# Expectations Traps and Monetary Policy with Limited Commitment

Himmels, Christoph and Kirsanova, Tatiana

University of Exeter

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# Expectations Traps and Monetary Policy with Limited Commitment*

Christoph Himmels† — University of Exeter

Tatiana Kirsanova‡ — University of Exeter

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**Abstract**

In this paper we study the existence and uniqueness properties of monetary policy with limited commitment in LQ RE models. We use a New Keynesian model with debt accumulation in the spirit of Leeper (1991) as a 'lab', because this model generates multiple equilibria under pure discretion, and under full commitment there are two distinct determinate regimes. We study how these properties change over the continuum of intermediate cases between commitment and discretion. We find that although multiple equilibria exist for high degrees of precommitment, even a small degree of precommitment selects a unique equilibrium for a wide range of parameters. We discuss the stability properties of policy equilibria which can be used to design an equilibrium selection criterion. We also demonstrate very different welfare implications for different policy equilibria.

Key Words: Limited Commitment, Commitment, Discretion, Multiple Equilibria

JEL References: E31, E52, E58, E61, C61

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†Address: Department of Economics, Business School, University of Exeter, Streatham Court, Rennes Drive, Exeter EX4 4PU; e-mail ch308@exeter.ac.uk.

‡Address: Department of Economics, Business School, University of Exeter, Streatham Court, Rennes Drive, Exeter EX4 4PU; e-mail t.kirsanova@exeter.ac.uk

# 1 Introduction

In this paper we study existence and uniqueness properties of monetary policy with limited commitment in the Blanchard and Kahn (1980) class of infinite-horizon discrete-time non-singular linear dynamic models that is typically used to study aggregate fluctuations in macroeconomics. Building on research in Schaumburg and Tambalotti (2007) and Debertoli and Nunes (2010) we contribute to the literature by showing the existence of expectations traps under quasi-commitment policy. We also demonstrate different dynamic properties of the arising equilibria that may help to select only one – the best – policy equilibrium. Models with multiple policy equilibria can help to explain the observed volatility of macroeconomic data and can help to suggest how control policies should be improved to avoid these traps.

The framework developed in Schaumburg and Tambalotti (2007) and Debertoli and Nunes (2010) allows us to study a continuum of intermediate cases between commitment and discretion and how the dynamic properties of the economy may change with degree of policy precommitment.[1] In particular, a 'quasi-commitment bridge' may link the economy with a (potentially) non-stabilizing policy under commitment and with multiple policy equilibria under discretion.

It is well known that in models with rational expectations (RE) commitment and discretion policies may imply very different dynamics of the economy. With full commitment the policy maker has complete control over the private sector's expectations about future policy and steers them in a way that furthers his stabilization goals. The policy maker can coordinate all future actions of consequent policy makers, which allows him to choose once, and apply indefinitely, an intertemporal contingency plan (Kydland and Prescott (1977)). In linear quadratic (LQ) models a commitment policy, if it exists, is always unique (Kwakernaak and Sivan (1972), Backus and Driffill (1986)).

With no commitment at all, i.e. under pure discretion, the policy maker does not control the expectations of the private sector and fails to coordinate the actions of consequent policy makers. Under discretion the policy maker optimizes in each period of time and the private sector knows that future policy makers will implement the same decision process in subsequent periods (Oudiz and Sachs (1985), Backus and Driffill (1986), Currie and Levine (1993)). However, in this framework expectation traps and multiple equilibria can arise, because the expectations of the private sector are shaped by anticipations about future policy behavior. Since the policy

[1] Originally, the framework is based on Roberds (1987). Schaumburg and Tambalotti (2007) term limited commitment 'Quasi-commitment' and Debertoli and Nunes (2010) use 'loose commitment'. In this paper we use these terms interchangeably.

maker cannot fully control private sector expectations, those expectations may trap the policy maker into implementing a policy that validates them. The trap is closed if it is less costly for the policy maker to validate the private sector beliefs about future policy than ignoring those expectations.[2]

Similar to the existing literature on limited commitment we study the properties of limited commitment by example, in a 'laboratory': we use a New Keynesian (NK) model with government debt accumulation.[3] The model describes economic behavior that is familiar from the literature on the fiscal theory of the price level in the spirit of Leeper (1991). We demonstrate that coordination failures can occur and multiple equilibria arise, because – similar to pure discretion – a policy maker with limited commitment can neither completely control the expectations of the private sector, nor can he coordinate the actions of all future policy makers. We also show that – similar to commitment – there can be gaps between stable regimes if the policy maker discounts the future.

However, despite the fact that multiplicity of policy equilibria survives for very high degrees of precommitment, even a small degree of precommitment eliminates the multiplicity for most realistic parameter values. We also find that, compared to the results in Schaumburg and Tambalotti (2007), the quantitative effect of a default on past promises is substantially reduced if there exists a predetermined variable. At the same time, the large gain due to even small degrees of precommitment, documented in Schaumburg and Tambalotti (2007), does not apply to all equilibria. We also show that if the private sector expects future re-optimizations which are never realized, the cost of controlling such an economy rises without bounds in all but the best equilibrium. The policy in the best equilibrium is robust to this sequence of events, and this might properly help to discriminate against other equilibria. We also compute optimal values of parameters that determine the fiscal stance in this model. Finally, we discuss a numerical approach to find a limited commitment equilibrium in a general LQ RE model with predetermined states and possible multiple equilibria.

The paper is organized as follows. In the next Section we introduce a simple NK model with debt accumulation. In Section 3 we review properties of discretion and commitment policies

[2] Models with multiple discretionary equilibria are presented in Lockwood and Philippopoulos (1994), King and Wolman (2004), Albanesi et al. (2003), Blake and Kirsanova (2008) and Dennis and Kirsanova (2009).

[3] To introduce the concept of monetary policy with limited commitment and to investigate its basic dynamic properties and the resulting welfare gains Schaumburg and Tambalotti (2007) use a simple NK model in LQ RE framework. Debertoli and Nunes (2010) generalize the concept to a general non-linear setting and look at the same properties in a simple flexible price model with government debt and fiscal policy. In contrast to the NK model in Schaumburg and Tambalotti (2007) our model has a predetermined state variable, government debt, which is crucial for the analysis.

for our model. We then study the properties of limited commitment policy and study welfare implications in Section 4. Section 5 concludes. Finally, the Appendix presents a numerical algorithm to find policy with limited commitment.

## 2 The Model

This Section presents a simple NK model with government debt. We use it as a laboratory to study the dynamic properties of an economy under monetary policy with limited commitment for several reasons. First, unlike the model in Schaumburg and Tambalotti (2007) this model has an endogenous predetermined state variable, government debt, which is affected by policy. The presence of such a variable is crucial to generate multiple equilibria under discretionary policy, which is a limiting case of quasi-commitment policy (Blake and Kirsanova (2008)). Second, the model is simple enough to derive most of our results analytically.

We adopt the model from Benigno and Woodford (2003).[4] The model describes an economic behavior familiar from the literature on the fiscal theory of the price level.[5] The economy consists of a representative household, a representative firm that produces the final good, a continuum of intermediate goods-producing firms and a monetary and fiscal authority. The intermediate goods-producing firms act under monopolistic competition and produce according to a production function that depends only on labor. Goods are combined via a Dixit and Stiglitz (1977) technology to produce aggregate output. Firms set their prices subject to a Calvo (1983) price rigidity. Households choose their consumption and leisure and can transfer income through time through their holdings of government bonds. All agents can observe and affect the accumulation of the real government debt. The accumulation of government debt must depend on the fiscal stance. Hence, in the model there is a non-optimizing fiscal authority that faces a stream of exogenous public consumption. These expenditures are financed by levying income taxes and by issuing one-period risk-free nominal bonds. We assume that the fiscal authority imposes a simple proportional rule for the tax rate: if the real debt is higher (lower) than in the steady state the tax rate rises (falls). We shall refer to the tax rate as 'taxes' and to the parameter of the proportional rule as the 'fiscal feedback'. The size of the fiscal feedback measures the strength of the fiscal stabilization of debt and, as we shall show, plays an important role in the model. The presence of the non-optimizing fiscal authority in the economy can be captured by this single parameter $\mu$.

[4] It was also used in Blake and Kirsanova (2008).

[5] See e.g. Leeper (1991), Woodford (2001), Davig and Leeper (2006), Favero and Monacelli (2005).

We assume that all public debt consist of riskless one-period bonds. The nominal value $\mathcal{B}_t$ of end-of-period public debt then evolves according to the following law of motion:

$$\mathcal{B}_t = (1 + i_{t-1})\,\mathcal{B}_{t-1} + P_t G_t - \tau_t P_t Y_t, \tag{1}$$

where $\tau_t$ is the share of national product $Y_t$ that is collected by the government in period $t$, and government purchases $G_t$ are treated as exogenously given and time-invariant. $P_t$ is aggregate price level and $i_t$ is interest rate on bonds. The national income identity yields

$$Y_t = C_t + G_t \tag{2}$$

where $C_t$ is private consumption. For analytical convenience we introduce the real value of debt at maturity $B_t = (1+i_{t-1})\mathcal{B}_{t-1}/P_{t-1}$, observed at the beginning of period $t$, so that (1) becomes

$$B_{t+1} = (1 + i_t)\left(B_t \frac{P_{t-1}}{P_t} - \tau_t Y_t + G_t\right). \tag{3}$$

We assume that fiscal authorities operate with simple mechanistic feedback rule that relates the tax rate $\tau_t$ and $B_t$

$$\tau_t = \tau_o \left(\frac{B_t}{B_o}\right)^{\mu \frac{B_o}{Y_o}}, \tag{4}$$

where $\tau_o$ and $B_o$ are steady state values of tax rate and real debt correspondingly.

Log-linearizing (3) and (4) yields

$$b_{t+1} = \frac{B_o}{Y_o}\iota_t + \frac{1}{\beta}\left((1 - \mu\tau_o)\, b_t - \frac{C_o}{Y_o}\tau_o c_t - \frac{B_o}{Y_o}\pi_t\right), \tag{5}$$

where $b_t = \frac{B_o}{Y_o}\ln\left(\frac{B_t}{B_o}\right)$, $c_t = \ln\left(\frac{C_t}{C_o}\right)$, $\iota_t = \ln\frac{(1+i_t)}{(1+i_o)}$ and subscript $o$ denotes steady state values of corresponding variables in zero inflation steady state. The private sector's discount factor $\beta = 1/(1 + i_o)$. To make the model particularly simple we assume $B_o = 0$, which eliminates the first-order effect of the interest rate and inflation on debt, and obtain the final version of linearized debt accumulation equation:

$$b_{t+1} = \rho b_t - \eta c_t \tag{6}$$

where the parameter $\rho = (1 - \mu\tau_o)/\beta$ is a function of the tax rate. From the definition of $\rho$ follows that the higher the fiscal feedback parameter $\mu$ the faster debt is stabilized. Parameter $\eta = C_o\tau_o/(\beta Y_o)$ describes the sensitivity of debt to the tax base.

The derivation of the appropriate Phillips curve that describes Calvo-type price-setting decisions of monopolistically competitive firms is standard (Benigno and Woodford (2003), Sec. A.5) and marginal cost is a function of output and taxes. A log-linearization of the aggregate supply relationship around the zero-inflation steady state yields the following (deterministic) New Keynesian Phillips curve

$$\pi_t = \beta\pi_{t+1} + \zeta\left(\left(\frac{1}{\sigma} + \frac{\theta}{\psi}\right)c_t + \frac{\tau_o}{(1-\tau_o)}\tau_t\right)$$

where $\zeta$ is the slope of Phillips curve, $\tau_t = \ln\left(\frac{\tau_t}{\tau_o}\right)$ and $\sigma$ and $\psi$ are parameters of the private sector utility function, and $\theta = C_o/Y_o$. Substituting in the log-linearized (2) and (4) yields

$$\pi_t = \beta\pi_{t+1} + \kappa c_t + \nu b_t, \tag{7}$$

where $\nu = \mu\kappa\tau_o/\left(1-\tau_o\right)$ and $\lambda = \kappa\left(1/\sigma + \theta/\psi\right)$.

Finally, the model is described by two equations, the debt accumulation equation (6) and Phillips curve (7). The aggregate agents' decision variable is inflation, $\pi_t$, and we assume that the policy maker chooses consumption $c_t$. Debt $b_t$ is the aggregate predetermined state variable in period $t$. The economy evolves according to (6) and (7) and the initial state $\bar{b}$ is known to all agents. In contrast to the standard NK model policy $c_t$ affects the predetermined state $b_{t+1}$.

The inter-temporal welfare criterion of the policy maker is defined by the quadratic loss function[6]

$$L = \frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left(\pi_t^2 + \lambda c_t^2\right). \tag{8}$$

The policy maker knows the laws of motion (6)-(7) of the aggregate economy and takes them into account when formulating policy.

# 3 Preliminaries: Discretion and Commitment

We shall compare the dynamics of the model under quasi-commitment policy with dynamics under the two limiting cases, discretion and commitment.[7] This Section gives all necessary definitions and presents solutions to these two limiting cases in a comparable form.

[6] The criterion is derived under the assumption of steady state labour subsidy. Here parameter $\alpha$ is a function of model parameters, $\alpha = \theta\lambda/\epsilon$, and $\epsilon$ is the elasticity of substitution between any pair of monopolistically produced goods.

[7] In this section we largely follow the approach and results in Blake and Kirsanova (2008) and in Kirsanova and Wren-Lewis (2011), but present results a the form convenient for our purposes.

In this and the next Section we shall work with the deterministic version of the model. After having solved the deterministic version of the LQ RE model it is easy to reinstate the stochastic components. Note that because of the certainty equivalence result the dynamics of the economy is fully determined by its deterministic part.

### 3.1 Discretionary Policy

Under discretion there is a sequence of policy makers: each period a new policy maker arrives in office. The new policy maker chooses the best policy knowing that he stays in office for only one period and the next-period policy maker will re-optimize again.[8] The law of motion of the aggregate economy (6)-(7) is known by the policy maker and taken into account when he formulates the optimal policy. Furthermore, the policy maker finds the best action every period and knows that future policy makers have the freedom to change policy, but will apply the same decision process. At every point in time $t$ the decision rules of each agent are linear functions of the current state

$$\begin{aligned} c_t &= c_b b_t, && (9) \\ \pi_t &= \pi_b b_t. && (10) \end{aligned}$$

Note that from

$$\pi_{t+1} \overset{eq.(10)}{=} \pi_b b_{t+1} \overset{eq.(6)}{=} \pi_b \left(\rho b_t - \eta c_t\right) \overset{eq.(7)}{=} \frac{1}{\beta}\pi_t - \frac{\kappa}{\beta}c_t - \frac{\nu}{\beta}b_t,$$

it follows that the private sector's decision can also be written as

$$\pi_t = \left(\beta\rho\pi_b + \nu\right) b_t + \left(\kappa - \beta\eta\pi_b\right) c_t. \tag{11}$$

The policy maker moves first within each period and the private sector observes the action of the policy maker. Thus, the private sector takes into account the 'instantaneous' influence of the policy choice measured by $\left(\kappa - \beta\eta\pi_b\right)$.

We can give now a more precise definition of discretionary policy: A policy determined by (9) is *discretionary* if the policy maker *finds it optimal* to follow it every period $s > t$, given the private sector (i) observes the current policy, (ii) knows that future policy makers re-optimize and use the same decision process, (iii) expects policy (9) will be implemented in all future periods.

[8] Our definition of discretionary policy is standard and follows Oudiz and Sachs (1985), Backus and Driffill (1986), see also Clarida et al. (1999).

We can write the criterion for optimality as

$$Sb_t^2 = \min_{c_t} \left( \left( \pi_t^2 + \lambda c_t^2 \right) + \beta S b_{t+1}^2 \right), \tag{12}$$

subject to constraints (6) and (11).

One can solve the problem using Lagrange multipliers. The Lagrangian can be written as

$$\mathcal{L}_t^d = \frac{1}{2} \left( \pi_t^2 + \lambda c_t^2 \right) + \beta \frac{1}{2} S b_{t+1}^2 + \xi_{t+1} \left( \rho b_t - \eta c_t - b_{t+1} \right) + \phi_{t+1} \left( \pi_t - \kappa c_t - \nu b_t - \beta \pi_b b_{t+1} \right). \tag{13}$$

This approach exploits the intertemporal representation (6)-(7) together with the underlying assumption that private sector expectations about its own future decisions will be necessarily a function of the future state, which is $\pi_{t+1} = \pi_b b_{t+1}$ for our model.

Only current period constraints matter for the policy maker and the first order conditions can be written as

$$0 = \beta S b_{t+1} - \xi_{t+1} - \beta \pi_b \phi_{t+1}, \tag{14}$$

$$0 = \pi_t + \phi_{t+1}, \tag{15}$$

$$0 = \lambda c_t - \eta \xi_{t+1} - \kappa \phi_{t+1}, \tag{16}$$

$$0 = \rho b_t - \eta c_t - b_{t+1}, \tag{17}$$

$$0 = \beta \pi_b b_{t+1} + \kappa c_t + \nu b_t - \pi_t. \tag{18}$$

These conditions yield the linear decision rule for the policy maker:

$$c_t = -\frac{\left( \left( \beta \rho \pi_b + \nu \right) \left( \kappa - \pi_b \eta \beta \right) - \beta \rho \eta S \right)}{\left( \left( \kappa - \beta \eta \pi_b \right)^2 + \lambda + \beta \eta^2 S \right)} b_t = c_b b_t. \tag{19}$$

The coefficient $c_b$ in (19) determines the optimal policy feedback on the predetermined state variable, $b_t$, where the feedback coefficient is a function of $S$. $S$ can be found from the optimality criterion (12):

$$S = \frac{\left( \left( \beta \rho \pi_b + \nu \right) + \left( \kappa - \beta \eta \pi_b \right) c_b \right)^2 + \lambda c_b^2}{\left( 1 - \beta \left( \rho - \eta c_b \right)^2 \right)}. \tag{20}$$

Hence the first order conditions (11), (19) and (20) define $c_b$, $\pi_b$ and $S$.

We can obtain all solutions in the following way. Suppose the policy maker guesses the response of the private sector to the state, $\pi_b$. Then the optimal discretionary policy is given by the pair (19) and (20). Given $c_b$ the optimal response $\pi_b^*$ of the private sector is given by (11). Therefore, for every – not necessarily optimal – $\pi_b$ we can compute a unique $\pi_b^*$ and plot the dependence $\pi_b^*(\pi_b)$, see the first panel in Figure 1. Clearly, if $\pi_b = \pi_b^*$ we have a solution to the discretionary problem.

For our base line calibration the graph of $\pi_b^*(\pi_b)$ intersects the $45^o$ degree line in three points labelled $A$, $B$ and $C$, so we have three discretionary policy equilibria.[9] A moderate inflation, set by the firms in response to a given debt level, $\pi_b$, increases the marginal return to a policy decision that increases consumption in response to this level of debt, $c_b$. Higher consumption raises demand and firms will increase their response to debt, $\pi_b^*$. This complementarity ensures steepness of $\pi_b^*(\pi_b)$ and three equilibria arise.

### 3.2 Commitment Policy

Under the full commitment policy the policy maker optimizes only once, in the initial moment. It chooses a contingency plan, which is than applied indefinitely but can be implemented sequentially. If there is a change of policy makers, the subsequent policy maker continues the policy of its predecessor; therefore we can assume that there is only one policy maker which takes office in period zero and stays infinitely.

When optimizing, the policy maker internalizes the effect of its choice on private sector's expectations and solves the following Lagrangian

$$\mathcal{L}^c = \sum_{t=0}^{\infty} \beta^t \left( \frac{1}{2}\left(\pi_t^2 + \lambda c_t^2\right) + \xi_{t+1}\left(\rho b_t - \eta c_t - b_{t+1}\right) + \phi_{t+1}\left(\pi_t - \kappa c_t - \nu b_t - \beta \pi_{t+1}\right) \right).$$

The corresponding first order conditions are:

$$\begin{aligned} 0 &= -\xi_t + \rho\beta\xi_{t+1} - \nu\beta\phi_{t+1}, && (21)\\ 0 &= \pi_t + \phi_{t+1} - \phi_t, && (22)\\ 0 &= \lambda c_t - \eta\xi_{t+1} - \kappa\phi_{t+1}, && (23)\\ 0 &= \rho b_t - \eta c_t - b_{t+1}, && (24)\\ 0 &= \beta\pi_{t+1} + \kappa c_t + \nu b_t - \pi_t, && (25) \end{aligned}$$

for $t \geq 0$; with initial conditions $b_0 = \bar{b}$ and $\phi_0 = 0$, and the transversality condition $\lim_{t\to\infty} b_t < \infty$.

If the system (6)-(7) is controllable, there always exists a unique path $\{c_t, \pi_t, b_t\}_{t\geq 0}$ which (i) satisfies system (21)-(25) and the initial conditions and (ii) all eigenvalues of the resulting

[9] The benchmark calibration follows Schaumburg and Tambalotti (2007) and Blake and Kirsanova (2008). The model's frequency is quarterly. The subjective discount rate $\beta$ is set to 0.99, the government share of total output $1-\rho$ is 0.25. The elasticity of intertemporal substitution $\sigma$ is 1/2, the Frisch elasticity of labor supply $\varphi = 1/2$, an elasticity of demand $\epsilon$ of 7. The Calvo parameter $\gamma = 0.75$. Fiscal feedback $\mu$ is set to 0.075.

transition matrix are less than $1/\sqrt{\beta}$ in modulus (see, e.g. Kwakernaak and Sivan (1972), Backus and Driffill (1986)).[10] For the rest of this paper we use the following definition: The economy is *stabilized* by a policy if all eigenvalues of the transition matrix are inside the unit circle. If the economy is stabilized by a policy we call such a policy *stabilizing*. In general, because $\beta < 1$ a stabilizing commitment policy may not exist in the LQ RE class of problems.

One way to solve the system (21)-(25) is to use the Schur decomposition, see e.g. Söderlind (1999). Alternatively, and more convenient for our purpose, we can also solve the system using an iterative scheme. The solution to the LQ commitment problem can be written in the following form

$$\pi_t = \pi_b b_t + \pi_\phi \phi_t, \tag{26}$$

$$c_t = c_b b_t + c_\phi \phi_t, \tag{27}$$

$$\xi_t = \xi_b b_t + \xi_\phi \phi_t. \tag{28}$$

System (21)-(25) yields the following matrix discrete algebraic Riccatti equation[11]:

$$\begin{bmatrix} c_b & c_\phi \\ \pi_b & \pi_\phi \\ \xi_b & \xi_\phi \end{bmatrix} = \begin{bmatrix} -\left(\lambda + \eta^2 \xi_b\right) & \left(\eta \xi_\phi - \kappa\right) & 0 \\ \kappa - \beta \eta \pi_b & \beta \pi_\phi - 1 & 0 \\ -\beta \eta \rho \xi_b & \beta\left(\nu + \rho \xi_\phi\right) & -1 \end{bmatrix}^{-1} \begin{bmatrix} -\eta \rho \xi_b & \eta \xi_\phi - \kappa \\ -\left(\nu + \beta \rho \pi_b\right) & \beta \pi_\phi \\ -\beta \rho^2 \xi_b & \beta\left(\nu + \rho \xi_\phi\right) \end{bmatrix} \tag{29}$$

Therefore, we can guess all feedback coefficients in (26)-(28) and thus in the right hand side of (29). Then, equation (29) gives an update of these coefficients. In the next step we update the right hand side of (29) and iterate until convergence. The algorithm will converge (Lancaster and Rodman (1995)).

This method allows us to compare the solution of the commitment problem with the discretionary solution. Again, suppose the response of the private sector to debt, $\pi_b$, is given. We can guess the other feedback coefficients in the system (26)-(28) and iterate the Riccati equation (29), *but do not update* $\pi_b$. If the procedure converges, we have obtained the optimal response of the policy maker to the private sector decision, provided that the private sector responds to the Lagrange multiplier (set by the policy maker) in an optimal way. Then, we iterate the Riccati equation *once* to obtain $\pi_b^*$. A solution to the commitment problem implies $\pi_b^* = \pi_b$. The graph of $\pi_b^*(\pi_b)$ intersects the $45^o$ degree line in one point labelled $A$, see the second panel in Figure 1, and we can verify with standard methods (Söderlind (1999)) that this point is, indeed, a solution. For the base line calibration the economy is stabilized by policy in the unique equilibrium $A$.

[10] System (6)-(7) is controllable if $\rho \neq 0$ and $\eta \neq 0$.

[11] See Appendix A.

Although the base line calibration delivers a stabilizing solution, note that if the fiscal feedback is weak, $0 < \mu < \mu^*$, where $\mu^* = (1 - \tau_o)(1 - \beta)\kappa / (\tau_o((1 - \tau_o)\kappa - \zeta\theta\tau_o))$, the economy is not stabilized by policy. The optimal monetary policy still delivers a finite value of the loss function (8), but all variables exhibit slow explosion with a rate of explosion less than $1/\sqrt{\beta}$. This solution should be disregarded as it violates the assumption of a finite working week.[12]

Finally, note that equation (22) implies price stability: if $\phi_t = 0$ and $\lim_{t\to\infty} \phi_t = 0$ it follows that $\sum_{t=0}^{\infty} \pi_t = 0$.

# 4 Quasi-Commitment

This Section studies monetary policy within a limited commitment framework. We discuss the continuum of intermediate cases between commitment and discretion. We want to understand (i) how a 'quasi-commitment bridge' links the economy with a (potentially) non-stabilizing policy under commitment and multiple policy equilibria under discretion, and (ii) whether quasi-commitment helps to eliminate some of the (multiple) equilibria.

## 4.1 Policy Equilibria

The quasi-commitment policy, as introduced in Schaumburg and Tambalotti (2007), also assumes sequential policy making. A new policy maker is appointed with a constant and exogenous probability $\alpha$ every period. When a new policy maker takes office, he reneges on the promises of his predecessor and commits to a new policy plan that is optimal at the time of the change. All agents understand the possibility and the nature of this change and form expectations accordingly. The private sector knows that a new policy maker will re-optimize, therefore it doubts the reliability of outstanding promises.

As in Schaumburg and Tambalotti (2007) and Debertoli and Nunes (2010) we assume that the policy maker's tenure in office depends on a sequence of exogenous i.i.d. Bernoulli signals $\{\eta_t\}_{t\geq 0}$ with $E[\eta_t] = \alpha$. If $\alpha = 1$ the policy authority acts under full discretion and every period a new policy maker arrives in office and re-optimizes the planning problem. If $\alpha = 0$ the policymaker stays in office infinitely long and keeps his promises.

Schaumburg and Tambalotti (2007) and Debertoli and Nunes (2010) demonstrate that the optimization problem under limited commitment can be expressed by the following Lagrangian

[12] This result was shown in a similar model in Schmitt-Grohe and Uribe (2004) and in Kirsanova and Wren-Lewis (2011).

$$\mathcal{L}^{qc} = \sum_{t=0}^{\infty} (\beta(1-\alpha))^t \Big( \frac{1}{2} \left( \pi_t^2 + \lambda c_t^2 + \beta\alpha S b_{t+1}^2 \right) + \xi_{t+1} \left( \rho b_t - \eta c_t - b_{t+1} \right) \\ + \phi_{t+1} \left( \pi_t - \kappa c_t - \nu b_t - \beta(1-\alpha)\pi_{t+1} - \beta\alpha\pi_b b_{t+1} \right) \Big) \tag{30}$$

for $0 \leq \alpha < 1$. The first order conditions are

$$0 = \beta\alpha S b_t - \xi_t + \rho\beta(1-\alpha)\xi_{t+1} - \nu\beta(1-\alpha)\phi_{t+1} - \beta\alpha\pi_b\phi_t, \tag{31}$$

$$0 = \pi_t + \phi_{t+1} - \phi_t, \tag{32}$$

$$0 = \lambda c_t - \eta\xi_{t+1} - \kappa\phi_{t+1}, \tag{33}$$

$$0 = \rho b_t - \eta c_t - b_{t+1}, \tag{34}$$

$$0 = \beta(1-\alpha)\pi_{t+1} + \beta\alpha\pi_b b_{t+1} + \kappa c_t + \nu b_t - \pi_t, \tag{35}$$

for $t \geq 0$, initial conditions $b_0 = \bar{b}$ and $\phi_0 = 0$, and the transversality condition $\lim_{t\to\infty} b_t < \infty$.

These first order conditions are similar to those for commitment, but depend on parameters, $\{\pi_b, S\}$, that describe the solution to the corresponding optimization problem under discretion.

We can plot the solutions to this system using the approach as in Section 3.2. The solution to (31)-(35) can be written in form of (26)-(28). The corresponding matrix Riccatti equation is similar to (29), but its coefficients depend on $\{\pi_b, S\}$. We can use a similar solution method to find the number of equilibria: suppose we guess the response of the private sector to the state variable, $\pi_b$. Then, we can solve the policy maker's problem under discretion and find the 'first guess' of $S$. In the next step we iterate the Riccati equation, but do not update $\pi_b$. If the procedure has converged, we iterate it once to obtain the update $\pi_b^*$. Solutions to the system (31)-(35) will be among the points where $\pi_b^* = \pi_b$.

For the base line calibration of $\alpha = 1/2$ (which implies an average regime duration of two quarters) the graph of $\pi_b^*(\pi_b)$ intersects the $45^o$ degree line in three points labelled $A$, $B$ and $C$, see the third panel in Figure 1.[13] Therefore, if we move from pure discretionary policy to a policy maker who stays in office on average for two periods all three equilibria surive.

We now discuss how this result arises. Note that if $\alpha = 1$, the policy maker defaults with certainty every period. Then, the Lagrangian (30) takes the form of (13) and we have the problem of discretionary optimization. However, the first order conditions for the limited commitment optimization problem (31)-(35) are left-discontinuous at point $\alpha = 1$. System (31)-(35) does not collapse to (14)-(18), because the past-period constraints still bind for any $\alpha < 1$. Taking

[13] Again, we can verify with alternative methods that these are indeed solutions to the optimization problem.

the limit $\alpha \to 1$ in system (31)-(35) does not eliminate $\phi_s$ – the Lagrange multiplier on the previous-period constraint – in equation (32). Because for any $\alpha < 1$ the private sector does not expect the occurrence of default with certainty in the next period, this property holds at the limit and implies discontinuity of the first order conditions.

In a next step we investigate the dynamic properties and the uniqueness of the solution at the limit $\alpha \to 1$. By continuity these properties will hold for large enough $\alpha < 1$. After the straightforward substitution of $\xi_t$, $c_t$ and $\pi_t$ the first order conditions (31)-(35) collapse to the following system

$$b_{t+1} = \frac{\lambda\rho + (\kappa\rho + \nu\eta)(\kappa - \beta\eta\pi_b)}{\lambda + S\beta\eta^2 + (\kappa - \beta\eta\pi_b)^2} b_t - \frac{\eta(\kappa - \beta\eta\pi_b)}{\lambda + S\beta\eta^2 + (\kappa - \beta\eta\pi_b)^2}\phi_t \tag{36}$$

$$\phi_{t+1} = -\frac{\nu(\lambda + S\beta\eta^2) + \beta\rho(\lambda\pi_b + S\kappa\eta)}{\lambda + S\beta\eta^2 + (\kappa - \beta\eta\pi_b)^2} b_t + \frac{\lambda + S\beta\eta^2}{\lambda + S\beta\eta^2 + (\kappa - \beta\eta\pi_b)^2}\phi_t \tag{37}$$

for $t \geq 0$, where the coefficients depend on the solution to the corresponding discretionary problem, $\pi_b$ and $S$.

All variables in system (36)-(37) are predetermined with initial conditions $\phi_0 = 0$ and $b_0 = \bar{b}$. Therefore for a given solution to the discretionary problem $D = \{\pi_b, S\}$ we can construct a unique path $P = \{b_t, \phi_t | D\}_{t \geq 0}$. The limiting case of first order conditions to the quasi-commitment optimization problem will have as many solutions as the corresponding discretionary optimization problem. Because there are three different discretionary equilibria there are three distinct sets $D^j = \left\{\pi_b^j, S^j\right\}$, $j = 1, 2, 3$ for the base line calibration. Therefore, three paths $P^j = \left\{b_t, \phi_t | D^j\right\}_{t \geq 0}$ satisfy the system (36)-(37). We plot the case $\alpha \to 1$ in the fourth panel in Figure 1. The $\pi_b^*(\pi_b)$ line intersects the $45^o$ degree line in three points, which are the same points as under pure discretion.[14]

However, system (31)-(35) describes the dynamics of the economy in which, although it is expected that new policy makers arrive in office with probability $\alpha$ and renege on the promises of their predecessors, defaults never happen in the realized history and therefore the Lagrange multiplier $\phi_s$ is never reset to zero for $s > t$. The left-discontinuity of the first order conditions at $\alpha = 1$ arise because for any $\alpha < 1$ the realized reoptimization may never happen, but it happens with certainty for $\alpha = 1$. If the consequent policy makers do reset $\phi_s$ to zero with probability $\alpha$, the dynamic properties of the economy are continuous at point $\alpha = 1$.

For a given $\alpha < 1$ the probability of the realized history with no default occurring in the past $K$ periods tends to zero with growing $K$. In this case the stability properties of system (31)-(35)

[14] The shape of $\pi_b^*(\pi_b)$ is different than in Panel I because we take into account the Lagrange multipliers when computing $\pi_b^*(\pi_b)$.

in each quasi-commitment equilibrium are different from the stability properties of the system that describes the expected evolution of the economy in the same equilibrium. In particular, in the limiting case $\alpha \to 1$ the system (36)-(37) has in two of the three equilibria one eigenvalue outside the unit circle. However, because the policy maker almost surely resets the Lagrange multiplier $\phi_s$ to zero in every period $t > 0$, the expected evolution of the economy is described by the following system:

$$b_{t+1} = \frac{\lambda\rho + (\kappa\rho + \nu\eta)(\kappa - \beta\eta\pi_b)}{\lambda + S\beta\eta^2 + (\kappa - \beta\eta\pi_b)^2} b_t, \tag{38}$$

$$\phi_t = 0. \tag{39}$$

We obtain equation (38) which also describes the evolution of debt under pure discretion. The evolution of the economy is a stationary process in every discretionary equilibrium $D^j$.

More generally, the expected evolution of the economy following an initial disturbance is obtained by taking averages of all possible evolutions, integrated over the distribution of the corresponding re-optimization draws. The expected evolution of the economy is described by the following transition matrix: $\alpha \begin{bmatrix} M_{yy} & 0 \\ 0 & 0 \end{bmatrix} + (1-\alpha) M$ in (46), see the Appendix for details. The stability properties of this transition matrix, which represent the dynamic properties of the economy under a limited commitment policy, are different from the stability properties of the system (36)-(37). This system describes only one of many possible re-optimization histories.[15]

This inconsistency between expected and observed paths can destabilize the economy (and ruin the agent's finances). However, this does not happen in all equilibria. If defaults do happen and for every time $t > 0$ there is at least one period $s > t$ when $\phi_s$ is reset to zero, then there are no issues with dynamic instability of the economy in any equilibrium. These defaults also ensure that if there is less-than-full precommitment, there is no price stability under limited commitment policy.

If no reoptimizations happen, while they are expected by the private sector our numerical analysis shows that the policy maker can control the economy only in equilibrium $A$ for all relevant paramter values. In equilibrium $C$ the absence of re-optimizations implies an unbounded cost of controlling the inflation expectations of the private sector, as we illustrate in the next section. Although this instability can be used as a reason to discriminate against all but equilibrium $A$, such a criterion would eliminate equilibrium $C$ for all $\mu > 0$, despite its empirical relevance documented in e.g. Davig and Leeper (2006) and Favero and Monacelli (2005). Mak-

[15] Using an analogy with a roulette game, system (36)-(37) describes the history when 'red' never realizes while it is expected – and it is bet on – with probability 1/2.

ing the probability of default endogenous might produce a less crude stability-based criterion which would not necessarily rule out equilibrium $C$ for the whole range of relevant parameters. The observed property only suggests that policy in equilibrium $A$ is robust to a (infinitely) long sequence of unexpected events.

Finally, the expected evolution of the economy under limited commitment policy is described by a stationary process in all three equilibria, $A$, $B$ and $C$ in Panel III. Panels II, V and VI in Figure 1 demonstrate that with smaller probability of default only one limited commitment equilibrium survives. Figure 1 plots equilibria for a particular value of the fiscal feedback parameter, $\mu = 0.075$. We shall demonstrate in the next section that multiple equilibria survive for a substantial degree of precommitment only if $\mu$ is relatively small.

## 4.2 Impulse Responses

In this Section we look at the impulse response functions to understand the dynamics of our model under a limited commitment policy better. We now use the stochastic version of the model to illustrate the effects of limited commitment on the shock transmission mechanism. As a benchmark we also plot the impulse responses of the two discretionary equilibria, $A$ and $C$, and under full commitment.

In Figure 2 we show the responses of key variables to a positive unit cost push shock. Under commitment (the blue dotted line) the policy maker engineers a fall in private consumption, which will dampen marginal costs. However, in contrast to discretion, the policy maker keeps consumption for several periods below the steady state. This policy allows the policy maker to lower expected future inflation and ensures price stability in the long run. Government debt initially increases due to the fall in consumption, but is brought back to the steady state with higher taxes.

Discretionary equilibria $A$ and $C$ can be described as policy regimes under active and passive monetary policy. If the fiscal feedback parameter $\mu$ is relatively large then an increase in public debt is practically eliminated by fiscal policy within few periods and the policy maker can focus on inflation stabilization. The equilibrium behavior of the discretionary monetary policy maker and of the private sector is, therefore, similar to the one in the standard New Keynesian model. The policy maker cuts consumption to lower marginal costs today and to place downward pressure on inflation. Due to the decrease in consumption, government tax revenues will fall, which leads to a rise in government debt. In subsequent periods the tax rate increases to guarantee fiscal solvency. This is a low-inflation-volatility equilibrium as the firms set relatively low inflation anticipating low consumption in the future. In equilibrium $C$ the fiscal feedback parameter $\mu$ is relatively small. Therefore fiscal policy can not ensure fiscal solvency and monetary policy remains passive. In this case monetary policy cannot decrease marginal costs by much in the initial period, because the accompanied fall in consumption would result in a large accumulation of government debt, due to the lower tax base. Because in this equilibrium the monetary policy maker has to ensure fiscal solvency, he rises consumption after the first period and therefore also tax revenues. This policy ensures that government debt will be stabilized. However, this implies a high-inflation-volatility equilibrium as firms set inflation relatively high, reacting to anticipated high demand in the future.

Following Schaumburg and Tambalotti (2007) we plot three different types of impulse responses under quasi-commitment policy. We set $\alpha = 1/2$, which implies an average regime duration of two quarters.

Panel I of Figure 2 shows the impulse response functions of Type (i). These impulse responses demonstrate the evolution of the economy if no reoptimization happens over the horizon of interest, while the private sector expects them to happen every period with probability 1/2. In this scenario a central banker stays in office unexpectedly long, which becomes more and more unrealistic over time. To generate impulse responses we use the transition matrix given by conditions (31)-(35). Similar to discretion we plot the two quasi-commitment equilibria $A$ and $C$. We use solid and dash-dotted lines correspondingly. Compared to the full commitment policy, quasi-commitment policy in the active monetary policy equilibrium $A$ delivers a stronger and longer lasting decrease in consumption. As reoptimizations are expected to happen the price setters expect future policy makers to increase consumption and therefore expect a high inflation in the future. Therefore, if the policy maker wants to exploit private sector expectations he has to pay a higher cost in from of a stronger recession. In the absence of reoptimizations this results in stronger future deflation and higher debt, compared to commitment.

Type (i) impulse responses under quasi-commitment policy in equilibrium $C$ are explosive. In this case the 'passive' monetary policy is not able to stabilize inflation, while trying to keep debt under control. After the shock occurred the policy maker cannot move consumption by much, since he has to avoid excessive debt accumulation. This is a similar behaviour as in discretionary equilibrium $C$. Because the private sector expects defaults in the future and hence high future inflation, inflation can only be controlled with low demand. However, lower consumption would result in excessive debt accumulation. Therefore negative consumption unwinds the attempt of the central bank to ensure fiscal solvency and the economy exhibits explosive behavior. As the fourth chart in the first panel shows, the Lagrange multiplier $\phi_s$ which measures the shadow price of controlling the private sector inflation expectations is much higher in equilibrium $C$ and explodes with time.[16] The result is not surprising, given that the monetary policy maker has to control debt in the passive equilibrium. This task becomes incompatible with inflation stabilization if expected defaults do not happen.

Impulse responses of Type (ii) in Panel II of Figure 2 characterize a more typical behavior of the economy under quasi-commitment. Suppose reoptimizations happen in periods 2, 3, 6 and 8 after the initial shock. In each of these periods the reoptimizing policy maker reneges on the plan of its predecessor. When the policy maker defaults on the promises of his predecessor, he resets the predetermined Lagrange multiplier to zero. The policy maker takes this opportunity to end the promised recession of his predecessor and raises consumption back to its initial level. The increase in consumption also leads to a faster reduction of government debt.

Interestingly, while in Schaumburg and Tambalotti (2007) the quantitative effect of a reoptimization is comparable to the effect of the initial shock, our model generates much smaller effects in both quasi-commitment equilibria; the jumps are much smaller. When the policy maker reneges on previous promises and reoptimizes, it faces the accumulated level of debt. The stock of debt serves as consumption smoothing vehicle and accumulates very slowly under full commitment. The ability of the (quasi-) committing policy maker to manipulate private sector expectations to some extent reduces the need to cut debt abruptly. In other words, the presence of the debt stock works as a commitment device and this results in relatively small values of the Lagrange multiplier, and relatively small costs of its resetting.

Type (iii) impulse responses (Panel III in Figure 2) are the ex ante averages of all the possible conditional IRFs integrated over the distribution of the corresponding reoptimization draws. Therefore they demonstrate the expected evolution of the system following the initial shock. Naturally, they are in between the IRF of the respective discretionary equilibria and the IRF under full commitment.

### 4.3 Welfare Analysis

The first panel in Figure 3 plots the level of the welfare loss as function of the fiscal feedback parameter $\mu$ for several degrees of precommitment $\alpha$. Following Schaumburg and Tambalotti (2007) we re-scale the welfare values by normalizing the welfare loss under the best discretionary equilibrium to one. This gives a clear picture of the relative gain in welfare for different values

[16] This Lagrange multiplier is set on the Phillips curve in the optimization problem of the policy maker.

of $\alpha$ and makes our results directly comparable.

The line for $\alpha = 1$ demonstrates that for $\mu = 0.075$ we have three discretionary equilibria labelled $A^d$, $B^d$ and $C^d$ and the loss is the smallest in equilibrium $A^d$. If $\mu$ is greater than some threshold, i.e. fiscal policy controls debt tightly, then only the best equilibrium $A$ survives. If fiscal policy does not control debt or controls it too weakly then only the worst equilibrium $C$ survives (see Blake and Kirsanova (2008)).

Lines for $\alpha = 1/2, \alpha = 1/8$ and $\alpha = 1/100$ plot welfare losses for the respective quasi-commitment regimes. We use solid lines if the quasi-commitment policy is stabilizing and dotted lines otherwise. It is apparent that if the fiscal feedback $\mu$ is very small than the economy is not stabilized in equilibria $A$ and $B$, but stabilized in equilibrium $C$. Under full commitment, $\alpha = 0$, the economy is non-explosive in $\mu = 0$ and is stabilized for every $\mu > \mu^*$. So, for every degree of precommitment there are at least two determinate regimes, for small and large values of the fiscal feedback parameter $\mu$.

Schaumburg and Tambalotti (2007), who solve a standard NK model without government debt, demonstrate that only a small increase in the degree of precommitment leads to a substantial welfare gain. In our model a reduction of the probability of default to $\alpha = 1/2$ does not eliminate any equilibria for $\mu = 0.075$ and their relative welfare-related ranking remains the same. The initial gap between the loss in the best discretionary equilibrium $A$ and commitment is nearly halved. A further reduction in $\alpha$ demonstrates that our result is consistent with Schaumburg and Tambalotti (2007): the gains from even minimal levels of credibility are substantial and the effect of credibility is clearly non-linear. Furthermore, 95% of the gains are produced after about seven years, see panels II and III in Figure 3.

Debertoli and Nunes (2010) solve a non-linear model with flexible prices and find a qualitatively different result: the loss is reduced only slowly with higher degrees of precommitment. Although our model is quite different from theirs, our results for the *worst* equilibrium $C$ are similar. For example, if $\mu = 0.015$ then all three equilibria still exist for $\alpha = 1/50$. For this degree of precommitment the initial gap between the loss in the *worst* discretionary equilibrium $C$ and under commitment is reduced only by 5%. Panels II and III demonstrate that an increase in the degree of precommitment does not always result in large welfare gains, so the results in Schaumburg and Tambalotti (2007) and Debertoli and Nunes (2010) do not necessary contradict each other.

Finally, we compute the optimal value of the fiscal feedback parameter $\mu^{opt}$. We define $\mu^{opt}$ as the value which delivers the minimum social loss if the economy is hit by a cost-push shock. We mark $\mu^{opt}$ with stars in Figure 3. Under the full commitment the minimum loss is achieved at the left boundary of the stable regime , $\mu^{opt} = \mu^*$. If some (limited) precommitment is possible then $\mu^{opt}$ is an interior point of regime $A$, but only slightly bigger than the point of discontinuity between regimes $\mu^*(\alpha)$. Overall, optimal fiscal feedback is small. This extends the results in Kirsanova and Wren-Lewis (2011) to a wider class of policy equilibria.

# 5 Conclusion

In this paper we study monetary policy with limited commitment using a simple New Keynesian model with debt accumulation. We demonstrate the existence of expectations traps similar to those existing under pure discretionary policy. Because the private sector expects eventual re-optimizations the current policy maker formulates its policy based on the forecast of the private sector about future policy makers' policies. We find that there can be at least as many limited commitment policy equilibria as in the corresponding discretionary policy problem.

We demonstrate that although multiple equilibria can survive a substantial degree of precommitment, nearly for all parameter values only a small degree of precommitment is required to select among them and achieve uniqueness.

Depending on which equilibrium prevails, an increase in the degree of precommitment may result in large or small welfare gains. Furthermore, we find that in the presence of a predetermined state variable the cost of default is relatively small.

Finally, we find that different equilibria imply different stability properties of the economy if an infinitely long sequence of no re-optimizations realizes. In this case, we demonstrate that the policy maker is only able to control the economy in the Pareto-dominant equilibrium. A further research might generalize this property to the whole class of LQ RE models with multiple equilibria under limited commitment policy.

# A Commitment FOCs in Form of Riccati Equation (29)

System (21)-(25) can be written as

$$\begin{bmatrix} 0 & 0 & \eta \\ 0 & \beta & 0 \\ 0 & 0 & \rho\beta \end{bmatrix} \begin{bmatrix} c_{t+1} \\ \pi_{t+1} \\ \xi_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & -\kappa \\ -\nu & 0 \\ 0 & \nu\beta \end{bmatrix} \begin{bmatrix} b_t \\ \phi_t \end{bmatrix} + \begin{bmatrix} \lambda & \kappa & 0 \\ -\kappa & 1 & 0 \\ 0 & -\nu\beta & 1 \end{bmatrix} \begin{bmatrix} c_t \\ \pi_t \\ \xi_t \end{bmatrix} \tag{40}$$

$$\begin{bmatrix} b_{t+1} \\ \phi_{t+1} \end{bmatrix} = \begin{bmatrix} \rho & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} b_t \\ \phi_t \end{bmatrix} + \begin{bmatrix} -\eta & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} c_t \\ \pi_t \\ \xi_t \end{bmatrix} \tag{41}$$

Substitute (26)-(28) into both sides of (40) and use (41) to substitute out $b_{t+1}, \phi_{t+1}$. We obtain

$$\begin{bmatrix} -\left(\lambda+\eta^2\xi_b\right) & -\left(\kappa-\eta\xi_\phi\right) & 0 \\ \kappa-\beta\eta\pi_b & \beta\pi_\phi-1 & 0 \\ -\beta\eta\rho\xi_b & \beta\left(\nu+\rho\xi_\phi\right) & -1 \end{bmatrix}\begin{bmatrix} c_t \\ \pi_t \\ \xi_t \end{bmatrix} = \begin{bmatrix} -\eta\rho\xi_b & -\kappa+\eta\xi_\phi \\ -\left(\nu+\beta\rho\pi_b\right) & \beta\pi_\phi \\ -\beta\rho^2\xi_b & \beta\left(\nu+\rho\xi_\phi\right) \end{bmatrix}\begin{bmatrix} b_t \\ \phi_t \end{bmatrix}$$

Substitution of (26)-(28) yields (29).

# B Limited Commitment Policy in General LQ RE Framework

We assume a non-singular linear deterministic rational expectations model, augmented by a vector of control instruments. Specifically, the evolution of the economy is explained by the linear system

$$\begin{bmatrix} y_{t+1} \\ E_t x_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} y_t \\ x_t \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}[u_t] + C\begin{bmatrix} \xi_{t+1} \\ 0 \end{bmatrix}, \tag{42}$$

where $y_t$ is an $n_1$-vector of predetermined variables with initial conditions $y_0$ given, $x_t$ is $n_2$-vector of non-predetermined (or jump) variables with $\lim_{t\to\infty} x_t = 0$, $u_t$ is a $k-$vector of policy instruments of the policy maker, and $\xi_t$ is a vector of i.i.d. shocks with covariance matrix $\Sigma$. For notational convenience we define the $n$-vector $z_t = (y_t', x_t')'$ where $n = n_1 + n_2$. We assume $A_{22}$ is non-singular.

The inter-temporal policy maker's welfare criterion is defined by the quadratic loss function

$$L_0 = \frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t g_t'\mathcal{Q}g_t = \frac{1}{2}\sum_{t=0}^{\infty}\beta^{s-t}\left(z_t'Qz_t + 2z_t'Pu_t + u_t'Ru_t\right). \tag{43}$$

The elements of vector $g_s$ are the goal variables of the policy maker, $g_t = \mathcal{C}(z_t', u_t')'$. Matrix $\mathcal{Q}$ is assumed to be symmetric and positive semi-definite.[17]

Schaumburg and Tambalotti (2007) and then Debertoli and Nunes (2010) demonstrate that the optimization problem can be written as

$$\min E_0\sum_{t=0}^{\infty}(\beta\omega)^t\left(z_t'Qz_t + 2z_t'Pu_t + u_t'Ru_t + \beta\left(1-\omega\right)y_{t+1}'Sy_{t+1}\right) \tag{44}$$

where $\omega = 1-\alpha$, subject to

$$\begin{aligned} y_{t+1} &= A_{11}y_t + A_{12}x_t + B_1u_t + C\xi_{t+1} \\ \omega E_t x_{t+1} + (1-\omega)Hy_{t+1} &= A_{21}y_t + A_{22}x_t + B_2u_t \end{aligned}$$

[17] It is standard to assume that $R$ is symmetric positive definite (see Anderson et al. (1996), for example). However, since many economic applications involve a loss function that places no penalty on the control variables, we note that the requirement of $\mathcal{Q}$ being positive definite can be weakened to $\mathcal{Q}$ being positive semi-definite if additional assumptions about other system matrices are met (Clements and Wimmer (2003)). The analysis in this paper is valid for $R \equiv 0$.

where $H$ and $S$ are components of solution to the corresponding discretionary problem, $x_t = Hy_t$ and the loss is $L_t(y_t) = \frac{1}{2} y_t' S y_t$.

The first order conditions to the appropriate Lagrangian

$$\begin{aligned}
\mathcal{L}^{qc} &= \sum_{t=0}^{\infty} (\beta\omega)^t \left( z_t' Q z_t + 2 z_t' P u_t + u_t' R u_t + \beta (1-\omega) y_{t+1}' S y_{t+1} \right. \\
&\quad + 2\varphi_{t+1}' \left( A_{21} y_t + A_{22} x_t + B_2 u_t - \omega x_{t+1} - (1-\omega) H y_{t+1} \right) \\
&\quad \left. + 2\psi_{t+1}' \left( A_{11} y_s + A_{12} x_s + B_1 u_s + \xi_{t+1} - y_{s+1} \right) \right)
\end{aligned}$$

can be written as

$$\begin{aligned}
&\begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & \beta A_{22}' & 0 & 0 & \beta A_{12}' \\ 0 & B_2' & 0 & 0 & B_1' \\ (1-\omega) H & 0 & 0 & \omega I & 0 \\ 0 & \beta\omega A_{21}' & 0 & 0 & \beta\omega A_{11}' \end{bmatrix} \begin{bmatrix} y_{t+1} \\ \varphi_{t+1} \\ u_{t+1} \\ x_{t+1} \\ \psi_{t+1} \end{bmatrix} \qquad (45) \\
&= \begin{bmatrix} A_{11} & 0 & B_1 & A_{12} & 0 \\ -\beta Q_{12}' & I & -\beta P_2 & -\beta Q_{22} & 0 \\ -P_1' & 0 & -R & -P_2' & 0 \\ A_{21} & 0 & B_2 & A_{22} & 0 \\ -\beta\left(\omega Q_{11} + (1-\omega) S\right) & (1-\omega) H' & -\beta\omega P_1 & -\beta\omega Q_{12} & I \end{bmatrix} \begin{bmatrix} y_t \\ \varphi_t \\ u_t \\ x_t \\ \psi_t \end{bmatrix}
\end{aligned}$$

Solution to this system (using Schur decomposition, for example, or iteration Riccati equation as we do in the text) can be written in the form

$$\begin{aligned}
\begin{bmatrix} u_t \\ x_t \\ \psi_t \end{bmatrix} &= \begin{bmatrix} X_{uy} & X_{u\varphi} \\ X_{xy} & X_{x\varphi} \\ X_{\psi y} & X_{\psi\varphi} \end{bmatrix} \begin{bmatrix} y_t \\ \varphi_t \end{bmatrix}, \quad \begin{bmatrix} y_{t+1} \\ \varphi_{t+1} \end{bmatrix} = \begin{bmatrix} M_{yy} & M_{y\varphi} \\ M_{\varphi y} & M_{\varphi\varphi} \end{bmatrix} \begin{bmatrix} y_t \\ \varphi_t \end{bmatrix}, \qquad (46) \\
W_t(y_t, \varphi_t) &= \frac{1}{2} \left( \begin{bmatrix} y_t \\ \varphi_t \end{bmatrix}' \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \begin{bmatrix} y_t \\ \varphi_t \end{bmatrix} \right).
\end{aligned}$$

Equation (44) yields

$$\begin{aligned}
\begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} &= \begin{bmatrix} I & 0 \\ X_{xy} & X_{x\varphi} \\ X_{uy} & X_{u\varphi} \end{bmatrix}' \begin{bmatrix} Q_{11} & Q_{12} & P_1 \\ Q_{12}' & Q_{22} & P_2 \\ P_1' & P_2' & R \end{bmatrix} \begin{bmatrix} I & 0 \\ X_{xy} & X_{x\varphi} \\ X_{uy} & X_{u\varphi} \end{bmatrix} \qquad (47) \\
&\quad + M' \begin{bmatrix} \beta\omega U_{11} + \beta(1-\omega) S & \beta\omega U_{12} \\ \beta\omega U_{21} & \beta\omega U_{22} \end{bmatrix} M
\end{aligned}$$

A possible iterative scheme is (different order of updates is possible):

1. Guess $M, X, U$, as part of them we have $H = X_{xy}$, $S = U_{11}$
2. Compute an update of $U$ using (47)

3. Solve (45) using Schur decomposition (with stability threshold as $1/\sqrt{\beta\omega}$) to find an update for $X$ and $M$.

Finally, the loss is found as in Schaumburg and Tambalotti (2007).

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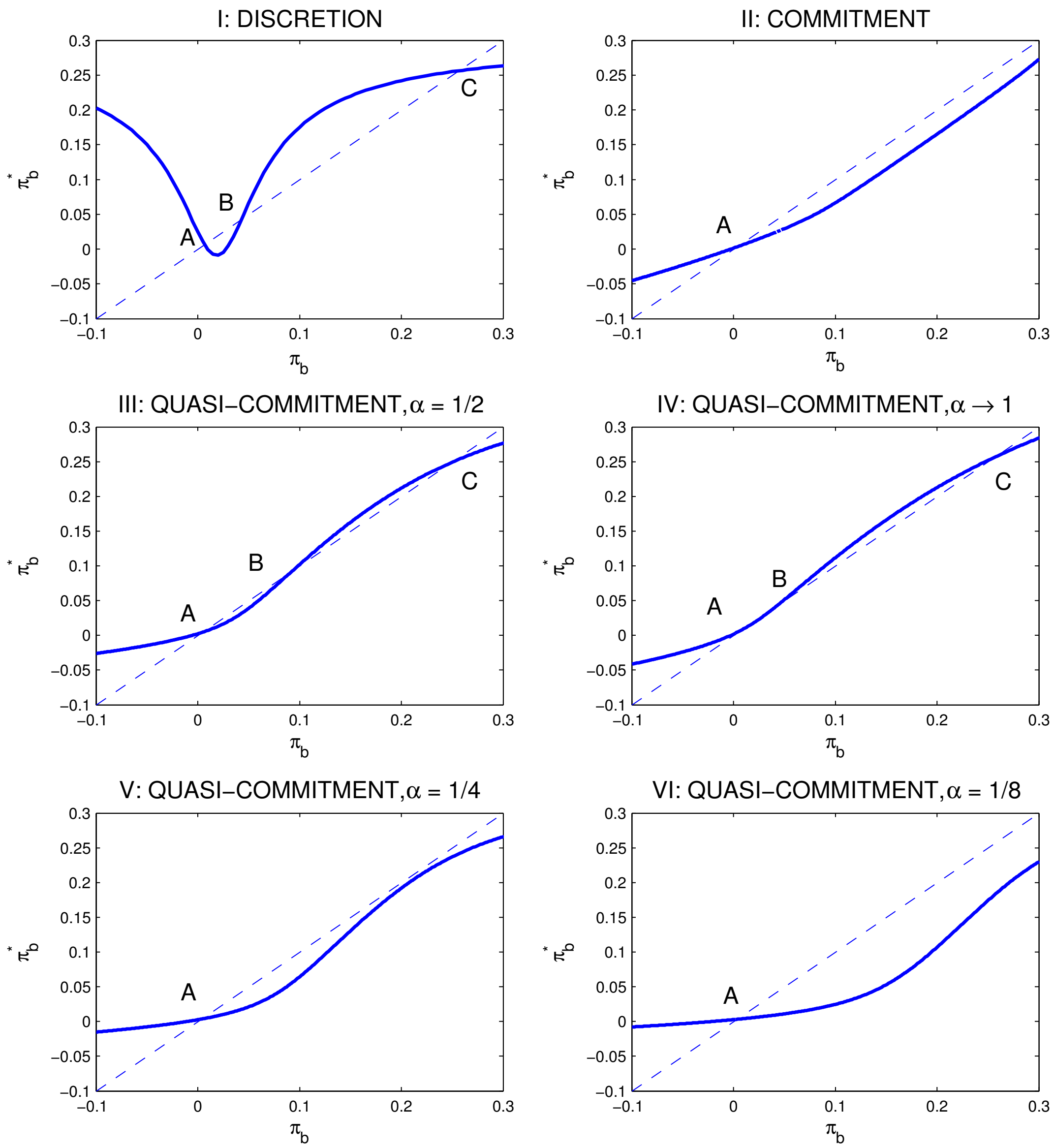


Figure 1: Multiple policy equilibria for different degrees of precommitment

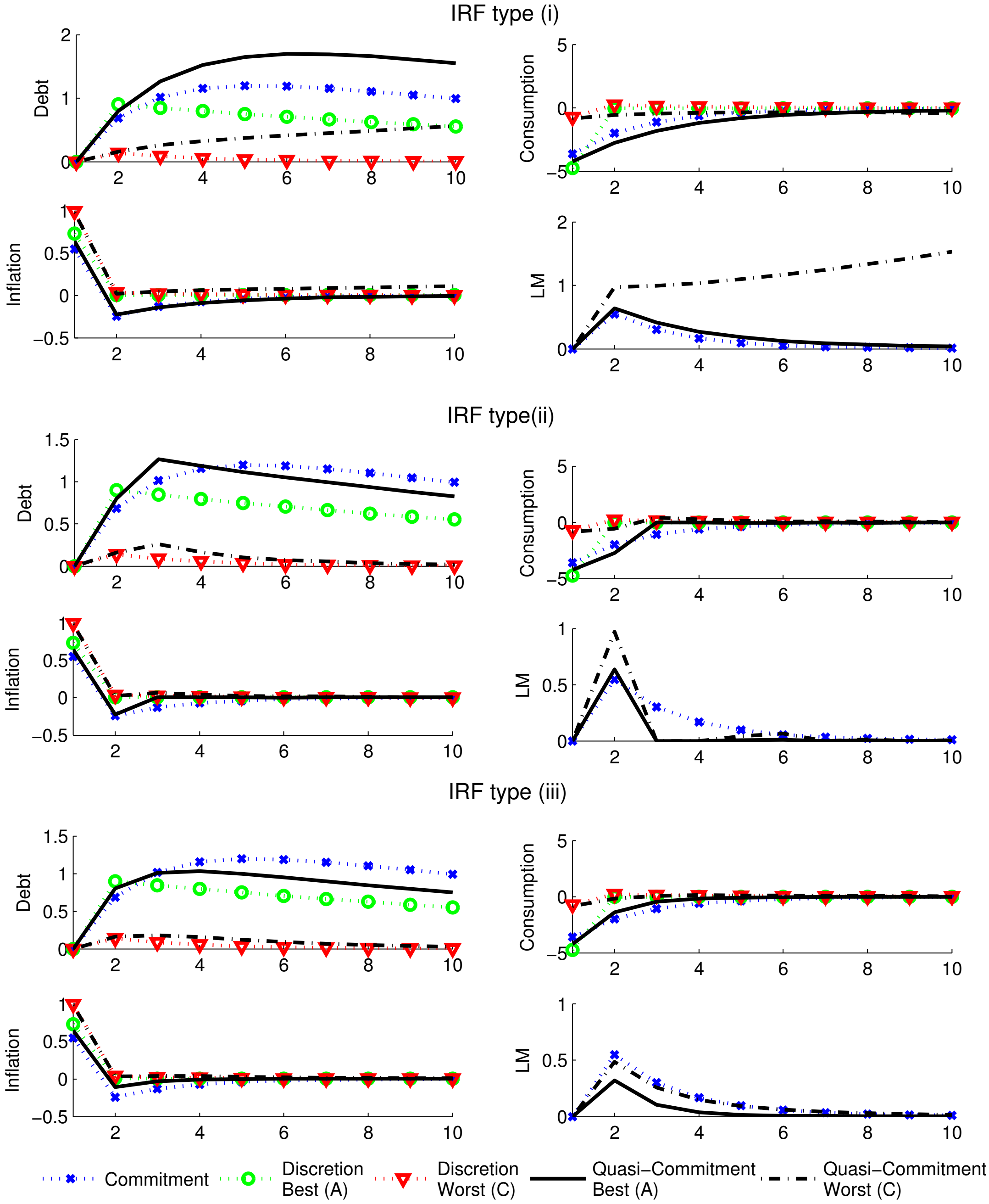


Figure 2: Impulse Responses to a 1% cost push shock

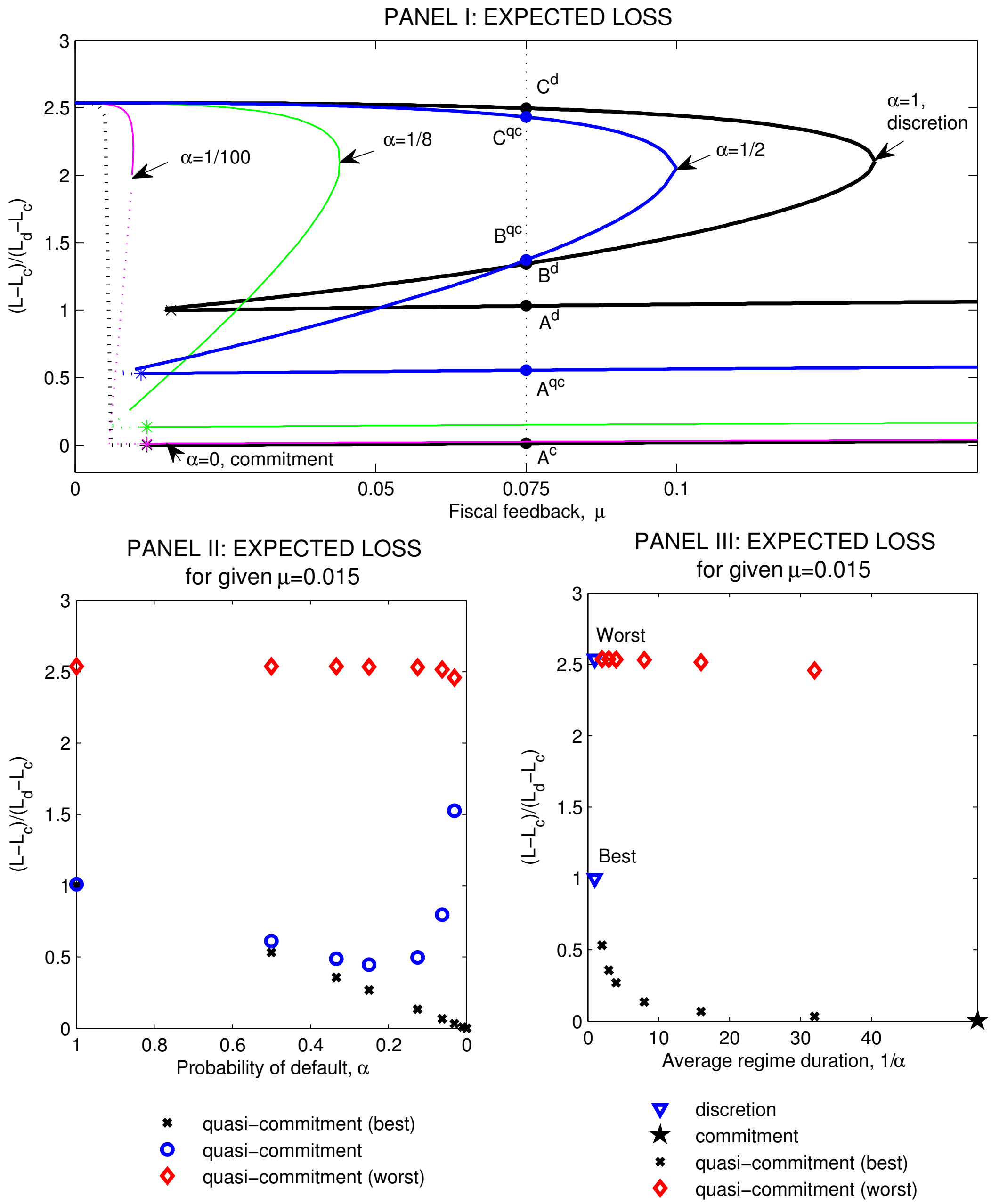


Figure 3: Welfare loss