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Posthumous tributes to Benoit Mandelbrot (1922-2010) have highlighted its remarkable influence on the natural sciences, from geometry to meteorology, to theories with non-Euclidean spaces and geospatial models approach. Mandelbrot has closed a series of major thinkers going back to classical Greece. A Greek among Romans (Estrada 2010a). The trajectory of his considerable influence is also related to theoretical and experimental economics. More specifically: the theory of financial markets and the psychology of economic behavior, the theory of fractals powerful enough explanations to understand a day of panic in the international stock exchanges.

Indeed, besides its known work: «The fractal geometry of nature» (1982), their contributions to economic theory comes from work recognized (Mandelbrot 1997, 2004, 2005). And his extraordinary work: «The (mis) Behavior of Markets: A Fractal View of Risk, Ruin and Reward» (2004). Mandelbrot developed for financial theory is an analogy of the physical world in the three states of matter: solid, liquid and gas. Three stages of randomness: «docile», «quiet», «wild». Three times in the markets that can be contrasted with the mathematics of fractal geometry.

Conventional economic theory assumes that the price variation can be modeled by random processes that are consistent with the pattern «docile», «docile», more simple, like every rise and fall came as determined by the toss of a coin. However, according to Mandelbrot, the fractal shown us is that the actual price behavior deviates substantially from this standard. A more accurate model, the variation multifractal «wild» price, provides the foundation for a new economic theory of greater scope and more reliable.

Mandelbrot offers an analytical framework for understanding the fractal randomness runaway rough forms in different manifestations, from the turbulence caused by a hurricane, electrical noise or the path of the stock quotes. This is not a theory of getting rich, but its fractal theory helps to see the emergence, development and evolution of disasters. Mandelbrot knocked prevailing myths in the modern economy that lead to underestimate the real risk in financial markets.

Taleb yields the best posthumous tribute to Benoit Mandelbrot to call it: «A Greek among Romans». In the second half of the twentieth century few mathematicians achieved universal influence as remarkable as Mandelbrot. The international scientific community will not cease to mourn the death of one of its finest sons.

In this brief note describes the trajectory of the fractal models/multifractal F/M by Benoit Mandelbrot (Estrada 2010b). The promise was discovered by the geometry of Mandelbrot covers a broad area of research fields, from meteorology and mathematical physics to the individual and collective behavior in society, besides his contributions to the analysis of the financial crisis in his wonderful essay on «The (mis) Behavior of Markets. A fractal view of Risk, Ruin and Reward» (2004). Mandelbrot's arguments have revealed significant anomalies in the prevailing paradigms. Is this a new paradigm in Kuhn's sense as stated by the same Mandelbrot?

Mandelbrot's hypothesis has been extended creatively to the field of financial modeling, the mathematical approach to the markets since the fractals geometric structures of various levels of size, each of which repeats a small scale the overall structure. The hypothesis suggests that Mandelbrot fractal rough measures reflect the nature and allow an approach non-Euclidean: geometric structures such as clouds, coast, wind gusts and conferences in the bags. To Mandelbrot the main feature in a market price, as in various phenomena of culture and nature, is the fractal geometry.

Summarizing his argument is as follows:

> The fractal geometry of roughness measured intrinsically. Thus marking the beginning of a specific quantity theory: the roughness in all its manifestations
>
> (Mand05)

The roughness behavior translates ubiquitous in nature and in culture (including financial markets). Fractality is scattered everywhere as fractal geometry objects are multiform. An overview of fractals and multifractal is described by Mandelbrot in a long chapter (Mand02). Despite its unquestionable antiquity, the study of roughness has been quite behind when compared with older concepts of physics as a slope (of a road or trend), weight, tone, warmth, color and similar categories. The problem, as emphasized by Mandelbrot is that the density of roughness was not measured. Work in this direction had to wait until the author.

A brief explanation of the author can see the contrast between the implications of fractal geometry in two seemingly divergent worlds:

the physical and financial. The inclination of a slope is defined by the derivative of the height *h (x)* along the slope. In theory, this definition implies that increasing the radius *dh / dx* tends to the limit *dx* ¬ *0*. Custom has made this explanation to be 'normal'. In practice, just *dh/dx* is almost constant. However, by definition, rugged profiles and surfaces are such that *dh/dx* vary in an unlimited extension. By contrast, a basic factor in many models of price variation – in Bachelier model and the derivative models fractal/multifractal – is not a suitable instrument. However, there is for height *(d/h)* radius length 'anomalous' (d/x). Its existence is a manifestation of 'escalation'. According to the model of Bachelier this radio has a limit $\alpha = ½$.The same in all instances for all financial data as a fundamental property, but an advantage and a disadvantage because a maximum of the ½ is not a valid parameter for the data set. On the contrary, fractal models / multifractal allow

$$\alpha\_ = ½.$$

First, the language of nature does not always correspond with the statement of the theoretical schemes. Normal science is established on ideal conditions. But the opposite is true in both its physical and cultural forms of reality are often a result of increasing complexity. At a distance without limit given values need to be constant. The case applies for extension inertia in the financial sector with the Bachelier model. Secondly, the measurement criterion is inefficient, according to Mandelbrot. The derivatives are not suitable when you have an irregular behavior of prices. The explanation in this case demands a measure Popper. The M/F contribute to measures that may be out of a dominant factor for the standard explanation: the prediction.

To explain problems of forecasting, Mandelbrot estimated values can be constant but different from ½. Both nature and culture in general decreasing situations occur, the author refers to similar cases (Mand02). The value of α may vary in a specific way in a matter of moments, that characterizes the multifractal. The value of α may be the same at all points in time but different from ½. This characterizes the fractal or HHM model (Hurst-Hölder-Mandelbrot). There is also an important intermediate event called «mesofractalidad» or PLM model (Pareto-Lévy-Mandelbrot).

In the case of judges Mandelbrot derived the mathematical intuitive concept predates the slope. That is, a quantitative measure of the intuitive notion of 'slope' would come before his mathematical concept, the concept of α takes the opposite road. Mandelbrot is originally who modified a concept introduced in 1870 Holderlin, a purely mathematical and totally separated from intuition. However, Mandelbrot's work is identified with the Hölder exponent a central aspect of the roughness. This proximity can be seen in the work of Mandelbrot fractal geometry applied to the cartoons (Mand02). Both Mandelbrot Hölder as one can see how intuition perceived roughness levels are different.

In summary, fractal geometry begins by measuring the roughness by means of α and / or concepts related fractals. With the value and/or distribution of α is directly observable. This is not an elusive concept that has to be discovered indirectly from many observations. The predominant role that has this exponent is a Mandelbrot aspect relates to the principle of parsimony.

For space reasons not explained numerous details related to financial theory (Mand97). Mandelbrot developed the arguments against the Brownian promotion are as follows: First, the value $\alpha = ½$ roughness characterized a form of 'mild'. Second, the roughness found in financial data takes an expression «wild» that excludes $\alpha = ½$.

While substitutes for Brownian theses were rejected, Mandelbrot's objections were heard and many alternative models reacted to save the apparent 'anomalies' discovered. The pioneering observations of the discontinuity of the dominant paradigm were the same author. In light of the fractal approach / multifractal (F/M) reviews had something in common: local roughness reestablished the ½.

In the field of financial theory often required adjustment is variable – stochastic – volatility. The postulate states that short records follow a Brownian motion, but the variance of the movement changes intermittently. Mandelbrot's critique is that the model boundaries are not only not, eliminated but that pushes the rules vary the volatility process. If these variations are fast, the Brownian effect is dissolved, «and in the homeopathic medicine» (Mandelbrot).

If the variance changes slowly, a second criticism that is activated by Mandelbrot is: only if the volatility measures the level of Brownian motion this aspect changes the 'variance variable in the model. Whether to proceed with the generally accepted premise that $\alpha = 1/2$. Mandelbrot argues that this conclusion has not been verified. The evidence suggests the opposite (Mand05).

Mandelbrot points out that in any case their work respond to a different epistemological framework. Their models are definitely not a model of volatility variable.

In the short term, Mandelbrot said, the patches can be defended as an immediate response to urgent needs. However, the explanation in science can not do likewise. When the number of arrangements in a formula exceeds its limits, it falls under its own weight and must begin again. The affirmation of scoliosis in Mand05 is an important statement for those working in the philosophy of science:

Thomas Kuhn brilliantly described this process as a «paradigm shift». What I introduced myself as a good alternative paradigm fractal/multifractal (F/M)

The Scholium Mandelbrot (Mand05) helps to highlight a phenomenon until now invisible to a majority of financial experts, namely that the explanatory models in analyzing investments, making portfolios, option pricing or risk management not contribute to solving fundamental problems, because the models themselves are part of the problem. Financial theory continues to work within traditional models of classical physics. Anomalies within the explanatory schemes are due to transfer of linear response to problems whose budgets are not linear.

The evolution and dynamics of the concept of paradigm in Kuhn taught that we can tell with relative caution when theoretical functions are extended within a particular field of scientific knowledge (Estrada 2010c). But when the application set is irrelevant for the precariousness of their paradigmatic exemplars. Fits within a paradigm may not be forever, there are conditions that require fundamental changes. Classical Mechanics Newton Particle (MCP) ended up being displaced in some areas of theoretical physics as their explanations were limited and disadvantages (Moul02). Key components of the concept of paradigm in Kuhn are discussed in greater scope in the reconstruction by Stegmüller (Steg83). Structuralism version allows you to extend the scope of the paradigm in several aspects: the role of theoretical concepts, the paradigmatic exemplars, methodological values and rules within the scientific community. The hypothesis with which operates Stegmüller reprocessing can be linked to the need to explore the paradigm proposed by Mandelbrot, hitherto unpublished aspects: what does it finance the core K of a theory? What paradigmatic exemplars offer a feature extensive description in the same jobs Mandelbrot?

Financial theory pragmatism necessary to estimate the nature of the paradigms of Kuhn, conventional financial models get caught in the standard descriptions of scientific knowledge. Become dangerously wrong in conditions that can affect people's basic income. An observation of the relationship between scientific community and the financial discipline paradigm suggests a separate section of epistemology of science.

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