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Defense Expenditure and Economic Growth under External Predation

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Abstract: This paper develops a growth model of a country under a Hobbesian environment with international conflicts where national defense is the only way to prevent external predation. The long run growth path is determined by the equilibrium of a dynamic game with three players, the external predator, the government and the family. The equilibrium growth path has three phases, submissive equilibrium, tolerant equilibrium and full-protected equilibrium. Different defense strategies result in different growth prospects and sustainable growth will endogenously induce adjustment of defense strategies.

Key words: economic growth; predate; defense expenditure

I. Question and Literature Review

In the matter of the external environment of economic growth, the mainstream economic growth theoretical models pay more attention to international trade and transnational capital floating, while conflict behavior among countries is usually out of the consideration.[1] In fact, the conflict behaviors between countries have never stopped, more than that, it is one of the most important research topics on the research of international relationship. Empirical studies also have demonstrated its significance to country's economic growth.[2] Based on the Conflict Growth Theory, this paper makes a long term growth model of a country under the environment of international conflicts. Meanwhile, as a basic measure to prevent the external predation, the national defense is introduced into the model. This conflict problem is formed by the external predation and the internal defense. The focal point of this model is the interrelationship of international conflict, national defense and economic growth, and the new discovery after the international conflict and national defense are integrated in the economic growth model.

Many scholars have researched on the international conflict by economics approach, especially game theory. In economics, the main thought about the explanation of international structure is: the emergence of nation-state is the equilibrium outcome of the interaction among the allocated resources of defense and

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[1] See the discussion about the open economy in Barro and Sala-i-Martin (1992).

[2] See Nye. J. S. (2002) "Understanding International Conflicts: An Introduction to Theory and History". Shanghai: Shanghai People Publishing House. pp. 1-16.

offense.[3] The resource allocation in equilibria is an important characteristic which shows the balanced relationships among countries, and it has inevitable restriction and effect in turn on countries' development. In *Friedman* (1977) and *Boulding* (1963), the equilibrium between predation and defense had been analyzed geographically and technically. Whenever the equilibrium takes place, it relies on the two actions' relative technical advantages, returns to scale, usable resources and other factors. *Hirshleifer* (1991、2000) had discussed these questions in a generalized framework. And other economists (*Grossman and Kim, 1995; Skaperdas, 1992; Garfinkel, 1990*) have done in-depth study on the micro foundation of resource distribution between predation and defense in some more special models.

In the plenty of research on the national defense, both the political and economic factors have been emphasized by economists, but they are not going on under the framework of economic growth theory. According to economists' research of defense, national defense is public product, whose demand function is similar to the common public product, and is different from the private product. There are three categories of models of military expenditure demand: the general empirical model, neoclassical model and decision making process model.[4] The first category is mainly from the empirical study, while the third category is mainly from study of the micro decision making process of military expenditure. It is the second category which provides theoretical reference for the relationship between economic growth and national defense, and tells us that, as the result of economic growth, the national income or government revenue is the budget constraints of the military expenditure. In this kind of model, the national income and outside threat is fixed as external variables. The military expenditure's effect on economic growth is ignored, and it is viewed as the investment to produce "security", which enters the utility function directly and determines the total utility together with consumption.

In this paper, the root of the international conflicts is attributed to the economic interest, and it is supposed that the only purpose to pay the military expenditure is to protect the native property from being plundered. Besides, the problems about security and growth are also brought into the economic growth model. The basic idea of this paper is: suppose under a Hobbesian environment in which the countries face the possibility of being plundered all the time. The only way to prevent the external predation is to invest for the national defense, and the possibility for one country's total income of being protected and being plundered is decided by the success function. The economic growth rate depends on the factor accumulation. At

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[3] See Sandler, Todd. & Hartley, Keith. (2001) The Economics Of Defense. Beijing: Economic Science Press, pp. 37.

[4] The characters of the three kinds of models are: the basic method of a general empirical model try to take account of all kinds of factors of national defense, including economy, politics and strategy and so on, and it usually applies them directly into empirical research. Murdoch and Sandler(1985), Looneyand Mehay(1990), Kollias(1994、1996) have mainly adopted and developed this kind of models, and then they did extensive researches in the U.S., Australia, Greece and Turkey and so on. The neoclassical model's foundation is the assumption of rational government, which considers the purpose of government is to maximize social welfare. When the total resource is finite, it needs to be distributed optimally to private department and public department. Besides, in general, assume that the welfare consists of national security and consumption. The defense expenditure demand model which bases on decision process doesn't agree with the basic assumption which considers government as an independent rational man. It pays more attention to the actual process for one country to make decision on the defense expenditure. (See Hartley & Sandler, 2001, pp. 70-85)

equilibrium, the extent of being protected influences the marginal return, and therefore influences the factor accumulation. Then, the country has to face the problem of how to optimally allocate resources for consumption, production and national defense. It decides the long term equilibrium growth path that attains both international conflict equilibrium and the balance among consumption, production and national defense. The first part of this paper is on the basic question and literature review. The second part describes the structure and character of the basic model. In the next part, a dynamic equilibrium solution and relevant analysis are given, and the fourth part elaborates the theoretical meaning of model's result.

II. Basic Model

The model analyses in the international conflict, how a country which is in a protection position attains the equilibrium between the international conflict and the internal economic growth, and what its economic growth path looks like. Suppose the world political and economical system is under the Hobbesian Rule, there are full of conflicts among countries and the only purpose of the international conflicts is to gain the economic interest.[5] Under this kind of international environment, a country has to face many other countries predations all the time. There is a country C which is under this system. For simplicity, suppose country C could not plunder other country but try to prevent the external predation. In other words, the conflict between country C and other countries is a predation-defense problem[6]. It is a reasonable assumption when the country is a developing country or it couldn't pose a threat to other countries. The only way to prevent predation for country C is to invest to the defense department to protect the national property. There are three players in this model: external predator, the government and representative family of country C. what follows is an explanation about their actions.

A. International Conflict

Country C is facing the conflict with the predators all the time. Assume that the proportion that country C's property is protected from being plundered is decided by the predation or defense investment by both sides through the Contest Success Function. At time t, the defense expenditure is $F_d(t)$, other countries' investment to plunder C's property is $Fp(t)$,and assume the Contest Success Function[7] as

[5] Though it is controversial to describe the international order in Hobbesian Rule, there are still many scholars who adopt and accept it, and they consider it as the most important difference between international society and domestic society. (See Nye. J. S. (2002), pp.4)

[6] For the meaning of predation-defense problem see Section 3.8.

[7] The conflict economics considers that the conflict process has some essential economic features as same as production process. Just as the most basic means to analyze production process is production function, in the formal economics literatures on conflict, they generally adopts some functions to simulate the relationship between input and output of conflict behavior, and these functions are called Contest Success Function. Hirshleifer (1989, 1994, 2000) introduce some general function forms. For the application of some functions, see Loury (1979), Tullock (1980), Rosen(1986), Appelbaum and Katz (1987), Dixit (1987), Hirshleifer (1989), Skaperdas (1996), Baik and Lee(2001), Hirshleifer (1989), Baik (1998) and so on.

$$p(F_d, F_p) = \frac{\theta F_d + \overline{F}}{\theta F_d + \overline{F} + F_p}.$$

Whenever not inducing confusion, we drop the time variable *t*. Here *p* is the proportion of C's property is under protection, θ and $\overline{F}$ are technical parameters of conflict.[8] $F_d(t)$ and $F_p(t)$ is the both sides' conflict investment at time *t*. As shown in the figure, $p > 0$, when $F_d(t)$ is *0*, which means country C would not lose all of its property even it gives up defense investment. After all, the predation is condemned morally, and it is limited by kinds of factors, such as the international environment, the prey and the predator's internal conditions and cost of wealth transition. Therefore, even if the prey is quite weak, the predator could not take all of its property away, and $\overline{F}$ is used to measure the strength of natural protection when country C spends no money on defense.

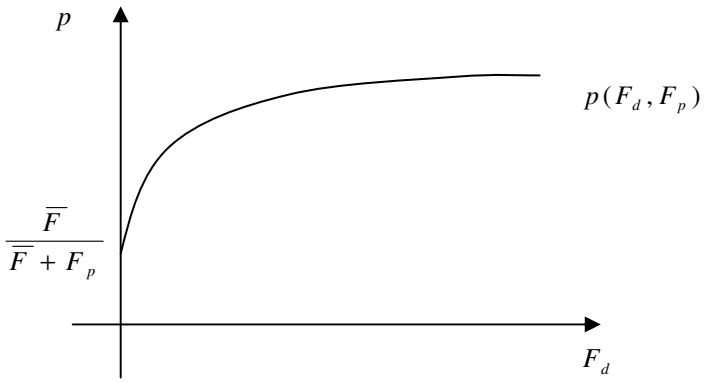

Fig.1 The Contest Success Function

Denote the per capita predation investment, per capita defense investment and per capita strength of natural protection respectively by *F*, f and *f*. The Contest Success Function is:

$$p(f, F) = \frac{\theta f + \bar{f}}{\theta f + \bar{f} + F} \qquad (1)$$

This function is called the intensive form of the Contest Success Function. The following mainly uses this form. Obviously, the proportion of country C being plundered at the time *t* is:

$$1 - p = \frac{F}{\theta f + \bar{f} + F}$$

B. The Actions of the Government and Family

Country C is made up of the government and many symmetrical families. As the

[8] It is more reasonable that *p* is decided by the stocks of bilateral conflict inputs. But this has no substantial effect on the theoretical conclusions of this model. For simplicity sake, we treat conflict inputs as flows.

model is not concerned with the predator's internal government and family, the government and family mentioned in this paper refer only to country C which is plundered. At time *t*, country C invests resources to production as well as defense. It needs to balance between the cannon and butter. The gross income in each period is divided into consumption, investment and defense expenditure, among which, defense expenditure is decided by the government while consumption and investment are decided by the families.

Each family population is standardized to be 1. As the total amount of families is quite large, we think that the action of a single family has no effect on the aggregate variables. So the symmetry of the families makes the solving process much simpler. Assume that the government which is responsible for maximizing the family lifetime utilities is a rational agent of the public, and finance for defense expenditure by lump-sum taxation.[9] Because the families are symmetrical, each family contributes f to defense expenditure. The national defense is public product, so the protected proportion of each family's output is p. The total capital stock of country C is K, and the aggregate production function is $Y(K)$. Each family's capital, namely the per capita capital, is k. The per capita production function is $y(k)$, and is assumed to have the linear form Ak. Because of the existence of the external predator, the family could not get the overall output, only $py(k)$, the remaining output after being plundered[10]. Therefore, at time t, the family budget constraint, i.e. the capital accumulation equation is:

$$\dot{k} = p(f, F)y(k) - f - c - \delta k \qquad (2)$$

In expression (2), δ is the capital depreciation rate. Expression 2 shows that defense expenditure's influence on economic growth is mainly on two aspects: on one hand, the defense expenditure occupies a part of current revenue, so that it may suppress consumption and investment; on the other hand, defense expenditure has effect on the proportion of the product being protected, so that it influences the long term economic growth. Assume that the defense expenditure per capita and the capital stock cannot be negative. Each period's consumption is also not negative as well. However, just as other growth models, equilibrium solution will not be negative, so it can be ignored here. So

$$f \geq 0 \qquad (3)$$

$$k(t) \geq 0 \qquad (4)^{11}$$

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[9] The assumption of lump-sum taxation can avoid the influence that the marginal tax rate may decrease the production marginal return.

[10] There is a minor question that what is the object being plundered. For example, in the model of Grossman and Kim (1995, 1996c), productive capital, defense barricade and the weapons are the objects which can be plundered, while the economic output will not be plundered. However, in the model of Grossman and Kim(1996a), output and productive capital can be plundered, but the defense barricade and the weapons are a kind of expenditure as same as consumption which couldn't be plundered. Here we consider that output is the only object that can be plundered.

[11] For the single family, the capital stock can be negative theoretically. But there is a symmetry assumption, and we don't consider the possibility that the whole country has net debt in the environment with international conflicts, therefore the capital stock per capita is more than 0.

The family tries to maximize the objective function:

$$U = \int_0^{\infty} u(c(t)) \cdot e^{-\rho t} dt \qquad (3)$$

Assume that the current utility function is:

$$u(c) = \ln c \qquad (6)^{12}$$

In a standard growth model, *k(0)* is a necessary condition. In order to obtain meaningful solutions, assume that the parameters satisfy the following conditions:

$$\theta > A/\rho \quad \text{and} \quad \theta(2\rho + \delta) > \theta A > (1+\theta)(\rho + \delta) \qquad (4)$$

In the following text, at first, we don't take into account the rationality assumption for the public government. We try to analyze the characteristics of economic growth under the government's different defense strategy assumption. Then, we explain the dynamic equilibrium path when the government's purpose is to maximize the family's lifetime total utility

C. The Predator's Action

Assume that many predators prey on country C's property and are ready to plunder all the time. For simplicity, assume that these predators take the chance randomly to plunder country C, and at one time there is one and only one predator. There are too many predators that it is difficult to collude with each other. So once a predator gain the chance, it will try to maximize the profit from this period's predation since in the next period the predation chance may be gained by another predator. Therefore, in this game, the predator's purpose is to maximize each period's profit, not like country C who considers the infinite horizontal problem. The proportion of the property every predator gains from country C is decided by the Contest Success Function. Therefore, the predator's optimization problem is to maximize the net revenue.

$$\text{Max} \quad [1 - p(F_d, F_p)]Y - F_p$$

Because of the assumption of family's symmetry, the objective function can also be written in the form of per capita (of country C), which is:

$$\text{Max} \quad [1 - p(f, F)]y - F \qquad (5)$$

$$\text{s.t.} \quad F \geq 0 .$$

Therefore, the model in this paper looks like a partial equilibrium mode. Just as Mejia and Posada (2002) said, if there are many external predators and they haven't colluded with each other, it is reasonable to explain the action by partial equilibrium.[13]

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[12] The logarithmic utility function is adopted here, which is only for simplicity. To adopt other utility function, like $u(c) = (c^{1-\alpha} - 1)/(1-\alpha)$, $\alpha > 0$, we can also come to this paper's crucial conclusion.

[13] Thompson (1974) has put forward an idea that every country needs to distribute its wealth into production and security area rationally.

III. The Dynamic Equilibrium

Even though we have set some simplified assumption to the players, the dynamic equilibrium still involves complicated technical problems. In the general setting, the model in this paper is not a repeated game model in continuous time. It is because, for country C, the production department and conflict department have effect with each other in every period, and different investment or output in every period can also change the conflict problem in every period. The closest concept to this model is differential game, in which the most extensively adopted solution concept is MPE, the Markov Perfect Equilibrium.[14] The key idea of MPE is: when the players' profit in each period is just related with some of the current state variables and players' current actions, the players' strategy is assumed related only with these state variables and these state variables are called return relevant variable. In other words, the games in the past time influence the coming games only through some state variables. Whenever the return relevant variables are equal, the following games are totally equivalence.

In this model, the action variables of country C's government and family are the defense expenditure and consumption in each period, namely $\{f_t\}$ and $\{c_t\}$. The predator's action variable is $\{f_t\}$,[15] the predation investment per period. The key point is the choosing of the return relevant variables in the model. Here it is country C's capital stock per capita, namely k_t. The reason why it just consist of country C's capital stock but doesn't consist of capital stock of predators is that the optimization problems of defender and predator are different in this one-to-many game. The predators pursue the maximization of profits, while the defenders pursue the maximization of the family lifetime utilities. According to expression (8), the predator's return per period only has relationship with the defender's output in that period which is related to the current capital stock and the current action variables (*f, F*). Therefore the predator's return relevant variable only needs to consider the defender's capital stock *k*, but not its own capital stock. Obviously, the current return per period (consumption's current utility) of family in country C is only decided by the current consumption. So, in the game, the return relevant variable only consists of country C's capital stock. Assume that the strategies of both sides are just the return relevant variable's function, i.e. *c(k)*, *f(k)* and *F(k)*. Meanwhile, in order to avoid being confused, the values of *c, f* and *F* at time *t* are denoted as c_t, f_t and F_t, and *k* is still the function of time, denoted as *k(t)*.

When other players' MPE strategies are given, the predator's problem is to solve the problem in expression (8), and the problem of country C's government is to choose f_t to maximize the family utility, and the family's problem is to maximize expression (5) under the constraints of expression (2), (3) and (7).

The following paragraphs are primary analyses on family *i*'s optimization problem under the condition that other players' strategies have been given. The current-value Hamilton Equation:

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[14] For the differential game and MPE's detail illustration, see Fudenberg and Tirole (1991).

[15] Because of the assumption that the predators gain randomly the chance to plunder, the predation input decision in different periods may be decided by different predators.

$$H_i(c_i,k_i,\mu_i)=u(c_i)+\mu_i[p(f,F)y(k_i)-f-c_i-\delta k_i]+\lambda f \tag{6}$$

The first-order conditions are:

$$\frac{\partial H_i}{\partial c_i}=0;\quad \dot{\mu}_i=\rho\mu_i-\frac{\partial H_i}{\partial k_i}$$

According to the assumption of the predator and government's MPE strategies, f and F are only related to the per capita capital k. While the change of a single family's capital has no effect on k, i.e. $\partial f(k)/\partial k_i=0$ and $\partial F(k)/\partial k_i=0$. From the symmetry, at the equilibrium, we have $k=k_i$. Therefore, the first-order conditions for a representative family can be written as:

$$\begin{cases} u'(c)-\mu=0 & (7)\\ \dot{\mu}=\rho\mu-\mu[p(f,F)A-\delta] & (8)\end{cases}$$

At the same time, slackness condition and transversality condition are written:

$$\lambda\geq 0,\quad \lambda f=0 \tag{9}$$

$$\lim_{t\to\infty}\mu(t)\cdot e^{-\rho t}k(t)=0 \tag{10}$$

According to the expression (11), the economic system's evolution is related to the predator and government's strategies. In the following paragraphs, to solve the economic growth path, assume directly that the government and predator adopt corresponding three kinds of typical dynamic strategies (MPE strategies). And then the following paragraphs explain how these strategies form a dynamic game equilibrium solution, and analyze the corresponding equilibrium growth path.

A. Conflict Strategy and the Growth Path

1. The growth path under tolerant strategy

Assume directly that the predator and government's tolerant strategies under the dynamic game are as following (The superscript "*" shows the value under tolerant strategy):

$$\begin{cases} F^*(k)=\dfrac{\theta}{(1+\theta)^2}y(k)\\ f^*(k)=\dfrac{\theta}{(1+\theta)^2}y(k)-\dfrac{\bar{f}}{\theta}\end{cases}$$

Obviously, in order for the existence of the government's strategy, assume that the following inequality holds all the time.

$$y(k)>\frac{(1+\theta)^2}{\theta^2}\bar{f} \tag{11}$$

From expression (1), we have:

$$p(f,F) = p^* = \theta/(1+\theta)$$

In other words, under the assumed condition that both the predator and government adopt the strategy $f^*(k)$ and $F^*(k)$, the protected proportion of country C's output is always p^*. It is clear that under the tolerant strategy, country C can spend part of its resources on national defense, but part of its output is still plundered. Therefore, its defense expenditure is limited. This is what the word "tolerant" really means.

Theorem 1. Assume that the government and predator adopt the tolerant strategy, and $k(0) > (1+\theta)^2 \bar{f}/(A\theta^2)$, then there is the unique equilibrium growth path. On the path, the capital and consumption always have positive growth rate. Moreover, consumption's growth rate is fixed, and expressions (15) to (17) hold.

$$\gamma_c^* = \gamma^* = A\theta/(1+\theta) - \rho - \delta \qquad \textbf{(12)}$$

$$c^*(k) = (\rho - \frac{\theta A}{(1+\theta)^2})[k(t) + \frac{\bar{f}}{(\gamma^* + \rho - \frac{\theta A}{(1+\theta)^2})\theta}] \qquad \textbf{(13)}$$

$$c_0^* = (\rho - \frac{\theta A}{(1+\theta)^2})[k(0) + \frac{\bar{f}}{(\gamma^* + \rho - \frac{\theta A}{(1+\theta)^2})\theta}] \qquad \textbf{(14)}$$

For the proof, see the appendix.[16] The initial consumption is given in expression (17), and expression (16) gives a policy function of c with respect to k on the equilibrium path. Therefore, the family's optimal MPE strategy is given as well as the predator and government's strategies are given as *f*(k)* and *F*(k)*. Although the chosen return relevant variable is capital per capita when we define the MPE strategy, accurately speaking, in the process to get expression (16), *k* should be the capital of the considered family. But under the family symmetry condition, these two things are equal.

Since there is only one equilibrium growth path, we can get a policy function *c*(k)* according to this path, as expression (16).

2. Growth path under the submissive strategy

The superscript "&" shows the value under the subject strategy. Assume directly that the predator and government's MPE strategies are as follows:

$$f^{\&}(k) = 0 \qquad (15)$$

[16] Due to limitations on space, the appendix is left out here. Someone who needs it can get in touch with the author.

$$F^{\&}(k) = \sqrt{\bar{f}Ak} - \bar{f} \tag{16}$$

Now country C's defense expenditure is always zero, this is what the word "submissive" means. In order to guarantee the validation of expression (19), assume always that:

$$y(k) \geq \bar{f} \tag{17}$$

Theorem 2. Assume that the government and predator adopt the submissive strategy, and $k(0) \geq \bar{f} / A$, then there is the unique equilibrium growth path, which is tending to the stable status. In the stable status, the values of capital and consumption are determined by expression (21) and (22).

$$k^{\&} = \frac{A\bar{f}}{(\rho + \delta)^2} \tag{18}$$

$$c^{\&} = \frac{\rho A\bar{f}}{(\rho + \delta)^2} \tag{19}$$

For the proof, see the appendix. Since there is only one equilibrium growth path, we can get a policy function $c^{\&}(k)$ according to this path, as expression (22)**.**

3. The growth under the friendly strategy.

The idea here is that there is no conflict under the friendly strategy. Both the predator and the government don't invest anything into conflict. The superscript "#" shows the value under the friendly strategy. Assume directly that the predator and government's MPE strategies are as follows:

$$\begin{cases} F(k) = F^{\#}(k) = 0 \\ f(k) = f^{\#}(k) = 0 \end{cases} \tag{20}$$

According to the expression (1), $p=1$. Now, the model is a standard neoclassical *Ak* growth model, meanwhile the following theorem is presented:

Theorem 3. Assume that the government and predator adopt the friendly strategy, then the economic system has the unique equilibrium growth path, on which both the consumption and investment have constant growth rate, and the expressions (24) and (25) hold.

$$\gamma_c^{\#} = \gamma_k^{\#} = \gamma^{\#} = A - \rho - \delta \tag{21}$$

$$c^{\#}(k) = \rho k(t) \tag{22}$$

For the proof, see the appendix. Since there is only one equilibrium growth path, we can get a policy function $c^{\#}(k)$ according to this path, as expression (25).

4. The growth path under the fully-protected strategy

The superscript " $ " shows the variable's value under the fully-protected

strategy. Assume that the predator and government adopt the fully-protected strategy, as follows:

$$\begin{cases} F^{\$}(k) = 0 \\ f^{\$}(k) = Ak/\theta - \bar{f}/\theta \end{cases} \tag{23}$$

The basic idea to give this strategy is: country C chooses a proper defense input, so that the predator's best response is to give up predation given the defense input. Obviously, $p=1$ under the strategies described in expression (26), which means that the output is protected completely.

Theorem 4. Assume that the government and predator adopt the fully-protected strategy, then the economic system has the unique equilibrium growth path, on which the consumption has constant positive growth rate, capital growth rate tends to the consumption growth rate, and there are equation (27) and (28).

$$\begin{cases} \gamma_c^{\$} = A - \rho - \delta \\ \lim\limits_{t\to\infty} \gamma_k^{\$} = \gamma_c^{\$} = A - \rho - \delta \end{cases} \tag{24}$$

$$c^{\$}(k) = (\rho - \frac{A}{\theta})[k(t) + \frac{\bar{f}}{(A - \frac{A}{\theta} - \delta)\theta}] \tag{25}$$

For the proof, see the appendix. Consumption grows at a constant rate, the same as the one under the friendly strategy, under which there is no conflict input. This is because that under both of the situations, the outputs are full protected. The difference is that country C has to input part of its resources into defense here. The capital growth rate tends to the consumption growth rate asymptotically, and the policy function for c, $c^{\$}(k)$ has its new form as well.

B. MPE Dynamic Equilibrium Growth Path

According to the above discussion, with different conflict strategies assumptions of the government and predator, there are different equilibrium growth paths for the economy. When the government and predator adopt the tolerant strategy and friendly strategy, consumption has a constant positive growth rate all along, and the growth rate under friendly strategy is higher. While under the submissive strategy, the economy tends to a stable state in which both consumption and capital are fixed. However, for these cases, the equilibria are still only equilibria for resources allocation between consumption and investment. The complete equilibrium needs to consider that whether the international conflicts are at the state of equilibrium. Here is a MPE strategy which attains equilibrium in both international and domestic level, and all of the above growth paths may appear in this equilibrium.

Theorem 5 The following strategy is a MPE equilibrium:

$$F(k)=\begin{cases} F^{\#}(k)=0 & if\ \ k(t)\le \bar{f}/A \\ F^{\&}(k)=\sqrt{\bar{f}Ak}-\bar{f} & if\ \bar{f}/A<k(t)\le \dfrac{(1+\theta)^2}{A\theta^2}\bar{f} \\ F^{*}(k)=\dfrac{\theta}{(1+\theta)^2}y(k) & if\ k(t)>\dfrac{(1+\theta)^2}{A\theta^2}\bar{f} \end{cases};$$

$$f(k)=\begin{cases} f^{\#}(k)=0 & if\ \ k(t)\le \bar{f}/A \\ f^{\&}(k)=0 & if\ \ \bar{f}/A<k(t)\le \dfrac{(1+\theta)^2}{A\theta^2}\bar{f} \\ f^{*}(k)=\dfrac{\theta}{(1+\theta)^2}y(k)-\dfrac{\bar{f}}{\theta} & if\ k(t)>\dfrac{(1+\theta)^2}{A\theta^2}\bar{f} \end{cases};$$

$$c(k)=\begin{cases} c^{\#}(k) & if\quad k(t)\le \bar{f}/A \\ c^{\&}(k) & if\quad \bar{f}/A< k(t)\le \dfrac{(1+\theta)^2}{A\theta^2}\bar{f} \\ c^{*}(k) & if\quad k(t)>\dfrac{(1+\theta)^2}{A\theta^2}\bar{f} \end{cases}$$

For the proof, see the appendix.

The equilibrium path should consist of three phases. In the "c-k" diagram (shown in Fig. 2), corresponding to the heavy line in the figure, obviously we have the conclusions:

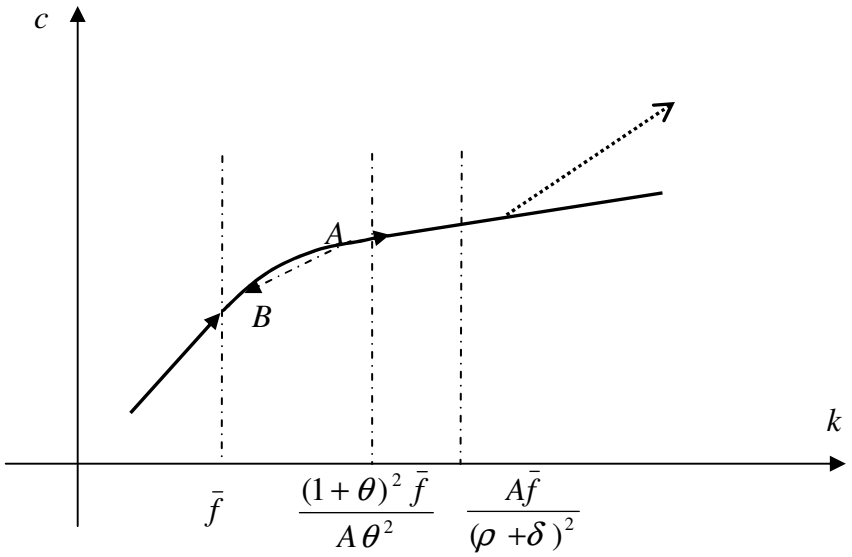

Fig. 2 The growth path in the dynamic equilibrium

(1) When $k \le \bar{f}/A$, all players' strategies refer to the situation of friendly strategy. Therefore, there is no conflict and no property being plundered. Both the consumption per capita and capital per capita have constant positive growth rate.

(2) When $k > (1+\theta)^2 \bar{f}/(A\theta^2)$, all players' strategies refer to the situation of tolerant strategy. A fixed proportion of the output is plundered, consumption per capita has a constant positive growth rate, and *c* is linear with *k*.

(3) When $\bar{f}/A < k \le (1+\theta)^2 \bar{f}/(A\theta^2)$, all players' strategies refer to the situation of submissive strategy. Consumption per capita and capital per capita tend to be a fixed value asymptotically. According to expression (7) and (21), we know the capital stock in the stable state is:

$$k^{\&} = \frac{A\bar{f}}{(\rho+\delta)^2} = \frac{A^2}{(\rho+\delta)^2}\frac{\bar{f}}{A} > \frac{(1+\theta)^2}{A\theta^2}\bar{f}$$

Therefore, the economy must reach the growth path under the tolerant strategy before it reaches the stable state.

Obviously, the above-mentioned phases take place in sequence. The key point is that the growth rates in all phases since beginning is greater than *0*, so that k can keep going up from *k(0)* until the time when it is greater than $k^{\&}$. In the equilibrium path's first phase, namely the friendly equilibrium phase, the condition to ensure a positive growth rate is the same to the classical growth model. More important is the condition that ensures the economy to evolve from the second phase, namely submissive equilibrium phase into the tolerant equilibrium phase. As part of the expression (7), $\theta A > (1+\theta)(\rho+\delta)$, i.e. $\theta > (\rho+\delta)/(A-\rho-\delta)$. This condition not only ensures the tolerant equilibrium phase's growth rate is positive, but also ensures the above expression holds. Therefore, the economy can reach the growth path under the tolerant strategy before the economy reaches the stable state. As mentioned previously, θ is used to measure the both sides' relative level of technology in conflict, in other words, it is possible to reach the tolerant equilibrium path only if country C's defense technology is above a particular level. Besides, the greater the ρ and δ are, the higher the lowest-needed value of θ. It means that when the future is less important, there must be a higher value of θ to ensure the motivation to pursue the long term positive growth rate.[17]

C. Fully-protected Dynamic Equilibrium

In part B, we give a dynamic MPE equilibrium of the model, and on the equilibrium growth path, country C's economy will tend to a constant positive growth

[17] For how these parameters influence the equilibrium in different phases, it can be illustrated by solving with numerical values. Due to limitations on space, it is left out here. Someone who needs it can get in touch with the author.

rate at last. But in this MPE equilibrium, only when the capital stock is extremely low can the economy be free from being plundered, and as the output increases, there is a fixed proportion of output being plundered all along after the economy reaches the tolerant equilibrium growth path. It is likely that the conclusion is inconsistent with our intuition, because after a country grows strong enough, it should have ability to protect its output completely. The following paragraphs will give a subgame perfect dynamic equilibrium, called the fully-protected equilibrium, in which the output is protected completely, and then prove that this equilibrium can only appear after k is large enough. Meanwhile, the subgame perfect dynamic equilibrium in part B is called the basic MPE equilibrium. The following theorem can be proved.

Theorem 6. When θ is high enough and *k(0)* is large enough, there is a subgame perfect equilibrium, as follows:

The predator's strategy: if country C's government always chooses $f = f^{\$}(k)$ in the whole history, it chooses $F = F^{\$}(k)$ in the current period; if country C's government has ever violated it before, it chooses the basic MPE strategy.

The country C government's strategy: if its own decision meets $f = f^{\$}(k)$ in the whole history, it keeps choosing $f^{\$}(k)$; if it has ever violated $f = f^{\$}(k)$ before , it chooses the basic MPE strategy.

The family's strategy: if the government always chooses $f = f^{\$}(k)$ in the whole history, it chooses $c = c^{\$}(k)$; if the government has ever violated it before, it chooses the basic MPE strategy.

For the proof, see the appendix.

It should be pointed out that this equilibrium has not been a strict MPE, for the strategies of both sides are related not only to k (Return Relevant Variable), but also to what have happened before. On the equilibrium path, country C's output is protected completely. this is what the fully-protected strategy and fully-protected equilibrium means. This equilibrium is interpreted as that in order to obtain a better position, country C promises a defense input, and predator can judge whether the promise is credible or not by the history and makes its choice. The crucial step in the proof is to illustrate country C's promise is enforceable by itself (namely credible). Because that it will turn to the basic MPE if the promise is violated, and compared with the fully-protected equilibrium, in the basic MPE, k and c will have higher level in the current period but the growth rate will be always lower from this period on. Therefore, when *k(0)* is large enough, the loss will always outweigh the gain whenever the government violates $f^{\$}(k)$ and turn to the basic MPE.

Theorem 7. When θ is high enough, the fully-protected equilibrium is better than the basic MPE equilibrium for country C (including government and family).

For the proof, see the appendix. This theorem illustrates that, if the government is a rational agent of families, when the capital stock reaches a certain level, the government prefers the fully-protected equilibrium growth path to the basic MPE growth path. As showed in Fig. 2, the last part of the heavy line (the basic MPE

growth path) is replaced by the dashed line with arrow.

While the key condition to realize the fully-protected equilibrium is a high enough θ, which makes country C realize fully-protected with less proportion of resources. In this way, though the proportion of consumption in the gross output is lower than the proportion in the tolerant equilibrium growth path, what is gained is more than what is lost in the long term, because a higher growth rate makes the future gross output larger. However, if the value of θ could not reach the requirement, the proportion of input to realize full protection of the output is too high and the proportion of consumption in the output is too low. Though it makes fully-protected equilibrium growth path has a higher growth rate, it still worsens the welfare than tolerant equilibrium growth path.

IV. Theoretical implications

This paper provides an analytical framework in which the international conflict and domestic growth are integrated in one model. It explains how to optimize the distribution of resources between the production and defense activities and the possible economic growth paths. This model has following theoretical implications at least.

A. Determinant of Long Term Growth

Nowadays, international circumstance is full of Hobbesian Rule and the conflicts have never disappeared. This model illustrates that the existence of external predator may make country C's growth in a very different way. Even if a country has good enough interior market environment and production technology so that it can achieve stable economic growth without international conflict, it will have a lower growth rate, and even couldn't achieve long term stable growth while there is external predator.

Therefore, in the environment of international conflict, a country's long term growth is the result of the interaction of international factors and internal factors. Long term equilibrium growth path is co-determined by the equilibrium of international conflicts and the balance between domestic consumption and investment. This country has to decide how to distribute its resource reasonably between the production and national defense all the time. The external predation may reduce economic marginal output, so what growth phase does this country locate in is decided by this country's position in the international conflict equilibrium.

In the submissive equilibrium, this country doesn't pay the defense expenditure, or the defense expenditure is kept in a low level which can be ignored by the predator. (In reality, it can also be illustrated as that the limited military expenditure is only used to keep the internal social order.) In this situation, though the country has a positive growth rate in the primary phase, as the increase of its output, its temptation to the external world becomes larger. Then there are more external predation and the level of economic security reduces. If it doesn't adjust security strategy, this country has to pay more and more "rent" to other country in exchange for its peace. Therefore, the proportion of being plundered increases, actual capital marginal return decreases and economic growth rate decreases gradually. If the government's national defense

strategy refuses to adjust, the economy will reach a fixed stable state at last. In the tolerant equilibrium, the government has to balance carefully all along between the security policy and growth policy. On one hand, national defense expenditure and this country's gross output show relatively stable linear relation, i.e., the defense expenditure climbs with the increase of gross output. On the other hand, this country will still tolerate part of its property is being plundered by predator. Under the condition of high enough defense technology parameter and production technology parameter, the economy can realize long term stable growth. But the growth rate is lower than the growth rate in the neoclassical model without external predation. In the fully-protected equilibrium, the government invests a larger portion's output to the national defense, and the defense expenditure is positive correlated with gross output. The property of this country is totally safe. If the defense technology parameter and production technology parameter are high enough, the economy can also attain a stable growth rate which is higher than the one in the tolerant equilibrium and is the same as the one under the environment without conflict. However, there is still a part of resource which is distributed to the national defense department all along. The direct cause leading these phases' transition in equilibrium is the capital stock's accumulation, which makes the defender has ability to realize a higher defense level. The mechanism is that higher defense technology parameter makes the realization of higher defense level become a better choice when the capital accumulation has reached a certain level.

B. The Mechanism of Interaction between Growth and Defense Expenditure.

In terms of defense expenditure's influence on economic growth, the existing research mainly involves two aspects' mechanisms: the one is defense expenditure causes a part of resource flow into defense department from production department, the other one is that as the defense department itself is productive, it can contribute product and its activities has positive externality to the productive department. In this paper's model, the first one still exists. In the tolerant equilibrium and fully-protected equilibrium, there is always a part of resource which flows into defense department. The second influence is ignored in this model. Empirical research shows that for many developing countries, there are no sufficient evidences to show that the defense expenditure has positive externality on production. More important, the model in this paper describes a third influence mechanism, which is also one of the crucial innovations of this model, namely defense expenditure's influence on domestic marginal production return. In terms of the modern economic growth theory, the key point to the long term growth rate is the factors' marginal return. Therefore, the third influence mechanism described by this model is more essential to a country's long term economic growth path. The basic process is that the external predation allowed by economic insecurity will decrease the expected factors marginal return. The increase of defense expenditure improves the economic security, that is to say the expected factors marginal return increases, so that the long term growth rate is also improved. There is no doubt that, defense expenditure of the country that is plundered

and the predator's predation expenditure interacts with each other. In the equilibrium, the factors marginal return decides the long term growth rate.[18]

In terms of economic growth's influence on defense expenditure, we can analyze the fundamental influence way from the sides of supply and demand. In terms of supply, the improvement of gross output relaxes the agent's budget constraint, so that the country has the ability to input more resource to national defense. In terms of demand, every improvement of the output makes the defense become more valuable. The temptation for the external predators is increasing, which cause much more predation and raise the degree of economic insecurity. So the demand to defense expenditure is raised. In the document on the defense expenditure, though they have paid attention to the external threat, generally, the external threat's seriousness is considered as exogenous one. While in the model of this paper, by the conflict's equilibrium analysis, the exogenous threat is endogenized. In other words, changes of this country's wealth have effect on the external threat so that the process in which the economic growth influences the economic growth has been explored deeper. Meanwhile, from the model, we can get a more important new insight: because the sustainable economic growth is companied by continuous accumulation of the capital stock, so it may cause one country's adjustment of defense strategy. This point will be discussed specifically in the following paragraphs.

C. The Optimal Growth: Balance between Safety and Development

Since different equilibriums appear while the capital stock accumulates continuously, in different developing phases of a country, its government may face different options. When the capital stock is very low, according to theorem 1, having no other choice, the government has to choose the submissive strategy and give up the defense input. Because at that time, this country is so poor that the predators are not very interested in it. So the country's best choice is to rely only on the natural protection state. At that time, though the economy is unsafe, it can realize positive growth. When the gross capital stock increases to meet the requirement in Theorem 1, it is better for the government to choose to invest in national defense because if it adheres to the submissive strategy, economy will stop growing at last. If it switches to tolerant strategy, its safety will not become worse though it is still unsafe,, and it still has the possibility to realize continuous stable growth. As the capital stock becomes larger, though tolerant equilibrium still exists theoretically, Theorem 7 illustrates that the whole society's welfare will be improved if it adopts fully-protected strategy. So this model predicts that the best growth path needs government to balance between security policy and development policy. A rational government should adopt different defense strategies in different development phases. When it is poor and less developed, it is more likely to choose the submissive strategy; after it has developed to a certain level, it is more likely to choose the tolerant strategy; and after it is strong enough, fully-protected strategy is a better choice.

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[18] If defense expenditure is financed by marginal taxation, the increase of defense expenditure will also bring down the private investment's marginal return. This influence is involved in many literatures. However, in this paper, we assume that the defense expenditure is financed by lump-sum taxation. For details, see the illustration in Section II-B.

The external environment also limits one country's policy choice. In this model, the quality of the external environment is mainly measured by $\bar{f}$. First of all, in all of the equilibriums, the higher $\bar{f}$ is, the less the defense expenditure is, and the more the resource will be used in consumption and production. Therefore, the social welfare will be improved. Secondly, according to Theorem 5, the more important meaning of $\bar{f}$ is that: it determines what kind of equilibrium growth path will arise as the amount of capital stock lies in some interval. For example, the higher the value of $\bar{f}$ is, the earlier the government can adopt fully-protected strategy, so that it can obtains a higher growth rate.

V. Conclusion

The paper tries to build a long term growth model in the environment with international conflicts. The analysis here shows that:

A country's long term growth is the result of the interactions of international factors and domestic factors. The long term equilibrium growth path is co-determined by the equilibrium of international conflicts and the balance between domestic consumption and investment..

According to the different characteristics of conflict equilibrium, equilibrium growth path may shows into three phases, namely submissive equilibrium growth phase, tolerant equilibrium growth phase and fully-protected equilibrium growth phase. When the initial capital stock is very low, it is in submissive equilibrium growth phase in which there is always a part of wealth being plundered without defense investment, and the higher the output is, the lower the economic growth rate is. When the capital stock increase to a higher level, it enters tolerant equilibrium growth path, in which there is positive defense expenditure and a fixed proportion's output being plundered while the economy still has a positive growth rate in stable state. When the capital stock is high enough, the fully-protected equilibrium growth path may exist and defense expenditure is increased until external predation is prevented completely, and the stable growth rate is higher than in the tolerant equilibrium.

The government's different defense strategies lead to different growth scenarios. The equilibrium growth path needs the government to balance security policy and growth policy. Continuous economic growth will cause endogenous adjustment of defense strategy, because a rational government will choose different strategies in different development phases. When the country is poor and backward, it chooses the submissive strategy. After it develops to a certain level, it switches to the tolerant strategy. When it is strong enough, the fully-protected strategy is a better choice.

Reference

[1] Appelbaum, E. and E. Katz, "Seeking Rents by Setting Rents: The Political Economy of Rent Seeking", *Economic Journal,* 1987, 97, 685-99.

[2] Baik, K. H., "Difference-Form Contest Success Functions and Effort Levels in Contests", *European Journal of Political Economy,* 1998, 14, 685-701.

[3] Baik, K. H. and S. Lee, "Strategic Groups and Rent Dissipation", *Economic Inquiry,* 2001, 39, 672-84.

[4] Barro, R. J. and X. Sala-i-Martin, "Public Finance in Models of Economic Growth", *Review of Economic Studies*, 1992, 59, 645-661.

[5] Boulding, K.E., *Conflict and defense: A General Theory*, York: Harper and Row, 1962.

[6] Dixit, A., "Strategic Behavior in Contests", *American Economic Review* , 1987, 77, 891-98.

[7] Friedman, D., "The Size and Shape of Nations", *Journal of Political Economics,* 1977, 85, 59-77.

[8] Fudenberg, D. and J. Tirole, *Game Theory*, Cambridge: The MIT Press, 1991.

[9] Garfinkel, M. R., "Arming as a Strategic Investment in a Cooperative Equilibrium", *The American Economic Review,* 1990, 80, 50-68.

[10] Grossman, H. I. and M. Kim, "Swords or Plowshares? A Theory of the Security of Claims to Property", *Journal of Political Economy*, 1995, 103, 275-288.

[11] Grossman, H. I. and M. Kim, " Predation and Accumulation", *Journal of Economic Growth*, 1996, 1, 333-351.

[12] Grossman, H. I. and M. Kim, "Morality, Predation, and Welfare", Unpublished Workpaper, 1996.

[13] Grossman, H. I. and M. Kim, "Predation and Production", in Michelle R.G. and S. Skaperdas eds., *the Political Economy of Conflict and Appropriation,* New York: Cambridge University Press, 1996.

[14] Grossman, H. I. and M. Kim, "Conflict and Rent-Seeking Success Functions: Ratio vs. Difference Models of Relative Success", *Public Choice*, 1989, 63, 101-112.

[15] Grossman, H. I. and M. Kim, "The Paradox of Power", *Economics & politics*, 1991, 3, 177-200.

[16] Grossman, H. I. and M. Kim, "The Dark Side of The Force", *Economic Inquiry*, 1994, 32, 1-10.

[17] Grossman, H. I. and M. Kim, "The Macrotechnology of Conflict", *The Journal of Conflict Resolution*, 2000, 44, 773-791.

[18] Loury, Glenn C., "Market Structure and Innovation", *Quarterly Journal of Economics*, 1979, 93, 395-410.

[19] Macnail, E. S., J. C. Murdoch, C. R. Pi and T. Sandler, "Growth and Defense: Pooled Estimates for the NATO Alliance, 1951~88", *Southern Economic Journal*, 1995, 61, 846~860.

[20] Mejia, D. and C. E. Posada, "Capital Destruction, Optimal Defense and Economic Growth", Workpaper, 2002.

[21] Rosen, S., "Prizes and Incentives in Elimination Tournaments", *American Economic Review*, 1986, 76, 701-715.

[22] Skaperdas, S., "Cooperation, Conflict and Power in the Absence of Property Rights", *American Economic Review*, 1992, 82, 720-739.

[23] Skaperdas, S., "Contest Success Functions", *Economic Theory*, 1996, 7, 283-90.

[24] Thompson, E. A., "Taxation and National Defense", *Journal of Political Economy*, 1974, 82, 755-782.

[25] Skaperdas, S., “Efficient rent-seeking”, In J. M. Buchanan, R.D. Tollison and G. Tullock eds., *Toward a theory of the rent-seeking society*, College Station, TX: Texas A&M University Press, 1980, 97-112.

[26] Sandler, Todd. & Hartley, Keith, *The Economics Of Defense*. Beijing: Economic Science Press, 2001.

[27] Angang Hu, Taoxiong Liu, the National Defense Develops much more Slowly than Economy （1952－2001）, *China’s Defense Economy*， 2003, 2.

[28] Angang Hu, Taoxiong Liu, the Comparison of Defense Capabilities of China, USA, Japan and India, *Strategy and Management*, 2003, 6, 40－45.

[29] Nye. J. S., *Understanding International Conflicts: An Introduction to Theory and History*. Shanghai: Shanghai People Publishing House, 2002, 1-16.