 5 have been discussed above, here the focus is on the comparison of the two models. The easiest way to compare them is to read table 2 horizontally; that is, for given parameter values, to compare the subsidy level, the choice of  $\theta$ , the expected subsidy, and social welfare.

In configuration 1,  $s_c^R$  is optimal, while  $s_c$  is suboptimal. This results in a situation where  $E_s^R > E_s$ ,  $W^R > W$  and  $\theta^R = \theta^W > \theta$ , where  $\theta^W$  denotes the socially optimal level of  $\theta$ . Hence, in this case, the reverse conditional subsidy is preferred to the (success based) conditional subsidy. Because the parameter values are such that the equilibrium and optimal choices of  $\theta$  are rather low, this makes it impossible (in model 5) to set  $s_c$  high enough to achieve the optimal expected subsidy, which is fixed.

In configuration 2, exactly the opposite occurs.  $s_c$  is optimal, while  $s_c^R$  is suboptimal. It follows that  $\theta = \theta^W > \theta^R$  and  $W > W^R$ . And this is true even though  $E_s^R > E_s$ . But because the endogenous probability of success is lower with the reverse subsidy, welfare is lower. In this case the equilibrium choices of  $\theta$  are quite high; this reduces the probability of obtaining the subsidy in the reverse subsidy model, forcing the government to raise  $s_c^R$  to its maximum.

From configurations 1 and 2 we observe that the model with the higher subsidy (100%)

results in lower welfare. This is because even a subsidy of 100% may be suboptimal if it does not induce the efficient level of effort by the firm. In that case, such a high subsidy is an indication that the government would like to set the subsidy at an even higher level to achieve the optimum, but is unable to. Whereas when the subsidy is strictly below 1, the constraint is not binding.

Finally, in configuration 3, the two models yield identical levels of  $\theta$  and  $W$ . Both models are at an interior solution, and both are equivalent. But this identical outcome is achieved differently. In this case the choice of  $\theta$  is intermediate. This allows the government to achieve the optimal  $\theta$  by setting a reverse conditional subsidy which is quite high, but lower than 1. At the same time, it allows the government to achieve the optimal  $\theta$  in model 5. Note that for this configuration  $s_c^R > s_c$ : a higher reverse conditional subsidy is required.

This comparison between regular conditional subsidies and reverse conditional subsidies with  $\theta$  endogenous yields results which are of the same spirit as those obtained from the comparison of models 2 and 7, where  $x$  was the choice variable. There it was found that for low (exogenous) values of  $\theta$ , the reverse conditional subsidy is preferred; for high values of  $\theta$ , the regular conditional subsidy is preferred; and for intermediate values of  $\theta$ , the two models are equivalent. The comparison performed in the current section between models 5 and 8 goes in the same direction, and shows that this intuition extends to the case where  $\theta$  is endogenous.

**Proposition 9.** *When the probability of success is endogenous and the government uses a reverse subsidy scheme such that the subsidy is paid only when the R&D project fails, the optimal subsidy increases with the size of the project and demand, and declines with production costs and innovation costs. The socially optimal level of innovation is induced iff the probability of success chosen by the firm is sufficiently small.*

**Proposition 10.** *With an endogenous probability of success, comparing only conditional subsidies, the subsidy based on failure generates more (less) R&D and welfare when the probability of success chosen by the firm is low (high). The two models are equivalent when the probability of success chosen by the firm is intermediate.*

## 6. Conclusions

This paper has analyzed how R&D subsidies can be made conditional on the success or failure of R&D projects, and how these conditional subsidies can be used on a stand-alone basis, or in conjunction with more traditional unconditional subsidies.

In general, the model warns against the use of conditional R&D subsidies alone. For a wide range of project sizes and degrees of riskiness, conditional subsidies alone may yield suboptimal levels of innovation. This problem can be avoided by combining conditional and unconditional R&D subsidies to achieve the optimal expected subsidy. In some cases, conditional subsidies yield a lower expected cost of the subsidy to the government. As for the choice between conditional subsidies based on success and those based on failure, the former are preferred for safer R&D projects, while the latter are superior for risky R&D projects.

The result that the optimal conditional subsidy varies negatively with the probability of success sheds light on the debate of when R&D subsidies contribute to increasing R&D, and when they finance research activities firm would undertake anyway. The argument that the subsidy should be positively correlated with the social profitability from innovation (size of innovation, market size, probability of success, etc.) overlooks the fact that many of the factors which increase the social value of the innovation also increase its private value to the innovator, increasing the privately optimal investment. Hence, the innovator already accounts for a large number of these factors, and it may be redundant for the government to subsidize R&D projects because they are promising or socially beneficial; the innovator's actions already reflect partly those benefits. Rather, it is when the private value of the innovation is small (due to a small market size, a low probability of success, etc.) that government intervention is most needed.

Tassey (1996) cites evidence from surveys by the Industrial Research Institute and others, which "show a definite decline in recent years in industry's willingness to fund longer-term, higher-risk technology research" (p. 581). R&D subsidies which reward success can contribute to alleviating this particular market failure. Alternatively, to provide more safety for firms, a reverse conditional subsidy could be used, which results in a high probability that the firm keeps the subsidy, given the high probability of failure.

In the current model the problem of overinvestment in R&D has not arisen. This is due to the restrictive monopolistic market structure adopted here. However, the same limitation of

conditional subsidies shown in the current model should extend to models with a more general market structure. For instance, where it is optimal to tax R&D because of overinvestment, a conditional (negative) subsidy alone may not always be able to achieve the optimal expected R&D tax, and hence may not induce the optimal level of innovation.

Even though in many of the cases studied above conditional subsidies did not yield results which were superior to unconditional subsidies, the results are still useful given that governments are currently using (not always explicitly) conditional subsidies, alone or in parallel to unconditional subsidies. This makes it important to investigate the consequences of using such subsidies. Hence, even if from a normative perspective conditional subsidies are not always needed,<sup>17</sup> from a positive perspective their use is widespread, and it is the role of economic research to clarify their strengths and weaknesses.

The limitations of the current work should be seen as avenues for future research. The success or failure of a R&D project is not always easy to establish, even ex post; the project may be multidimensional and may succeed on some dimensions but fail on other dimensions; even in unidimensional cases, success may be a continuous rather than a discrete variable. Moreover, some projects may succeed technologically, but fail commercially. Firms may have private information about the true quality, and hence the probability of success, of their R&D projects.<sup>18</sup> The interactions between conditional R&D subsidies and the incentives for innovation, cost overruns, moral hazard and adverse selection are also important areas left for future research. Small firms may be risk averse, and explicit modeling of risk attitudes may be required; conditional R&D subsidies increase the risk facing the firm, while reverse conditional subsidies reduce it, because they reduce the variability in the firm's profits. Finally, the relationship between conditionality, crowding-out of private R&D and the political process can be complex. Gretzmacher (2002:4) notes that "government bureaucrats are under pressure exhibiting short-term success in allocating public funds, which can lead to a funding of R&D projects with higher success probability. These are projects, which the firm would have undertaken anyway and private expenditures can be crowded out...".

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<sup>17</sup>Even though in a more general setting (risk aversion, budget constraints, etc.) they could dominate unconditional subsidies.

<sup>18</sup>One way of alleviating this asymmetry of information would be to let firms bid for the subsidy, to extract information about the firms' true assessment of their projects (Martin and Scott, 2000).

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**Table 1. Summary of results**

Model #	Firm's choice	Policy instrument	Value(s) of instrument(s)	Expected subsidy	Social optimum achieved
1	$x$	$s_u$	$s_u=1/3$	$E_s=1/3$	Yes
2	$x$	$s_c$	$s_c=\min\{1/3\theta,1\}$	$E_s=\min\{1/3,\theta\}$	Yes if $\theta \geq 1/3$
3	$x$	$s_u$ and $s_c$	$s_u+\theta s_c=1/3$	$E_s=1/3$	Yes
4	$\theta$	$s_u$	$s_u=1/3$	$E_s=1/3$	Yes
5	$\theta$	$s_c$	$s_c=\min\{2/(9\theta(.)),1\}$	$E_s=\min\{2/9,\theta(.)\}$	Yes if + - + - $\theta(x,\lambda,A,\alpha) \geq 2/9$ <sup>(1)</sup>
6	$\theta$	$s_u$ and $s_c$	$s_u+1.5\theta(.)s_c=1/3$	$E_s \in [2/9, 1/3]$	Yes
7	$x$	$s_c^R$	$s_c=\min\{1/[3(1-\theta)],1\}$	$E_s=\min\{1/3,1-\theta\}$	Yes if $\theta \leq 2/3$
8	$\theta$	$s_c^R$	+ - + - $s_c=\min\{s_c(x,\lambda,A,\alpha),1\}$ <sup>(1)</sup>	$E_s=\min\{(1-\theta(.))s_c(.),1-\theta(.)\}$	Yes if + - + - $\theta(x,\lambda,A,\alpha)$ is not too high <sup>(1)</sup>

(1) The signs above the arguments of functions denote the signs of the partial derivatives.

**Table 2. Reverse conditional subsidies and conditional subsidies with endogenous probability of success**

Parameter changed	Model 8					Model 5			
	$s_c^R$	$\theta$	$1-\theta$	$E_s=(1-\theta) s_c^R$	$W$	$s_c$	$\theta$	$E_s=\theta s_c$	$W$
<b>Configuration 1: A=1000,R=50,x=5,<math>\lambda</math>=30000</b>									
None	0.3660	0.0595	0.9405	0.3442	338544	1	0.0424	0.0424	338535
A=2000	0.4080	0.1220	0.8780	0.3582	1426380	1	0.0948	0.0948	142636
R=100	0.3641	0.0564	0.9436	0.3436	303845	1	0.0400	0.0400	303837
x=10	0.4060	0.1194	0.8806	0.3576	338865	1	0.0924	0.0924	338843
$\lambda$ =40000	0.3573	0.0447	0.9554	0.3413	338517	1	0.0312	0.0312	338510
<b>Configuration 2: A=1000,R=50,x=5,<math>\lambda</math>=3000</b>									
None	1	0.5144	0.4856	0.4856	339481	0.3733	0.5953	0.2222	339501
A=1500	1	0.6352	0.3648	0.3648	790687	0.2448	0.9078	0.2222	790910
R=100	1	0.5007	0.4993	0.4993	304692	0.3940	0.5641	0.2222	304704
x=7	1	0.6089	0.3911	0.3911	340373	0.2664	0.8343	0.2222	340526
$\lambda$ =3500	1	0.4762	0.5238	0.5238	339345	0.4355	0.5103	0.2222	339349
<b>Configuration 3: A=1000,R=50,x=5,<math>\lambda</math>=4500</b>									
None	0.8237	0.3969	0.6031	0.4948	339146	0.5600	0.3969	0.2222	339146
A=1100	0.9741	0.4385	0.5615	0.5469	414303	0.5067	0.4385	0.2222	414303
R=100	0.7646	0.3760	0.6240	0.4771	304386	0.5910	0.3760	0.2222	304386
x=5.5	0.9662	0.4367	0.5633	0.5443	339296	0.5089	0.4367	0.2222	339296
$\lambda$ =5000	0.7181	0.3572	0.6428	0.4616	339075	0.6221	0.3572	0.2222	339075

Figure 1. Different combinations of  $s_u$  and  $s_c$  (Model 3)

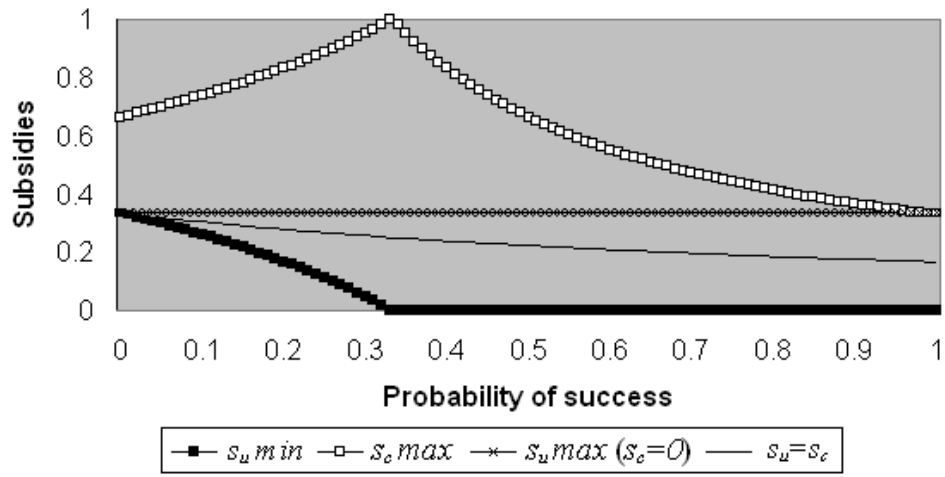


Figure 2. Conditional subsidies

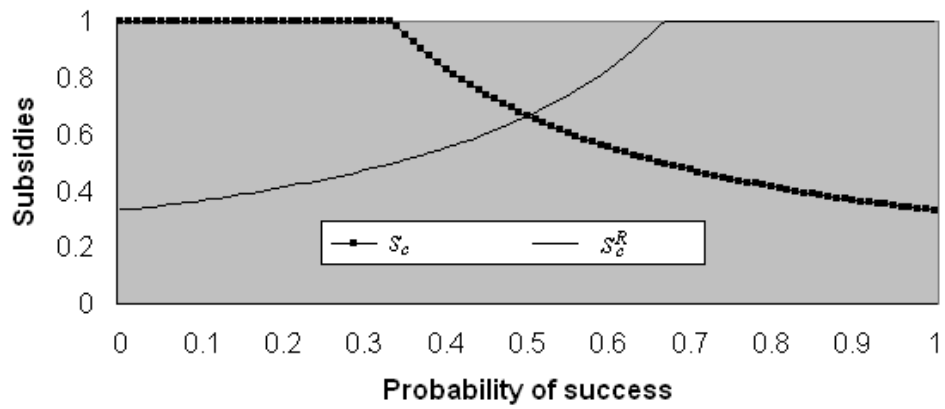
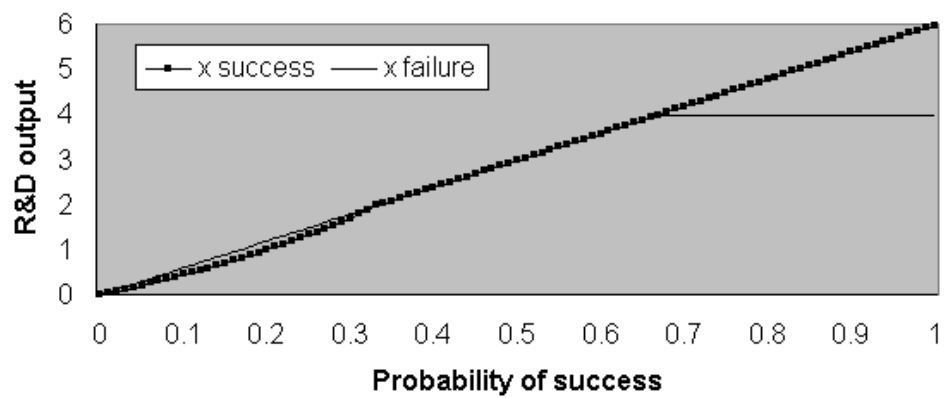


Figure 3. R&D output



## Appendix. Second-order conditions

Model	R&D stage	Subsidy stage
1	$\gamma > \theta/4(1-s_u)$	$\gamma > \theta/8s_u$
2	$\gamma > \theta/4(1-\theta s_c)$	$\gamma > 1/8s_c$
3	$\gamma > \theta/4(1-s_u-\theta s_c)$	$\gamma > \theta/8(s_u+\theta s_c)$
4	$s_u < 1$	$s_u > 0$
5	$\theta s_c < 1/3$	$\lambda > f_5(A, \alpha, x, s_c)$
6	$\lambda > x(2(A-\alpha)+x)/4\theta(2-2s_u-3\theta s_c)$	$\lambda > f_6(A, \alpha, x, s_u, s_c)$
7	$\gamma > \theta/4(1-(1-\theta)s_c^R)$	$\gamma > \theta/8(1-\theta)s_c^R$
8	$\theta > -(1-s_c^R)/3 s_c^R$	$f_8(A, \alpha, x, \lambda, s_c^R) < 0$

Note: For models 5 and 8, the soc of the subsidy stage are not presented in full due to their length. It is easy to verify that all soc are satisfied given our assumptions on parameters and given the solutions derived in the paper. Most of these conditions require that the cost of R&D be sufficiently high, and a few require that the subsidy level or the probability of success be in the interval (0,1).