A Quality of Play Model of a Professional Sports League

Cyrenne, Philippe

Department of Economics, The University of Winnipeg

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Philippe Cyrenne*

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Abstract

Assuming that consumers value both the absolute and relative quality of play, I compare the choice of ticket prices, team qualities, and number of games played in a noncooperative outcome versus that chosen by a social planner. I find that the nature of consumer preferences regarding the quality of play determines whether the demand for talented players are strategic complements or substitutes. A strong preference by fans for a superior team makes players strategic substitutes while a concern for a high quality of play and competitive balance make players strategic complements. Moreover, when fans only value the relative quality of play, there is an overemployment of talented players in the noncooperative outcome versus the socially optimal outcome; when they only value the absolute quality of play then there is an underemployment of talented players in the noncooperative outcome.(JEL L1,L2,L4,L83)

1 Introduction

It is perhaps an understatement to suggest that economic issues have gained an increasing prominence in the operation of professional sport leagues. All professional sports leagues have had protracted labour disputes which have been fairly long lived and acrimonious.[1] Many observers

*Associate Professor, Department of Economics, the University of Winnipeg, Winnipeg, Manitoba, Canada. Helpful comments were received from seminar participants in the Policy Analysis Division of the Faculty of Commerce, the University of British Columbia; the Canadian Economic Association meetings in St.John's, Nfld.; the Western Economic Association meetings, in Seattle Washington, two anonymous referees, and the Managing Editor, William Nielson. Special thanks to Tom Ross for a number of helpful discussions. The usual caveat applies (sportei.tex).

[1]For an entertaining and informative discussion of the economic history of sports leagues see Quirk and Fort (1992).

of professional sports leagues question whether the actions taken by league owners are consistent with social welfare, or whether league rules are designed solely to promote the interests of owners, at the expense of either consumers, players, or both.

The first formal model of a professional sports league, set in a dynamic framework was Quirk and El Hodiri (1974).[2] In a static version of what is termed the *win model*, Fort and Quirk (1995) examine a noncooperative game in which players are chosen by teams to maximize profits, where profits are increasing in a team's relative win percentage as well as a number of shift parameters including drawing potential and local and national television opportunities. In evaluating a number of instruments used by leagues - the reserve option clause, salary caps, rookie draft, revenue sharing, and franchise location decisions - Fort and Quirk find that with the exception of salary caps, "the methods in use provide no profit incentives for improving competitive balance and, in certain cases, actually harm it".[3]

While the win model has been useful in thinking about a number of issues regarding the operation of sports leagues, it seems clear that it or variations of the model are not designed to answer the question posed by Rottenberg (1956) of whether league rules are consistent with social welfare.[4] For that to be achieved, it is necessary to start with some idea of consumer preferences regarding the output of the professional sports industry. Existing models seem to equate social welfare with competitive balance without explaining the underlying primitives regarding consumer preferences. It is argued here that a useful approach is to consider a quality of play model, a model that has

[2]The model included the depreciation of player skills, the drafting of players, bonuses paid to players, as well as television and gate revenue.

[3]Fort and Quirk (1995:1266). For an alternative approach based on the win model, see Vrooman (1995).

[4]In examining the baseball player's labour market, Rottenberg (1956) maintained that league restraints such as the reserve clause, waiver rules, and other labour mobility restrictions, were not designed to and did not achieve competitive balance among teams. In his view, their sole purpose was to lower players' salaries.

not received as much attention as the win and championship models.[5] According to Scully (1995), the overall quality of play is important to the operation and success of sports leagues. While Scully suggests a number of ways in which quality of play issues might affect the operation of a sports league, the full implications of this assumption are not derived. This is the model developed in this paper.[6]

In the model developed here, I emphasize the quality of the contest or game as perceived by fans as a useful approach to understanding the operation of sports leagues. Specifically, I assume that teams are involved in producing contests, which results in both an absolute and relative quality of play. I view the unique aspect of sports leagues to be the fact that teams (firms) compete indirectly through their choice of team qualities (input levels) which effects both the quality of contest (output) provided and the salary costs of rivals. The subsequent quality of play influences the demand for tickets in much the same way that quality considerations influence the demand for any other product. While the approach taken here yields a number of results obtained elsewhere, a principal advantage of the approach is the use of a general model which nests a number of specific models. For example, a number of characteristics of contests felt important by observers of sports leagues can be incorporated as determining the quality of play as perceived by fans. For example, fans may consider a contest of high quality if its team is more likely to win. Or fans may view a game that has a large number of talented players as being of high quality. Alternatively, they may consider a more closely contested game as being of high quality implying that fans have a concern for competitive balance.

[5]For an exposition of the championship model see Whitney (1988, 1993).

[6]For a model of a sports league based on imperfect information see Atkinson, Stanley and Tschirhart (1988). The model developed here abstracts from information problems.

Using this approach, I show that the team qualities chosen in the noncooperative versus social welfare maximizing outcome, depend on a number of factors, including (*i*) the degree of monopsony power possessed by teams, (*ii*) consumer preferences regarding the quality of play, (*iii*) the distribution of fans by their willingness to pay and (*iv*) the degree of local monopoly power held by teams. One strong result is that the noncooperative choice of team qualities is inconsistent with social welfare maximization, implying that leagues must internalize the effect of talent choices by teams on the welfare of fans of rival teams. In addition, I show that players are strategic substitutes if fans only value winning, while players are strategic complements if competitive balance and the absolute quality of play are important to fans. I also show how comparative statics regarding changes in the market size of rivals can allow one to distinguish between these two possibilities. Perhaps more importantly, I show using a specific formulation of the model, that when fans only value the relative quality of play, there is an overemployment of talented players in the noncooperative outcome versus the socially optimal outcome; when they only value the absolute quality of play then there is an underemployment of talented players in the noncooperative outcome.

A recent paper related to the approach taken here is Hausman and Leonard (1997) who develop an economic model based on the positive externality generated by superstars.[7] One significant difference is that the goal of Hausman and Leonard (1997) is to develop a positive model as a basis for their empirical work, while the objective of the present approach is to develop a normative model for comparison purposes. [8] In addition, Hausman and Leonard's model emphasizes the

[7]The analysis contained here was developed independently of the work of Hausman and Leonard (1997). I thank an anonymous referee for alerting me to this paper.

[8]Hausman and Leonard (1997) show that the presence of Michael Jordan on the Chicago Bulls of the National Basketball Assocation raises television ratings increasing revenues for both the Bulls and their opponents.

absolute quality of play, while the model developed here isolates the separate effects of the relative and absolute quality of play.

The paper is organized as follows. Section 2 outlines the basic model which has two stages and is solved recursively. Section 3 presents a specific formulation of the model allowing for closed form solutions. Section 4 concludes the paper.

2 The Model

As in Fort and Quirk (1995), I consider a two team league. The preferences of a fan i of team 1 are defined as $U_{1i} = U_{1i}(g; V^{12})$ where g is the number of home games played and V^{12} is the quality of the games between team 1 and team 2 as perceived by fans of team 1, which is abbreviated as V^1, with $\partial U_{1i}/\partial g > 0$ and $\partial^2 U_{1i}/\partial g^2 < 0$ $\partial^2 U_{1i}/\partial g \partial V^1 > 0$.[9] The derivative $\partial U_{1i}/\partial g = f_{1i}(V^1, g)$ is viewed as the willingness to pay for a game between team 1 and 2, with the conditions $\partial f_{1i}/\partial V^1 > 0$, $\partial^2 f_{1i}/\partial^2 V^1 \leq 0$ and $\partial f_{1i}/\partial g < 0$, $\partial^2 f_{1i}/\partial g^2 \geq 0$.

Fan i's total willingness to pay for a given number of home games g_o between the two teams (in other words a season ticket) is defined as $p_{1i} = \int_0^{g_o} f_{1i}(V^1, x)dx$. The market demand for season tickets facing team 1 is the summation of these individual demands defined as $p_1(n_1, V^1, g_o)$, where n_1 is the number of season ticket sales by team 1. Alternative specifications of p_1 imply a different distribution of fans regarding willingness to pay.[10] The term $p_1(y, V^1, g_o)$ is the price that makes the *yth* consumer indifferent between buying a seasons ticket for g_o home games between teams

[9] A specific utility function that would give rise to the above inverse demand function is $U^1 = \alpha V^1 g - \beta g^2/2$ with p_{1i} equal to $\partial U_{1i}/\partial g = \alpha V^1 - \beta g$.

[10] Tirole (1988:100) argues this specification can be thought of as representing the demand curve of a large number of consumers with unit demands, ranked in descending order of willingness to pay.

1 and 2 where the quality of play is given by V^1 and not buying at all.[11] The market demand function for team 2 is correspondingly, $p_2(n_2, V^2, g_o)$ where V^2 is the quality of the contest between team 1 and team 2 as perceived by a fan of team 2.

The quality of the contest is assumed to be a function of the quality of play of both teams, q_1 and q_2, that is, $V^1 = V^1(q_1, q_2)$. The quality of a team's play is assumed to be linearly related to the number of talented players it employs, that is $q_1 = \theta h_1$ and $q_2 = \theta h_2$.[12] The parameter θ, captures the inherent uncertainty of athletic contests, which I assume comes from a uniform distribution with mean equal to 1 and support $[1-\epsilon, 1+\epsilon]$. The expected quality of play for each team is then $Eq_1 = h_1$ and $Eq_2 = h_2$. Thus we can define the expected quality of the contest between team 1 and team 2, as perceived by fans of team 1 as $V^1 = V^1(h_1, h_2)$, with fans of team 2 perceiving a contest between these two teams as $V^2 = V^2(h_1, h_2)$. [13] In section 3 I discuss forms V^1 and V^2 might take.

While talented players are assumed to be equivalent in their ability to affect team performance, reservation wages for players may vary if they have different outside opportunities. This implies the supply of talented players is less than perfectly elastic, which is captured by a wage equation of the form $w = w(h_1 + h_2)$, where it is assumed that each player supplies one unit of playing time for the series of games defined as a season.[14] The total salary bills for the two teams are $S^1 = wh_1$

[11] Dixit (1979) and Gal-Or (1982) examine the outcomes which result when each firm chooses price and the quality of its own output. In the setting examined here, each firm chooses its output price, as well as the level of an *input* which effects the quality of the product provided by the other firm.

[12] Depending on the sport, one could envision, decreasing, constant or increasing returns to the quality of play from the number of talented players on a team. As in Quirk and Fort (1995) roster limits are ignored in this paper.

[13] It might be useful to consider these functions as the *valued added to the contest* produced by the talented players. For the rest of the paper, I drop the adjective expected and refer to the quality of the contest.

[14] This implies that players' salaries are independent of the number of games played, which is assumed to hold locally given the number of games that are scheduled, g_o.

and $S^2 = wh_2$. With no loss in generality, constant marginal costs other than salary costs, are set to zero.[15]

The timing of the model is as follows: in the first stage, teams decide on the number of games to be played cooperatively, given the equilibrium in the second stage. In the second stage, two alternative outcomes are examined. The first is the noncooperative outcome, in which teams choose talent levels and ticket sales noncooperatively to maximize their individual revenues.[16] The second is the social welfare maximizing outcome, in which talent levels and ticket sales are chosen to maximize social welfare. [17]

2.1 The Determination of Ticket Prices and Team Qualities

(i) The Noncooperative Outcome

In this outcome, each team chooses its ticket price and team quality to maximize profits given as $\pi^j = p_j n_j - wh_j$, $j = 1, 2$.[18] Profits are maximized subject to a constraint on the number of tickets that can be sold, with $\overline{N_1}$ and $\overline{N_2}$ being the capacities of the two teams. The Lagrangean for the respective teams are $\mathcal{L} = p_j n_j - wh_j + \lambda_j(\overline{N_j} - n_j)$ for $j = 1, 2$ where $w = w(h_1 + h_2)$. The Kuhn-Tucker conditions for team j are

$$\partial\mathcal{L}/\partial n_j = p_j + n_j \frac{\partial p_j}{\partial n_j} - \lambda_j \leq 0 \quad if < 0,\ n_j = 0 \qquad for\ j = 1, 2 \tag{1}$$

[15] While not incorporated in this model, the level of fixed costs for teams would undoubtedly play a role in determining the optimal number of teams in a league.

[16] Alternatively the number of games to be played could be determined by bargaining, with or without side payments, or by voting.

[17] An earlier version incorporating a three stage game revealed a much higher analytic burden in addition to obscuring the central points raised in this paper.

[18] For a defence of the profit maximization assumption regarding team owners, see Ferguson et al. (1991). It is possible to have teams choose ticket prices rather than ticket sales, but the expressions are simpler if ticket sales is the choice variable.

$$\partial\mathcal{L}/\partial h_j = \frac{\partial p_j}{\partial V^j}\frac{\partial V^j}{\partial h_j}n_j - (w + h_j\frac{\partial w}{\partial h_j}) \leq 0 \quad if < 0,\ h_j = 0 \quad for\ j = 1,2 \tag{2}$$

$$\partial\mathcal{L}/\partial\lambda_j = (\overline{N_j} - n_j) \geq 0 \quad if > 0,\ \lambda_j = 0 \quad for\ j = 1,2 \tag{3}$$

Condition (1) shows that each team sets the monopoly ticket price in its own market, the level determined by whether or not the capacity constraint is binding. [19] Condition (2) indicates that team j hires talented players until the marginal revenue product of increased team quality $\frac{\partial p_j}{\partial V^j}\frac{\partial V^j}{\partial h_j}n_j$ equals the marginal factor cost, $(w + h_j\frac{\partial w}{\partial h_j})$.

(ii) The Social Welfare Maximization Outcome

To maximize social welfare, ticket prices and team qualities are chosen to maximize the sum of consumer and producer surplus, where the latter includes the rent to players.[20] For team 1, consumer surplus plus profits can be written as $CS_1 + \pi_1 = [\int_0^{n_1} p_1(V^1, g_o, y)dy - p_1 n_1] + [p_1 n_1 - w h_1]$ with the corresponding expression for team 2. The rent earned by players is defined as $\psi = wH - \int_0^H w(z)dz$ where $H = h_1 + h_2$. Combining and cancelling terms yields the Lagrangean $\mathcal{L} = \int_0^{n_1} p_1(V^1, g_o, y)dy + \int_0^{n_2} p_2(V^2, g_o, y)dy - \int_0^H w(z)dz + \lambda_1(\overline{N_1} - n_1) + \lambda_2(\overline{N_2} - n_2)$. The Kuhn-Tucker conditions for team 1 are

$$\partial\mathcal{L}/\partial n_j = p_j(V^j, g_o, n_j) - \lambda_j \leq 0 \quad if < 0,\ n_j = 0 \quad for\ j = 1,2 \tag{4}$$

$$\partial\mathcal{L}/\partial h_1 = \int_0^{n_1} \frac{\partial p_1}{\partial V^1}\frac{\partial V^1}{\partial h_1}dy + \int_0^{n_2} \frac{\partial p_2}{\partial V^2}\frac{\partial V^2}{\partial h_1}dy - w \leq 0 \quad if < 0,\ h_1 = 0 \tag{5}$$

$$\partial\mathcal{L}/\partial h_2 = \int_0^{n_1} \frac{\partial p_1}{\partial V^1}\frac{\partial V^1}{\partial h_2}dy + \int_0^{n_2} \frac{\partial p_2}{\partial V^2}\frac{\partial V^2}{\partial h_2}dy - w \leq 0 \quad if < 0,\ h_2 = 0 \tag{6}$$

$$\partial\mathcal{L}/\partial\lambda_j = (\overline{N_j} - n_j) \geq 0 \quad if > 0,\ \lambda_j = 0 \quad for\ j = 1,2 \tag{7}$$

[19] It is possible to include television revenues or price discrimination by team owners at this stage.
[20] This is equivalent to the objective of maximizing the surplus earned by fans, team owners and players.

There are four principal differences between the noncooperative and social welfare maximizing outcome. First, assuming that the capacity constraints are nonbinding, (4) shows that social welfare maximization requires that tickets be priced at marginal cost rather than the local monopoly prices set by noncooperative teams implied (1). Given marginal cost is zero and no capacity constraints, this means that each team should allow all potential fans to attend the game to maximize social welfare. Second, social welfare maximization requires that teams act like perfect competitors rather than monopsonists in their demand for players. This can easily be seen if we assume a specific inverse labour supply curve, $w = R_o + r_o(H)$ then $\int_0^H w(z)dz = R_oH + r_oH^2/2$ with the derivative $\int_0^H \frac{\partial w(z)}{\partial h_1}dz$ equal to the wage equation $w = R_o + r_o(h_1 + h_2)$. This implies that the social welfare maximizing outcome maximizes the total number of high quality players employed by the league.[21] Third, by ignoring the effect of its team quality on the quality of contest as perceived fans of its rival, a team's noncooperative choice of talented players is inconsistent with the maximization of social welfare. [22] Fourth, similar to other treatments of quality issues, social welfare maximization requires that teams consider the effect of an increase in team quality on all fans, whereas the noncooperative firm considers the effect on its marginal fans. A unique aspect to the problem considered here is that in choosing its team quality, each team must internalize the effects on the average fan of *both* teams. [23]

For comparison, a monopoly league would set ticket prices in each city to maximize joint

[21] This outcome generates a greater total economic rent than the noncooperative, monopsonistic outcome. It is clear that players associations may seek a number of alternative objectives in their collective agreements with leagues, such as the maximization of total player payments, or the maximization of the economic rent to high quality players.

[22] The effect of each team's choice on the *quality of the contest* provided by other teams can be considered a production externality.

[23] This well known result is due to Spence (1975, 1976). Tirole points out that for a given output, the "social planner looks at the effect on an increase in quality on all consumers; the monopolist considers the effect of an increase in quality on the marginal consumer" (Tirole (1988: 101)).

profits, and talent levels would be chosen to internalize the pecuniary externality on rival teams. It is easily seen that to replicate the social welfare outcome, the monopolist would have to perfectly price discriminate among fans in addition to act as a price taker in the player market.[24]

Related to the issue of a league monopoly is gate sharing, in which each team chooses its ticket sales and team quality to maximize a weighted average of its gate revenues (s) and the gate revenues of its rival (1-s). [25] One reason that gate sharing does not allow the full monopoly outcome to be achieved is that only a fraction of the effect of a team's talent choices on the revenues of its rival is internalized. A second reason is that gate sharing is not *profit sharing* in that the effect of each team's choice of players on the salary costs of other teams is ignored.

2.2 Determining the Number of Games

I consider the optimal number of games from two perspectives: (1) the fully private outcome, which includes antitrust immunity from collusion on the number of games being played and (2) the social planner outcome in which both team qualities and the number of games are chosen to maximize social welfare.

(i) Collusive Number of Games

In this case, the number of games are chosen to maximize joint profits, which are $\pi^m = p_1 n_1 + p_2 n_2 - w(h_1 + h_2)(h_1 + h_2)$. Using the envelope theorem and equations (1),(2) for $j = 1, 2$, the first

[24] An important issue that arises in the case of a monopoly league is that control over team qualities of the sort required may lead fans to suspect that game outcomes are also set in order to maximize league profits. This would require imperfect information on the part of fans, which is absent from the model developed here.

[25] In the National Basketball Association (NBA) and the National Hockey League (NHL), the home team receives all the revenues from the gate. In contrast, in the National Football League (NFL) the home team's share of the gate is 60%, while in Major League Baseball (MLB) the American League teams receive 80% of the gate while the share of their National League counterparts is 90%.

order condition (with capacity contraints nonbinding) is

$$\partial\mathcal{L}/\partial g_o = [\frac{\partial p_1}{\partial V_1}\frac{\partial V^1}{\partial h_2}n_1 - h_1\frac{\partial w}{\partial h_2}]\frac{\partial h_2}{\partial g_o} + [\frac{\partial p_2}{\partial V_2}\frac{\partial V^2}{\partial h_1}n_2 - h_2\frac{\partial w}{\partial h_1}]\frac{\partial h_1}{\partial g_o} + \frac{\partial p_1}{\partial g_o}n_1 + \frac{\partial p_2}{\partial g_o}n_2 = 0 \qquad (8)$$

There are essentially four terms involved in the collusive choice of the number of games played. The first is the effect on team 1's marginal profitability of an increase in team 2's demand for players brought about by an increase in the number of games. The second is the corresponding effect on team 2's marginal profitability while the third and fourth terms are the direct effect of the increased number of games on each teams' profits.

(ii) Social Welfare Maximizing Number of Games

If the league's objective was social welfare maximization, the socially optimal number of games, given the socially optimal choice of team qualities, is determined by the condition

$$\partial\mathcal{L}/\partial g_o = \int_0^{n_1} \frac{\partial p_1}{\partial g_o}dy + \int_0^{n_2} \frac{\partial p_2}{\partial g_o}dy = 0 \qquad (9)$$

In comparing the last term two terms of (8) and (9), it is easily seen that the social welfare choice of games must take into account the valuation of the average fan of both teams whereas for the noncooperative case, it is the effect on the valuation of the marginal fan of both teams that is important. For the social welfare case, only the direct effect of the number of games on valuation of the fans for both teams must be taken into account since all other external effects have already been internalized by the league.

3 A Specific Formulation of the Model

In order to compare the noncooperative and social welfare outcomes, I use the following specific formulation of the model which yields closed form solutions. I define the quality of contests as

$V^1 = \alpha(h_1+h_2)+d(h_1-h_2)$ and $V^2 = \alpha(h_1+h_2)+d(h_2-h_1)$ where $d = (\omega-\gamma)$. [26] The first term for each describes the absolute quality of play while the second describes the relative quality of play as perceived by the fans of the respective teams. To capture the idea of competitive balance I assume that a lesser concern for wins, or a greater concern for competitive balance implies a smaller $d = (\omega - \gamma)$. The demand functions are $p_1 = (m_1 + V^1 + g_o) - n_1$ and $p_2 = (m_2 + V^2 + g_o) - n_2$, where m_1 and m_2 might be considered market size parameters.[27] Thus increases in the number of games g_o, or their quality V^1 will shift out the market demand for tickets. For the wage equation, I specify the functional form $w = R_0 + r_o(h_1 + h_2)$ where R_0 is a lower bound of the reservation wage and r_0 is the disutility of labour.

(i) The Noncooperative Outcome

Substituting the demand functions into (2) for $j = 1, 2$ and taking the required derivatives, and substituting for the definitions of V^1 and V^2 and solving yields

$$h_{1n}* = [(2R_o - (\alpha+d)g_o)(\pi^2_{2,2} - \pi^1_{1,2}) - (\alpha+d)(m_1\pi^2_{2,2} - m_2\pi^1_{1,2})]/D \tag{10}$$

$$h_{2n}* = [(2R_o - (\alpha+d)g_o)(\pi^1_{1,1} - \pi^2_{2,1}) - (\alpha+d)(m_2\pi^1_{1,1} - m_1\pi^2_{2,1})]/D \tag{11}$$

A maximum requires that $\pi^1_{1,1} = [(\alpha+d)^2 - 4r_o] < 0$, $\pi^2_{2,2} = [(\alpha+d)^2 - 4r_o] < 0$ and $D = \pi^1_{1,1}\pi^2_{2,2} - \pi^1_{1,2}\pi^2_{2,1} > 0$ where $\pi^1_{1,2} = [(\alpha-d)(\alpha+d) - 2r_o]$ and $\pi^2_{2,1} = [(\alpha-d)(\alpha+d) - 2r_o]$.

With these results we can determine a number of comparative statics.

[26] A more general form for the quality of the contest variable, is $V^1 = \alpha(h_1+h_2)+\omega(h_1-h_2)-\gamma(h_1-h_2)^2$. Using this form, the choice of players by team 1 that maximizes the quality of contest for its fans, given the talent choice of its rival, is $h_1(h_2) = h_2 + (\alpha+\omega)/2\gamma$. This suggests that as wins become more importamt to fans, the greater is the talent advantage desired. This form, however, does not allow for closed form solutions.

[27] An alternative form that can be considered is if the market size parameter enters *multiplicatively* for example, $p_j = m_jV^j + g_o - n_j$ for $j = 1, 2$. This formulation complicates the comparative statics considerably since the parameters m_1 and m_2 now enter the determinant, as well as the numerator of the solutions.

Proposition 1 *When fans are concerned with both the absolute and relative quality of play, and talented players are in less than perfectly elastic supply, the number of high quality players demanded by a team is: (i) increasing in its market size m_1; and (ii) increasing (decreasing) in the market size of its rival m_2 if players are strategic complements (substitutes);(iii) decreasing (and possibly increasing) in the minimum reservation wage R_o if players are strategic complements (substitutes) and (iv) decreasing (increasing) in the number of games g_o if players are strategic complements (substitutes).*

Proof: For (i) the comparative static is $\frac{\partial h_1^n}{\partial m_1} = -[(\alpha + d)\pi^2_{2,2}/D > 0$, *for (ii) it is* $\frac{\partial h_1^n}{\partial m_2} = [(\alpha + d)\pi^1_{1,2}]/D$, *where* $D = \pi^1_{1,1}\pi^2_{2,2} - \pi^1_{1,2}\pi^2_{2,1} > 0$. *The sign of* $\frac{\partial h_1^n}{\partial m_2}$ *is positive for strategic complements* ($\pi^1_{1,2} > 0$), *and negative for strategic substitutes. For (iii) the comparative static is* $\frac{\partial h_1^n}{\partial R_o} = 2(\pi^2_{2,2} - \pi^1_{1,2})/D$ *which is* < 0 *if* $\pi^1_{1,2} > 0$, *but possibly* > 0 *if* $(\pi^2_{2,2} - \pi^1_{1,2}) > 0$ *which is possible if* $\pi^1_{1,2} < 0$, *and* $\pi^1_{1,2} < \pi^2_{2,2}$ *given* $D > 0$ *For (iv) the comparative static is* $\frac{\partial h_1^n}{\partial g_o} = -[(\alpha+d)(\pi^2_{2,2} - \pi^1_{1,2})]/D$ *which is positive if players are strategic complements* ($\pi^1_{1,2} > 0$), *but could be negative if players are strategic substitutes* ($\pi^1_{1,2} < 0$) *and* $\pi^1_{1,2} < \pi^2_{2,2}$ *given that* $(\alpha + d) > 0$. ‖

The comparative statics clearly depend on the preferences regarding absolute and relative quality of play. For example, if players are strategic substitutes ($\pi^1_{1,2}, \pi^2_{2,1} < 0$) which is implied by a strong concern for a superior team (α is small and $d = \omega - \gamma$ is large) then an increase in team $1's$ market size would decrease team $2's$ demand for talented players. When quality of play and competitive balance are important (α is large and $d = \omega - \gamma$ is relatively small), players are strategic complements ($\pi^1_{1,2}, \pi^2_{2,1} > 0$), which suggests that an increase in team $1's$ market size would increase the demand for talented players by team 2.[28] An increase in the minimum reservation

[28]The win model suggests that the reaction functions are downward sloping in (h_1, h_2) space while the more general

wage, R_o will reduce the number of high quality players demanded if they are strategic complements ($\pi^1_{1,2} > 0$) while the opposite is possible for strategic substitutes. Increases in the number of games will increase the number of talented players demanded by a team if they are strategic complements, with the opposite possible for strategic substitutes.

(ii) The Social Welfare Maximizing Outcome

Social welfare for the specific formulation can be defined as $SW = (m_1 + V^1 + g_o)n_1 - n_1^2/2 + (m_2 + V^2 + g_o)n_2 - n_2^2/2 - (R_oH + r_oH^2/2)$. If the first order condition for the social welfare maximizing price, $p_j = (m_j + V^j + g_o) - n_j = 0, for \quad j = 1,2$ as in (4) is substituted into the objective function, we obtain $SW = (m_1 + V^1 + g_o)^2/2 + (m_2 + V^2 + g_o)^2/2 - (R_oH + r_oH^2/2)$.[29] Taking the derivatives with respect to h_1 and h_2, and solving simultaneously yields the social welfare maximizing talent levels,

$$h_{1s}* = [(R_o - 2\alpha g_o)(\Pi^s_{2,2} - \Pi^s_{1,2}) - (m_1(\alpha+d) + m_2(\alpha-d))\Pi^s_{2,2} + (m_2(\alpha+d) + m_1(\alpha-d))\Pi^s_{1,2}]/D \quad (12)$$

$$h_{2s}* = [(R_o - 2\alpha g_o)(\Pi^s_{1,1} - \Pi^s_{2,1}) - (m_2(\alpha+d) + m_1(\alpha-d))\Pi^s_{1,1} + (m_1(\alpha+d) + m_2(\alpha-d))\Pi^s_{2,1}]/D \quad (13)$$

A maximum requires the conditions $\Pi^s_{1,1} = [(\alpha + d)^2 + (\alpha - d)^2 - r_o] < 0$ $\Pi^s_{2,2} = [(\alpha + d)^2 + (\alpha - d)^2 - r_o] < 0$ $E = \Pi^s_{1,1}\Pi^s_{2,2} - \Pi^s_{1,2}\Pi^s_{2,1} > 0$ where $\Pi^s_{1,2} = \Pi^s_{2,1} = [2(\alpha - d)(\alpha + d) - r_o]$.

(iii) Comparing the Outcomes

While a comparison of the noncooperative and social welfare outcomes for the case of asymmetric market sizes requires simulation techniques, it is possible to make analytic comparisons for the case of equal market sizes, $m_1 = m_2 = m$. Using (10) and (12), the respective talent levels for the

quality of play model allows for the possibility they are upward sloping.

[29] We are assuming here that the capacity constraints are non-binding.

noncooperative and social welfare maximizing cases are

$$h_1^n = h_2^n = [(\alpha + d)(g_o + m) - 2R_o]/2(3r_o - \alpha^2) \tag{14}$$

with the necessary restriction for an interior solution $(2R_o - d(g_o + m))/(g_o + m) < \alpha < (3r_o)^{1/2}$.

$$h_1^s = h_2^s = [2\alpha(g_o + m) - R_o]/2(r_o - 2\alpha^2) \tag{15}$$

with the necessary restriction for an interior solution $R_o/2(g_o + m) < \alpha < (r_o/2)^{1/2}$.

Proposition 2 *When fans only value the relative quality of play,* $(\alpha = 0, d > 0)$ *the noncooperative choice of talent level is higher than the socially optimal choice of talent levels,* $(h_n^1 > h_s^1)$*; when fans only value the absolute quality of play* $(\alpha > 0, d = 0)$*, then the noncooperative choice of talent is less than the socially optimal choice of talent levels,* $(h_n^1 < h_s^1)$*. Proof: Comparing (14) and (15) it is easy to see that when* $(\alpha = 0, d > 0)$ *the* $\lim_{R_o \to 0}(h_n^1 - h_s^1) = d(g_o + m)/6r_o$ *which is positive since* $(d, g_o, m, r_o > 0)$*. For the case* $(\alpha > 0, d = 0)$*, the* $\lim_{R_o \to 0}(h_n^1 - h_s^1) = [\alpha(g_o + m)(-5r_o)/2(r_o - 2\alpha^2)(3r_o - \alpha^2)$ *which is less than zero since the numerator is less than zero, and the terms* $(r_o - 2\alpha^2), (3r_o - \alpha^2)$ *are positive.* $\|$.

The intuition for these results is as follows. The first result, that the noncooperative solution results in too many talented players being employed when the absolute quality of play is unimportant, has been independently observed by Becker (1994).[30] Perhaps more importantly, if fans don't value the absolute quality of play, then social welfare maximization demands that no talented players be employed. The second result that the noncooperative outcome results in too few talented

[30] The view that the relative performance feature of sports is key to understanding sports leagues is also shared by Barro (1996:154) who argues that "to a considerable extent, a team's or athlete's output is measured not so much by absolute skill - how far a ball is hit or how fast a race is run - but by comparisons with the skills of other performers."

players being used when the absolute quality of play is important, is unique to the analysis here.[31] This is because for the noncooperative case, each team does not internalize the effect of its choice of talented players on the quality of the contest offered by rivals, resulting in too few talented players being employed by teams. In contrast, inspection of (14) and (15) reveals that the social welfare outcome is independent of the relative quality of play parameter d since this is internalized in the symmetric case. This leaves the valuation of the absolute quality of play α as the key determinant of the number of talented players that should be employed to maximize social welfare.

(iv) Institutional Responses to the Misallocation Problem

Apart from the league restraints already described, professional sports leagues have recently introduced a number of measures designed to affect the allocation of talent and the distribution of league revenues. Originally adopted in the National Basketball Association, a salary cap is a league revenue sharing agreement between owners and players. Each team is required to spend a specified minimum amount on player salaries, but is prohibited from spending an amount larger than $Cap = fY/t$, where f is players' share of league revenues Y, and t is the number of teams in the league.[32] For the noncooperative outcome, it is clear that when the salary cap is binding that the total demand for players will be affected, which will reduce player salaries and allow teams that are unconstrained to add to their talent pool.[33]

More recently Major League Baseball and its Players' Association have agreed to the use of a luxury tax system. For the years 1997, 1998, and 1999, teams with payrolls in excess of the tax

[31] Hausman and Leonard (1997) show that when the superstar effect is important, an inefficient distribution of talent results because small market teams free ride on the talent level choices of large market teams. Hausman and Leonard (1997) did not compare the noncooperative result with the outcome that maximizes social welfare.

[32] See Fort and Quirk (1995:1279-1282).

[33] It is relatively straightforward to add a salary cap constraint to the model developed here.

threshold specified for the year will be taxed by the league.[34] Becker (1994) argues that a luxury tax, if coupled with a second tax on teams that perform below average, is better than a salary cap for restraining the inefficient competition for players. He argues that a salary tax is too rigid and that the tax approach would result in a superior allocation of talent.

4 Conclusion

The most significant difference between the analysis developed here and the win and championship models is that these models do not incorporate the absolute quality of play. This omission does not allow one to explain the existence of leagues of varying skill levels or provide insight into the differences between college and professional sports. A second major difference is that the approach taken here begins with a specification of consumer preferences which allows one to compare the welfare properties of the noncooperative result with the social welfare outcome. A natural extension of the model developed here would be to include the effect of winning and losing on the subsequent reallocation of players. Attention has been focussed on the initial allocation of players based on the expectations of owners regarding the importance of relative versus absolute quality of play to their fans. It is possible to consider a subsequent stage, that is, once the games are played and winning percentages are determined. If wins are the key determinant of the demand for tickets, team rosters should be very sensitive to changes in win-loss records, regardless of whether the poor record is the result of bad luck or insufficient talent. However, if the quality of play is more important, then team rosters should be less effected by the short term fortunes of teams.

[34]The tax thresholds and tax rates are for 1997 ($51 million, 35%); for 1998 ($55 million, 35%); and for 1999 ($58.9, 34%). For details on the luxury tax system see *1997-2000 Basic Agreement* between Major League Baseball and the Players' Association.

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