Does a Rising Tide Lift All Boats?
Welfare Consequences of Asymmetric Growth

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Abstract: A common presumption is that increased growth in the aggregate enhances the welfare of both the rich and the poor. I show that instead, as the rich get richer, the welfare of the poor may decline if the underlying growth is asymmetric. There are two distinct and complementary explanations: First, sector-biased, skill-biased technological change, and second, efficiency improvements in the government sector. In the first case, skill-biased technological change in sectors consumed by the skilled rich increases their income beyond the increase in economic wealth, causing a decline in the consumption and welfare of the low-skilled poor. This result stands in contrast to the standard model of skill-biased technological change. In the second case, growth takes the form of improved efficiency in a government sector that is financed by rich taxpayers. The welfare of the low-skilled poor will decline whenever the consumption bundle of the skilled rich embodies more skill intensity than does the production of government services. This analysis demonstrates that a rising tide need not lift all boats and that the exact nature of consumption patterns is important not only for growth and inequality, as has been emphasized in earlier literature, but also for welfare.

JEL: H50, I31, J20, J24, O11, O39

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1. Introduction

There is a growing consensus that new technologies complement skill, either directly or through productivity growth in the production of skill-complementing capital. Models of skill-biased technological change and capital-skill complementarity offer explanations for the rising skill premium in the latter half of the twentieth century and predict that wage inequality is likely to continue to increase. This paper expands these models to incorporate an additional insight, that new technologies appear to be directed not only toward factors of production (skilled labor), but also toward goods consumed predominantly by the rich. The result of this asymmetric growth is a fall in the welfare of the low-skilled poor in addition to rising wage inequality. This finding contrasts with the implications of the canonical one-sector model of skill-biased technological change, in which welfare increases despite rising wage inequality.¹

This insight also has important policy implications. Welfare is inherently difficult to measure, but some evidence suggests that well-being of the least-skilled poor has in fact fallen or stagnated in the face of economic growth. In the U.S., real GDP per capita increased 73% between 1970 and 2000, while the real wages of the lowest quintile earners decreased by over 20%. Likewise, Brazilian GDP per capita increased over 46% during the same period, yet the living conditions of the poorest residents have not improved.² Thus the “rising tide” of economic growth did not by necessity “lift all boats”, as JFK famously predicted in 1963. My analysis provides a formal explanation of how the Poor’s welfare may fail to improve in the presence of aggregate economic growth even in advanced countries such as the United States.

The analysis differs from earlier studies that have examined circumstances in which economic growth may reduce welfare. Some notable examples include models of the Dutch Disease, as discussed in Corden and Neary (1982) and Krugman (1987), and of Immiserising Growth (see Baghwati 1958). Both Dutch Disease and Immiserising Growth rely on specific circumstances that need not hold in general. In contrast, the explanation I provide in this paper dispenses with these special conditions. It is applicable to labor markets in a closed economy, but can also be reinterpreted to apply to a two-country international trade setting. Furthermore, the model presented below directly addresses the “trickle down phenomenon” often heard in policy debates. An extensive literature has documented the failure of U.S. economic growth to

¹ See Acemoglu (1998, 2003)
² According to the World Bank Development Research Group, in 2000, over 10% of the population continued to live on less than $1.25 a day.
“trickle down” to the lowest quintile of wage earners, but theoretical explanations to date remain inadequate. Beaudry and Green (2003), for example, propose a model of organizational change that can generate falling real wages but relies on a counterfactual increase in the price of capital. In contrast to Beaudry and Green, in the model presented below the focus is explicitly on welfare rather than the real wage, and the welfare implications do not rely on any assumptions about the existence or price of capital. Furthermore, the model with sector-specific, skill-biased technological change is consistent with some interesting features of the macroeconomy during the last half of the Twentieth Century, including 1) increasing expenditure shares of high-end services, 2) an increasing skill premium, 3) increasing skill intensity in high-end service sectors, and 4) a fall in the price of capital.

The first part of this paper extends the one-sector, two-factor model in Acemoglu (1998) to an economy with two sectors (Yachts and Potatoes) and two agent types (Rich and Poor). The Rich agents own an endowment of high-skilled labor, while the Poor own an endowment of low-skilled labor. The key assumptions are, first, demand for Yachts is increasing in income; second, skill-biased technological improvements are sector specific; and third, the elasticity of substitution between high skilled labor and low skilled labor is greater than unity. If technology improves in the Yacht sector, the wage of the skilled Rich increases. The Rich in turn use their increased income to demand more Yachts, which requires skilled labor to flow out of the Potato sector and into the Yacht sector. The result is a fall in the supply of Potatoes. If preferences are strongly nonhomothetic such that the Poor consume only Potatoes, their welfare will decline.

The assumption that technological growth been biased toward the goods the rich consume has some empirical support in the macroeconomic literature. Buera and Kaboski (2009) document that as income has grown in the latter half of the twentieth century, there has been a substantial increase in the expenditure share of skill-intensive services such as finance, insurance, real estate, and architectural services. I thus interpret Yachts to be skill intensive services. Furthermore, Jorgenson and Stiroh (2000) argue that the majority of TFP growth has

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3 Galor and Moav (2000) also propose a model of ability-biased technological change that can account for rising wage inequality. Their model generates a temporary fall in welfare for the low-skilled due to the assumption on the production function that technological progress erodes the weight given to unskilled labor.

4 Appendix C extends the capital-skill complementarity model in Krusell, Ohanian, Rios-Rull, and Violante (2000), which has three factors of production, and derives the same welfare implications. This paper considers capital-skill complementarity to be consistent with skill-biased technological change and therefore refers to the two interchangeably. In contrast to Beaudry and Green (2003), the results here are consistent with a fall in the price of capital over time.
been in the production of computers and IT, and Bosworth and Triplett (2000) show that the most intensive users of computer technology have included high-skill services such as finance, insurance, and communications.

This supporting empirical evidence is consistent with arguments in Acemoglu (1998, 2003) that technological change responds to market forces. As the rich demand more financial services, for example, the returns to the inputs in financial service production increase, which in turn increases the incentive to create software for the finance industry. The implication of this form of asymmetric growth, according to the model presented below, is a bifurcation of the economy: skilled labor flows from sectors consumed by the Poor to those consumed by Rich, depriving the Poor of goods and services.

This pattern of bifurcation may be most salient among goods or services within the same sector. Broda and Romalis (2009) document that low-income households consume a basket of goods that is entirely different from the basket of high-income individuals, even though the goods are similarly classified. Their evidence is based on scanner data for consumer goods such as Maxwell coffee and Starbucks, but I assume that a similar pattern holds for the nontraded service sector as well. For example, low-income households use basic medical services at local clinics while the wealthy undergo plastic surgeries. If we reinterpret Yachts to be high-end services such as cosmetic plastic surgeries, the model offers insights into the implications of a plastic surgeon’s office obtaining state-of-the-art operating equipment: Skilled nurses leave the clinic in the poor neighborhood to earn a higher wage at the plastic surgeon’s office in the wealthy neighborhood, driving up prices or reducing quality at the clinic.

This phenomenon also is consistent with the chronic underdevelopment of the poorest neighborhoods in America, South Africa, and elsewhere. If technological improvements have been biased toward investments in products for the wealthy, skilled labor and capital will flow into the provision of goods and services for the wealthy, leaving fewer productive inputs to provide for the poor. In the poorest neighborhoods, where goods and services are consumed exclusively by the low-skilled residents, only low-quality services provided by primarily low-skilled workers will remain. Imagine a state-of-the-art auto repair shop built near a gated community in Cape Town, South Africa. Skilled mechanics will earn a high return using the new equipment, leaving the low-skilled auto workers to repair cars for the poor out of shacks in the townships. Since the low-skilled mechanics work with inferior capital equipment their
marginal product remains low, as does their income. Low income implies that demand for goods and services in low-income neighborhoods remains insufficient to attract new investments that would, in turn, increase wages and wealth.

Sector-specific, skill-biased technological change is not the only source of asymmetric growth that has implications for income, demand patterns, and welfare. The second part of the paper examines another type of asymmetric growth in the form of improved efficiency in a government sector that is financed exclusively by taxes on the wealthy. This model is complementary to the earlier model in that either channel could be in operation without the other, or both channels could be active. The purpose of the second model is to understand the implications of tax cuts at the upper end of the income distribution. For tractability, I focus on a model in which the government balances its budget every period. I assume that government services do not enter agents’ utility functions, and that an exogenous level of government services is necessary. The government sector could represent military or defense services, for example. As the threat to national security falls, so does government spending, rendering the provision of security by the government more efficient in the sense that it can provide the same level of security at a lower cost. An alternative interpretation of this model of the government sector is simply spending on wasteful political projects. Government efficiency improves when wasteful projects are eliminated, leaving only necessary services (such as food inspection or defense) to be financed by taxes on the skilled Rich.

With the inclusion of the government sector, the “rising tide” takes the form of improved government efficiency. As government efficiency improves, the tax burden of the Rich falls and their wealth increases. If the consumption bundle of the Rich embodies more skill intensity than does the production of government services, the welfare of the low-skilled Poor will decline. As in the first model with sector-biased, skill-biased technological change, the wealth transfer to the Rich is greater than the increase in private economic wealth; the Poor are worse off as a result. In contrast to the model of sector-biased, skill-biased technological change, in the model with the government sector the direction of the welfare change for the Poor depends on sectoral differences in skill intensities rather than input elasticities of substitution. Furthermore, the model with the government sector does not require that the Rich and the Poor consume different bundles of goods for the Poor to be worse off. However, the welfare decline for the Poor will be greater when the consumption bundles differ such that consumption of the Rich embodies more
skill intensity. In other words, even if the Rich use more low-skilled cleaning services, the welfare decline of the poor can be substantial if the Rich disproportionately hire skilled architects in addition to low-skilled cleaning services.

One implication of this model is that government waste is actually beneficial to the Poor when the production of goods consumed by the Rich is skill intensive relative to production in the government sector. Furthermore, taxes are important for more than their ability to pay for government services or to redistribute income: There are secondary effects on demand patterns that feed back into production decisions, the distribution of income, and welfare. In the calibrated baseline model the effects are admittedly small because the sectors within the government that are low-skill intensive are a small share of the whole economy. Therefore efficiency improvements in these sectors have a small wealth effect on the Rich (and in turn a small welfare impact on the Poor). Despite these small calibrated effects, the intuition from the model has implications for a more realistic setting in which tax cuts for the high-income coincide with budget deficits rather than efficiency improvements in the government sector: Tax cuts increase the disposable income of the skilled Rich, increasing the demand for high-end goods and services, which has an immediate effect on the supply of low-end goods and services available to the Poor. Although I do not explicitly incorporate endogenous technology, dynamic effects may operate as well. The arguments in Acemoglu (1998, 2003) imply that technology should flow to the high-end sectors as demand for these products increases. If this endogenous technology response is skill-biased (as in Section 3), then the welfare decrease for the Poor may be persistent and self-reinforcing in the absence of countervailing forces in the economy (such as Hicks-neutral growth and technology spillovers).

The remainder of the paper proceeds as follows: Section 2 reports the evidence that technological improvements have been biased toward goods predominantly consumed by the wealthy. Section 3 details the baseline model and illustrates the welfare effects of sector-specific skill-biased technological change. Section 4 incorporates a Government sector to demonstrate the welfare effects of falling tax rates due to the end of a tax-financed war. Section 5 concludes.

2. Macroeconomic Evidence of Sector-Biased Technological Change
A near consensus has emerged that U.S. economic growth, especially in the 1990s, has primarily been due to productivity growth in the production and the use of information technology (IT)
equipment. To the extent that IT use is unevenly distributed across sectors, technological progress will be asymmetric. The questions we address in this section are first, whether there has been substantial asymmetry in the use of IT equipment (and therefore economic growth), and second, whether this asymmetry is related to consumption demand patterns.

Triplett and Bosworth (2000) note that IT use has, indeed, been concentrated in a handful of industries. The 1992 capital flow tables show that five industries (financial services, wholesale trade, business services, insurance, and communications) alone accounted for over half of new purchases of computers. If the measure of IT includes communications equipment in addition to computers and peripheral equipment, the air transportation industry also is included as a primary user of IT. The pattern based on the 1997 capital flow table is remarkably similar: At a more aggregated industry level, the three primary users of computers, software, and communications equipment are information, finance and insurance, and professional and technical services.

Of the IT-intensive industries mentioned above, four can be linked to NIPA consumption categories: finance, insurance, professional services, and air transportation. The expenditure share of each of these categories has increased in the latter part of the Twentieth Century; their combined share increased by over 57% between 1970 and 2000. As Buera and Kaboski (2009) document, each of these is a relatively skill-intensive service industry, and consumption categories that experienced increasing expenditure shares are almost exclusively skill-intensive services. Other categories, such as food, clothing, and low-skill services have fallen or stagnated as a share of personal consumption expenditures. I interpret this evidence as indicative of nonhomothetic preferences: As income rises, demand shifts toward skill-intensive services, including those that are the most intensive users of IT.

In this paper I therefore interpret evidence of productivity growth in the use and production of IT capital as technological change that is biased toward goods consumed by the Rich. Technological change is also assumed to be skill-biased based on the overwhelming evidence in support of skill-biased technological change (including capital-skill

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6 The fact that the services demanded by the Rich are be skill-intensive is irrelevant for the substantive results presented in Section 3 when there is sector-biased, skill-biased technological change. However, the skill intensity of the consumption bundle of the Rich will be important in the model with a government sector (Section 4).
complementarity) in the latter part of the Twentieth Century. Section 3 models skill-biased technological change in the simplest form by allowing IT technology to augment skill in production functions with two factors (skilled and unskilled labor). Appendix C treats IT equipment as an additional factor in production functions in which IT capital and skill are relative complements.

3. **Baseline Model**

The baseline model consists of two factors (high-skilled labor $H$ and low-skilled labor $L$), two agent types (Rich and Poor), and two goods (Yachts and Potatoes) in a static economy. $H$ Rich agents each inelastically supply one unit of high-skilled labor and $L$ Poor agents each supply a unit of low-skilled labor. Here technology is taken as exogenous, and all markets are competitive.

3.1 **Consumer Preferences**

Rich (R) and Poor (P) consumers have identical nonhomothetic preferences over Yachts and Potatoes of the form

$$U_i(F_i, Y_i) = \max(a \times \log(F_i + b), Y_i)$$

where $i \in R, P$ and $Y_i$ is consumption of Yachts by consumer type $i$. We use $F_i$ to denote consumption of Potatoes by consumer $i$ ($P$ already refers to Poor agents). This form of preferences has the useful property that consumption switches from exclusively Potatoes to exclusively Yachts as wealth crosses a certain threshold determined by the scale parameters $a$ and $b$. It captures the fact documented in Broda and Romalis (2009) that low-income households consume a basket of goods that is entirely different from the basket of high-income individuals, even though the goods may be similarly classified. For example, the Rich consume high-quality Starbucks coffee while the Poor consume Maxwell instant coffee. The evidence in Broda and Romalis is based primarily on scanner data and applies mainly to different brands of

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7 See, for example, Bound and Johnson (1992), Autor, Levy and Murnane (2003), and Autor, Katz, and Kearney (2008)

8 A more common form of preferences in the structural change literature takes the form

$$U_i(F_i, Y_i) = \begin{cases}  F_i & \text{if } F_i < \bar{F} \\ \bar{F} + \gamma_i & \text{if } F_i > \bar{F} \end{cases}$$

in which the wealthy consume both Potatoes and Yachts but only once they’ve satiated their demand for Potatoes. The welfare implications are robust to this form of preferences, but they are analytically inconvenient.
goods within a sector, but we make the additional assumption that the Yacht bundle includes skill-intensive service sectors that are not included in the Potato bundle, such as financial planning services and architectural services. Since the skill-intensive services have experienced the majority of technological improvements in the form of IT use, we will assume that technological growth occurs primarily in the yacht sector (see section 3.4).

An interesting quality of the consumer preferences is that if the wealth of the Poor were to increase, they would initially consume more Maxwell coffee and Mickey’s Malt Liquor (referred to collectively as Potatoes). At some point their wealth may be high enough that they instead purchase fine wines, airline tickets, and financial services (Yachts). I assume that endowments and technologies are such that the low-skilled Poor remain low-income and thus consume only Potatoes, while the high-skilled rich consume only Yachts. We can thus rewrite preferences as

\[ U_R(\cdot) = Y_R \quad U_P(\cdot) = a \times \log(F_P + b). \]

If the Rich were handed a Potato, it would not increase their utility. This seems reasonable; wealthy households likely have little use for malt liquor since it would take up cabinet space reserved for higher quality alcoholic beverages. Similarly, if the Poor were handed a Yacht their utility would not increase. This is clearly a less palpable assumption but may be appropriate in some contexts. If the low-income poor were given a claim on architectural services they could not use it without owning a home (which they may not be able to afford). Rather than actually use the service they would exchange it for a good or service that will provide them with utility.

### 3.2 Production

Potatoes \((F)\) and Yachts \((Y)\) are competitively produced with a constant-returns-to-scale technology using high-skilled labor and low-skilled labor:

\[ F = F(z_F H_F, L_F) \]

and

\[ Y = Y(z_Y H_Y, L_Y) \]

where \(H_j\) and \(L_j\) are high-skilled labor and low-skilled labor employed in sector \(j \in F, Y\) and \(z_j\) is the skill-augmenting technology parameter in sector \(j\). Here we assume that production has the
same constant elasticity of substitution (CES) functional form as the models in Acemoglu (1998, 2003):

\[
F = \left[ \eta(z_F H_F)^{\frac{\sigma_F-1}{\sigma_F}} + (1-\eta)L_F^{\frac{\sigma_F}{\sigma_F}} \right]^{\frac{\sigma_F}{\sigma_F-1}}
\]

and

\[
Y = \left[ \mu(z_Y H_Y)^{\frac{\sigma_Y-1}{\sigma_Y}} + (1-\mu)L_Y^{\frac{\sigma_Y}{\sigma_Y}} \right]^{\frac{\sigma_Y}{\sigma_Y-1}}.
\]

Appendix B examines equilibrium effects when production functional forms are not specified, and Appendix C incorporates IT capital into a nested CES functional form similar to that used in Krusell, Ohanian, Rios-Rull, and Violante (2000).

3.3 Equilibrium and the Effects of Asymmetric Growth

In the static competitive equilibrium consumers maximize utility subject to their budget constraints; firms maximize profits, and labor markets clear. The \( H \) Rich agents’ collective budget constraint is

\[ Hw_H \geq F_R p_R + Y_R p_Y \]

where \( w_H \) is the wage for high-skilled labor, \( p_j \) is the price of good \( j \), and \( j_R \) is consumption of good \( j \) by Rich agents. Since endowments and technology are such that the Rich have enough wealth to exclusively purchase Yachts, their budget constraint can be written as

\[ Hw_H \geq Y p_Y. \]

Furthermore, since production is competitive and exhibits constant returns to scale, \( p_Y \) will equal the cost-minimizing bundle of inputs necessary to produce one Yacht. Thus

\[ Y p_Y = H_Y w_H + L_Y w_L \]

and we can rewrite a representative Rich agent’s problem as

\[
\max \left[ \mu(z_Y H_Y)^{\frac{\sigma_Y-1}{\sigma_Y}} + (1-\mu)L_Y^{\frac{\sigma_Y}{\sigma_Y}} \right]^{\frac{\sigma_Y}{\sigma_Y-1}}
\]

s.t. \( Hw_H \geq H_Y w_H + L_Y w_L \).

Likewise, the representative Poor agent’s problem is
\[
\text{max } .5 \times \log \left( \frac{\eta (z_Y H_F)^{\sigma_Y^{-1}}}{\sigma_Y} + (1 - \eta) L_F^{\sigma_F^{-1}} \right)
\]

s.t. \( L w_L \geq H_F w_H + L_F w_L \).  

(2)

Viewing the consumers’ problem as a choice over consumption of the two labor types is helpful for understanding the comparative static effects of an increase in \( z_Y \), which in equilibrium will depend on the substitution elasticities \( \sigma_Y \) and \( \sigma_F \). Equilibrium is fully characterized by the budget constraints (equations (1) and (2)), utility maximization by the Rich:

\[
\frac{w_H}{w_L} = \frac{\mu}{1 - \mu} \frac{z_Y}{K_Y} \left( \frac{L_Y}{\sigma_Y} \right)^{\frac{1}{\sigma_Y}},
\]

(3)

utility maximization by the Poor:

\[
\frac{w_H}{w_L} = \frac{\eta}{1 - \eta} \frac{z_F}{K_F} \left( \frac{L_F}{\sigma_F} \right)^{\frac{1}{\sigma_F}},
\]

(4)

and market clearing:

\[
L_F + L_Y = L,
\]

(5)

\[
H_F + H_Y = H.
\]

(6)

As noted above, technological improvements have been biased toward high-end services, which are modeled here as Yachts. Therefore the object of interest is skill-biased technology in the Yacht sector, \( z_Y \).

Proposition 1: If high-skill labor and low-skilled labor are substitutes in the production of Yachts (\( \sigma_Y > 1 \)), then an increase in skill biased technology in the Yacht sector (\( z_Y \)) will cause a decrease in the amount of Potatoes produced and therefore a decline the welfare of the Poor. If labor types are substitutes in the production of Potatoes (\( \sigma_F > 1 \)) the decline will be due to an outflow of high-skill labor from the Potato sector to the Yacht sector. If labor types are complements (\( \sigma_F < 1 \)) in the production of Potatoes the decline will be due to an outflow of both inputs from the Potato sector.

Proof: See Appendix A.
The empirically relevant case is when labor types are substitutes \((\sigma_Y > 1 \text{ and } \sigma_F > 1)\), although the crucial assumption for a fall in the welfare of the Poor is simply \(\sigma_Y > 1\). An increase in \(z_Y\) drives up the wage premium, increasing the income of the Rich. Rich agents use their income to effectively purchase bundles of high-skill labor and low-skill labor. Since the \(z_Y H_Y\) bundle is a substitute for \(L_Y\) in the Rich’s utility function, the increase in \(z_Y\) increases \(z_Y H_Y\), causing the Rich to desire a substitution of \(H_Y\) for \(L_Y\). Since the increase in \(z_Y\) also increases the return to skilled labor and therefore the wealth of the Rich, the Rich are able to meet their desire for more skilled labor by purchasing skilled labor from the Poor. Skilled labor therefore flows from the Poor to the Rich (from the Potato sector to the Yacht sector).

The effect on the allocation of low-skilled labor, \(L\), depends on the elasticity of substitution in the Potato sector. If \(\sigma_F < 1\), labor types are complements for the Poor and the \(z_Y\)-induced decline in \(H_F\) lowers the value of \(L_F\), which in turn diminishes the income of the Poor relative to the income of the Rich. In this case, the Rich have enough wealth to purchase more low-skilled labor in addition to high-skilled labor. If \(\sigma_F > 1\), which is likely the empirically relevant case, the outflow of \(H\) from the Poor’s consumption bundle causes a desire to substitute \(L\) for \(H\), which increases the value of \(L\) relative to the case of complements. The Poor then are able to retain enough wealth to purchase low-skilled labor from the consumption bundle of the Rich.

When inputs are substitutes in each sector, the net effect is a fall in the utility of the Poor. This is because the effect of the outflow of high-skilled labor from the Potato sector outweighs the effect of the inflow of low-skilled labor (see Appendix A). Figure 1 illustrates the net effect of an increase in \(z\) using an Edgeworth Box in which the representative agents trade high-skilled labor and low-skilled labor. The isoutility lines are identical to isoquants in the production of Potatoes for the Poor and Yachts for the Rich. Note that the original endowment of \((L, H)\) to the Rich is \((0,1)\). Figure 1 decomposes the change in allocations into what are labeled a substitution effect and an income effect. The substitution effect is defined as the change in allocations induced by an increase in \(z_Y\) when the economy’s endowment point is assumed to be the original equilibrium (rather than \((0,1)\)). The remaining distance from the original equilibrium to the actual new equilibrium is the income effect.
The key point in Figure 1 is that the income effect, rather than the substitution effect, drives down the utility of the Poor. In fact, the substitution effect places both the Poor and the Rich on a slightly higher isoulety line. The income effect captures the fact the Rich are endowed with high-skilled labor, which has increased in value. The Rich are able to use their increased wealth to purchase additional skilled labor for the production of Yachts.

An alternative way to understand the mechanism driving down the welfare of the Poor is through prices. If we normalize the price of low-skilled labor to unity, then the price of a Potato is

$$p_F = \left[ \eta^{\sigma_F} \left( \frac{w_H}{z_F} \right)^{1-\sigma_F} + (1 - \eta)^{\sigma_F} \right]^{\frac{1}{1-\sigma_F}}$$

and the price of a Yacht is
\[ p_Y = \left[ \mu_Y \left( \frac{w_H}{z_Y} \right)^{1-\sigma_Y} + (1-\mu)\sigma_Y \right]^{\frac{1}{1-\sigma_Y}}. \]

When \( z_Y \) increases the marginal product of skilled labor increases, driving up \( w_H \). The price of Yachts falls because the increase in \( z_Y \) is greater than the increase in \( w_H \) \((dw_H/dz < 1)\). Potatoes, meanwhile, do not benefit from price-reducing technological change, and thus the price of Potatoes increases because of the increase in \( w_H \). Therefore the Poor do not benefit from higher wages but must pay a higher price for their consumption good.

The Poor would benefit if technological change augments either factor in the Potato sector or augments low-skilled labor in the Yacht sector. For example, technological change biased toward low-skilled labor in the Yacht sector pulls down \( w_H \) and the price of Potatoes relative to the return on low-skilled labor, thus improving the welfare of the Poor. The Poor likewise benefit from skill-biased technological change in the Potato sector: An increase in \( z_F \) increases \( w_H \) relative to \( w_L \), but the overall effect is a fall in the price of Potatoes.

The greater is the elasticity of substitution in the Yacht sector, the greater is the consumption loss for the Poor in response to an increase in \( z_Y \). Define \( \hat{F} = dF/F \) and \( \hat{z}_Y = dz_Y/z_Y \). Then total differentiation of equations 1 through 6 yields the response of Potato production to a small change in skill-biased technology in the Yacht sector:

\[
\hat{F} = -F^{-\frac{\beta}{\beta-1}} \left[ \frac{\sigma_Y H_Y (\sigma_Y - 1) (1-\eta) L_F \frac{1}{\sigma_F} L_Y}{(\sigma_F H_F + \sigma_Y H_Y) L_F (\sigma_Y - 1) + (\sigma_F L_F + \sigma_Y L_Y)(H_F + \sigma_Y H_Y)} \right] \times \hat{z}_Y,
\]

the magnitude of which is increasing in \( \sigma_Y \) when \( \sigma_Y > 1 \). Most estimates of the elasticity of substitution between skilled labor and low-skilled labor are between 1.4 and 2, implying that an increase in skill-biased technology in the Yacht sector drives down the supply of Potatoes.\(^9\) Note that the direction of the change in the supply of Potatoes does not depend on factor intensities in the two sectors.

### 3.4 Calibration

To get a sense of the magnitude of the consumption loss for the Poor we calibrate the model by choosing \( \sigma_Y = \sigma_F = 1.4 \), which is at the lower end of the empirical estimates of the elasticity of substitution between skilled labor and low-skilled labor.\(^9\)

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substitution between skilled and low-skilled labor. We set the skill ratio, $\frac{H}{L}$, equal to 0.7, which is close to the 2000 relative supply in Buera and Kaboski (2009) and the 1996 relative supply in Acemoglu (2002). The starting values for $z_Y$ and $z_F$ are equal to one. Finally, we choose $\eta = \mu = 0.62$ to match the wage premium in 2000, which is approximately 2.1 (see Acemoglu and Autor 2010). With these parameter values, a percent increase in $z$ from a starting value of 1 causes a change in the supply of Potatoes of $-0.1\%$.

Acemoglu (2002) postulates that skill biased technology increased almost tenfold in the U.S. between 1970 and 1990, based on a one-sector model with an elasticity of substitution equal to 1.4. The model here shows that if the full extent of technological improvements had been specific to sectors exclusively consumed by wealthy college graduates, the consumption loss for low-skilled workers would have been around a magnitude of 22% in the absence of hicks-neutral technological improvements, increases in the relative supply of high-skilled labor, technology spillovers, and other sources of economic growth.

The assumption that technological improvements are confined to the Yacht sector is illustrative but not realistic. If technological improvements occur in both sectors, the net effect on the supply of Potatoes will depend on the relative magnitude of skill-biased technological change in the Yacht sector. Table 1 shows different combinations of increases in $z_Y$ and $z_F$ that achieve the same increase in the wage premium, along with the percent change in Potatoes per low-skilled worker. With an elasticity of substitution equal to 1.4 in both sectors, an 85% increase in $z_Y$ requires a 13% increase in $z_F$ to ensure that the supply of Potatoes does not fall. When the elasticity is 2, a 75% increase in $z_Y$ will depress the Poor’s consumption of Potatoes even if $z_F$ increases by 22%. When growth is more symmetric, the consumption and welfare of the Poor improves.
Table 1: Response of Consumption of the Poor to Skill Biased Technological Improvements

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Note: Each row generates an equivalent increase in the skill premium for a given value of the elasticity of substitution.

As in the canonical one-sector model in Acemoglu (1998), inequality increases in all cases. For example, when $z_Y$ doubles the skill premium increases by 23.5%. Furthermore, the expenditure share of Yachts increases by over 15%. The model therefore matches both the increasing trend in the skill premium and the trend documented in Buera and Kaboski (2009) of an increasing expenditure share of high-end services over time. Finally, the model predicts that the skill intensity of the Yacht sector should increase in response to the increase in $z_Y$ due to the inflow of skilled labor. This is exactly the pattern observed in the high-end sectors mentioned in Section 2. Between 1940 and 2000, the average skill intensity over all sectors increased almost 70%, while the average skill intensity in the high-end service sectors increased over 250%.10

4. Government Spending and Welfare

The previous section examined growth in the form of sector-specific, skill-biased technological change. Here we examine growth in the form of increased government efficiency. As in the previous model, economic growth will depress the welfare of the low-skilled poor because the wealth increase of the skilled Rich is greater than the increase in private economic wealth. Unlike the model above, the welfare decline does not rely on differential consumption patterns

10 Based on IPUMS data used in Buera and Kaboski (2009). The high-end industries include Security and commodity brokerage and investment companies, Banking and credit, Legal services, Engineering and architectural services, Real estate, Insurance, and Air transportation.
between the Rich and the Poor. However, the magnitude of the decline will be greater if the consumption of the Rich embodies more skill intensity than the consumption bundle of the Poor.

Rather than define efficiency to be output per units of input, in which case “government efficiency” may strike readers as a bit of an oxymoron, I define improvements in government efficiency as a lower necessary level of government spending on anything over which agents do not derive utility. Spending on war is the clearest example: War may be necessary for security, but citizens are assumed to not care about wars for their own sake. As threats to national security decline, so can war spending, freeing resources for use in the private sector.

If we model war as the production and destruction of tanks by the government, then one would expect that less tank production should be welfare-improving. The model below shows that it is not welfare-improving for low-skilled workers when certain assumptions hold. Specifically, the model imposes a progressive tax system in which only skilled workers pay taxes. If a war ends (fewer tanks are produced and destroyed), the savings are passed on to the tax-paying skilled who then demand more private sector goods and services. If the consumption bundle of the skilled workers embodies more skill intensity than the war production function, the wages and welfare of the low-skilled will fall. The mechanism is similar to that in Section 3: The income increase for the skilled Rich is greater than the increase in economic wealth available for private consumption, allowing the Rich to purchase productive resources that would otherwise have been purchased by the Poor.

4.1 Model

$H$ skilled Rich agents and $L$ low-skilled Poor agents have identical, nonhomothetic preferences over Potatoes (F) and Yachts (Y). Consumer preferences are identical to those in section 3, and we again assume that the income of Poor agents is low enough that they consume only Potatoes, while the Rich agents consume only Yachts. Reduced form preferences are therefore

$$U_R(\cdot) = Y_R \quad U_P(\cdot) = a \times \log(F_P + b).$$

Yachts and Potatoes are again produced with a constant returns technology over high-skilled labor and low-skilled labor, although here we assume a general functional form and ignore the effects of factor-augmenting technology:

$$F = F(H_F, L_P)$$

and
\[ Y = Y(H_Y, L_Y). \]

We introduce a government sector, which also takes high-skilled labor and low-skilled labor as inputs in a constant-returns-to-scale production function:

\[ G = G(H_G, L_G). \]

Agents do not derive utility from the government sector. Instead, we assume an exogenously given level of necessary government services \( \bar{G} \) that is financed by taxes on the skilled Rich:

\[ \tau w_H H = p_G \bar{G}, \]

where \( \tau \) is the income tax rate on the Rich and \( p_G \) is the price of government services.

### 4.2 Equilibrium

The representative skilled Rich agent’s budget constraint is

\[ (1 - \tau) w_H H = p_Y Y_R, \]

which we can rewrite as

\[ w_H H = p_Y Y + p_G \bar{G} \tag{7} \]

The low-skilled Poor agent’s budget constraint is

\[ w_L L = p_F F \tag{8} \]

The remaining equations that characterize the static competitive equilibrium are cost minimization in the three sectors,

\[ c_Y(w_H, w_L) = p_Y, \tag{9} \]

\[ c_F(w_H, w_L) = p_F, \tag{10} \]

\[ c_G(w_H, w_L) = p_G, \tag{11} \]

market clearing conditions,

\[ H_Y + H_F + H_G = H, \tag{12} \]

\[ L_Y + L_F + L_G = L, \tag{13} \]

and factor demands given by Shepard’s Lemma:

\[ \frac{\partial c_Y}{\partial w_H} = \frac{H_Y}{Y} \tag{14} \]

\[ \frac{\partial c_Y}{\partial w_L} = \frac{L_Y}{Y} \tag{15} \]

\[ \frac{\partial c_F}{\partial w_H} = \frac{H_F}{F} \tag{16} \]
\[
\frac{\partial c_F}{\partial w_L} = \frac{L_F}{F} \tag{17}
\]
\[
\frac{\partial c_G}{\partial w_H} = \frac{H_G}{G} \tag{18}
\]
\[
\frac{\partial c_G}{\partial w_L} = \frac{L_G}{G}. \tag{19}
\]

Note that the cost function for sector \( Y \) is
\[
c_Y(w_H, w_L) = \min\{w_H H_Y + w_L L_Y : Y(H_Y, L_Y) = 1\},
\]
and the cost functions for sectors \( F \) and \( G \) are similarly defined.

For any variable \( x \), define \( \hat{x} = dx/x \). Then log-linearizing equations (7) through (19) gives the responses of endogenous variables to a change in \( \hat{G} \):
\[
\hat{w}_H = s_Y(\hat{Y} + \hat{p}_Y) + s_G(\hat{G} + \hat{p}_G) \tag{20}
\]
\[
\hat{w}_L = \hat{F} + \hat{p}_F \tag{21}
\]
\[
\theta_H \hat{w}_H + \theta_L \hat{w}_L = \hat{p}_Y \tag{22}
\]
\[
\phi_H \hat{w}_H + \phi_L \hat{w}_L = \hat{p}_F \tag{23}
\]
\[
\psi_H \hat{w}_H + \psi_L \hat{w}_L = \hat{p}_G \tag{24}
\]
\[
\lambda_{H_Y} \hat{H}_Y + \lambda_{H_F} \hat{H}_F + \lambda_{H_G} \hat{H}_G = 0 \tag{25}
\]
\[
\lambda_{L_Y} \hat{L}_Y + \lambda_{L_F} \hat{L}_F + \lambda_{L_G} \hat{L}_G = 0 \tag{26}
\]
\[
\hat{H}_Y = \hat{Y} + \theta_H \sigma_Y(\hat{w}_H - \hat{w}_L) \tag{27}
\]
\[
\hat{L}_Y = \hat{Y} + \theta_L \sigma_Y(\hat{w}_H - \hat{w}_L) \tag{28}
\]
\[
\hat{H}_F = \hat{F} + \phi_H \sigma_F(\hat{w}_H - \hat{w}_L) \tag{29}
\]
\[
\hat{L}_F = \hat{F} + \phi_L \sigma_F(\hat{w}_H - \hat{w}_L) \tag{30}
\]
\[
\hat{H}_G = \hat{G} + \psi_H \sigma_G(\hat{w}_H - \hat{w}_L) \tag{31}
\]
\[
\hat{L}_G = \hat{G} + \psi_L \sigma_G(\hat{w}_H - \hat{w}_L) \tag{32}
\]

We denote cost shares of labor in the Potato sector as \( \phi_H \equiv w_H H_F / F p_F \) and \( \phi_L \equiv w_L L_F / F p_F \).

Cost shares in the Yacht sector (\( \theta_H \) and \( \theta_L \)) and in the Government sector (\( \psi_H \) and \( \psi_L \)) are similarly defined. The shares of labor types in each sector \( j \in \{Y, F, G\} \) are \( \lambda_{H_j} = \frac{H_j}{H} \) and \( \lambda_{L_j} = \frac{L_j}{L} \). The cost shares of each good in the Rich agent’s budget constraint are \( s_Y = \frac{Y p_Y}{Y p_Y + G p_G} \) and \( s_G = \frac{G p_G}{Y p_Y + G p_G} \). As in section 2, \( \sigma_j \) is the elasticity of substitution in sector \( j \).
We can normalize \( w_L = 1 \) so that \( \tilde{w}_L = 0 \). Then the above system of equations has thirteen equations and twelve unknowns. By Walras’ Law, one of the equations is redundant, so we will choose to ignore equation (1). The remaining system of equations is

\[
\begin{bmatrix}
0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\theta_H & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\phi_H & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\psi_H & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \lambda_{Hc} & 0 & \lambda_{Hy} & 0 & \lambda_{Hy} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \lambda_{Lc} & 0 & \lambda_{Ly} & 0 & \lambda_{Ly} & 0 \\
-\theta_H \sigma_C & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
\theta_L \sigma_C & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
-\phi_H \sigma_Y & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\
\phi_L \sigma_Y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
\psi_H \sigma_G & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
\psi_L \sigma_G & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\tilde{w}_H \\
\tilde{p}_Y \\
\tilde{p}_F \\
\tilde{p}_G \\
\tilde{g} \\
\tilde{h}_Y \\
\tilde{h}_F \\
\tilde{h}_G \\
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
1 \\
0 \\
\end{bmatrix}
\]

The Poor’s welfare is monotonically increasing in the production of Potatoes, \( F \). Therefore the variable of interest is \( \hat{F} \), which has a simple relationship with the change in the wage premium, \( \hat{w}_H \), given by

\[
\hat{F} = -\phi_H \hat{w}_H. \tag{33}
\]

The above equation derives from the Poor’s budget constraint and cost minimization in the Potato sector. As the price of skilled labor increases, so does the price of Potatoes, which drives down the purchasing power of the Poor. Therefore to understand the effects of changes in the size of the government sector on the welfare of the Poor, we only need to determine its effects on the price of skilled labor.

The relationship between the change in the skilled wage and the change in government production is

\[
\hat{w}_h = \frac{(\lambda_{H_g} \lambda_{L_Y} - \lambda_{L_g} \lambda_{H_Y})}{\lambda_{L_g} \lambda_{H_Y} \psi_L \sigma_G + \lambda_{H_g} \lambda_{L_Y} \psi_H \sigma_G + \lambda_{L_g} \lambda_{H_Y} [\phi_L \sigma_F + \phi_H] + \lambda_{L_Y} [\lambda_{H_Y} \sigma_Y - \lambda_{H_F} \phi_H (1 - \sigma_F)]}
\]

The denominator is positive whenever \( \sigma_F \geq 1 \), which we again assume here. Then the skilled wage will be inversely related to the size of the government sector whenever \( \lambda_{L_g} \lambda_{H_Y} > \lambda_{H_g} \lambda_{L_Y} \), which is equivalent to \( \frac{H_Y}{L_Y} > \frac{H_G}{L_G} \). This condition states that whenever the yacht sector is more skill intensive than the government sector, a decrease in the size of the government sector will drive
up the skilled wage and drive down the welfare of the Poor. The mechanism is straightforward: Lowering $\tilde{G}$ allows the Rich to use their income to purchase Yachts instead of government services. If Yachts are relatively skill-intensive, the demand for skilled labor increases, pushing up the skilled wage. The increase in $w_H$ pushes up the price of Potatoes relative to $w_L$, which diminishes the purchasing power of the low-skilled Poor. The effect of the increase in $w_H$ on the fall in the consumption of the Poor is more intense the larger is the cost share of skilled labor in the Potato sector.

An increase in $\tilde{G}$ has the opposite effect: if the government hires primarily low-skill workers for any task, however useless, the welfare of all low-skill workers will increase. One implication is that wasteful pork projects may be beneficial to all low skill workers, not just those directly employed by the projects. If low-skilled labor is mobile, the government-induced demand for low-skilled workers will drive up their wage while the skilled Rich foot the bill.

4.3 Calibration
To get a sense for the sensitivity of wages and welfare to changes in the government sector, we treat the government sector as the Armed Forces, the size of which increases during times of war and decreases during times of peace. For example, in 2001 approximately 0.5% of workers were in the Armed Forces, while in 2008 over 0.8% of workers were in the Armed Forces, an increase that we attribute to the wars in Iraq and Afghanistan. In this section we will approximate the wage and welfare consequences of an end to these wars and a 50% reduction in the size of the Armed Forces.

The following calculations are based on American Community Survey data provided by the Integrated Public Use Microdata Series (IPUMS). Skilled workers are defined as those who have completed four or more years of college. In 2008, the skill ratio, $\frac{H}{L}$, was .43 and the skill premium, $\frac{w_H}{w_L}$, was 2.14. The proportion of skilled workers in the Armed Forces, $\lambda_{HG}$, was 0.0065 and the proportion of low-skilled workers in the Armed Forces, $\lambda_{LG}$, was .009. To get a lower bound for the magnitude of the effect of changes in government size on wages and welfare, we first assume that the Yacht and Potato sectors have the same skill intensity (in which case the model above is equivalent to a model with only one consumption sector). Table 2 shows the implied parameter values. A one percent decrease in the size of the Armed Forces will increase
the wage premium by 0.0028% and decrease low-skilled workers’ consumption by 0.0014%. If the Armed Forces are halved to just below their 2001 size, low-skilled workers’ consumption will fall by around 0.07%.

These magnitudes increase when we allow the skill intensities in the Yacht and Potato sectors to differ. Although there is no direct data on the embodied skill intensity of the consumption bundle of the Rich, we can place an upper bound based on the skill intensities in the service sectors mentioned in Section 2. The most skill intensive industry, “Security and Commodity Brokerage and Investment Companies”, has a skill ratio of 0.66. We therefore conservatively set \( \frac{H_Y}{L_Y} \) to 0.6 and assume that the Yacht sector uses half of the total supply of high-skilled labor. Table 1 shows the resulting parameter values and the sensitivity of \( \bar{w}_H \) to \( \bar{G} \):

A percent decrease in \( \bar{G} \) increases the wage premium by 0.0028% and decreases the size of the Potato sector by 0.007%. If \( \bar{G} \) is halved, the wage premium will increase by around 1.0% and the consumption of Potatoes by the Poor will decline by around 0.36%.

| Table 2: Parameter Values and Effects of a Change in the Size of the Government Sector |
|----------------------------------------|-------------------------------|-------------------------------|-------------------------------|----------------|----------------|----------------|----------------|
| Lower Bound                           | \( \lambda_{HY} \) | \( \lambda_{HF} \) | \( \lambda_{HG} \) | \( \lambda_{LY} \) | \( \lambda_{LF} \) | \( \lambda_{LG} \) | \( \phi_H \) | \( \psi_H \) | \( \bar{w}_H/\bar{G} \) | \( \bar{F}/\bar{G} \) |
| Lower Bound                           | 0.497          | 0.497          | 0.007          | 0.496          | 0.496          | 0.009          | 0.48          | 0.42          | -0.0028          | 0.0014          |
| Upper Bound                           | 0.500          | 0.494          | 0.007          | 0.214          | 0.846          | 0.009          | 0.35          | 0.40          | -0.0206          | 0.0072          |

Note: Bounds for the response of \( F \) to a change in \( G \) are based on varying \( \lambda_{LY} \) so that the ratio of skilled labor to unskilled labor in the Yacht sector is 0.43 for the lower bound and 0.6 for the upper bound.

In the example above the Government sector is small relative to the rest of the economy. If the Government sector is initially large and intensive in low-skill workers, the welfare loss from a decrease in \( \bar{G} \) will be even more substantial. The effects will have the opposite sign when the government sector is skill intensive relative to the Yacht sector.

5. Conclusion
Rich individuals consume a bundle of goods and services that is different from the bundle of their Poor counterparts. Some goods and services may be higher quality versions of the same item (see Bils and Klenow (2001) and Broda and Romalis (2009)), while other services are
exclusive to the consumption bundles of the wealthy. The exact nature of consumption patterns is important not only for growth and inequality patterns, as has been emphasized in the earlier literature, but also for welfare.

As the Rich get richer, productive resources are increasingly devoted to the production of goods and services demanded by the Rich. If the source of the income increase of the Rich is asymmetric growth, their welfare increase may come at the expense of the welfare of the Poor. This is because the Rich’s increased income allows them to consume bundles of productive resources that would otherwise have been purchased by the Poor. If the Rich’s income increase is higher than the increase in economic wealth, the Poor not only fail to benefit from the growth, but they are actually worse off. Inequality increases and welfare falls.

This paper illustrated this mechanism through asymmetric growth of two forms: first, sector-biased, skill-biased technological change, and second, efficiency improvements in the government sector (or less government waste). In the first case, the welfare decline of the low-skilled Poor is greater the more biased is growth toward high-end goods and services. The welfare loss is also greater the less important are high-end goods and services in the consumption bundle of the Poor. In the extreme, the Poor do not consume any goods that experience productivity gains, and their consumption/welfare loss is substantial.

In related work, Murphy, Schleifer, and Vinshy (1989), Zweimuller (2000), Matsuyama (2000), Matsuyama (2002), and Buera and Kaboski (2009) examine growth patterns in economies with heterogeneous agents that have nonhomothetic preferences. Here I assumed a similar economic environment, but focused specifically on growth in the form of skill-biased technological change. My analysis therefore provides a link between the structural change literature and the models of skill-biased technological change in Acemoglu (1998, 2003). As in Baumol (1967) the technological change is assumed to be sector-specific. I demonstrated that Baumol’s cost disease, combined with the additional assumption of differential consumption by heterogeneous agents, not only increases the price of skilled labor, but also reduces the welfare of the Poor.

The second form of asymmetric growth modeled in this paper is a decline in government waste under a progressive tax system. In this scenario, a more efficient government depresses the welfare of the Poor whenever the consumption bundle of the Rich embodies more skill-intensity than the production of government services. This is true even if the consumption
bundles of the Poor and the Rich are the same. However, the effect is stronger when the bundle consumed by the Rich embodies higher-than-average skill intensity. Thus the end of a tax-financed war which employed low-skilled soldiers will hurt other low-skill workers most when the skilled Rich use their tax savings to purchase architectural, financial, and other skill-intensive services.  

Evidence suggests that sector-biased, skill-biased technological change is important at the sector level: Productivity growth in the US in the latter part of the Twentieth Century was primarily confined to the use and production of IT capital. The sectors that use the majority of IT capital are skill-intensive services that have upward-sloping Engel curves, including financial services, insurance, professional services, and air transportation. The increase in productivity in the production or use of IT employed in these industries actually deprives other sectors that cater to the Poor of resources, lowering their supply and expenditure share over time. Thus sector-biased, skill-biased technological change offers an explanation for the increasing expenditure shares of skill-intensive services in the latter part of the Twentieth Century. Skill-biased technological change in service sectors demanded by the skilled Rich increases the expenditure share of skill-intensive services and lowers the shares of other sectors.

The focus of this paper is on illustrating mechanisms rather than on developing methods to test these models empirically. A rigorous empirical test of the model would require matching consumption of disaggregated goods and services to the inputs used in production of the specific goods and services. Existing data clearly are inadequate for such an analysis. The development of suitable datasets for testing this proposition is beyond the scope of this paper.

An interesting avenue for empirical research would be to document productivity gains and the adoption of IT by service establishments at the neighborhood level. The differential nature of consumption between services in high-income and low-income neighborhoods is perhaps the starkest implication of the models presented in this paper when applied to the U.S. economy. If service establishments in high-income neighborhoods experience skill-biased

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11 This model is similar to the 2-factor, 2-sector setup in Naito (1999), which extends the Harbinger (1962) model to show that under a nonlinear income tax system inefficiencies in the government sector can Pareto-improve welfare by relaxing the skilled agents’ incentive compatibility constraint.
13 Buera and Kaboski (2009) propose an additional/alternative explanation for the rise of the service economy based on the substitution of market production of services for home production of services in response to economic growth. Ngai and Pissarides(2007) demonstrate that balanced growth is possible when technological change is sector-biased.
technology improvements or utilize more IT capital than their counterparts in low-income areas, then the model in this paper may help explain the chronic underdevelopment of some of the poorest neighborhoods.
References


Appendix A

Proof of Proposition 1: Total differentiation of equations (1) through (6) yields

\[
\frac{dH_F}{dz} = -\frac{(\sigma_Y - 1)(\sigma_F L_F + L_Y)H_Y}{H_F(\sigma_F L_F + \sigma_Y L_Y)} + \frac{H_Y((\sigma_F + \sigma_Y - 1)L_F + \sigma_Y L_Y)}{\sigma_Y - 1} \tag{A1}
\]

where \( \dot{x} = dx/x \) for any variable \( x \). Equation (A1) implies that \( dH_F/dz \) is negative if and only if \( \sigma_Y > 1 \). Assuming this is the case, (A2) implies that \( dL_F/dz \) is negative if and only if \( \sigma_F < 1 \).

If we assume that the elasticity of substitution is greater than unity in both sectors, then an increase in \( z \) will cause an outflow of skilled labor from the Potato sector and an inflow of unskilled labor. We can determine the net effect on the supply of Potatoes by total differentiation of the Potato production function:

\[
\frac{dF}{F} = \frac{1}{\sigma_F - 1} \eta K_F \frac{1}{\sigma_F} dK_F + (1 - \eta) \frac{1}{\sigma_F} dL_F.
\]

Substituting in (A1) and (A2) yields

\[
\dot{F} = -F \left[ \frac{\sigma_Y H_Y L(\sigma_Y - 1)(1 - \eta)L_F^{-\sigma_F} L_Y^{-\sigma_F}}{(\sigma_F H_F + \sigma_Y H_Y) L_F (\sigma_Y - 1) + (\sigma_F L_F + \sigma_Y L_Y)(H_F + \sigma_Y H_Y)} \right] \times \dot{z},
\]
which states that if $\sigma_Y > 1$ the supply of Potatoes decreases whenever there is an improvement in skill-biased technological change in the Yacht sector.

Appendix B
Here we alter the model in Section 3 to allow production of Yachts and Potatoes to use a general constant-returns-to-scale functional form. In the static competitive equilibrium consumers maximize utility subject to their budget constraints; firms maximize profits, and labor markets clear. The H Rich agents solve

$$\max Y_R$$
$$\text{s.t. } Hw_H = Yp_Y$$ \hspace{1cm} (B1)

Likewise, the Poor agents solve

$$\max 1.5 \times \log(F_P + 1)$$
$$\text{s.t. } Lw_L = Fp_F$$ \hspace{1cm} (B2)

Prices of Potatoes and Yachts are equal to unit costs $c_F$ and $c_Y$:

$$c_F(w_H, w_L) = p_F$$ \hspace{1cm} (B3)
$$c_Y\left(\frac{w_H}{z}, w_L\right) = p_Y.$$ \hspace{1cm} (B4)

Market clearing implies

$$H_F + H_Y = H$$ \hspace{1cm} (B5)
$$L_F + L_Y = L.$$ \hspace{1cm} (B6)

Shepard’s Lemma determines conditional factor demands in the Food sector:

$$\frac{\partial c_F}{\partial w_H} = \frac{H_F}{F}$$ \hspace{1cm} (B7)
$$\frac{\partial c_F}{\partial w_L} = \frac{L_F}{F}.$$ \hspace{1cm} (B8)

and relative factor demands in the Yacht sector are derived setting marginal rates of technical substitution equal to the ratio of input prices:

$$\frac{\partial Y}{\partial H} / \frac{\partial Y}{\partial L} = \frac{r}{w}$$ \hspace{1cm} (B9)

Equations A1 through A9 characterize the equilibrium. We can log-linearize the equilibrium equations to determine the effects of an increase in $z$ on all endogenous variables:

$$\hat{w}_H = \hat{Y} + \hat{p}_Y$$ \hspace{1cm} (B11)
\[ \hat{w}_L = \hat{F} + \hat{p}_F \]  
\[ \phi_H \hat{w}_H + \phi_L \hat{w}_L = \hat{p}_F \]  
\[ \theta_H \hat{w}_H + \theta_L \hat{w}_L = \theta_2 \hat{z} + \hat{p}_Y \]  
\[ \lambda_{HF} \hat{H}_F + \lambda_{HY} \hat{H}_Y = 0 \]  
\[ \lambda_{LF} \hat{L}_F + \lambda_{LY} \hat{L}_Y = 0 \]  
\[ \hat{H}_F = \hat{F} + \phi_H \sigma_F (\hat{w}_L - \hat{w}_H) \]  
\[ \hat{L}_F = \hat{F} + \phi_L \sigma_F (\hat{w}_H - \hat{w}_L) \]  
\[ \hat{L}_Y - \hat{H}_Y + (\sigma_Y - 1) \hat{z} = \sigma_Y (\hat{w}_H - \hat{w}_L) \]  

For any variable \( x \) above, \( \hat{x} = \frac{dx}{x} \). We denote cost shares of labor in the Potato sector as \( \phi_H \equiv \frac{w_H F}{F_p F} \) and \( \phi_L \equiv \frac{w_L L_F}{F_p F} \). Likewise in the Yacht sector \( \theta_H \equiv \frac{w_H Y_U y}{Y_p Y} \) and \( \theta_L \equiv \frac{w_L L_Y}{Y_p Y} \). The shares of labor types in each sector are \( \lambda_{HF} = \frac{F_H}{H} \), \( \lambda_{HY} = \frac{Y_H}{H} \), \( \lambda_{LF} = \frac{F_L}{L} \), and \( \lambda_{LY} = \frac{L_Y}{L} \). The elasticity of substitution between the labor types is \( \sigma_F \) in the Potato sector. In the Yacht sector \( \sigma_Y \) in the elasticity of substitution between \( Z_H \) and \( L \). Solving the above system of equations yields the percentage change in Potatoes in response to a percentage increase in Yacht-specific, skill-biased technological change:

\[ \hat{F} = - \frac{(\sigma_Y - 1) \lambda_{HY}}{\left\{ (1 + \sigma_F) \lambda_{HF} + \lambda_{HY} \left[ \frac{\sigma_Y}{\phi_H} + \frac{\lambda_{LF}}{\phi_H} \left( \frac{\sigma_F \phi_L}{\phi_H} - 1 \right) \right] \}^2} \]

Potato production will fall in response to an increase in \( z \) whenever \( \sigma_Y > 1 \) and
\[ \frac{\sigma_Y}{\phi_H} + \frac{\lambda_{LF}}{\phi_H} \left( \frac{\sigma_F \phi_L}{\phi_H} - 1 \right) > 0 \]. This latter condition will hold when production functions are of the CES form as in Section 3.

### Appendix C

Here we extend the model in Section 3 to include equipment capital, \( K \). Production of Potatoes and Yachts takes the nested CES form:

\[ F = \left[ \eta \left( \lambda \left( K_F \frac{\sigma_F - 1}{\sigma_F} + (1 - \lambda) F_H \right) \right)^{\frac{\sigma_F - 1}{\sigma_F} \frac{\beta - 1}{\beta}} + (1 - \eta) L_F^{\frac{\beta - 1}{\beta}} \right] \]
\[ Y = \left[ \mu \left( \lambda (zK_Y)^{\frac{\sigma_Y - 1}{\sigma_Y}} + (1 - \lambda)H_Y^{\frac{\sigma_Y - 1}{\sigma_Y}} \right)^{\frac{\gamma - 1}{\gamma}} + (1 - \mu)L_Y^{\frac{\gamma - 1}{\gamma}} \right]^{\frac{\gamma}{\gamma - 1}}, \]

which is similar to the production function estimated by Krusell, Ohanian, Rios-Rull, and Violante (2000). The technology parameter, \( z \), augments capital in the Yacht sector only. Alternatively, we could assume that capital is sector-specific, and that productivity improvements are unique to the production of capital used in the yacht sector. With competitive markets the effects on factor demands and prices will be the same; the only difference is that capital in the Yacht sector would be measured as \( zK_Y \) instead of \( K_Y \). Krusell et al. implicitly assume that the unit of measurement of capital is \( zK \) (theirs is a one-sector model) in order to account for the fall in the price of equipment capital during the latter part of the Twentieth Century. However, this assumption is not necessary: in the calibrated general equilibrium model the price of \( K \) (and the price of \( zK_Y \)) can fall in response to an increase in \( z \), as we demonstrate below.

Preferences are the same as in the baseline model, and we assume that the Rich own the economy’s endowment of capital \( K \) in addition to high-skilled labor \( H \). The representative Rich agent therefore solves

\[ \max \ Y \]
\[ \text{s.t. } rK + w_H H \geq rK_Y + w_H H_Y + w_L L_Y \]  
\[ \text{(C1)} \]

and the Poor agent solves

\[ \max \ 0.5 \times \log(F) \]
\[ \text{s.t. } w_L L \geq rK_F + w_H H_F + w_L L_F, \]
\[ \text{(C2)} \]

where \( r \) is the price of capital.

In the competitive equilibrium the marginal rates of technical substitution must equal input prices. This consists of two equations in the Yacht sector,

\[ \frac{\mu}{1 - \mu} \lambda z^{\frac{\sigma_Y - 1}{\sigma_Y}} \left( \lambda (zK_Y)^{\frac{\sigma_Y - 1}{\sigma_Y}} + (1 - \lambda)H_Y^{\frac{\sigma_Y - 1}{\sigma_Y}} \right)^{\frac{\gamma - 1}{\gamma}} \left( \frac{1}{L_Y^{\frac{\gamma - 1}{\gamma}}} \right)^{\frac{1}{\sigma_Y}} = \frac{r}{w_L} \]  
\[ \text{(C3)} \]

and

\[ \frac{\mu}{1 - \mu} (1 - \lambda)z^{\frac{\gamma - 1}{\gamma}} \left( \lambda (zK_Y)^{\frac{\sigma_Y - 1}{\sigma_Y}} + (1 - \lambda)H_Y^{\frac{\sigma_Y - 1}{\sigma_Y}} \right)^{\frac{\gamma - 1}{\gamma}} \left( \frac{1}{L_Y^{\frac{\gamma - 1}{\gamma}}} \right)^{\frac{1}{\sigma_Y}} = \frac{w_H}{w_L}, \]  
\[ \text{(C4)} \]

and two equations in the Food sector,

\[ \frac{\eta}{1 - \eta} \lambda \left( \lambda K_F^{\frac{\sigma_F - 1}{\sigma_F}} + (1 - \lambda)H_F^{\frac{\sigma_F - 1}{\sigma_F}} \right)^{\frac{\beta - \sigma}{\beta(\sigma - 1)}} \left( \frac{1}{L_F^{\frac{\beta - 1}{\beta}}} \right)^{\frac{1}{\sigma_F}} = \frac{r}{w_L} \]  
\[ \text{(C5)} \]
\[
\eta \frac{1}{1 - \eta} (1 - \lambda) \left( \lambda K_F^{\sigma_F - 1} \frac{\sigma_F}{\sigma_F} + (1 - \lambda) H_F^{\sigma_F - 1} \right)^{\frac{\beta - \sigma}{\beta (\sigma - 1)}} L_F^{1 - \frac{1}{\sigma_F}} = \frac{WH}{w_L}.
\]

(C6)

Equations (C1) through (C6), in addition to market clearing conditions

\[K = K_F + K_Y, \quad H = H_F + H_Y, \quad L = L_F + L_Y,
\]

fully characterize the competitive equilibrium.

We calibrate the model using the parameter estimates in Krusell et al (2000):

\[\beta = \gamma = 1.67, \quad \sigma_Y = \sigma_F = 0.67, \quad \lambda = 0.553, \quad \eta = 0.587.
\]

As in section 3, we set \(\frac{H}{L} = .7\) to match its value in 2000. We set \(\mu = 0.65\) instead of 0.587 to help match the 2000 skill premium, \(\frac{WH}{w_L} = 2.1\), and because a higher value of \(\mu\) increases the relative skill intensity in the Yacht sector, consistent with the evidence in Buera and Kaboski (2009). The capital stock, \(K = 7\), is chosen to match the skill premium. The starting value for \(z\) is 1.

Table C1 shows the response of endogenous variables to a 10% increase in \(z\). The supply of Potatoes, \(F\), falls by 0.43%, due entirely to an outflow of skilled labor. Unskilled labor and capital actually flow into the Potato sector. When \(z\) increases, the technology-capital bundle \(zK_Y\) increases in the Yacht sector. Since \(zK_Y\) and \(H_Y\) are relative complements (determined by the magnitude of \(\sigma_Y\) relative to \(\gamma\)), the Rich demand more skilled labor in the Yacht sector, which increases \(w_H\) and \(H_Y\). The Rich also demand less capital because the level of \(zK_Y\) is high relative to \(H_Y\), which lowers the price of capital. The result is an outflow of capital from the Yacht sector and into the Potato sector. The stronger the relative complementarity between capital and skill, the stronger is the fall in \(r\) and in inflow of capital to the Potato sector. If we change our baseline calibration slightly to decrease the relative complementarity (through either an increase in \(\sigma_Y\) or a decrease in \(\gamma\)), the sign of the change in \(r, K_F\), or both can reverse. All other variable changes are robust to a wide range of parameter values.

| Table C3: Response of Endogenous Variables to a 10% Increase in z |
|---------------|--------|------|-------|--------|--------|------|
| \(\hat{w}_H\)  | -1.59% | \(\hat{r}\) | -1.70% | \(\hat{K}_F\) | 0.51%  | \(\hat{H}_F\) | -1.45% | \(\hat{L}_F\) | 0.29%  | \(\hat{\rho}\) | -0.43% | \(\hat{Y}\) | 2.07% |

Note: The price of low-skilled labor, \(w_L\), is normalized to 1.