Analysis and measurement of the uncertainty in Mini-Dms model for the French economy

Carlo Bianchi and Jean-Louis Brillet and Giorgio Calzolari

IBM Scientific Center, Pisa, Italy., INSEE, Service des Programmes, Paris, France.

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Centro Scientifico IBM, Pisa.
Service des Programmes, INSEE, Paris.

1. INTRODUCTION

It is purpose of this work to evidence, in the behaviour of the Mini-DMS model for the French economy, some stochastic properties which may confirm, strengthen or sometimes contradict the results obtained from the standard simulation analysis, which is purely deterministic. In particular, the model's capabilities in forecasting, in economic policy experiments, and the model's dynamic behaviour will be faced by regarding forecasts, multipliers and characteristic roots as point estimates and associating with them a measure of dispersion, like a standard error.

Some results will be obtained through the use of stochastic simulation techniques (Monte Carlo), in some case with the aid of variance reduction methods (antithetic variates) which allow for considerable improvement of the computational accuracy. Other results will be obtained by means of analytic simulation techniques, based on first order approximations which are asymptotically exact under standard hypotheses.

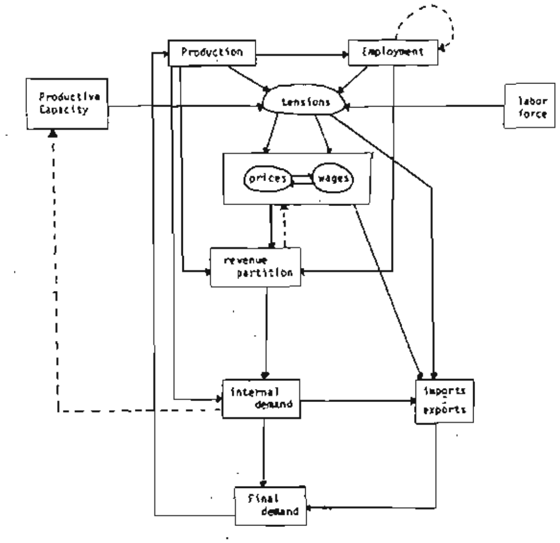
The plan of the paper is as follows. In section 2 a description of the Mini-DMS model is briefly outlined and section 3 summarizes the main assumptions and notations used in the paper. Section 4 is concerned with the problem of evaluating the uncertainty of forecasts produced by the model and results are displayed and commented in section 5. Section 6 deals with some statistical properties of multipliers, when their estimate is obtained, as usual, from deterministic simulation of the model with estimated parameters; some experimental results are presented and comments are focussed on the problem of reliability versus effectiveness of economic policy actions. The last section (7) deals with the computation of characteristic roots of a local linearization of the model, and provides experimental results of the computation of asymptotic standard errors of the roots.

2. SOME REMARKS ON MINI-DMS MODEL

The Mini-DMS model (Brillet 1981a) constitutes a smaller version of the Dynamic Multi Sectorial model of the French economy (Fouquet et al., 1978) built in 1974-1976 at INSEE (National Institute for Statistics and Economic Studies) to be used as a medium term forecasting tool, in particular for national planning studies conducted through the Commissariat General au Plan (General Planning Agency). Largerly reduced in size (the present version contains 154 equations, 65 of which are behavioural, as compared to more than 2400 for the larger version) Mini-DMS nevertheless preserves the same economic structure as well as most of the theoretical mechanism of the original model. The economic equilibrium is reached through two simultaneous iterative processes: a Keinesian process on demand (a given value of demand induces a level of production from which a new value is determined as the given of its individual elements) and the price wage rate loop.

Figure 1 gives a very schematic view of the process: from final demand the model deduces production and desired employment level, to which the effective level adjusts only partially; comparison between availabilities (predetermined production capacity, labor force, job supply) and the quantities actually used produces disequilibrium or tension variables, which determine the level reached by the iterative loop between wage rate and price index; the subsequent partition of the revenue between business firms and households gives their respective demand elements: investment (through an accelerator-profit formulation) and consumption, thus global domestic demand, which, corrected of the external trade elements (influenced, besides demand itself, by available productive capacity and competitivity) produces a new value for final demand, allowing a reinitialization of a process which hopefully leads, after some iterations, to an equilibrium value.

In its present state, the Mini-DMS model can be considered as being half way between an operational-forecasting tool: its acceptable forecasting



SIMPLIFIED ARCHITECTURE OF THE MINI-DMS MODEL

(dotted lines are associated with lagged iteration)

Fig.1

qualities, as well as its rather detailed set of decisional variables, can lead to its use for simple enough macro-economic studies, and an instrument for carrying out mathematical economic experiments, some of which have already been made in the near past, concerning in particular multiplier analysis, optimal control problems or dynamic properties of alternate formulations (Brillet, 1981b).

The estimates of the structural parameters of the model have been obtained by means of a straightforward extension of Brundy and Jorgenson's (1971) instrumental variables method (limited information) to the case of nonlinear models. The method has been applied iteratively, till convergence has been reached, so that the final estimates of coefficients are not affected by the choice of the initial coefficients values. In each iteration, the instrumental variables are computed as deterministic solution values of the system. Since the number of stochastic equations in the model is considerably larger than the sample period length, the estimate of the covariance matrix of the disturbance process would be singular, and the standard system estimation methods could not be applied.

3. NOTATIONS AND GENERAL STATEMENTS

The following notations will be adopted in this paper. Let the structural econometric model be represented as

$$f(y_t, y_{t-1}, x_t, a) = u_t; \qquad t=1,2,\dots,T \tag{3.1}$$

where $f = (f_1, f_2, \dots, f_m)'$ is a vector of functional operators, continuously differentiable with respect to the elements of current and lagged y, x and a, at least till the second order; $y_t = (y_{1t}, y_{2t}, \dots, y_{mt})'$, $x_t = (x_{1t}, x_{2t}, \dots, x_{nt})'$ and y_{t-1} are the vectors of current endogenous, exogenous and lagged endogenous variables, respectively; $a = (a_1, a_2, \dots, a_s)'$ is the vector of the structural coefficients to be estimated (all the other known coefficients of the model are excluded from this vector and included in the functional operators); $u_t = (u_{1t}, u_{2t}, \dots, u_{mt})'$ is the vector of structural stochastic disturbances (or error terms) at time t, having zero mean and being independently and identically distributed over time, with finite contemporaneous covariance matrix, and independent of all the predetermined variables. In all the experiments described in this paper, the contemporaneous distribution of the error terms is assumed multivariate normal: $u_t \sim N(0, \Sigma)$.

It is usually assumed that a simultaneous equation system like (3.1) implicitly defines a single inverse relationship (reduced form) for relevant values of the coefficients, the predetermined variables, and any values of the disturbance terms:

$$y_t = g(y_{t-1}, x_t, a, u_t). \tag{3.2}$$

Of course, the vector of functions g implicitly defined is usually unknown, but can be assumed continuously differentiable, like f.

Processing the sample data by means of a suitable estimation method, we get an estimated vector of coefficients, $\hat{a}$, an estimated covariance matrix of the structural disturbance process, $\hat{\Sigma}$, and an estimate of the coefficients covariance matrix which will be indicated as $\hat{\Psi}/T$. Four remarks may be helpful at this point.

1) Few assumptions in addition to those listed above are usually enough to ensure consistency and asymptotic normality of $\hat{a}$, produced by suitable estimation methods, in case of linear dynamic or nonlinear static models. In these cases, it is usually ensured that, asymptotically as $T\to\infty$,

$$T^{\frac{1}{2}} (\hat{a} - a) \sim N(0, \Psi). \tag{3.3}$$

2) Unfortunately, to our knowledge, there are no general theoretical tools to prove that (3.3) holds when the nonlinear model includes lagged endogenous variables among the predetermined variables. It can, however, be assumed that (3.3) holds under heuristic considerations, as in Gallant (1977, pp.73-74). If (3.3) holds, then several results which will be derived are asymptotically exact; if (3.3) does not hold exactly, the results which will be derived are not asymptotically exact, but simply "reasonable" approximations.
3) If $\hat{\Psi}$ is a consistent estimate of Ψ, an estimate of the covariance matrix of a multinormal distribution which approximates the small sample distribution

of $\hat{a}$ is obtained as $\hat{\Psi}/T$, that is dividing $\hat{\Psi}$ by the actual length of the sample period (see Schmidt, 1976, p.254).

4) $\hat{\Psi}/T$ is, together with $\hat{a}$, a standard outcome of system estimation methods. For example, in case of FIML estimation, $\hat{\Psi}/T$ is nothing but the inverse of the Hessian (with minus sign) of the concentrated log-likelihood, calculated at the point which maximizes the likelihood. When limited information estimation methods are applied, as in our case, this matrix must be built block by block, after the estimated $\hat{a}$ has been obtained, and the resulting matrix may be singular, in case of undersized samples. In our case, the blocks of matrix $\hat{\Psi}/T$ have been computed as in Brundy and Jorgenson (1971, p.215). Mini-DMS model involves 155 unknown structural coefficients, so that $\hat{\Psi}/T$ is a 155×155 full matrix (symmetric, of course).

Let us now introduce an additional simplification, which is crucial: we disregard from misspecification. Of course, the results which will be obtained could be used for tests of misspecification or, more simply, for empirical measurement of the misspecification effects (e.g. Fair, 1980), however we shall work, in the rest of the paper, as if equation (3.1) represents the "true" structure of the economic system and that, therefore, the "true" process which generates endogenous variables is represented by equation (3.2). An assumption like this is, of course, considerably hazardous and unrealistic, but quite helpful and useful. We should not forget, in fact, that we are going to analyze and measure the degree of uncertainty associated with some simulation experiments. Roughly speaking, if a measure of uncertainty is obtained under the assumption of correct specification, it should be interpreted as the "minimum" degree of uncertainty associated with a given simulation result. In other words, we get in this way a kind of empirical upper bound for the reliability of model's results.

Let us now use the model with estimated parameters in simulation experiments, such as ex-post prediction, forecasting, evaluation of impact multipliers for economic policy, etc. Results obtained with the usual

simulation procedure, which is deterministic, are in some way related to the reduced form notation (3.2) where a is replaced by its estimate, $\hat{a}$, and the random error terms u_t are set to their expected value (zero). For example, if we are interested in evaluating the model's goodness of fit over the sample period, we calculate the values of the endogenous variables $\hat{y}_t$ such that $f(\hat{y}_t, y_{t-1}, x_t, \hat{a})=0$, that is, resorting to the reduced form notation,

$$\hat{y}_t = g(y_{t-1}, x_t, \hat{a}, 0). \tag{3.4}$$

The same is done if we are using the model to produce forecasts one period ahead, conditional on the exact knowledge of all the predetermined variables. Generalization to dynamic simulation is straightforward (see section 4).

Whichever analysis of the economy is done by means of model's simulation, we must not forget that, while (3.2) is the process which generates the real data, what we are using is the simulation process (3.4). The differences in the two processes are given by the presence, in the latter, of the random vector of estimates, $\hat{a}$, instead of the vector of constant "true" coefficients a, and by the presence of the random error terms u_t in the former. These two differences, which cause the uncertainty of simulated results, are the two error sources whose effects will be measured throughout this paper, in several cases.

We shall try to isolate the effects of these two error sources, as far as possible. When separation of effects can be made, the two error sources can be treated with different computation procedures. Although both error sources have, in some sense, a common origin, it is extremely important to distinguish between their different behaviour.

The structural disturbance vector, u_t, is a vector of zero mean random error terms embodied in the model by the very nature of the endogenous variables, which are random variables. Whichever sample period length we have at our disposal, and whatever estimation method we apply, we cannot reduce the size of the error caused by u_t. All we can get, from using more and more efficient estimation methods, is a more accurate estimate of the error process covariance matrix, Σ, which however remains a nonzero matrix. Even if all the other variables and parameters of the model were known with certainty, the simulation process would differ from the real data generation process by a nonlinear function of u_t,

The simplest (and perhaps the only possible, in case of medium-large models) way of treating nonlinear transformations of random variables with finite variances and covariances is surely stochastic simulation. We may proceed as follows.

First of all a vector of pseudo-random error terms must be generated for each period of time to which simulation results are related (one only, for instance, if we are dealing with one-period forecasts or impact multipliers). The distribution of these pseudo-random vectors must be as close as possible to the distribution of u_t; a suitable choice is the multinormal distribution with zero mean and covariance matrix equal to the available estimate $\bar{\Sigma}$. With Mini-DMS model we are dealing with a case of undersized samples, for which a suitable generation method has been proposed in McCarthy (1972). The generated error terms are inserted into the model, and the usual simulation procedures are applied to produce the desired result (forecast, for example). The whole procedure is repeated from the beginning a given (possibly large) number of times, in such a way as to produce a sample of outcomes, from which we calculate sample means and variances. A number of replications more and more increasing is, usually, expected to produce more accurate values of means and variances.

The nature of the error involved in the estimated coefficients, $\hat{a}$, is rather different and, if we are able to isolate its effects from those due to the structural disturbances, we can process and measure such effects in a completely different way. In many cases, in fact, we could even apply full analytical methods or, if simpler in practice, some simulation procedures

which are straightforward numerical applications of the analytical methods, therefore called "analytic simulation".

We notice that the error involved in the estimated coefficients is only due to the availability of data for a short number of time periods. If an infinitely large sample could be available, the consistency of the estimation method would produce a coefficient vector without errors. For finite sample lengths, we now observe that, conditional on the exact knowledge of predetermined variables, the simulation process is nothing but a function of the estimated coefficients (3.4) and that this function, even if usually unknown, is continuously differentiable. We can, therefore, apply a well known theorem on the limiting distribution of functions of sample statistics, which states as follows (see Rao, 1973, p.388).

<u>Theorem</u>. Let $\hat{a}$ be an s-dimensional statistic, such that the asymptotic distribution of $T^{\frac{1}{2}}(\hat{a}-a)$ is s-variate normal with mean zero and covariance matrix Ψ. Let q be an m-dimensional vector of functions of the s variables and each q_i be totally differentiable. Then the asymptotic distribution of $T^{\frac{1}{2}}[q(\hat{a})-q(a)]$ is m-variate normal with zero means and covariance matrix $Q\Psi Q'$, where $Q=\partial q/\partial a'$.

A practically equivalent statement, which is suitable for our case (see again Rao, 1973, p.388), is that, if $\hat{a}$ is distributed approximately as s-variate normal with mean a and covariance matrix Ψ/T, then $q(\hat{a})$ is distributed approximately as m-variate normal with mean $q(a)$ and covariance matrix $Q\Psi Q'/T$. The accuracy of the approximation, of course, increases as the sample size increases.

Whenever possible, we shall resort to the above theorem to measure the degree of uncertainty due to errors in estimated coefficients. We must only remark that, if $q(\hat{a})$ is the simulation process (3.4), then the matrix of its derivatives, Q, can be computed analytically even if the functional operators g are unknown. For the implicit functions theorem, in fact, we have the equality

$$(3.5) \qquad \partial g/\partial a' = -\ (\partial f/\partial y')^{-1}\ \partial f/\partial a'$$

which involves only the functional operators of the structural form of the model. This holds when we are evaluating the effects of coefficients errors on forecasts produced by the model, because in such a case $g(\hat{a})$ is simply the simulation process (3.4); but also in other cases, when simulation is used to compute multipliers or characteristic roots, the above considerations on the possibility of getting analytically our results still hold, even if they become more complicated.

In all cases, however, we have preferred to calculate derivatives only numerically, as ratios of finite differences. This method (analytic simulation) seems to be computationally simpler and has proved to be sufficiently accurate in most cases.

4. FORECAST ERROR AND ITS DECOMPOSITION

Let h be a time period not belonging to the sample estimation period 1,2,...,T, and let the model be used to forecast at times h+1, h+2, ..., h+r. Given the values of the endogenous variables at time h, y_h, and the values of the exogenous variables in the forecast periods, x_{h+1}, x_{h+2}, ..., x_{h+r}, then the values of the endogenous variables in the forecast periods can be obtained recursively as:

$$(4.1) \qquad \begin{aligned} y_{h+1} &= g(y_h, x_{h+1}, a, u_{h+1}); \\ y_{h+2} &= g(y_{h+1}, x_{h+2}, a, u_{h+2}) \\ &= g(g(y_h, x_{h+1}, a, u_{h+1}), x_{h+2}, a, u_{h+2}) \\ &= g_2(y_h, x_{h+1}, x_{h+2}, a, u_{h+1}, u_{h+2}); \\ &\ \ \vdots \\ y_{h+r} &= g(y_{h+r-1}, x_{h+r}, a, u_{h+r}) \\ &= g(g(\ldots), x_{h+r}, a, u_{h+r}) \\ &= g_r(y_h, x_{h+1}, \ldots, x_{h+r}, a, u_{h+1}, \ldots, u_{h+r}). \end{aligned}$$

Several sources of error could be identified at this point (like, for instance, the uncertainty in the forecast values of exogenous variables, etc.); however, to get a measurement of a "minimum" degree of uncertainty, we can perform the analysis conditional on exact knowledge of the initial values of the endogenous variables in the forecast period, y_h, and on the values of the exogenous variables x_{h+1}, x_{h+2}, ..., x_{h+r}, and confine ourselves to the two error sources discussed in section 3. To further help understand these, we summarise how model builders use their models to produce forecasts.

The model builder must choose a starting point for the simulation experiment. Such a point (h) is usually the last time period for which "sure" information is available; in many cases it is h=T, the last point of the sample estimation period. For the purposes of this paper, it is simpler to start from a period h not belonging to 1,2,...,T (for example, h=T+1); if we shall choose h inside the sample period a slight approximation will occur. The model builder next introduces values for y_h, and x_{h+1}, x_{h+2}, ..., x_{h+r}, sets the random error terms u_{h+1}, u_{h+2}, ..., u_{h+r} to their expected value (zero) and solves simultaneously the dynamic system (3.1) at time h+1, h+2, ..., h+r. Using the reduced form notation (3.2), forecasts are obtained as

$$(4.2) \qquad \begin{aligned} \hat{y}_{h+1} &= g(y_h, x_{h+1}, \hat{a}, 0); \\ \hat{y}_{h+2} &= g(\hat{y}_{h+1}, x_{h+2}, \hat{a}, 0) = g(g(y_h, x_{h+1}, \hat{a}, 0), x_{h+2}, \hat{a}, 0) \\ &= g_2(y_h, x_{h+1}, x_{h+2}, \hat{a}, 0, 0,); \\ &\ \ \vdots \\ \hat{y}_{h+r} &= g(\hat{y}_{h+r-1}, x_{h+r}, \hat{a}, 0) = g(g(\ldots), x_{h+r}, \hat{a}, 0) \\ &= g_r(y_h, x_{h+1}, \ldots, x_{h+r}, \hat{a}, 0, \ldots, 0). \end{aligned}$$

Forecasts $\hat{y}_{h+1}$, $\hat{y}_{h+2}$, ..., $\hat{y}_{h+r}$ differ from the values of the endogenous variables in the forecast period y_{h+1}, y_{h+2}, ..., y_{h+r}, because the estimated $\hat{a}$ is used instead of the unknown coefficients vector a, and due to the existence of the random error terms u_{h+1}, u_{h+2}, ..., u_{h+r}.

In order to assign forecasts an estimate of their degree of uncertainty, it would be useful to get, at least, an estimate of the first two moments of the forecast errors $\hat{y}_{h+1}-y_{h+1}$, $\hat{y}_{h+2}-y_{h+2}$, ..., $\hat{y}_{h+r}-y_{h+r}$.

In reverse order, we start from the estimation of the second order moment which usually proved to be the more interesting, in practice; standard errors allow one, in fact, to obtain confidence intervals for the single forecasted endogenous variables, while an estimate of the covariance matrix allows the construction of joint confidence regions or the testing of hypotheses. The random error terms have been assumed serially independent; therefore, assuming exact knowledge of all the predetermined variables (y_h, x_{h+1}, x_{h+2}, ..., x_{h+r}), the vector of coefficients, $\hat{a}$, which is obtained from an estimation procedure applied to the data of the sample period, is independent of the random error terms in the forecast period (which is outside the sample estimation period). We can now decompose the vector of forecast errors, in the generic forecast period h+k, as follows:

$$\begin{aligned}(4.3)\qquad \hat{y}_{h+k} - y_{h+k} &= g_k(y_h,x_{h+1},\ldots,x_{h+k},\hat{a},0,\ldots,0) \\ &\quad - g_k(y_h,x_{h+1},\ldots,x_{h+k},a,u_{h+1},\ldots,u_{h+k}) \\ &= [g_k(y_h,x_{h+1},\ldots,x_{h+k},\hat{a},0,\ldots,0) \\ &\quad - g_k(y_h,x_{h+1},\ldots,x_{h+k},a,0,\ldots,0)] \\ &\quad + [g_k(y_h,x_{h+1},\ldots,x_{h+k},a,0,\ldots,0) \\ &\quad - g_k(y_h,x_{h+1},\ldots,x_{h+k},a,u_{h+1},\ldots,u_{h+k})].\end{aligned}$$

Having assumed exact knowledge of all the predetermined variables involved in our forecast, the two components of the forecast error vector are independent, since the former depends on $\hat{a}$, while the latter depends on u_{h+1}, ..., u_{h+k}. We can , therefore, calculate the variances or the covariance matrices of the two components separately, and sum them to get variances-covariances of the final results (see Bianchi and Calzolari, 1980).

The variances-covariances of the second component, which is due to the random error terms u_{h+1}, ..., u_{h+k}, can be approximately computed as sample means and variances-covariances of replicated stochastic simulations in the forecast period.

As far as the first component is concerned, its covariance matrix can be computed by means of the linear approximation discussed in section 3 (asymptotically exact, or not, depending on assumption 3.3). If we define G_{h+k} as the (m×s) matrix of first order partial derivatives of the vector of functions g_k with respect to the elements of a, computed at the point (y_h, x_{h+1}, ..., x_{h+k}, a, 0, ..., 0), then, asymptotically,

$$\begin{aligned}(4.4)\qquad T^{\frac{1}{2}}\,&[g_k(y_h,x_{h+1},\ldots,x_{h+k},\hat{a},0,\ldots,0) \\ &- g_k(y_h,x_{h+1},\ldots,x_{h+k},a,0,\ldots,0)] \sim N(0,G_{h+k}\Psi G'_{h+k}).\end{aligned}$$

Calculating G_{h+k} at the point (y_h, x_{h+1}, ..., x_{h+k}, $\hat{a}$, 0, ..., 0), replacing Ψ with the available estimate $\hat{\Psi}$, and dividing $\hat{G}_{h+k}\hat{\Psi}\hat{G}'_{h+k}$ by the actual length of the sample period, T, we get the approximate covariance matrix of the first component of the forecast errors.

Standard errors of forecasts are derived from the covariance matrix obtained from summing the covariance matrices of the two components.

Let us now consider the first order moment (mean) of the vector of forecast errors. As far as the first component is concerned, the asymptotic normal approximation already adopted suggests that we can regard such component as a zero mean vector. Much more troublesome is the computation of the mean vector of the other component. Such component is a function of the random error terms, which have mean zero but finite variances-covariances; therefore, its mean is zero if the function is linear (linear models), but may be nonzero if the function is nonlinear (as for Mini-DMS model). The mean of this component (which can be interpreted as the bias of the vector of forecasts), can be computed as sample mean of the same set of stochastic simulation results already used to compute the variances-covariances. However, with Mini-DMS and with many other macroeconomic models it happens that the mean is, for most variables, much smaller than the corresponding standard deviation; therefore, if we want to compute an accurate value of the mean, we should use a very large number of replications, and the cost of computation would be very high (for most variables, a few thousands replications are not enough to produce a significant value of the mean).

A method which produces accurate results with a drastic reduction of the computation costs has been applied. This method is based on stochastic simulation with antithetic variates; it works as follows. Rather than performing all stochastic simulation replications independently of one another, we perform couples of replications: each couple is independent of the others, but inside one couple we use the same set of pseudo-random disturbances with the opposite sign in the two replications. If the results of the two replications are negatively correlated (and they are strongly so, in our case), the arithmetic average of the two replications in each couple has a variance which is considerably smaller than for the average of two independent simulation runs (see Calzolari, 1979, or Cheng, 1982).

The experimental results described in the following section have been obtained with 500 couples of simulations. Comparison of computed values of the first moment (bias of deterministic simulation) with its experimental standard deviation ensures, with sufficiently high probability for all the displayed variables, that results are exact at least in the first digit.

5. PRACTICAL USE OF THE METHODS

In this section we will apply the set of methods described above to the Mini-DMS model, used for forecasting over a period beginning in 1981 (first period outside the sample period) and ending in 1985 or 1986, which corresponds to the normal use of the model; even if it is quite possible to simulate it over a much wider period, such as thirty or fourty years, without any major stability problems, however the costs of tests would then become prohibitive, and we feel that the results presented below will already be

quite significant. In our presentation we will try to use the perspective of the practical forecaster rather than that of the theoretician: we shall try to show how these methods can be used to gain additional information on the practical characteristics of individual forecasts.

Taking into account the information about the error characteristics (structural disturbances, error on estimated coefficients) of the whole set of equations allows us to derive two sets of interesting statistics: the bias due to the use, as forecast, of the deterministic solution, and the standard error of the forecast itself.

5.1. Bias introduced by the deterministic solution

We shall now study the temporal evolution of the bias for some of the main variables of the model over a 6 year forecasting period (1981-1986). Figure 2 shows the results for the gross domestic product (PIBZ) and its main components: PIBZ shows an increasing bias. Although this bias could be considered small, it is not negligible from an operational point of view: in 1986 the overestimation reaches 0.25%.

To interpret this evolution, let us consider figures 2 and 3. Figure 2 shows that the bias on the gross domestic product is, in fact, the result of the addition of larger biases, one positive on household consumption, one negative on investment, while exports and imports are less concerned. In 1986 the bias on consumption reaches now 0.9%, and -2.6% on investment: it seems that the main error lies on overestimation of the share of wages in the results of production. Indeed, figure 3 confirms this idea: the trajectory for the bias on wage rate lies much higher than that on consumption price, inducing a positive error on purchasing power. The profit rate, increased at first by the rise in activity, worsens later continuously. Of course the model interactions do not allow to establish a rigorous causal sequence in these influences: part of the bias on purchasing power is of course due to the bias on activity, thus on the tensions on the labour market; but, by observing the relative sizes, we can assume that the main sources of the bias are a positive error on purchasing power and a negative one on prices, these two errors inducing, through different channels, an overestimation of activity.

Another interesting criterion for forecast users is the evolution of balances (government and trade); figure 4 shows that the bias on these variables, coherent in sign with the previous observations, is not small: for instance an average variation of 3.5 billions francs on government balance could only be reduced through a significant variation of some policy instrument, such as a 3% increase of the amount of internal revenue tax.

These observations, showing the relative influence of the bias on the whole economic equilibrium, cannot be extended to the whole set of operational forecasting models: not because the precision of this particular model is questionable, or because its forecasting properties are insufficient, but because, the global bias being the result of the algebric sum of many individual biases, we have no way (except by experience) to know how these elements will compensate each other. But if the bias on the Mini-DMS forecasts should prove representative, it would be good to keep that problem in mind. Although it is obviously not practical to ask forecasters to replace each time a single deterministic simulation by a large number of stochastic ones, one idea could be to compute, for one standard forecast, the bias on each behavioural variable which is not due to explanatory variables of the associated equation, and to add it as a set of constant adjustments in the subsequent simulations.

5.2. Error on forecasts

We will now study the evolution over the years 1981-1985 of the standard error of forecast first through the structural disturbance terms, then through the uncertainty on estimated coefficients.

Figures 5-10 show these characteristics associated with a number of selected variables. These figures show first a number of constant characteristics: the error due to disturbance terms is higher in the first period than that due to errors in estimated coefficients; but, as these two errors rise with time, the quicker growth of the latter allows it to become the more important after a few periods. This seems quite logical: indeed for one particular period the global error due to the structural disturbances is the effect of current disturbance terms and of the previous ones through the dynamic equations. The increase in the error is only due to the increase in the number of independent sources, as it is assumed no correlation exists between structural disturbances of different periods; while obviously the error on coefficients produces for each individual equation a highly correlated sequence of errors which means that for autoregressive equations (provided that, as is almost always the case, in practice, the influence of lagged variables is positive), the correlation between lagged and current elements of the error due to coefficients is highly positive. This effect is particularly evident for those variables which are defined through autoregressive formulations (such as prices or wage rates), but also on global equilibrium and its definition (added value).

Another general consideration is that the precision on industrial product variables is always lower than on non-industrial: this is certainly due to the added uncertainty introduced by the sensitivity of this sector to external trade, itself characterized by rather imprecise variables. Simulation of the model over the estimation period shows indeed that keeping external trade variables exogenous increases precision greatly.

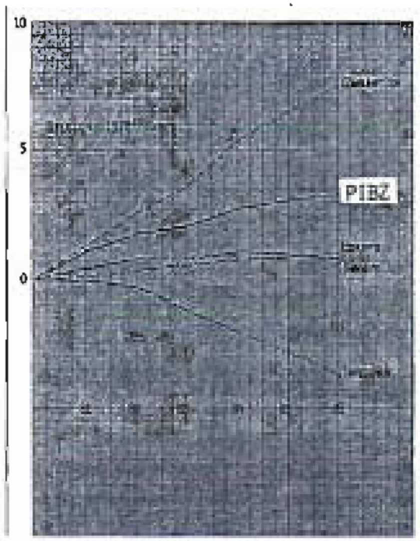


Fig.2

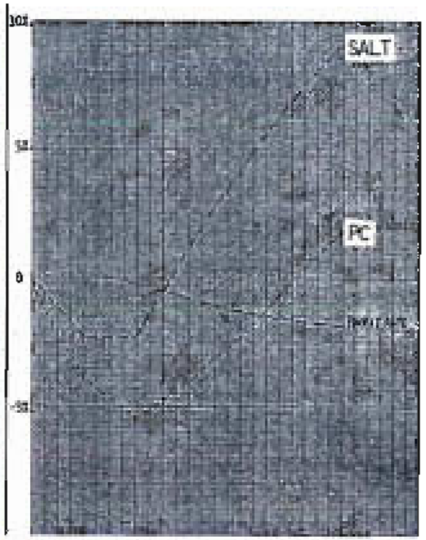


Fig.3

Bias introduced by the deterministic solution

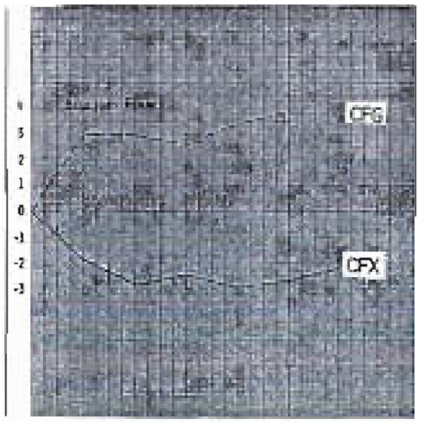


Fig.4

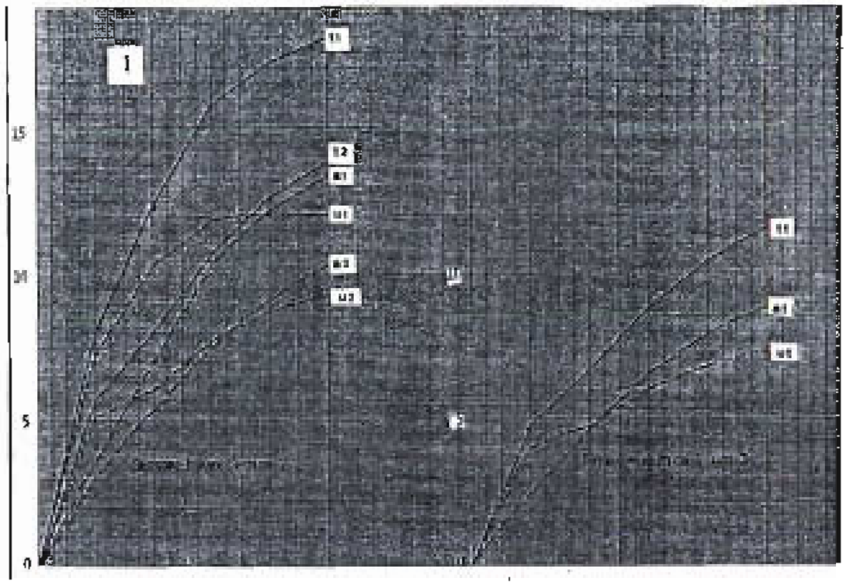


Fig.5

Standard error of forecast. When not otherwise specified, standard errors are given as percentages of the forecast values.

1 = sector 1; 2 = sector 2;
t = total (1 & 2).

u = error due to random error terms;
a = error due to estimated coefficients;
t = total (u & a).

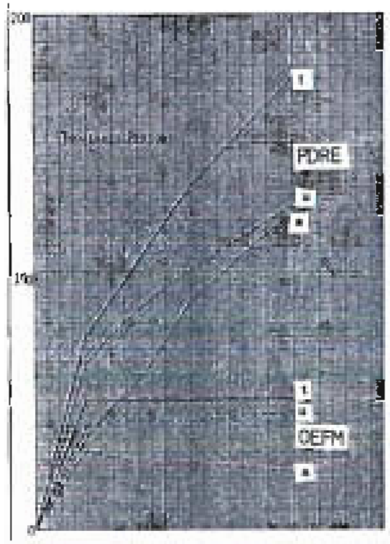


Fig.6

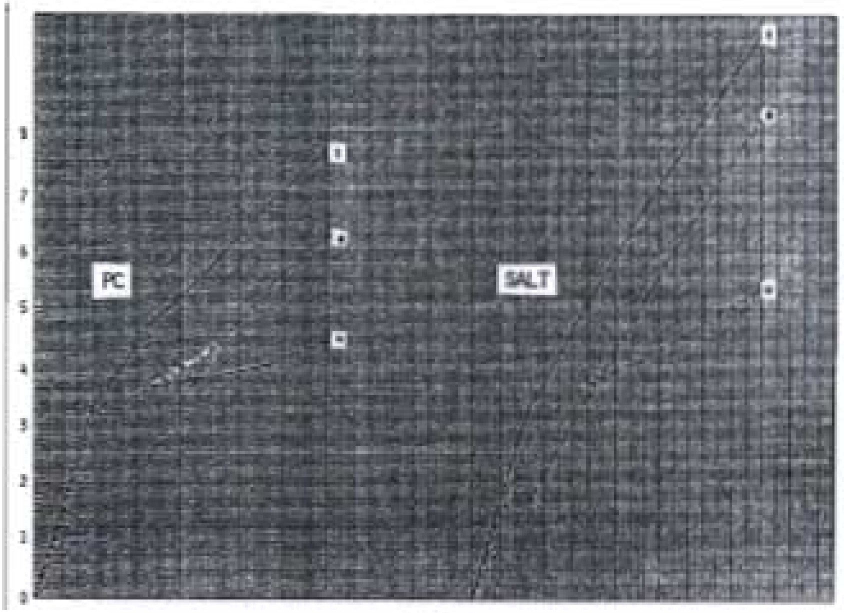


Fig.7

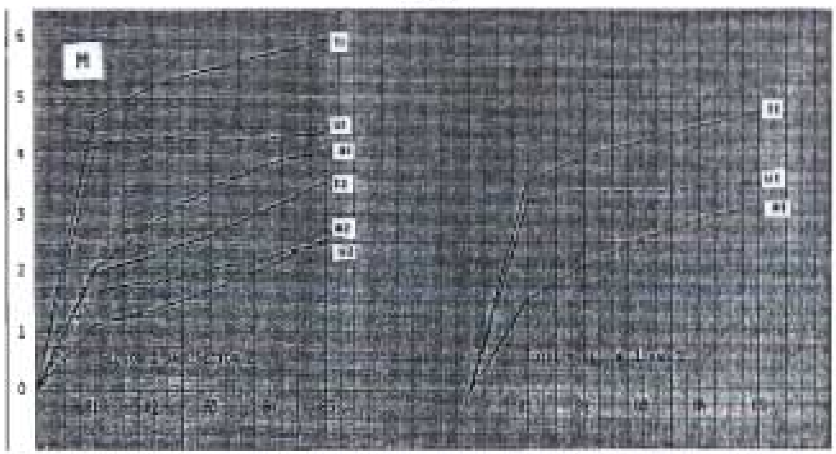


Fig.8

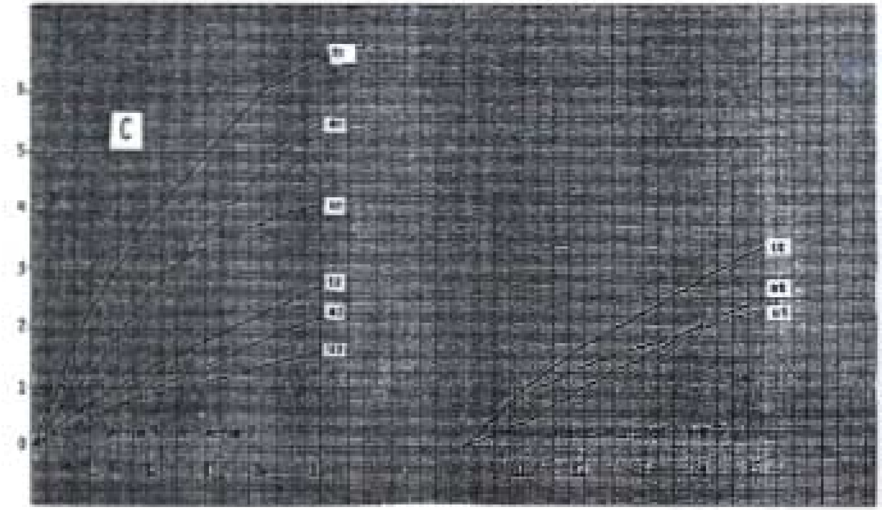


Fig.9

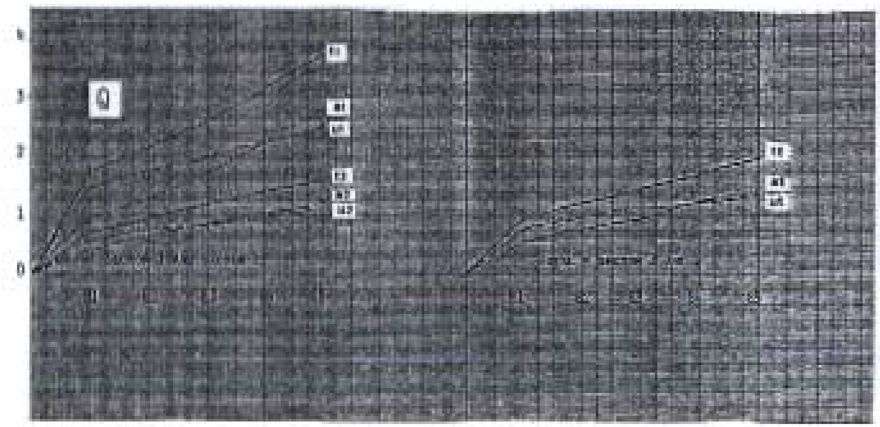


Fig.10

We can also see that, for each type of error, the correlation between the two sectors is in general very high; we do not profit, as could be expected, by the aggregation to increase precision, except for investment, where the mechanisms of the two equations are quite different.

Finally, if we use these results to judge the forecasting quality of the model, this judgment would be somewhat mixed: the precision shown is not too good, expecially if we compare it with the same measure taken over the estimation period, but not too bad either. Now, if we concern ourselves with individual cases, the observations we can make are coherent with the nature of the associated variables: the steadiness of the error increase with time grows with the dynamicity of the equation. A striking example is the comparison (fig.6) between the evolution of errors on unemployment (job demands) and job offers: while the error on unemployment, defined by a first order equation, increases almost linearly, for job offers it reaches a stable level at the first period; the decomposition shows that the error due to the structural disturbance terms increases a little, while error due to coefficients decreases; indeed the main determinant of the level of job offers is the difference between actual level of employment and a desired value, based on constant productivity; as the equation is not dynamic, the error due to the equation itself has no reason to increase, while the effect of a slowly growing divergence of economic equilibrium (of constant sign) decreases as the actual level of employment adapts itself to the disturbance.

The same mechanism explains the sharp inflexion in the evolution of the error on imports (fig.8), as the disequilibrium on production capacity utilization is an important explanatory component for this variable.

6. UNCERTAINTY OF IMPACT AND DYNAMIC MULTIPLIERS

If impact and dynamic multipliers, calculated from the model, are used to evaluate the results of policy actions, it would be useful if the estimate of each multiplier could be accompanied by an estimate of its degree of uncertainty. This may give, to the policy maker, an indication about how much reliance he can place on the results of the policy action. Natural indicators of the degree of uncertainty in a multiplier are obtained by estimating its first and second moments. This can be accomplished as follows.

A multiplier is usually defined as the first order partial derivative, with respect to an exogenous variable (current or lagged), of the conditional expectation of an endogenous variable, all other predetermined variables being kept constant. This gives the theoretical value of a multiplier.

As soon as we move from the theoretical definition to the computational practice, we observe that model builders calculate a multiplier simply as first derivatives (with finite differences, as in Evans and Klein, 1968, p.49) of an endogenous variable, with respect to an exogenous, in a deterministic solution point of the model. The so obtained value may be called the "estimated" multiplier. The computation of first derivatives with finite differences can be very accurate; nevertheless estimated multipliers may differ from the corresponding theoretical value for the usual two reasons.

1) The structural form parameters are unknown; therefore, the estimate of a multiplier is obtained from the model with estimated parameters.
2) Even if the structural parameters were known with certainty, the deterministic solution of an endogenous variable may be different from the conditional expectation of the same variable, in nonlinear models. This consideration, which is immediately extendable to multipliers (see Howrey and Kelejian, 1971, p.308), implies that the model builder, working in a standard way, would get for the multipliers values different from the theoretical values, even if he could know the "true" parameters of the

model.

As far as the first point is concerned, it was shown in Bianchi, Calzolari and Corsi (1981) that each estimated impact multiplier converges in probability to the impact multiplier that would be obtained with deterministic simulation from the model with "true" coefficients, and that the difference between the estimate and its limit is asymptotically normally distributed with mean zero and variance easily obtainable from applying the general theorem of section 3. From the estimated variance of this asymptotic distribution, as usual, we get the approximated standard error which can be interpreted as a measure of the degree of uncertainty associated with the given multiplier.

As far as the second point is concerned, we have evidenced the existence of a systematic error which may be called the "inconsistency" of multipliers: even if the sample size increases to infinite, the probability limit of the estimated multiplier may differ from the theoretic multiplier. As for the case of forecasts, the magnitude of this type of error could be quantified by means of stochastic simulation with antithetic variates (but this investigation will not be accomplished in this paper).

The extension to dynamic multipliers (delay or sustained) is straightforward; details can be found in Bianchi et al. (1981), while a full analytical treatment of the problem for linear models is given in Gill and Brissimis (1978).

6.1. Standard errors of multipliers in Mini-DMS model

We shall now study the evolution of the precision of multipliers associated to some of the most important exogenous variables of the model, measuring their influence on the main endogenous variables. To avoid having to study a too large set of results, we have in fact as much as possible gathered the

exogenous variables of similar nature into groups, restricting the variation of each variable of the group to an identical percentage change; indeed most of these groups contain variables which are bound to move simultaneously, such as tax rates similar in nature but differentiated by the agent to which they are applied.

This study will be made over a four year period (1981-1984) first observing the results on the impact multipliers in the first period, then the effects of a constant change maintained from 1981 to 1984 (sustained multiplier). The synthetic exogenous variables will be the following:

DM1 - foreign demand in industrial product
PX - foreign price index in Francs
RR - real interest rate
DH - weekly work duration
AG - government real demand
NAG - government employment
TCSS - social security rate (wage earners)
TCSE - social security rate (business firms)
TACP - VAT rate on consumption
TAI - VAT rate on investment.

Table 1 gives the value and the standard error of each impact multiplier in the first year, measured for the main macroeconomic endogenous variables:

C - consumption
I - investment
M - imports
X1 - exports
Q - added value
PIBZ - gross national product
N - employment
PDRE - unemployment
OEFM - job offers
PC - consumption prices
SALT - wage rate
CFG - government balance
CFX - trade balance.

Table 2 gives, for the fourth year, the effect of a sustained change from 1981 to 1984 (sustained multipliers). After having checked (which is not the

Table 1

Impact multipliers at 1981 and asymptotic standard errors

	DM1	PX	RR	DH	AG	NAG	TCSS	TCSE	TACP	TAI
C	.116E+5 (.47E+4)	-.184E+5 (.16E+5)	-59.4 (14.)	.177E+5 (.19E+5)	.177E+5 (.34E+4)	13.9 (4.1)	-.214E+6 (.54E+5)	-.701E+5 (.13E+5)	-.453E+5 (.57E+4)	-.199E+5 (.22E+4)
I	.717E+4 (.35E+4)	-.722E+4 (.98E+4)	-335. (65.)	.231E+4 (.76E+4)	.120E+5 (.27E+4)	1.18 (.39)	-.211E+5 (.66E+4)	-.493E+5 (.96E+4)	-.222E+5 (.43E+4)	-.112E+5 (.20E+4)
M	.344E+5 (.69E+4)	.161E+5 (.17E+5)	-174. (36.)	.266E+5 (.13E+5)	.380E+5 (.27E+4)	5.97 (1.7)	-.942E+5 (.24E+5)	-.650E+5 (.13E+5)	-.329E+5 (.53E+4)	-.157E+5 (.24E+4)
X1	.413E+5 (.64E+4)	.826E+5 (.17E+5)	81.9 (17.)	.447E+5 (.13E+5)	-.149E+5 (.26E+4)	-2.60 (.95)	.382E+5 (.12E+5)	-.208E+5 (.11E+5)	-.271E+4 (.41E+4)	-.231E+4 (.21E+4)
Q	.401E+5 (.75E+4)	.794E+5 (.20E+5)	-153. (36.)	.618E+5 (.17E+5)	.585E+5 (.38E+4)	6.26 (1.7)	-.102E+6 (.25E+5)	-.963E+5 (.17E+5)	-.444E+5 (.68E+4)	-.217E+5 (.30E+4)
PIBZ	.445E+5 (.80E+4)	.850E+5 (.21E+5)	-185. (41.)	.659E+5 (.19E+5)	.705E+5 (.39E+4)	36.0 (2.2)	-.133E+6 (.32E+5)	-.108E+6 (.18E+5)	-.501E+5 (.73E+4)	-.248E+5 (.32E+4)
N	411. (94.)	808. (.23E+3)	-1.59 (.46)	-.193E+4 (.24E+3)	616. (.11E+3)	.658E-1 (.20E-1)	-.107E+4 (.30E+3)	-995. (.23E+3)	-460. (99.)	-225. (45.)
PORE	-188. (57.)	-405. (.13E+3)	.544 (.23)	.150E+4 (.31E+3)	-155. (80.)	-.185 (.16)	301. (.14E+3)	391. (.13E+3)	168. (58.)	86.3 (28.)
OEFM	140. (37.)	309. (92.)	-.360 (.11)	92.4 (.17E+3)	84.8 (38.)	.107E-1 (.50E-2)	-178. (82.)	-275. (76.)	-114. (32.)	-58.1 (15.)
PC	.784E-1 (.60E-1)	.887 (.18)	.152E-3 (.17E-3)	-.485 (.12)	-.403E-1 (.43E-1)	-.190E-5 (.60E-5)	.724E-1 (.82E-1)	.778 (.18)	.513 (.68E-1)	.162 (.29E-1)
SALT	3.53 (2.5)	24.1 (6.5)	-.831E-3 (.66E-2)	-2.87 (4.2)	.227E-2 (1.8)	.104E-3 (.28E-1)	-.374 (3.6)	14.4 (5.6)	10.8 (2.4)	2.99 (1.1)
CFG	.119E+5 (.20E+5)	-.924E+5 (.53E+5)	-174. (73.)	-.110E+6 (.37E+5)	-.182E+6 (.14E+5)	-40.4 (7.9)	.115E+7 (.36E+5)	.178E+6 (.44E+5)	.779E+5 (.19E+5)	.916E+5 (.86E+4)
CFX	.302E+5 (.11E+5)	-.321E+5 (.35E+5)	583. (.12E+3)	-.498E+5 (.31E+5)	-.136E+6 (.73E+4)	-21.1 (5.9)	.328E+6 (.81E+5)	.189E+6 (.32E+5)	.101E+6 (.14E+5)	.475E+5 (.61E+4)

Ratios = multipl. / std. err.

	DM1	PX	RR	DH	AG	NAG	TCSS	TCSE	TACP	TAI
C	2.45	-1.16	-4.18	.919	5.16	3.40	-3.96	-5.58	-8.01	-9.09
I	2.07	-.737	-5.14	.303	4.42	3.01	-3.21	-5.12	-5.11	-5.73
M	4.99	.942	-4.90	1.97	14.1	3.56	-3.99	-4.96	-6.23	-6.53
X1	6.46	4.74	4.78	3.43	-5.73	-2.73	3.30	-1.92	-.659	-1.10
Q	5.36	3.99	-4.27	3.70	15.3	3.78	-4.09	-5.70	-6.57	-7.23
PIBZ	5.58	4.00	-4.51	3.55	18.2	16.1	-4.15	-5.91	-6.89	-7.75
N	4.35	3.51	-3.48	-7.94	5.77	3.27	-3.57	-4.30	-4.65	-4.94
PORE	-3.33	-3.11	2.35	4.80	-1.95	-1.17	2.19	2.97	2.89	3.04
OEFM	3.76	3.37	-3.42	.552	2.26	2.14	-2.18	-3.63	-3.59	-3.79
PC	1.31	4.94	.903	-3.96	-.943	-.315	.882	4.44	7.84	5.65
SALT	1.40	3.72	-.126	-.679	.0012	.513	-.105	2.59	4.56	2.76
CFG	.602	-1.75	-2.37	-2.97	-12.8	-5.14	32.0	4.06	4.12	10.6
CFX	2.76	-.908	5.07	-1.61	-18.6	-3.60	4.06	5.92	7.18	7.80

Table 2

Sustained multipliers at 1984 and asymptotic standard errors
(dynamic simulation from 1981)

	DM1	PX	RR	DH	AG	NAG	TCSS	TCSE	TACP	TAI
C	.307E+5 (.17E+5)	.130E+5 (.39E+5)	-.111E+4 (.29E+3)	.129E+6 (.37E+5)	.469E+5 (.13E+5)	23.3 (3.0)	-.576E+6 (.40E+5)	-.131E+6 (.43E+5)	-.754E+5 (.17E+5)	-.403E+5 (.10E+5)
I	-.347E+4 (.72E+4)	-.627E+5 (.19E+5)	-.160E+4 (.34E+3)	.284E+5 (.16E+5)	.152E+5 (.66E+4)	1.20 (1.5)	-.505E+5 (.22E+5)	-.846E+5 (.24E+5)	-.362E+5 (.10E+5)	-.214E+5 (.50E+4)
M	.272E+5 (.83E+4)	-.228E+5 (.12E+5)	-.134E+4 (.25E+3)	.792E+5 (.15E+5)	.515E+5 (.36E+4)	8.70 (1.8)	-.233E+6 (.17E+5)	-.984E+5 (.15E+5)	-.482E+5 (.62E+4)	-.273E+5 (.37E+4)
X1	.337E+5 (.12E+5)	.516E+5 (.25E+5)	-330. (.19E+3)	.726E+5 (.21E+5)	-.159E+5 (.84E+4)	-3.79 (1.5)	.753E+5 (.28E+5)	-.471E+5 (.24E+5)	-.145E+5 (.10E+5)	-.845E+4 (.54E+4)
Q	.286E+5 (.74E+4)	.190E+5 (.22E+5)	-.179E+4 (.38E+3)	.142E+6 (.19E+5)	.721E+5 (.69E+4)	9.80 (2.7)	-.267E+6 (.33E+5)	-.155E+6 (.24E+5)	-.724E+5 (.11E+5)	-.402E+5 (.55E+4)
PIBZ	.357E+5 (.80E+4)	.294E+5 (.22E+5)	-.204E+4 (.41E+3)	.164E+6 (.21E+5)	.886E+5 (.68E+4)	41.9 (3.0)	-.345E+6 (.32E+5)	-.177E+6 (.25E+5)	-.828E+5 (.11E+5)	-.466E+5 (.60E+4)
N	492. (.11E+3)	440. (.32E+3)	-22.9 (5.1)	-.200E+4 (.28E+3)	.117E+4 (98.)	.160 (.38E-1)	-.422E+4 (.49E+3)	-.233E+4 (.35E+3)	-.110E+4 (.16E+3)	-600. (81.)
PORE	-193. (72.)	-227. (.15E+3)	8.50 (3.1)	.209E+4 (.50E+3)	-276. (.14E+3)	-.201 (.16)	.100E+4 (.52E+3)	841. (.27E+3)	370. (.13E+3)	207. (68.)
OEFM	13.1 (14.)	4.45 (45.)	-2.38 (.71)	154. (37.)	33.0 (9.8)	.222E-2 (.38E-2)	-125. (47.)	-162. (39.)	-65.6 (16.)	-38.7 (9.3)
PC	.425 (.18)	2.51 (.50)	.118E-1 (.51E-2)	-.556 (.34)	-.414E-1 (.15)	.175E-4 (.30E-4)	-.253E-2 (.50)	1.32 (.55)	.832 (.25)	.295 (.12)
SALT	14.4 (6.3)	67.7 (14.)	.191 (.14)	.865 (14.)	2.35 (5.3)	.117E-2 (.11E-2)	-11.9 (18.)	17.9 (18.)	14.3 (8.1)	3.98 (4.1)
CFG	.242E+5 (.26E+5)	-.145E+6 (.71E+5)	-.497E+4 (.15E+4)	-.184E+6 (.63E+5)	-.191E+6 (.22E+5)	-36.1 (4.9)	.103E+7 (.60E+5)	.178E+6 (.85E+5)	.720E+5 (.38E+5)	.101E+6 (.19E+5)
CFX	.726E+5 (.22E+5)	.887E+5 (.52E+5)	.378E+4 (.64E+3)	-.164E+6 (.46E+5)	-.198E+6 (.16E+5)	-36.5 (5.2)	.897E+6 (.53E+5)	.250E+6 (.53E+5)	.133E+6 (.22E+5)	.706E+5 (.14E+5)

Ratios = multipl. / std. err.

	DM1	PX	RR	DH	AG	NAG	TCSS	TCSE	TACP	TAI
C	1.82	.334	-3.78	3.47	3.54	7.85	-14.5	-3.05	-4.31	-3.91
I	-.485	-3.27	-4.66	1.82	2.29	.774	-2.26	-3.52	-3.47	-4.25
M	3.29	-1.90	-5.35	5.28	14.4	4.84	-13.8	-6.52	-7.73	-7.30
X1	2.73	2.09	-1.74	3.38	-1.88	-2.53	2.66	-1.94	-1.41	-1.56
Q	3.85	.869	-4.71	7.56	10.5	3.66	-8.17	-6.56	-6.81	-7.26
PIBZ	4.47	1.32	-4.96	7.85	13.0	14.0	-10.7	-7.18	-7.62	-7.79
N	4.35	1.38	-4.46	-7.06	12.0	4.26	-8.61	-6.65	-7.06	-7.42
PORE	-2.68	-1.47	2.77	4.14	-1.91	-1.27	1.92	3.07	2.88	3.03
OEFM	.924	.098	-3.34	4.13	3.35	.578	-2.66	-4.18	-4.01	-4.17
PC	2.36	5.05	2.32	-1.63	-.279	.590	-.005	2.41	3.37	2.42
SALT	2.28	4.77	1.38	.063	.440	1.09	-.666	.985	1.78	.978
CFG	.919	-2.04	-3.42	-2.91	-8.78	-7.40	17.2	2.08	1.89	5.71
CFX	3.31	1.72	5.95	-3.58	-12.5	-6.96	17.0	4.68	6.08	5.40

purpose of this study) that the signs of the multipliers produced are coherent with the teachings of economic theory, and that their size seems at first sight to be reasonable, we will concern ourselves with their relative precision, measured by the ratio between the value found and its estimated standard error.

To help the interpretation we can first establish a classification between global exogenous variables, depending whether they can be considered as resulting directly from a government decision (instruments), bearing either on demand (AG directly, NAG and TCSS through household consumption) or on supply characteristics (TCSE, TAI and also TACP[4]); or resulting from foreign (non-modelized) evolution (DM1 and PX); or being imperfectly controlled by government (DH and RR).

Using this classification, we can deduce from table 1 the following observations, which for the most part are coherent with preconceived ideas.

1) The largest part of the results can be considered as significant, at least regarding the sign of the multipliers; the only major exception concerns the effect of demand instruments on prices. However, in general, the actual values bear a rather high uncertainty, with wide confidence intervals.
2) As could be expected, the effect of direct demand variables (AG) is most precise on the equilibrium in real terms and on employment, while the supply (or price) variables affect more exactly the price equilibrium. The variables influencing demand only indirectly (NAG and TCSS) have the most imprecise multipliers, both on demand and prices, this characteristic being mostly associated with the first year effect, where to the uncertainty on additional revenue households add the one of their behaviour (savings or consumption) regarding its use.
3) A special case is represented by the effect of prices on exports, where the low precision of a lowering of taxes can be explained by the antagonistic effects of the increase in competitivity and the rise of tensions on production capacity utilizations.
4) Concerning the other variables, RR can be characterized as a slightly imprecise supply-side instrument. While the effect of DH is clear on employment and prices (the model assumes that a lowering of DH is totally compensated in terms of montly wage rate) and not on activity itself (expecially regarding the effect on wage-profits repartition). The precision characteristics of foreign trade variables (DM1 and PX) can be associated with those of the similar (demand/supply) domestic variables, although the effect of PX on real equilibrium is not significant. Another interesting feature is the fact that, while PX (which can be characterized as an exchange rate instrument) affects trade balance with the right sign, the result is not significant.

(4) According to the model, a lowering of VAT rate does not increase the purchasing power of the indexed wage rate, but decreases its nominal value, thus reducing production costs.

Table 2 gives us some information about how the previous considerations are affected by temporal evolution of multipliers.

1) The precision of supply-side instruments does not change much concerning real terms equilibrium: only consumption and investment are affected but not global equilibrium. But, to the direct effect on prices, is added the contrary effect transmitted through disequilibrium on demand, reducing precision.
2) For demand instruments also, uncertainty behaves much similarly, although it can be observed that the error on TCSS decreases in most cases (we have already observed that the impact multiplier is more subject to error than the sustained multiplier); in particular the effect on prices shows no sign of becoming significant.
3) The effect of DH becomes globally significant and this is probably due to the increase in the size of the multipliers themselves.

4) As to external trade variables, their significance decreases with time as the progressive addition of the secondary effects adds to the sources of error and limits the size of the initial effect; for instance, an increase in foreign price induces domestic inflation, which limits the gain in competitivity and the gain on exports.

To conclude, let us suggest a direction of study which could prove useful as a decision element for operational forecaster. In many cases the important criterion is not the raw efficiency of the instrument, but the effect of a variation of the instrument associated with a given ex-ante cost in terms of government deficit; in that case the criterion becomes $(\partial y/\partial x)/(\partial CFG/\partial x)$ where x is the instrument and y is the endogenous variable which we want to modify. The uncertainty of this criterion can be considered supposing $\partial CFG/\partial x$ either known with certainty, or affected by an error similar to the one produced above; in the latter case the problem arises of taking into account the correlation between errors on numerator and denominator. Let us give a very imperfect example. The following tables give the impact multipliers at 1981, and their standard errors, of TCSE and TCSS on PIBZ and CFG.

	CFG	PIBZ
TCSE	178068. (85467.)	-176965. (24660.)
TCSS	1028610. (59876.)	-344908. (32173.)

The values for $(\partial PIBZ/\partial x)/(\partial CFG/\partial x)$ and their standard errors, supposing (a) the denominator to be certain, or (b) independent from the numerator, are

	TCSE	TCSS
	.994	.335
(a)	(.138)	(.031)
(b)	(.497)	(.037)

For a decision maker (supposed risk-averting) the appeal of the higher average efficiency of TCSE should be somewhat moderated by the much greater uncertainty on this instrument.

7. SIGNIFICANCE OF THE CHARACTERISTIC ROOTS OF LINEARIZED MINI-DMS

The dynamic behaviour of an econometric model depends on the roots of the associated characteristic equation. As the structural estimated coefficients of the model are subject to sampling errors, likewise to the multipliers discussed in section 6, also the characteristic roots are affected by an error.

The estimation of the asymptotic standard errors of the characteristic roots in linear econometric models has been dealt with by Theil and Boot (1962), Neudecker and van de Panne (1966), Oberhofer and Kmenta (1973). By means of these asymptotic standard errors, it is possible to test the model for stability (however, the power of the test is open to question, because the null hypothesis must be always stability, rather than instability, as well pointed out in Oberhofer and Kmenta, 1973,fn.5 and Gustafson, 1978); moreover, it must be recalled that what is analyzed in this case is the behaviour of the linearized model, which can differ from that of the original nonlinear model, as discussed in Malgrange (1981).

In case of nonlinear models, an explicit linearization must be preliminarily performed. In this paper an analytic simulation procedure, similar to the one described in section 6 for the multipliers, has been applied to a linearization of Mini-DMS model performed in the last year of the sample period (1980).

The dynamic behaviour (and stability) of a local linearization of the model at time t is determined by the characteristic roots of the (m×m) matrix Π^{o} ($\hat{\Pi}^{o}$

Table 3

Characteristic roots of linearized Mini-DMS at 1980

	Modulus of eigenvalue	Std. err. of modulus	Period	Std. err. of period
λ(1) =	1.17352	.0380754	.0	.0
λ(2) =	1.14243	.0131210	.0	.0
λ(3) =	1.12300	.0157439	.0	.0
λ(4) =	1.12024	.0259693	.0	.0
λ(5) =	1.10580	.0120044	.0	.0
λ(6) =	1.05679	.0069107	.0	.0
λ(7) =	1.00825	.0306785	.0	.0
λ(8) =	1.00000	.0	.0	.0
λ(9) =	1.00000	.0	.0	.0
λ(10)=	1.00000	.0	.0	.0
λ(11)=	.984923	.0378938	68.7454	31.0409
λ(12)=	.984923	.0378938	-68.7454	31.0409
λ(13)=	.912069	.0358358	25.7913	7.28819
λ(14)=	.912069	.0358358	-25.7913	7.28819
λ(15)=	.905827	.0910048	.0	.0
λ(16)=	.872718	.1537460	.0	.0
λ(17)=	.857726	.0638728	.0	.0
λ(18)=	.844753	.1062740	.0	.0
λ(19)=	.836842	.0284487	.0	.0
λ(20)=	.833363	.0288249	.0	.0
λ(21)=	.803693	.0069597	.0	.0
λ(22)=	.800652	.0056815	.0	.0
λ(23)=	.723649	.0066726	.0	.0
λ(24)=	.717811	.0073336	.0	.0
λ(25)=	.695449	.0071044	.0	.0
λ(26)=	.596146	.0840680	.0	.0
λ(27)=	.556172	.0230588	.0	.0
λ(28)=	.526030	.0050466	.0	.0
λ(29)=	.495817	.1396630	.0	.0
λ(30)=	.447211	.0562543	.0	.0
λ(31)=	.431198	.0788047	.0	.0
λ(32)=	.292527	.0214940	15.1372	2.18560
λ(33)=	.292527	.0214940	-15.1372	2.18560
λ(34)=	.287171	.0404480	5.46722	.422743
λ(35)=	.287171	.0404480	-5.46722	.422743
λ(36)=	.235244	.0766252	.0	.0
λ(37)=	.126216	.0716399	-18.3010	3.15655
λ(38)=	.126216	.0716399	18.3010	3.15655
λ(39)=	.102149	.0205302	11.2109	2.53963
λ(40)=	.102149	.0205302	-11.2109	2.53963

is its estimate) of partial derivatives, in the neighborhood of the solution point at time t, of y_t with respect to y_{t-1}. Since the model contains endogenous variables lagged more than one period, it has been reconducted into the above scheme simply by "augmenting" the vector of endogenous variables.

Let λ be a real characteristic root of Π^*, and $\hat{\lambda}$ the corresponding characteristic root of $\hat{\Pi}^*$. From the assumption (3.3), since $T^{\frac{1}{2}}(\hat{a}-a)$ is asymptotically normally distributed as $N(0,\Psi)$, then $T^{\frac{1}{2}}(\hat{\lambda}-\lambda)$ is asymptotically normally distributed as $N(0,j'\Psi j)$ where j is the vector of partial derivatives of λ with respect to the elements of a. If the computation is performed for $\hat{\lambda}$, through the matrix $\hat{\Pi}^*$, we get a consistent estimate $\hat{j}$ of j and the square root of $(\hat{j}'\hat{\Psi}\hat{j})/T$ is the estimated asymptotic standard error of the given root.

If λ is complex, the above results hold both for the modulus and for the argument (or for the corresponding period); in this case we have to compute the two vectors of partial derivatives of the modulus and of the argument, respectively.

To interpret the results on eigenvalues (table 3 displays the 40 eigenvalues largest in modulus) we should have some some information about their association with individual lagged influences. Indeed such a study has been already done on the model, and although the year concerned was not the same, the results are quite comparable. The number and size of eigenvalues larger than 1 is the same, and the three unitary values associated with first difference identities appear here also. The additional information we get here is the relatively high precision of the first values, which should suggest that the process they describe is significantly divergent, while the large uncertainty on some of the subsequent figures (eigenvalues 15, 16, 18 and 29) seems to correspond to elements which have proved hard to interpret in the previous study (Brillet, 1981b).

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