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“Fire Sales” in Housing Market: Is the House-Search process similar to a Theme Park visit?1

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Abstract:

Three striking empirical regularities have been repeatedly reported: the positive correlation between housing prices and trading volume, between housing price and the time-on-the-market (TOM), and the existence of price dispersion. This short paper provides perhaps the first unifying framework which mimics these phenomena in a simple competitive search framework. In the equilibrium, sellers with heterogeneous waiting cost and buyers are endogenously segregated into different submarkets, each with distinct market tightness and prices. With endogenous search effort, our model also reproduces the well-documented price-volume correlation. Directions for future research are also discussed.

*JEL Classification Codes:* D830, E300, R210, R310

*Keywords:* housing market, competitive search, price dispersion, trading volume, time on the market.

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1. Introduction

Both casual observations and serious empirical research agree that the housing market is characterized by a strong decentralized pattern of exchange with severe search frictions. In sharp contrast to the prediction from the traditional Walrasian settings, empirical "anomalies" such as price dispersion in the real estate market, a nontrivial time on the market (TOM) positively associated with housing prices, the positive correlation between housing prices and trading volumes, etc., are repeatedly reported. This paper is among the first few efforts to develop a unifying competitive search framework with heterogeneous sellers to illustrate the behavior of the housing market reflected in the above empirical findings.

The modeling choice is indeed intuitive. The existence of price and rent dispersion naturally leads one to a search-theoretic setting (for instance, see Gabriel et al, 1992; Leung et al, 2006; Plazzi et al, 2008) to depict the decentralized pattern of exchange. Another necessary condition for price dispersion is the heterogeneity on the seller's and/or the buyer's side, which generates corresponding submarkets (Diamond, 1971). In most cases in reality, these submarkets are partially segregated since some of the sellers or buyers are free to flow between these submarkets. As a consequence, the competitive search framework may suit the issue better than the traditional search-theoretic settings.

To simplify the exposition, we focus on one-side heterogeneity and assume that sellers are different in terms of their waiting costs. The assumption of heterogeneity in the waiting costs variable attempts to capture the differing financing costs, as well as the search efforts and costs among different house sellers. Some sellers are more urgent to sell the house since the financing costs of the alternative funding recourses is high, for instance, when they are on the verge of bankruptcy. Other sellers may want to sell their houses as soon as possible since they would be relocated to another place and the pecuniary and opportunity costs to deal with the housing selling procedure are considerably large compared with their gains/losses from selling.

\footnote{Adding other heterogeneity in the setting would not change our principal results, but it will complicate the algebra significantly.}
the house. They are the "fire-sale" sellers in our model. Meanwhile, our model also contains other sellers who are willing to wait for better prices. On the buyer side, we include homogeneous waiting costs to reflect buyers' lodging costs and search-related costs when they are looking for a suitable house to purchase.

There are various ways to deal with the heterogeneity in housing search issues. For instance, one can consider identical and fully segregated submarkets to investigate vacancy issues. As a result, the prices and TOMs are the same in all the submarkets. In contrast, we adopt the competitive search framework based on the seminal work in Moen (1997) to investigate heterogeneous and partially segregated submarkets, where buyers are free to enter either submarket. As a result, the price and the expected TOM in one submarket may differ from those in another submarket. Moreover, in the case with exogenous shocks, this framework allows us illustrate how these submarket interact with each other due to the flavor of the ex ante competitiveness embodied in this framework.

This is in contrast to the earlier search-theoretic frameworks explaining price dispersion, such as Axell (1974), Butters (1977), Reinganum (1979), von zur Muehlen (1980), Burdett and Judd (1983), Diamond (1985), Rob (1985), Salop and Stiglitz (1985), Benabou (1988, 1992a,b, 1993), and Rauh (2001). They focus on commodity markets with take-it-or-leave-it offers. In such markets, all the goods are alike, and sellers can adjust their inventory easily. As a consequence, time on the market is not an important issue. In contrast, houses usually differ from each other in one way or another. Moreover, it takes a long time to build a house. Thus, inventory adjustment is much more difficult. Sellers have to sell what they have, and thus the tradeoff between the selling price and the speed of sales is crucial. Our continuous time framework also captures the fact that the negotiation with the buyers is more frequent than that in commodity markets.

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4 Needless to say, the model can be reformulated as having identical sellers and heterogeneous buyers. The principal results will not be changed.
5 For instance, see Wheaton (1990).
6 It is close to directed search, or directed matching framework originated in Peters (1991) and Montgomery (1991). See also Becsi et al (2005).
In our model, sellers with higher waiting costs, i.e. those in the "fire-sale" situation, are willing to accept lower prices, which attract a larger number of potential buyers so that the house would be sold faster. As a consequence, the prices in the two submarkets would differ, and lower (higher) prices would be associated with shorter (longer) TOMs. These theoretical predictions are consistent with the empirical findings. For example, Merlo and Ortalo-Magné (2004) find that sellers post different prices to target various types of consumers, while the submarket with a higher listing price has a lower matching rate and a longer time on the market. Leung, Leong and Wong (2006) find that price dispersion in the housing market is non-trivial. Moreover, the degree of price dispersion after controlling the traits of the house can be explained by the movements of the macroeconomic variables. In the context of commercial real estate, Plazzi, Torous and Valkanov (2008) also find significant rent dispersion. In this paper, the degree of dispersion is determined by the distribution of seller's waiting costs, which is in practice affected by the macroeconomic conditions. The positive correlation between TOM and transaction price are found in the work of Kang and Gardner (1989), Forgey et al (1996), Leung, Leong and Chan (2002), Anglin et al (2003), among others.

In addition, we can also demonstrate positive relationship between housing prices and trading volume in the cases with shocks in the demand-side variables, such as residential value of houses, buyer's waiting cost, and buyer's reservation value. This is in line with the traditional wisdom based on supply-demand analysis, as well as empirical findings in Fisher et al (2003), Leung, Lau and Leong (2002). Intuitively, in the cases with higher (lower) residential value, lower (higher) buyer's reservation value, or lower (higher) buyer's waiting costs, the houses are relatively more (less) attractive. As a result, the housing price would rise (fall), and potential buyers would inflow to (outflow from) the town, leading to a larger (smaller) transaction volume. In this regard, this paper provides an alternative search-theoretic explanation about the positive correlation between housing prices and trading volume, other than the down-payment explanation. More specifically, the down-payment effect model by Ortalo-Magne and Rady (2006) seems to capture the short-run dynamics while this paper focuses on the steady

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state relationship. It is consistent with the empirical finding of Leung, Lau and Leong (2002), which suggests that the short-run dynamics of the housing market is driven by down-payment effect, where the longer-run relationship between housing price and trading volume is due to search friction.

The remaining of the paper is structured as follows: The baseline model with competitive search and heterogeneous waiting costs will be introduced in the next section. Results will be presented and discussed in order. In the concluding remarks, we will compare the competitive search model presented here with a theme park visit in an intuitive ways. Future research directions will also be discussed.

2. A Baseline Model of Housing Price Dispersion

2.1. A Tale of Two Submarkets

This section outlines the formal model. The horizon of the model is infinite and time is continuous. There is a continuum of sellers who have different waiting costs, so that some of them are more urgent to sell the house than others. For simplicity, we focus on the case with only two types, impatient and patient sellers. The principal result however can be generalized to a more general setting. And without loss of generality, the waiting cost for the impatient sellers, \( c^H \), is higher than that of patient ones, \( c^L \), \( c^H > c^L > 0 \). Let \( S^i \) be the population (or measure) for the sellers with a flow waiting cost of \( c^i \), \( i=H,L \). Similarly, we define \( B^H \) and \( B^L \) as the measure of buyers focusing on the two "submarkets" respectively. Notice that the "submarkets" need not be separated geographically. They simply represent the trading which involve type \( i \) sellers, \( i=H,L \). The impatient sellers, who are urgent to sell the house, may post this information in the advertisement, so that the buyers can distinguish the two types of
sellers easily. For simplicity, we assume complete and perfect information between the agents. In the type-i submarket, the buyer's waiting cost $\kappa$.

In each submarket $i$, $i=H,L$, the number of successful matching in an infinitesimal period is governed by a random matching function, $M\left(B^i, S^i\right)$, which exhibits constant return to scale in $B$ and $S$, with positive but diminishing marginal returns in each argument. We define the market tightness $\theta^i = B^i / S^i$ in the sense that it is more difficult for a buyer to find a seller in a tighter market.\(^8\) For each submarket $i$, we can define $\eta^i$ as the flow matching rate for a buyer to find a seller in submarket $i$ such that

$$\eta^i = \frac{M\left(B^i, S^i\right)}{B^i} = M\left(1, \frac{1}{\theta^i}\right)$$

Similarly, the flow matching rate for a seller to find a buyer, $\mu^i$, satisfies

$$\mu^i = \frac{M\left(B^i, S^i\right)}{S^i} = M\left(\theta^i, 1\right) = \theta^i \eta^i$$

We can denote $M_j(.)$ as the first derivative of the matching function $M$ with respect to the $j$-th argument. Note that the assumed feature of the matching function suggests that $M_j\left(\theta^i, 1\right) = M_j\left(1, 1/\theta^i\right)$ since the first derivative of a constant-return-to-scale function must be homogeneous of degree zero. As a result, we use these two expressions interchangeably.

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\(^8\) Needless to say, we can also define the market tightness from a seller perspective. The results in this paper will not change under this alternative definition of tightness.
2.2. Housing Prices and Bellman Equations

In this model, sellers with different waiting costs could post different prices to differentiate each other. The actual price $P^i$ in the submarket $i$ would be determined by a Nash bargaining solution, which will be discussed in the next subsection. Let $\Pi^i$ denote the value for type-$i$ sellers, $V^i$ the value for type-$i$ buyers (who are still searching the market but have not owned houses), for $i=H,L$, and $\Omega$ the value of a house owner, which is independent of the waiting cost level. Since the buyers are free to enter each of the two submarkets, the values of all types of buyers $V^i$, are also the same as the reservation value (i.e. the value for outside options), $\overline{V}$, i.e. $V^i = \overline{V}$, for $i=H,L$. For simplicity, we assume that both $\overline{V}$ and $\Omega$ are exogenously determined.

As it is standard in the literature, we assume that buyers and sellers maximize the expected value of the sum of the periodic utility flow, which is constantly discounted by the rate $r$. Given the model structure outlined above, the dynamic optimization of the buyers and sellers can be summarized by the following Bellman equations, $i=H,L$,

$$r\Pi^i = -c^i + \mu^i \left( P^i - \Pi^i \right)$$  \hspace{1cm} (3)

$$rV^i = -\kappa^i + \eta^i \left( \Omega - P^i - V^i \right)$$  \hspace{1cm} (4)

From equation (3) we can obtain

$$\Pi^i = \frac{-c^i + \mu^i P^i}{r + \mu^i} = \frac{-c^i / \mu^i + P^i}{r / \mu^i + 1}.$$  \hspace{1cm} (5)
Note that $1/ \mu^i$ is actually the mean waiting time for the buyers. Hence, equation (5) means that the seller's value equals the discounted net gain from selling a house, while the net gain is the price net of the waiting cost during the waiting period.

Similarly, equation (4) yields

$$V^i = \frac{-\kappa + \eta^i (\Omega - P^i)}{r + \eta^i} \tag{6}$$

The intuition of (6) is analogous to that of equation (5).

2.3. The Bargaining Process

The housing price is determined by the Nash bargaining with seller's bargaining power as $\alpha$. This means that the seller and buyer will solve the following joint surplus maximization problem

$$\max_{\pi, \Omega} \left\{ (P^i - \Pi^i) \pi \left( \Omega - P^i - V^i \right)^{-\alpha} \right\}$$

The solution is

$$P^i = (1 - \alpha) \Pi^i + \alpha (\Omega - V^i) \tag{7}$$

Equation (7) says that price is a weighted average of two objects: one is the value of the seller, $\Pi^i$, and the other is the net gain from being a buyer (or a house-searcher) to a house owner between $(\Omega - V^i)$. Thus, to solve for the house price, it is necessary to solve out the seller value and buyer value at the equilibrium. From equations (5), (6), and (7), we can solve $\Pi^i$, $V^i$ and $P^i$, $i=H,L$. 

\[ \Pi^i = \frac{\alpha \mu^j (r\Omega + c^i + \kappa)}{r[r + \alpha \mu^j + (1 - \alpha)\eta^j]} - \frac{c^i}{r} \]  

(8)

\[ V^i = \frac{(1 - \alpha)\eta^j (r\Omega + c^i + \kappa)}{r[r + \alpha \mu^j + (1 - \alpha)\eta^j]} - \frac{\kappa}{r} \]  

(9)

\[ P^i = \frac{\alpha (r + \mu^j) (r\Omega + c^i + \kappa)}{r[r + \alpha \mu^j + (1 - \alpha)\eta^j]} - \frac{c^i}{r} \]  

(10)

2.4. **Buyer’s Free Entry and the Price-TOM Relation**

Since buyers are free to enter either submarket, the buyer's value should be the same as the reservation values for outside options. Hence,

\[ V^H = V^L = \overline{V}. \]  

(11)

Note that the right-hand side of equation (9) is increasing in \( c^i \), and decreasing in \( \theta^i, i=H,L \). While \( c^H > c^L \) and (11), we have

\[ \theta^H > \theta^L, \mu^H > \mu^L, \text{ and } \eta^H < \eta^L. \]  

(12)

Meanwhile, from equation (6), we can show that, \( i=h,L \),

\[ P^i = \Omega - \overline{V} = \frac{r\overline{V} + \kappa}{\eta^i}. \]
which suggest that the housing price is decreasing in $\theta^i$. Thus, the transaction prices of the two submarkets are indeed different, with the expected result that the submarket of impatient sellers (higher waiting cost) would sell at a lower price,

$$P^H > P^L.$$  \hspace{1cm} (13)

Note the expected Time On the Market (TOM) is $1/\mu^i$, $i=H,L$, we can combine (12) and (13) and obtain Proposition 1 about the price-TOM relation in the housing market.

**Proposition 1 (Price-TOM Relation):** In a competitive search framework with heterogeneous waiting costs for sellers and free entry for the buyers, the submarket with a higher (lower) price must have a longer (shorter) expected time-on-the-market.

This result is intuitive. In the case that the sellers are eager to sell the house, house price must be lower than usual. Observing the possible low price, more buyers would crowd in that market segment, which leads to a higher probability of matching, and a shorter time on the market. Empirically, Merlo and Ortalo-Magné (2004) find that sellers post different prices and the submarket with a higher listing price has a lower matching rate and a longer time on the market. The empirical evidence in Kang and Gardner (1989), Forgey et al (1996), Leung, Leong and Chan (2002), Anglin et al (2003), among others, support the proposition 1.

2.5. **Price Dispersion**

The following proposition simply repeats (13):
**Proposition 2 (Price Dispersion):** In the current competitive search framework, housing prices would be different even for identical houses. Specifically, the seller with higher waiting costs would ask for a lower housing price, in an effort to reduce the waiting costs by selling the house faster.

The above results are in line with the empirical findings. Leung et al (2006) find the price dispersion cannot be only attribute to randomness or econometric mis-specification as the degree of price dispersion systematically varies with some macroeconomic variables. In addition, Plazzi, Torous and Valkanov (2008) also find empirical rent dispersion in the commercial real estate market.

2.6. **Comparative Statics and the Price-Volume Correlation**

On top of the two major propositions, we can also derive the comparative-statics results, which are summarized by the following table:
\[ \begin{array}{|c|c|c|c|c|c|} 
\hline
i=H,L & c' \uparrow & \kappa \uparrow & \Omega \uparrow & \alpha \uparrow & \bar{V} \uparrow \\
\hline
\theta' & + & - & + & - & - \\
\hline
\mu' & + & - & + & - & - \\
\hline
\eta' & - & + & - & + & + \\
\hline
P' & - & - & + & + & - \\
\hline
\Pi' & - & - & + & + & - \\
\hline
M(B',S') & + & - & + & - & - \\
\hline
\text{Price-Volume} & - & + & + & - & + \\
\hline
\text{Co-movement} & & & & & \\
\hline
\end{array} \]

Table 1: Comparative Statics for the Model with Costly Search Effort

Note that the trading volume for a given period is proportional to the matching rate \( M(B',S') \), and therefore we can deduce the price-volume co-movements from the table above. While the intuitions are in fact straightforward, it may be instructive to present the explanations in a more systematic manner.

1. When sellers are more eager to sell the house, in the case that their waiting costs \( c' \) are higher, house prices would be lower. As a result, more buyers would be attracted to the economy, leading to a higher buyer-seller ratio (market tightness), a higher selling rate and a lower buying rate. In this case, a lower price would be associated with a higher trading volume.
2. On the other hand, given the outside value of the buyer (for instance, potential buyers may prefer to have leisure instead of involving in house-searching), a higher level of buyer's waiting cost ($\kappa'$) may discourage them in entering the market at all, which results in a lower market tightness (i.e. a lower buyer-seller ratio). House price would be lower, as a compensation for the larger difficulty in house-searching, so that the "return" of house search can match alternatives. Thus, we would observe a lower price and a smaller trading volume (and hence a positive price-trading volume correlation).

3. When the benefit from owning a house and living in the economy ($\Omega$) is larger, more buyers would be attracted to the economy, and the market tightness is higher. With more demand, the house price is driven higher. Thus, a higher price is associated with higher trading volume (and hence a positive price-trading volume correlation).

4. If the seller's bargaining power ($\alpha$) is relatively larger, the house price is higher. Meanwhile, some buyers would leave the market, leading to a lower market tightness and a lower trading volume.

5. While the buyer's entry value ($\bar{V}$) is higher, some buyers would not participate the house searching activities, resulting in a lower market tightness, and thus a lower price (and hence a positive price-trading volume correlation).

Some economists insist to frame the house market as if it has a downward sloping demand curve and an upward sloping supply curve. We can then rephrase the result in the price-volume co-movements in Proposition 3:

**Proposition 3 (Comparative Statics and Price-Volume Correlation):** In the baseline model with fixed entry value for the buyers and fixed number of sellers, housing prices and the trading volumes would move in the same direction as if the system is hit by a demand shock, such as changes in the buyer's waiting costs, the values for owning a house, or the buyer's entry value.
On the other hand, housing prices and the trading volumes would move in the opposite directions as if the system is hit by a supply shock, such as the changes in the seller's waiting costs, or the seller's bargaining power.

Empirically, Fisher et al (2003), Leung, Lau and Leong (2002), among others, find strong contemporary co-movements between housing prices and the trading volumes, while Leung, Lau and Leong (2002) also find that price would lead the trading volume by 24-48 months in the monthly data.

3. Conclusion

To a certain extent, the idea behind this paper is analogous to a theme park visit. In those famous theme parks, visitors do not know ex ante whether it will be very crowded and they would need to wait in a long queue before they can enjoy some machines, or a ghost house. They can choose to buy a more expensive "VIP-pass" and save some time for waiting, or buy a cheaper normal-pass and hope that they may not need to wait that long ex post. Normally, people with a higher "waiting cost" such as those foreign visitors would prefer the more expensive options. For some others, they prefer to wait. Thus, the time-on-the-queue (TOQ) will be negatively related to the price of the "pass".

Similarly, some sellers in this paper have higher waiting cost than the others, and they prefer to sell their houses in a submarket with higher "liquidity." Unlike the theme park visit, however, the supply side of the housing market is endogenous. The buyers will take the sellers' strategies as given and then self-select into different submarkets. Moreover, while pass-purchasing is certain, house-purchasing is not. Even within each sub-market, there is a random matching process among potential buyers and sellers. And while the price of the pass is given, the
housing price in each submarket will be determined through a Nash bargaining process, which will in turn depend on the market-tightness of the corresponding submarket.

Perhaps more importantly, this paper differs from the theme park visit example in that there are three stylized facts for this paper to mimic, namely, the existence of price dispersion, the positive correlation between the market price and trading volume, and that between the transaction price and the time-on-the-market (TOM). The empirical "anomalies" found against the Walrasian predictions can be explained within our competitive search framework. The free-entry assumption implies the positive correlation between housing prices and the time on the market. With the introduction of costly search efforts, the buyers with higher waiting costs are more eager to purchase a house, and hence provide more search efforts. The increase in search intensity would lead to higher trading volumes. It also holds in the case with a positive shock in the waiting costs. In addition, we show that price dispersion can exist easily even with perfect information and perfect competition in the ex ante sense, as long as the trades are decentralized.

The current model, of course, can be further improved. For instance, in this model, both the reservation value of a house buyer (or house searcher) and the value of a house owner are exogenously determined. Future work should endogenize these values in a more general model. The model also implicitly assumes that there is some “commitment” mechanism on the seller side. Recall that in the model sellers with different waiting costs are self-selected into different “sub-markets,” say, by advertisement. We also assume that once a match between a potential seller and potential buyer is made, the price will be determined by Nash Bargaining. In that case, sellers with higher waiting cost (“Fire Sale”) would sell at lower price. This attracts more potential buyers to that sub-market. Hence, from the seller point of view, the “Fire Sale sub-market” will have a higher matching rate. However, patient sellers (sellers with lower waiting cost) may find it profitable to enter that sub-market, pretending to be a Fire Sale. Thus, this
model demands the sellers to truthfully reveal their own types and commit to “stay” in that sub-market. Future research should relax such an assumption.9

Future work can also be extended in other directions. For instance, "middlemen" are missing in this analysis. Previous partial equilibrium analysis (such as Yavas, 1994, 1995) show that the introduction of intermediary may affect the equilibrium configuration, and efficiency under some conditions. Second, Zenou (2009) study the location of various types of workers in a search theoretic framework. It will be interesting to extend the analysis here to a model with both house market and labor market search.10

Third, the search friction in the housing market may influence the asset portfolio, as illustrated by Anglin and Gao (2010). It would be interesting to explore the general equilibrium implications for such consideration. And while this paper focuses on the heterogeneity in waiting costs, future research may explore the situation with the co-existence of financial constraint and search frictions. Another candidate for future research is to merge the current housing market model with the conventional neoclassical framework, as in Lagos and Wright (2005). These directions are indeed being pursued.

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9 Among others, see Tse and Leung (2011) for some earlier efforts on this.
10 See Coulson and Fisher (2007) for some related attempt along this line.
Reference


Appendix

Proof for the Comparative Statics

From (8) to (11), we know that the equilibrium is determined by the following equations, \(i=H,L\),

\[
V^H = V^L = \bar{V} = \left(\frac{1-\alpha}{r}\right)^{\frac{1}{\eta_i}} \left(\frac{\rho \Omega + c^i + \kappa}{r + \alpha \mu^i + (1-\alpha)\eta_i}\right) - \frac{\kappa}{r}
\]

\[
P^i = \frac{\alpha (r + \mu^i)(\rho \Omega + c^i + \kappa)}{r[r + \alpha \mu^i + (1-\alpha)\eta_i]} - \frac{c^i}{r}
\]

\[
\Pi^i = \frac{\alpha \mu^i (\rho \Omega + c^i + \kappa)}{r[r + \alpha \mu^i + (1-\alpha)\eta_i]} - \frac{c^i}{r}
\]

Notice that there is no direct dependence of market \(i\) variables (i.e. \(P^i, \Pi^i\)) on the market \(j\) variables (i.e. \(P^j, \Pi^j\)), \(i,j=H,L\), and \(i \neq j\). In this sense, the two sub-markets are “segmented.” Thus, we can first solve the market tightness, and then the values and prices, and worry less on the cross-market effects. Consequently, the effects from exogenous variables, including \(c^H, c^L, \kappa, \Omega, \alpha\) can be figured out.

Observe that the buyer’s value is increasing in \(c^i\), and decreasing in \(\theta^i, i=H,L\). For instance, when \(c^H\) increases, \(\theta^H\) would be larger, but \(P^H\) and \(\Pi^H\) would decline. Since

\[
P^i = \bar{V} - \frac{rV + \kappa}{\eta_i}
\]

and
\[
\Pi^i = \frac{\alpha \mu_i (r \Omega + c^i + \kappa)}{r \left[ r + \alpha \mu_i + (1 - \alpha) \eta^i \right]} - \frac{c^i}{r}
\]

\[
= P^i - \frac{\alpha r (r \Omega + c^i + \kappa)}{r \left[ r + \alpha \mu_i + (1 - \alpha) \eta^i \right]}
\]

\[
= \Omega - \bar{V} - \frac{r \bar{V} + \kappa}{\eta^i} = \frac{\alpha}{(1 - \alpha) \eta^i} \left[ (1 - \alpha) \eta^i (r \Omega + c^i + \kappa) \right]
\]

\[
= \Omega - \bar{V} - \frac{\alpha (\kappa + r \bar{V})}{(1 - \alpha) \eta^i}
\]

\[
= \Omega - \bar{V} - \frac{r \bar{V} + \kappa}{(1 - \alpha) \eta^i}
\]

\[
i = H, L. \text{ Recall that both } \bar{V} \text{ and } \Omega \text{ are exogenously determined. Thus, a change in } c^i \text{, will go only through a change in } \eta^i \ i = H, L. \text{ The variables in the other sub-market, } \theta^L, P^L, \Pi^L \text{ would not change.}
\]

In this case, lower price would drive higher trading volume. Similarly, when \( c^L \) increases, \( \theta^L \) would increase, but \( P^L, \Pi^L \) would be lower. The variables in the other submarket, \( \theta^H, P^H, \Pi^H \) would not change, and we also have higher trading volume accompanied with lower price. This is in line with responses to supply shocks.

Note that the buyer's value is decreasing in both \( \kappa \) and \( \theta^i, i = H, L. \) Thus, if the buyer's waiting cost, \( \kappa \), is larger, \( \theta^i \) would be smaller. Note that

\[
\bar{V} = \frac{(1 - \alpha) \eta^i (r \Omega + c^i + \kappa)}{r \left[ r + \alpha \mu^i + (1 - \alpha) \eta^i \right]} - \frac{\kappa}{r}
\]

\[
= \frac{(1 - \alpha) \eta^i (r \Omega + c^i)}{r \left[ r + \alpha \mu^i + (1 - \alpha) \eta^i \right]} - \frac{\kappa}{r \left[ r + \alpha \mu^i + (1 - \alpha) \eta^i \right]}
\]

Notice that the first term in the expression above is independent of \( \kappa \) directly. As \( (r + \alpha \mu^i) \), \( [r + \alpha \mu^i + (1 - \alpha) \eta^i] \) are all positive, \( \bar{V} \) is decreasing in \( \kappa \).
Furthermore,

\[
\frac{r\bar{V} + \kappa}{(1 - \alpha)\eta^i} = \frac{r\Omega + c^i + \kappa}{r + \alpha\mu^i + (1 - \alpha)\eta^i}
\]

\[
= \frac{r\Omega + c^i}{r + \alpha\mu^i + (1 - \alpha)\eta^i} + \frac{r(1 - \alpha)\eta^i(r\Omega + c^i)}{(r + \alpha\mu^i)(r + (1 - \alpha)\eta^i)} - \frac{r}{r + \alpha\mu^i}\bar{V}
\]

\[
= \frac{r\Omega + c^i - r\bar{V}}{r + \alpha\mu^i}
\]

Thus, we can re-write

\[
P^i = \Omega - \bar{V} - \frac{r\bar{V} + \kappa}{\eta^i}
\]

\[
= \Omega - \bar{V} - (1 - \alpha)\frac{r(\Omega - \bar{V}) + c^i}{r + \alpha\mu^i}
\]

\[
P^i = \Omega - \bar{V} - \frac{r\bar{V} + \kappa}{(1 - \alpha)\eta^i}
\]

\[
= \Omega - \bar{V} - \frac{r(\Omega - \bar{V}) + c^i}{r + \alpha\mu^i}
\]

Thus, both \( P^i \) and \( \Pi^i \) are lower due to smaller \( \theta^i, i=H,L \), in the case of higher value of \( \kappa \).

Now let's consider the effects from a higher \( \Omega \). Note that the buyer's value is increasing in \( \Omega \) and decreasing in \( \theta^i, i=H,L \). Hence, a higher \( \Omega \) would attract more immigrants, and raise \( \theta^i, i=H,L \). Since

\[
\frac{r\bar{V} + \kappa}{(1 - \alpha)\eta^i} = \frac{r\Omega + c^i + \kappa}{r + \alpha\mu^i + (1 - \alpha)\eta^i},
\]
we have

\[ P' = \Omega - \bar{V} = \frac{\bar{V} + \kappa}{\eta^i} = \Omega - \bar{V} - (1 - \alpha) \frac{\bar{V}}{r + \alpha \mu^i} + c^i \]

\[ = \left( \Omega - \bar{V} \right) \frac{(r + \alpha \mu^i) - (1 - \alpha) (c^i)}{(r + \alpha \mu^i)} - \frac{(1 - \alpha) (c^i)}{(r + \alpha \mu^i)} \]

Since

\[ \bar{V} = \frac{(1 - \alpha) \eta^i (r \Omega + c^i)}{r[r + \alpha \mu^i + (1 - \alpha) \eta^i]} - \frac{\kappa}{r[r + \alpha \mu^i + (1 - \alpha) \eta^i]} \cdot (r + \alpha \mu^i) \]

Therefore,

\[ \left( \Omega - \bar{V} \right) = \left( \Omega \right) \frac{[r + \alpha \mu^i]}{[r + \alpha \mu^i + (1 - \alpha) \eta^i]} + \frac{\kappa (r + \alpha \mu^i) - (1 - \alpha) \eta^i c^i}{r[r + \alpha \mu^i + (1 - \alpha) \eta^i]} \cdot \]

Thus, \( \left( \Omega - \bar{V} \right) \) is increasing in \( \Omega \), and hence is \( P', i=H,L \).

An alternative way is to see the positive correlation between \( \Omega \), and \( P', i=H,L \) is to notice that

\[ P' = \frac{\alpha (r + \mu^i) (r \bar{V} + \kappa)}{(1 - \alpha) \eta^i} - \frac{c^i}{r} \]

Since \( \mu^i = \theta^i \eta^i \), an increase in \( \theta^i \) (due to an increase in \( \Omega \)) would lead to an increase in \( P', i=H,L \).

Similarly,

\[ \Pi' = \frac{\alpha (\mu^i (r \bar{V} + \kappa))}{(1 - \alpha) \eta^i} - \frac{c^i}{r} \]

Therefore, an increase in \( \theta^i \) (due to an increase in \( \Omega \)) would lead to an increase in \( \Pi', i=H,L \).

Now we want to investigate the implication of a change in the bargaining power \( \alpha \). Observe that buyer's value is decreasing in both \( \alpha \) and \( \theta^i \). So the market tightness would be smaller when the seller's bargaining power is larger. Note also that

\[ P' = \Omega - \bar{V} = \frac{r \bar{V} + \kappa}{\eta^i} \]
\[ \Pi^i = \Omega - \bar{V} + (r\bar{V} + \kappa) \frac{1}{(1-\alpha)} \left( \frac{-1}{\eta^i} \right). \]

When \( \alpha \) increases, \( \eta^i \) will also increase, and hence the house price \( P^i \) will increase as well. At the same time, both \( 1/(1-\alpha) \) and \( -1/\eta^i \) increase with \( \alpha \). Thus \( \Pi^i \) increases, \( i=H,L \).

If the buyer's entry value \( (\bar{V}) \) is higher, market tightness must be lower. Both house price and sellers value would decline, since both \( P^i \) and \( \Pi^i \) are decreasing in \( \bar{V} \)

\[
P^i = \Omega - \bar{V} -(1-\alpha)\left(\frac{r(\Omega - \bar{V}) + c^i}{r + \alpha \mu^i}\right)
= \Omega - \frac{(r + \alpha \mu^i)\bar{V} + (1-\alpha)(r\Omega + c^i - r\bar{V})}{r + \alpha \mu^i}
= \Omega - \frac{\alpha (r + \mu^i)\bar{V} + (1-\alpha)(r\Omega + c^i)}{r + \alpha \mu^i},
\]

and

\[
\Pi^i = \Omega - \bar{V} - \frac{r(\Omega - \bar{V}) + c^i}{r + \alpha \mu^i}
= \Omega - \frac{r\Omega + \alpha \mu^i \bar{V} + c^i}{r + \alpha \mu^i}.
Q.E.D.