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10 February 2011

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February 10, 2011
[Preliminary and incomplete]

Abstract

In the present paper, I analyze how unobservable savings affect risk sharing and bankruptcy decisions in the financial system. I extend the Diamond and Dybvig (1983) model of financial intermediation to an environment with heterogeneous intermediaries, aggregate uncertainty and agents' hidden borrowing and lending. I demonstrate three results. First, unobservability imposes a burden on financial intermediaries, that in equilibrium are not able to offer a banking contract that balances insurance and incentive motivations. Second, unobservable markets do induce default, but only as long as insurance markets are incomplete. Therefore, their presence is not a rationale for government intervention on bankruptcy via "resolution regimes". Third, even in case of complete markets the competitive equilibrium is inefficient, and a simple tier-1 capital ratio similar to the one proposed in the Basel III Accord implements the efficient allocation.

Keywords: Financial intermediation, hidden savings, bankruptcy, insurance, optimal regulation

JEL Classification: E44, G21, G28

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1 Introduction

The issue I want to address in this paper is the influence of unobservable savings on risk sharing and default decisions in the financial system. Despite that being a "hot topic" among public authorities dealing with macroprudential policy, to the best of my knowledge no proper theoretical analysis has been developed so far. The present paper fills this gap.

Unobservability of actions, markets or contracts is an important feature that has characterized US' financial system in the last two decades, due to the presence of the so-called "shadow banking system". That is defined as a set of markets, institutions, intermediaries and instruments that provide banking services (in terms of liquidity, leverage and maturity transformation) without being regulated as such. Examples of what is considered part of the shadow banking system are Special Investment Vehicles (SIVs) like Asset-Backed Securities (ABS), or intermediaries like hedge funds, or repo markets.[1] No measure of the dimension of the shadow banking system existed until recently (current estimates argue the total amount of resources traded can be as large as US$ 12 Trillion), but to have a rough idea in figure 1 I report the evolution of total financial assets of US' commercial banks *vis-à-vis* those of the shadow banking system, using a generally accepted definition of the latter.[2] While both measures were not particularly high at the beginning of the Nineties, the graph shows that in the last twenty years the shadow banking system has literally exploded, reaching about US$ 24 Billion in 2007, at the beginning of the financial crisis. As a comparison, total financial assets of commercial banks peaked at around US$ 14 Billion, instead. From the point of view of the present work, the lesson I draw from the data is that a great part of the financial wealth of the U.S. has been conveyed in the economy through "off-the-counter" unobserved channels, hence the introduction of unobservability captures a fundamental feature of the current financial system that researchers cannot ignore.

I study the role of hidden markets in the standard environment for the positive and normative analysis of financial intermediation: the seminal work by Diamond and Dybvig (1983). In their model, each agent is hit by an idiosyncratic shock affecting her preference for liquidity, which is privately observed. An intermediated equilibrium, in which banks collect deposits and sign a withdrawal contract with their customers, emerges because banks are able to pool resources and provide insurance against such shocks.

To this set-up, the present paper adds the following features: first, I assume that the economy is ex ante divided into groups (or sectors, or countries) of different dimensions.

[1]For a more detailed description of the role of the shadow banking system, see Pozsar et al. (2010).
[2]See the note to the picture for details.

Each agent is assigned at the beginning of time to one of them, and exclusively signs a banking contract with a bank in her own group. In this way, I am able to study different international markets and how they affect bankruptcy. Second, the whole economy is affected by aggregate uncertainty: after contracts are signed, a state of the world is revealed which affects the return on assets and the distribution of the idiosyncratic shock. Third, depositors are allowed to engage in unobservable borrowing and lending with other agents across groups: once withdrawn from their own bank in accordance with the banking contract, agents trade a hidden amount of a risk-free bond, whose return is determined in equilibrium.

I use this environment to understand how default emerges as an equilibrium outcome in decentralized environments. More specifically, I will talk about "illiquidity" if banks do not have enough resources to repay their depositors, and about "insolvency" if illiquid banks cannot sell their assets in the market and are forced to liquidate their portfolios to fulfill their obligations. I show that as a consequence of unobservable markets the banking contract in competitive equilibrium does not improve over the autarky equilibrium with no banks. Moreover, the banking contract is incomplete, in the sense that its terms do not change with respect to the realization of the aggregate state of the world. That in turn leads to a budget imbalance, as banks may be too much (or too little) leveraged to repay according to the contract. Then I show that in an environment with incomplete insurance markets banks may turn out illiquid, and if the illiquidity is generalized they can easily become insolvent. On the contrary, in an environment in which banks can trade in a complete set of state-contingent claims I show that neither illiquidity nor insolvency emerge, because banks can fully insure against those risks. Therefore I conclude that the role of unobservable markets to explain bankruptcy crucially depends on the completeness of the insurance market.

As a second result, I also show that the competitive equilibrium with complete markets is not constrained efficient. This result does not follow from default, though. It lies on the fact that in a competitive equilibrium the price of borrowing and lending in the hidden market only reflects arbitrage conditions with respect to the observable market, and not the willingness of agents to insure against idiosyncratic risk. In that sense, optimal regulation can be imposed on the decentralized financial system, in order to implement to efficient solution, and I prove that such regulation takes the form of a simple tier-1 capital ratio. On the other side, there is no need to distort in any direction the decentralized default decisions of the intermediaries, as default is a consequence of market incompleteness. In that sense, the conclusions of my model strongly support liquidity regulation as the one proposed by the Basel III Accord, but find that hidden markets are not a theoretical rationale for government interventions at times of default, the so-called ”resolution regimes”.[3]

The present paper contributes to different topics in the literature. In general, it belongs to a wide and revamping literature on financial intermediation, which has in the already mentioned work by Diamond and Dybvig (1983) its cornerstone. In particular, it embraces the ”business cycle” view about the origin of financial crises, which has in Gorton (1988) its first and most important contributor. According to that view, financial crises are not the outcome of self-fulfilling prophecies, as in the standard Diamond-Dybvig model, but are consequences of real shocks affecting the economy, that is, an equilibrium outcome.

In introducing unobservable savings, the present work represents an extension of Allen and Gale (2004), in which they prove that once contracts are constrained to be incomplete it may be optimal for the intermediary to default in some states. It is also related to the analysis about hidden savings and financial intermediation proposed by Farhi et al. (2009), as I model unobservable borrowing and lending in the same fashion. With respect to Farhi et al. (2009) I add aggregate uncertainty, which allows me to study liquidation policy and default, too.

My paper is also related to those works that try to rationalize the 2007-2009 financial crisis in formal theoretical models. In the characterization of the intermediated equilibrium, it resembles a recent analysis by Farhi and Tirole (2010): the authors show that the anticipation of a possible public bailout leads banks to have the same risky investment strategy, making the probability of a bailout even higher. In that sense, they talk about ”strategic complementarities” among financial intermediaries. The parallel between that result and mine lies in the fact that also in the present paper strategic complementarities arise, as all banks endogenously offer an incomplete contract, regardless of the realization of the aggregate state of the world. Here though the main reason for that is that ”traditional” banks in all sectors face the very same unobservable market. This line of research is also connected to more recent attempts to study optimal regulation, in particular macroprudential policy. Acharya et al. (2010) and Hanson et al. (2010) are both examples of works which highlight how fierce government intervention might push the most part of financial intermediation in the shadow banking system.

[3]Resolution regimes are a set of institutions that regulates banks' bankruptcy in an orderly manner, and in particular frames government intervention. Examples of resolution regimes have been recently introduced into more comprehensive legislations trying to deal with the aftermath of the 2007-2009 financial crisis, like the UK's Banking Act of 2009 and the US' Dodd-Frank Wall Street Reform and Consumer Protection Act of 2010.

Finally, my paper contributes to the extending literature on social insurance and its implementation.[4] The seminal paper by Cole and Kocherlakota (2001) and the more recent work of Ales and Maziero (2010) are two examples of environments in which unobservability (of income or contract arrangements) is introduced. In this line of literature, the closest work to mine though is the one by Golosov and Tsyvinski (2007), in which they introduce unobservable borrowing and lending via hidden exchanges. In this set-up, they are able to study the effect of such friction on the trade-off between insurance and incentives in a standard dynamic Mirrleesian economy, and characterize the implementation of the efficient allocation through taxation.

The paper is organized as follows: in section 2 I describe the basic environment of the economy. Section 3 is devoted to the characterization of the decentralized intermediated equilibrium. In sections 4 I solve for the constrained efficient allocation and show that the decentralized equilibrium with complete markets is inefficient. This will create the space for government intervention: I derive optimal regulation in section 5. Finally, section 6 concludes.

2 Environment

The basic structure of the model is a standard Diamond and Dybvig (1983) model of financial intermediation with aggregate and idiosyncratic uncertainty, as in Allen and Gale (2004). The economy lasts for three periods, $t = 0, 1, 2$. The world is ex-ante divided into n groups populated by μ^i agents, with $\sum_i \mu^i = 1$.[5] Each agent is ex-ante identical, and receives at date 0 an endowment $e = 1$. Every group is also populated by a continuum of Bertrand-competitive financial intermediaries (or more commonly, banks), who operate in an official market with free entry, and offer deposit contracts to individuals. Assume the relationship between costumers and banks is exclusive, in the sense that agents can only deposit their endowment into banks in their own group.

There are two assets available in the economy: a short assets, which is essentially a storage technology that yields one unit of consumption good in $t+1$ for each unit invested in t, and a long asset, which delivers $\hat{R}(s) > 1$ units of consumption in period $t = 2$ in the

[4] For an overview of the literature on the *New Dynamic Public Finance*, see Kocherlakota (2010).

[5] In this context, groups may be seen as sectors of the same economy, regions of the same country, or different countries in the world economy. Throughout the text, I will indistinctly use sectors, regions and countries to refer to groups.

aggregate state s for each unit invested in $t = 0$.[6] In this respect, the short asset is "liquid", as it provides utility from one period to the following, while the long asset is "illiquid". Such illiquidity is only partial: in fact, the long asset can be sold before its natural maturity.[7] That comes at a cost, as each unit of the long asset yields only a positive amount $r < 1$ of the consumption good in $t = 1$.

2.1 Uncertainty

The economy is affected by two types of uncertainty. An aggregate shock is defined over a finite number of states of the world, labeled by $s = 1, \ldots, S$. The commonly known probability that the state s is realized is $\nu(s)$, with $\sum_s \nu(s) = 1$. Aggregate uncertainty is resolved at the beginning of period 1, and affects both the return on the long asset and the distribution of a preference shock. The latter is an idiosyncrasy affecting single agents in every group. Being ex-ante equal, in $t = 1$ every consumer draws a type $\theta \in \{0, 1\}$ which is private information to herself. Individual types affect the point in time at which agents enjoy consumption, according to the utility function $U(c_1^i, c_2^i, \theta) = (1 - \theta)u(c_1^i) + \beta\theta u(c_2^i)$. Clearly, if $\theta = 0$ the agent is willing to consume only at date 1, while if $\theta = 1$ she will consume only at date 2. As it is customary in this line of research, I then refer to type-0 and type-1 agents as early and late consumers, respectively. The felicity function $u(c)$ is increasing, twice continuously differentiable, concave, and satisfies Inada conditions. Moreover, I restrict myself to the class of felicity functions with relative risk aversion bigger than or equal to unity, that is $-\frac{u''(c)c}{u'(c)} \geq 1$. The discount factor β is such that $\beta\hat{R}(s) > 1$ in any state of the world. Define the probability of being of type θ in country i and state s as $\pi^i(\theta, s)$. I assume that the preference shocks are independent across agents, so that by the law of large number the cross-sectional distribution of types is equivalent to their probability distribution. Moreover, we must have that $\sum_\theta = \pi^i(\theta, s) = \mu^i$ in any state.

[6] Here, in order to keep focus on the intermediation system, I do not model the supply side of the economy. In general, we may argue that the yields on the short and long assets are fixed (albeit state-dependent) because of the presence of some form of deposit insurance. The case in which different countries have different regulatory environments, and therefore different returns from their national technologies, is analyzed in Panetti (2011).

[7] The existence of a liquidation technology makes a difference between the environment in the present paper and the one in Allen and Gale (2004). In their work, banks at default are forced to sell their holdings of the long asset to other intermediaries. In this way, Allen and Gale (2004) show that in equilibrium banks randomize, i.e. not every bank decides to go bankrupt. Here instead I focus only on symmetric equilibria.

2.2 The Banking Contract

At the beginning of date 0, agents deposit their endowment into banks in their own group, and sign a banking contract. Such contract states the amount of the consumption good that each depositor will be entitled to withdraw at dates 1 and 2, depending on the reported type and on the realization of the aggregate state of the world.

Once collected all the deposits, banks decide how to allocate them in a complex portfolio of securities. In particular, they can buy short and long assets (labeled X^i and Y^i, respectively), and trade in a complete market for Arrow-Debreu securities. The first two are needed to allocate resources across time between date 1 and 2, according to the possible realization of the idiosyncratic shocks. The third ones instead are state-contingent claims used to allocate deposits across different states of the world: after the payment of a price $q_t(s)$, they pay 1 unit of consumption good in period t if the state s is realized. Define the amount bought as $\tau_t^i(s)$ for any $t = 1, 2$.

A secondary market opens at $t = 1$, after all the uncertainty is realized and the aggregate state is publicly revealed. Banks then trade in a cross-sectoral secondary interbank market: I assume they can borrow or lend with full commitment an amount $Z^i(s)$ of non-contingent bonds (non-contingency comes from the fact that the aggregate state is already public knowledge), paying $\hat{R}(s)$ units of consumption at date 2 after the correspondence of a price $p(s)$ to be determined in equilibrium. The introduction of such market allows banks to adjust their holding of liquid assets with respect to the realization of the aggregate state, by shifting resources from date 1 to date 2 or vice versa.

Notice that at this point in time, banks also have other channels to move resources to and from date 2. In case of unexpected liquidity needs, banks can indeed choose whether to borrow from other banks through the interbank market (by giving up $\hat{R}(s)$ units of consumption at date 2 to get an amount $p(s)$ at date 1) or liquidate an amount $L(s) \geq 0$ of the long assets they hold in portfolio (by giving up $\hat{R}(s)$ units of consumption at date 2 to get an amount r at date 1). Similarly, if banks find themselves in need of giving away part of their liquidity, they can lend in the interbank market, or use the storage technology for an amount $M^i(s) \geq 0$. Since these channels are perfect substitutes, the bank will observe the price of borrowing/lending in the interbank market (emerging in equilibrium), compare it with the return on the liquidation technology or on storage, and choose the one with the highest yield.

If a bank is able to pay back to its depositors the amount promised in the contract it is said to be "liquid". If it has to borrow from other banks in the interbank market and pay back the loan at date 2, then it is said to be "illiquid but solvent". If in order to honor the promise it has to liquidate part of its portfolio, it is instead said to be "insolvent". Finally, if the bank has to liquidate all its holdings, it is said to be "bankrupt". Importantly, I assume that Arrow securities are completely default-free, i.e. in every state of the world banks will first clear their holdings and only afterwards engage in liquidation or storage. As discussed in Allen and Gale (2004), the assumption of preferential treatment - albeit restrictive - is pivotal for such insurance market to work properly.

Notice that in the following I will focus my attention on equilibria with pure strategy, where banks in the same group have symmetric investment strategies, both ex ante and ex post. Therefore, I can restrict myself to the analysis of a representative bank without loss of generality.

Before going on, it is useful to summarize the terms of the contract in the following definition:

Definition 1. *A banking contract* $C^i(\theta, s) = \{c_1^i(\theta, s), c_2^i(\theta, s), X^i, Y^i, \tau_t^i(s)\}$ *is an ex ante portfolio allocation, stating how much consumption depositors are entitled to receive, and the investment in short assets, long assets and state-contingent claims, and an ex-post policy vector* $J^i(s) = \{L^i(s), M^i(s), Z^i(s)\}$ *of liquidation policies, storage and amounts to borrow or lend in the interbank market.*

2.3 The Shadow Market

After having withdrawn the amount of consumption good stated in the contract, agents engage in trades in the shadow market. I model this feature of the economy as unobservable exchanges, through which individuals can freely borrow and lend across groups an uncontingent bond yielding a "shadow return" $R(s)$ to be determined in equilibrium. Notice that, despite the fact that agents can only deposit their endowment in banks in their own group, I allow them to trade across groups at a later stage.[8] More formally, in the

[8] I could have modeled the shadow banking system so that agents were allowed to write side contracts with financial institutions in other groups. Instead, I followed Farhi et al. (2009) and assume that they trade only with other individuals. That would not change the main results of the paper, while consistently simplifying the notation.

hidden market agents solve the following problem:

$$
\begin{aligned}
V(C^i(\theta,s), J^i(s), R(s), \theta, s) = & \max_{x_1^i, x_2^i, b^i, \theta'} U(x_1^i, x_2^i, \theta) \\
s.t. \qquad & x_1^i + b^i = c_1^i(\theta', s) \\
& x_2^i - R(s) b^i = c_2^i(\theta', s)
\end{aligned}
\tag{1}
$$

Given the terms of the banking contract, the return on the shadow investment $R(s)$, and the realizations of the idiosyncratic and aggregate shocks, each agent decides which type θ' to report, how much to consume in the two periods (x_1^i and x_2^i) and how much to borrow or lend (b^i) in order to maximize their welfare, subject to the budget constraint. By the assumptions of concavity of the felicity function u and linearity of the constraints, the utility function V is clearly concave. The fact that agents trade only uncontingent bonds is not an a priori restriction on the completeness of the hidden market, but an endogenous feature of the hidden market. I prove that in appendix A.

The environment so far describes a complex game of asymmetric information between banks and depositors. Nevertheless, by the Revelation Principle I can focus on direct mechanisms in which agents truthfully report their types to the banks. The incentive compatibility constraint then can be defined in the following way:

Definition 2. *A banking contract $\{C^i(\theta,s), J^i(s)\}$ is incentive compatible if:*

$$
V(C^i(\theta,s), J^i(s), R(s), \theta, s) \geq V(C^i(\theta',s), J^i(s), R(s), \theta, s) \tag{2}
$$

for any $\theta, \theta' \in \{0,1\}$ and any realization of the aggregate state $s = 1, \dots, S$.

Incentive compatibility states that each agent should find optimal to truthfully report her type, but given the presence of only two types, this can be simplified:

Lemma 1. *A banking contract $\{C^i(\theta,s), J^i(s)\}$ is incentive compatible if:*

$$
c_1^i(0,s) + \frac{c_2^i(0,s)}{R(s)} = c_1^i(1,s) + \frac{c_2^i(1,s)}{R(s)} \tag{3}
$$

for any realization of the aggregate state $s = 1, \dots, S$.

Proof. In Appendix C. ■

Truth-telling therefore implies that the banking contract should give to each type the same present discounted value of consumption, evaluated at the return on the shadow investment.

2.4 Timing

Before going on, it is useful to summarize the timing of actions: at time 0, each agent deposits her endowment into a bank in her own group, hence total deposits of a representative bank are μ^i. Banks then set up a full state-contingent incentive-compatible contract $C^i(\theta, s)$ with depositors, stating the consumption allocation and the ex ante investment strategy in short assets, long assets and Arrow securities. At $t = 1$, all uncertainty is resolved: the aggregate state is revealed to everyone, and each agent gets to know her private type. The market for Arrow-Debreu securities clears, and banks decide how much consumption to shift to or from date 1. That is, they decide whether to trade in the interbank market, and eventually how much to physically liquidate or store. Then, they pay consumption to those depositors who reported being of type 0. After that, agents can engage in unobservable trades across sectors. Finally, at date 2 again the market for state-contingent claims clears, then also the interbank market clears, agents are paid the amount stated in the banking contract by the banks, and eventually the return on their hidden investment.

2.5 Autarky Equilibrium

A preliminary step is helpful to understand the results in the next sections: here I characterize the equilibrium in autarky. In particular, autarky is defined as an environment in which banks do not exist, and agents have to directly invest in short (X) and long assets (Y). While such decision happens ex ante (i.e. before the realization of the aggregate shock), agents have the opportunity to rebalance their portfolios ex post by trading in a secondary competitive market at price $P(s)$. Clearly, if they come up to be of type 0, they will sell all their long assets and enjoy the proceedings, and similarly if they are of type 1 they will instead sell all their short assets and buy long assets on the market. More formally, each individual then solves:

$$\max_{X,Y} \quad \theta u(c_1) + \beta(1-\theta)u(c_2)$$

subject to:

$$X + Y = 1$$
$$X + P(s)Y = c_1$$
$$\hat{R}(s)\left(\frac{X}{P(s)} + Y\right) = c_2$$

Notice there are two ways for the agent to have consumption at date 2, and they must be equivalent. They might invest one unit of consumption at date 0 in the long asset and get $\hat{R}(s)$, or invest in the short asset, get 1 at date 1 and buy $1/P(s)$ units of the long asset in the market. Since there must be no arbitrage opportunities between these two strategies, in equilibrium it must be the case that $P(s) = 1$. Then from the budget constraint I derive the equilibrium consumption in autarky as: $c_1 = 1$ and $c_2 = \hat{R}(s)$.

3 Decentralized Intermediated Equilibrium

In this section, I define and characterize the market equilibrium of the environment described in the previous section. I solve for the consumption allocation and the portfolio strategy by backward induction. Therefore, I begin with solving the problem in (1) that agents solve in the market for hidden borrowing and lending. The second step instead relates to the ex-post investment problem. Taking as given the equilibrium return on hidden savings, and the ex-ante terms of the banking contract determined at date 0, each bank observes the aggregate state of the world, and decides how much to borrow or lend in the interbank market, and eventually how much to store or liquidate. More formally, a representative bank in group i solves:

$$W(C^i(\theta, s), R(s), \theta, s) = \max_{J^i(s)} \sum_{\theta} \pi^i(\theta, s) V(C^i(\theta, s), J^i(s), R(s), \theta, s) \tag{4}$$

subject to the incentive compatibility constraint in (3) and the budget constraints at date 1 and 2:

$$X^i + \tau_1^i(s) + rL^i(s) \geq \sum_{\theta} \pi^i(\theta, s) c_1^i(\theta, s) + p(s)Z^i(s) + M^i(s) \tag{5}$$

$$\hat{R}(s)(Y^i - L^i(s) + Z^i(s)) + M^i(s) + \tau_2^i(s) \geq \sum_{\theta} \pi^i(\theta, s) c_2^i(\theta, s) \tag{6}$$

The budget constraints deserve some more explanations. At $t = 1$, banks' assets are the return on the storage technology X^i, the state-contingent return $\tau_1^i(s)$ and whatever amount they physically liquidate from the long asset, whose return is $rL^i(s)$. With such amount, banks decide how much to store ($M^i(s)$) and how much to borrow or lend in the interbank market ($p(s)Z(s)$), given the amount of consumption they promised to early consumers at date 0. Finally, at $t = 2$ banks receive the return on their investment in the long technology, and eventually on their investment in the secondary market, and use those together with the storage and the return on the Arrow-Debreu security $\tau_2^i(s)$ to pay back consumption to late consumers.

At this point of the paper, it is useful to present a result about the price system, that will greatly simplify the analysis. The hypotheses of Bertrand competition and free entry imply that there must be no profits from holding the short or the long asset. This means there are three ways banks can invest one unit of consumption at date 0 to have some resources at date 2, and they must be equivalent. First, banks can invest in the long technology, which yields $\hat{R}(s)$. Second, they can invest in the short asset, have one unit of consumption in $t = 1$, and then lend in the interbank market: such strategy would yield $\hat{R}(s)/p(s)$ in $t = 2$. Third, they can invest in the short asset, have one unit of consumption in $t = 1$, and give it to the early consumers for them to operate in the shadow market. This would yield $R(s)$ units of consumption at date 2. Clearly, the following lemma must be true:

Lemma 2. *In any state of the world $s = 1, \ldots, S$, the return on the hidden investment is $R(s) = \hat{R}(s)$. Similarly, the price in the secondary interbank market is $p(s) = 1$.*

Proof. In the text above. ■

Notice that the prices that emerge in equilibrium exclusively reflect arbitrage relations among the different available markets, and do not consider insurance motivations or deep preference parameters at all. Moreover, since by assumption the liquidation technology can only be used to transfer resources from date 2 to date 1, and the storage only to transfer resources from date 1 to date 2, banks would always prefer to use the interbank market to shift resources to and from date 1, if at $p(s) = 1$ such market clears.

Finally, at date 0 each bank chooses the consumption allocation and the ex-ante portfolio. I will distinguish three different cases: one where groups are in autarky, one in which they can trade in incomplete markets, and finally one with complete markets.

3.1 No Insurance Markets

In the case of no insurance markets, I exogenously prevent groups from trading with each other, either at date 0 or at date 1. The constraint in (5) is thus modified by imposing $\tau_t^i(s) = Z^i(s) = 0$. Moreover, the shadow market only opens to domestic depositors, but arbitrage conditions still holds, hence it is still the case that $R(s) = \hat{R}(s)$.

At date 0 banks maximize the ex-post welfare of their customers (solution of the banking problem at date 1) by choosing the banking contract and the ex-ante portfolio allocation to maximize:

$$\max_{C^i(\theta,s)} \sum_s \nu(s) \sum_\theta \pi^i(\theta, s) W(C^i(\theta, s), R(s), \theta, s) \tag{7}$$

subject to the incentive compatibility in (3), the date-1 budget constraint in (5), the date-2 budget constraint in (6), and the date-0 budget constraint:

$$X^i + Y^i \leq \mu^i \tag{8}$$

This simply states that total deposits μ^i must be allocated only between short and long assets. The definition of a decentralized equilibrium is then straightforward:

Definition 3. *Given an endowment e and a probability distribution $\nu(s)$, a decentralized intermediated equilibrium with no insurance markets is a price $R(s)$, a banking contract $\{C^i(\theta, s), J^i(s)\}$ and an amount of bonds $b^i(C^i(\theta, s), J^i(s), R(s), \theta, s)$ for every $s = 1, \ldots, S$ $i = 1, \ldots, n$ and $\theta = \{0, 1\}$ such that:*

- *Given prices, the allocation solves the banking problem*
- *Given prices, the allocation solves the shadow problem for each agent*
- *Markets clear in any state:*

$$\sum_\theta \pi^i(\theta, s) b^i(C^i(\theta, s), J^i(s), R(s), \theta, s) = 0 \tag{9}$$

$$\sum_\theta \pi^i(\theta, s) \left[c_1^i(\theta, s) + \frac{c_2^i(\theta, s)}{\hat{R}(s)} \right] = \mu^i - (1 - r)L^i(s) - \left(1 - \frac{1}{\hat{R}(s)}\right) M^i(s) \tag{10}$$

Notice that the resource constraint of the economy states that the total available resources are the sum of deposits minus the loss incurred by liquidating the long asset and the loss implied by storage. Then three cases can happen in equilibrium:

Lemma 3. *The equilibrium ex ante investment strategy with no insurance markets is* $X^i = \sum_{k=1}^{S} \nu(k)\pi^i(0,k)$ *and* $Y^i = \sum_{k=1}^{S} \nu(k)\pi^i(1,k)$. *The optimal consumption allocation reads:*

$$c_1^i(1,s) = c_2^i(0,s) = 0 \qquad ; \qquad c_2^i(1,s) = \hat{R}(s)c_1^i(0,s)$$

where:

1. *If* $\pi^i(0,s) = X^i$, $c_1^i(0,s) = 1$, *and* $M^i(s) = L^i(s) = 0$
2. *If* $\pi^i(0,s) < X^i$, $c_1^i(0,s) = \frac{\mu^i - \left(1-\frac{1}{\hat{R}(s)}\right)X^i}{\mu^i - \left(1-\frac{1}{\hat{R}(s)}\right)\pi^i(0,s)} < 1$, *and* $L^i(s) = 0, M^i(s) = X^i - \pi^i(0,s)c_1^i(0,s) > 0$
3. *If* $\pi^i(0,s) > X^i$, $c_1^i(0,s) = \frac{\mu^i + \frac{1-r}{r}X^i}{\mu^i + \frac{1-r}{r}\pi^i(0,s)} < 1$, *and* $M^i(s) = 0, L^i(s) = \frac{1}{r}[\pi^i(0,s)c_1^i(0,s) - X^i]$

The equilibrium investment in the shadow market is $b^i = 0$.

Proof. In Appendix C. ■

The intuition for this result is straightforward. Given that banks do not know ex ante in which state of the world they are going to end up at date 1, they invest in short (long) assets an amount equal to the weighted average of all possible realization of the number of early (late) consumers. As far as consumption is concerned, banks optimally choose not to provide any consumption to early consumers at date 2 and to late consumers at date 1 (they do not enjoy utility from that). Moreover, they choose to allocate consumption between date 1 and 2 so that no type has incentive to deviate from truth-telling. In this way in equilibrium agents do not trade in the shadow market at all: early consumers do not have any collateral against which to borrow as $c_2^i(0,s) = 0$, and late consumers do not have resources to lend as $c_1^i(1,s) = 0$.

Consumption entitlements crucially depend on the realization of the aggregate state: if the bank receives a shock exactly equal to its expectation, then neither liquidation nor storage are needed, and the agents enjoy the very same consumption they might receive in autarky. To put it differently, the friction imposed by the presence of hidden savings remove the possibility for banks to provide additional risk sharing to their depositors, though the bank is liquid and solvent. If instead the bank receives a shock lower or higher than its expectation, it is forced to store or liquidate, respectively. In that case, the bank can become illiquid, and automatically insolvent. That has a negative effect on consumption, which is lower than in autarky because of the loss incurred by liquidating or storing assets.

3.2 Incomplete Markets

In the case of incomplete insurance markets, banks are exogenously limited to trade across groups only ex-post, in the secondary interbank market. The constraint in (5) is thus modified by only imposing $\tau_t^i(s) = 0$. As in lemma 2, arbitrage implies that $p(s) = 1$. Moreover, the shadow banking now goes across groups, and still $R(s) = \hat{R}(s)$. In the banking problem at date 0 both the objective function and the budget constraint are the same as in the previous case.

The only other difference with the case of no markets lies in the total resource constraint of the economy. Given that there must be market clearing in the secondary bond market, that is $\sum_i Z^i(s) = 0$, the resource constraint can be rewritten as:

$$\sum_{i=1}^{n} \sum_{\theta} \pi^i(\theta, s) \left[c_1^i(\theta, s) + \frac{c_2^i(\theta, s)}{\hat{R}(s)} \right] = 1 - \sum_i \left[(1-r)L^i(s) + \left(1 - \frac{1}{\hat{R}(s)} \right) M^i(s) \right] \quad (11)$$

The definition of decentralized intermediated equilibrium changes accordingly, with the new resource constraint and the required market clearing in the (now cross-groups) shadow market, that is $\sum_i \sum_\theta \pi^i(\theta, s) b^i = 0$. Moreover, the equilibrium should also include the price $p(s)$ of borrowing and lending in the interbank market. But the characterization of equilibrium changes:

Lemma 4. *The equilibrium ex ante investment strategy with incomplete insurance markets is $X^i = \sum_{k=1}^S \nu(k)\pi^i(0,k)$ and $Y^i = \sum_{k=1}^S \nu(k)\pi^i(1,k)$. The optimal consumption allocation reads:*

$$c_1^i(1,s) = c_2^i(0,s) = 0 \qquad ; \qquad c_2^i(1,s) = \hat{R}(s) c_1^i(0,s)$$

Define $\Pi(0,s) = \sum_i \pi^i(0,s)$. Then:

1. *If $\Pi(0,s) = \sum_k \nu(k)\Pi(0,k)$, $c_1^i(0,s) = 1$, $Z^i(s) = \sum_k \nu(k)\pi^i(0,k) - \pi^i(0,s)$ and $M^i(s) = L^i(s) = 0$ for every group i*
2. *If $\Pi(0,s) \neq \sum_k \nu(k)\Pi(0,k)$, the equilibrium goes back to the no market case*

The equilibrium investment in the shadow market is $b^i = 0$.

While the proof of the lemma is the same as the proof of the previous one, the intuition is crucially different. The interbank market, although incomplete, permits some risk sharing across groups. Hence, even if a bank faces a number of early consumers different than the one expected, it may be able to borrow from or lend to other banks, provided that market clears at $p(s) = 1$, that it is fixed by no-arbitrage. That happens if the economy-wide number of early consumers is equal to its expected value, which means that there is enough liquidity in the system to cover for the needs of any group. In that case, banks may be illiquid, but are still solvent because they can sell part of their portfolios and adjust to the unexpected shock. If instead the interbank market does not clear at the equilibrium price, banks are forced to go back to the case with no insurance markets, and eventually deal with their own illiquidity/insolvency.

3.3 Complete Markets

In this last case, banks are free to trade in a complete set of state-contingent claims at date 0. The banking problem at date 0 is the same as before, but now I substitute the time-0 budget constraint in (8) with:

$$X^i + Y^i + \sum_{s=1}^{S} q_1(s)\tau_1^i(s) + \sum_{s=1}^{S} q_2(s)\tau_2^i(s) \leq \mu^i \tag{12}$$

Here I allow total deposits μ^i to be allocated among short assets, long assets and a whole set of Arrow-Debreu securities: each bank pays $q_t(s)$ at time 0 to have an amount of the consumption good $\tau_t^i(s)$ at date t if the state s is realized (see the left hand side of (5) and (6)). The definition of decentralized intermediated equilibrium is the same as in the previous subsection (with the addition of prices $q_t(s)$ and quantities $\tau_t^i(s)$), hence I do nor repeat it here.

The presence of complete insurance markets in turn has a dramatic effect on the characterization of the banking contract:

Proposition 1. *The equilibrium consumption allocation with complete insurance markets is:*

$$\begin{aligned} c_1^i(1, s) &= c_2^i(0, s) = 0 \\ c_1^i(0, s) &= 1 \\ c_2^i(1, s) &= \hat{R}(s) \end{aligned} \tag{13}$$

in any state $s = 1, \ldots, S$ and group $i = 1, \ldots, n$. The price of the Arrow-Debreu securities *are:*

$$q_1(s) = \frac{\pi^i(0,s)\nu(s)}{\sum_k \pi^i(0,k)\nu(k)} \tag{14}$$

and $q_2(s) = q_1(s)/\hat{R}(s)$ for any state $s = 1, \ldots, S$. The ex ante investment strategy is:

$$X^i = \sum_{k=1}^{S} \nu(k)\pi^i(0,k) \tag{15}$$

$$Y^i = \sum_{k=1}^{S} \nu(k)\pi^i(1,k) \tag{16}$$

$$\tau_1^i(s) = \pi^i(0,s) - \sum_{k=1}^{S} \nu(k)\pi^i(0,k) \tag{17}$$

$$\tau_2^i(s) = \hat{R}(s)\left(\pi^i(1,s) - \sum_{k=1}^{S} \nu(k)\pi^i(1,k)\right) \tag{18}$$

The equilibrium ex post investment strategy is $Z^i = L^i(s) = M^i(s) = 0$, and the equilibrium investment in the shadow market is $b^i = 0$.

Proof. In Appendix C. ■

This is the crucial result of the section. Banks invest in the short (long) asset an amount equal to the average number of early (late) consumers they may face, weighted by the probability that each state of the world is realized. Then, in order to meet any unexpected excess or lack of demand, they find optimal neither to trade in the interbank market nor to use the storage and the liquidation technology. On the contrary, they buy or sell state-contingent claims. In that sense, market completeness ensures that banks are always able to meet the demand for liquidity of their depositors. Therefore, neither illiquidity nor insolvency emerge in the market equilibrium.

3.4 Discussion

The conclusion of this section is straightforward. The presence of unobservable markets imposes a distortion on the decentralized intermediated equilibrium: it limits the amount of risk sharing that banks can offer to their depositors to the level that the depositors can get in autarky. In that sense, contract incompleteness endogenously emerges in equilibrium. Such incompleteness is the reason why banks choose the very same ex ante portfolio of short and long assets, irrespective of the market structure they face. On the other side, the effect of unobservable markets on banks' default decisions crucially depends on the availability of insurance markets. When such markets are complete, banks are able to perfectly insure against illiquidity and insolvency, hence no default emerges in equilibrium. Incomplete market structures imply that illiquidity and insolvency may instead materialize as an equilibrium phenomenon.

In the next section, I characterize the planner problem in an environment with complete insurance market, and study whether the decentralized equilibrium is indeed constrained efficient.

4 Constrained Efficient Allocation

In this section, I solve the social planner problem in the economy with complete insurance markets. Before going into the details, it is important to highlight that the constrained efficient allocation with observable savings but with unobservable individual types is equivalent to the unconstrained first best allocation. In fact, the planner ensures perfect risk sharing both within and between groups. Moreover, Allen and Gale (2004) show that in this environment a version of the First Welfare Theorem holds, hence the decentralized intermediated equilibrium with private types is efficient. As a reference, I characterize the constrained efficient allocation without hidden savings in Appendix B.

The opening of the hidden market imposes a new constraint on the social planner problem. That is, the optimal banking contract must induce ex-ante utility for any type in such an amount that agents have incentives to truthfully report their own private types, and not operate at all in the shadow market. More formally:

$$U(c_1^i(\theta,s), c_2^i(\theta,s), \theta) \geq V(C^i(\theta,s), J^i(s)R(s), \theta, s) \tag{19}$$

must hold for any θ and in any state of the world. Thus, the problem of the planner reads:

$$\max_{\substack{\{c_1^i(\theta,s), c_2^i(\theta,s)\}_{s=1,\dots,S} \\ \theta\in\{0,1\} \\ i=1,\dots,n}} \sum_i \sum_s \nu(s) \sum_\theta \pi^i(\theta,s) U(c_1^i(\theta,s), c_2^i(\theta,s), \theta) \tag{20}$$

subject to (19) and the intertemporal resource constraint:

$$\sum_i \sum_\theta \pi^i(\theta,s) \left[c_1^i(\theta,s) + \frac{c_2^i(\theta,s)}{\hat{R}(s)} \right] \leq 1 \qquad \forall s = 1,\dots,S \tag{21}$$

Notice the dimensionality of the program: I need to simultaneously solve for $2nS$ unknowns. To overcome such difficulty, I follow Farhi et al. (2009) and reduce the program to a simple one. In order to do that, I use a feature of the problem that has been already mentioned in the previous sections: the fact that in presence of side trades individuals only care about the present discounted value of the consumption bundle they are entitled to receive. Then the following can be proved:

Lemma 5. *The problem in* (20) *is equivalent to:*

$$\max_{\substack{\{R(s),\mathcal{I}^i(s)\}_{s=1,\dots,S} \\ i=1,\dots,n}} \sum_i \sum_s \nu(s) \sum_\theta \pi^i(\theta, s)\tilde{V}(\mathcal{I}^i(s), R(s), \theta, s) \tag{22}$$

subject to:

$$\sum_i \sum_\theta \pi^i(\theta, s)\left[x_1^i + \frac{x_2^i}{\hat{R}(s)}\right] \leq 1$$

where:

$$\begin{aligned} \tilde{V}(\mathcal{I}^i(s), R(s), \theta) = & \max_{x_1^i, x_2^i} U(x_1^i, x_2^i, \theta) \\ s.t. \qquad & x_1^i + \frac{x_2^i}{R(s)} \leq \mathcal{I}^i(s) \end{aligned} \tag{23}$$

Proof. In the appendix C. ∎

The previous lemma ensures that I can solve the equivalent program (22), while having definitely less unknowns to solve for ($(1+n)S$ vs. $2nS$). I now use backward induction. For type-0 agents, problem (23) is:

$$\max_{x_1^i, x_2^i} \quad u(x_1^i) \qquad s.t. \qquad x_1^i + \frac{x_2^i}{R(s)} = \mathcal{I}^i(s) \tag{24}$$

Clearly, it must be the case that $x_1^i = \mathcal{I}^i(s)$, because type-0 agents only care about consuming at date 1. In the same way, I can solve type-1's maximization, and derive $x_2^i = R(s)\mathcal{I}^i(s)$. Planner's problem then simplifies to:

$$\max_{\substack{\{R(s),\mathcal{I}^i(s)\}_{s=1,\dots,S} \\ i=1,\dots,n}} \sum_i \sum_s \pi^i(0, s)u(\mathcal{I}^i(s)) + \beta\pi^i(1, s)u(R(S)\mathcal{I}^i(s)) \tag{25}$$

subject to:

$$\sum_i \mathcal{I}^i(s)\left[\pi^i(0,s)+\pi^i(1,s)\frac{R(s)}{\hat{R}(s)}\right]\leq 1 \qquad \forall s=1,\ldots,S$$

I am now ready to characterize the constrained efficient allocation with shadow banking in the following proposition:

Proposition 2. *In any $s = 1,\ldots,S$ and group $i = 1,\ldots,n$ the constrained efficient allocation reads:*

$$\begin{aligned}
&c_1^i(1,s)=c_2^i(0,s)=0 \qquad \forall i=1,\ldots,n\\
&c_1^i(0,s)=\mathcal{I}^{i*}(s) \qquad \forall i=1,\ldots,n\\
&c_2^i(1,s)=R^*(s)\mathcal{I}^{i*}(s) \qquad \forall i=1,\ldots,n
\end{aligned} \tag{26}$$

where $\{R^*(s),\mathcal{I}^{i*}(s)\}$ is the solution to:

$$\begin{aligned}
\frac{u'(\mathcal{I}^{i*}(s))}{u'(\mathcal{I}^{j*}(s))}=1=\frac{u'(R^*(s)\mathcal{I}^{i*}(s))}{u'(R^*(s)\mathcal{I}^{j*}(s))} \qquad \forall i,j\\
\beta\hat{R}(s)u'(R^*(s)\mathcal{I}^{i*}(s))=u'(\mathcal{I}^{i*}(s)) \qquad \forall i=1,\ldots,n\\
\sum_i \mathcal{I}^{i*}(s)\left[\pi^i(0,s)+\pi^i(1,s)\frac{R^*(s)}{\hat{R}(s)}\right]=1
\end{aligned} \tag{27}$$

Proof. In the appendix C ■

This proposition states that in presence of hidden borrowing and lending the planner optimally chooses to provide no consumption in the first period to late consumers and in the second period to early consumers. Moreover, she ensures perfect risk sharing between groups, by ensuring that agents of the same types are entitled with the same amount of consumption, regardless the group they belong to. Finally, the planner also ensures risk sharing within each group, by imposing an ex ante Euler equation. It is then trivial to see that such allocation is equivalent to the one that emerges in the constrained problem with asymmetric information, which in turn is equivalent to the unconstrained optimum. This is the multi-group version of the main proposition in Farhi et al. (2009), and states that the planner can indeed tilt incentives and prices so as to implement the first best. This in turn means that the planner does use neither interbank markets nor liquidation nor storage to clear the resource constraint and achieve the equilibrium allocation: she optimally chooses to write a state-contingent contract.

In order to understand this result, first notice that I can rearrange the Euler equation and prove that the efficient return on the hidden investment is:

$$1 < R^*(s) \leq \beta \hat{R}(s) < \hat{R}(s) \tag{28}$$

in any state of the world.[9] Moreover, the efficient amount of consumption provided by the planner is:

$$c_1^i(0,s) = \frac{1}{\sum_j \left[\pi^j(0,s) + \pi^j(1,s)\frac{R^*(s)}{\hat{R}(s)}\right]} > 1 \tag{29}$$

$$c_2^i(1,s) < \hat{R}(s) \tag{30}$$

This means that the planner, given that she marginally values more early consumers than late consumers, compresses agents' ex post consumption profile: she provides to type-1 depositors more than their endowments, and to the others less than what they would have if they invested all their income in the long asset. To put it differently, the planner makes agents' consumption less volatile. Moreover, she is also able to provide state-contingent consumption while enforcing the right incentives: in fact, the planner imposes a wedge between the return on the official technology and the return on hidden savings, so that the shadow market is completely shut down.

To sum up, I proved so far that the planner, despite the presence of unobservable trading, is still able to implement the unconstrained efficient allocation, characterized by perfect risk sharing both within and between groups. This is because she is able to completely shut the hidden market down. In order to do so, I showed that the planner imposes a wedge between the return on the official long asset and the return on the bond exchanged in the hidden market, so as to lower incentives for agents to trade among themselves. Given that the constrained efficient allocation is equivalent to the unconstrained efficient one, neither illiquidity nor insolvency materialize in equilibrium.

[9]The assumption about relative risk aversion is crucial to show this result. Rewrite $-\frac{u''(c)c}{u'(c)} \geq 1$ as $-\frac{u''(c)}{u'(c)} \geq \frac{1}{c}$. This in turn means that $-(log[u'(c)])' \geq (log[c])'$. Integrate this between z_1 and z_2 so as to obtain $log[u'(z_1)] - log[u'(z_2)] \geq log[z_2] - log[z_1]$. Once taken the exponent, the latter expression gives $\frac{u'(z_1)}{u'(z_2)} \geq \frac{z_2}{z_1}$.

4.1 Discussion

With the previous results in hand, I can finally compare the competitive equilibrium and the constrained efficient allocation. Given the presence of a complete set of Arrow securities and of a common hidden market, banks in the decentralized economy indeed ensure perfect risk sharing across groups: agents of the same type receive the same consumption bundle, regardless of the group they belong to. At the same time, it is evident that financial intermediaries are not able to provide more insurance against idiosyncratic risk than autarky: they do not shift resources towards the marginally more valuable early consumers as the planner does. As a general rule, a market allocation is inefficient when the price system is inefficient, and this case is not different. In fact, the inefficiency of the decentralized intermediated equilibrium arises because the return on the shadow market is too high with respect to the efficient return. This is because in a competitive equilibrium the price of the bond in the shadow market (i.e. the inverse of the return) does not reflect the willingness of individuals to hedge against idiosyncratic shocks or their risk aversion, but only arbitrage conditions between the official and the hidden markets.

A second result is implied, too. The allocation in the decentralized intermediated equilibrium is not inefficient because of default. Again, the inefficiency of the market solution only emerges because of the relation between official and hidden financial markets. In that sense, as long as insurance markets are complete there is no connection between the presence of hidden trades and default. Therefore I conclude that the presence of unobservable markets is not a rationale for the introduction of resolution regimes in the banking sector to distort default decisions.

In the next section, I characterize the regulatory intervention that the government can set to implement the efficient allocation.

5 Optimal Regulation

In this section, I want to discuss an intervention such that banks autonomously implement the constrained efficient allocation in competitive equilibrium. I argue that such a mechanism is an ex-ante group-specific tier-1 capital requirement. The formal definition of tier-1 capital includes the most liquid resources among banks' financial assets: retained earnings and equity. In the model, the counterpart of such measure is the sum of short asset holdings (deposits stored from date 0 to date 1, similar to retained earnings) and investment in Arrow-Debreu securities paying at date 1 (short term assets paying in specific states of the world, similar to equity). Hence the tier-1 capital requirement takes the form of:

$$\frac{X^i + \sum_s q_1(s)\tau_1^i(s)}{\mu^i} \geq F^i \tag{31}$$

The intuition for the need of a tier-1 capital requirement is the following. In the new competitive equilibrium that emerges in the presence of regulation - that I call "regulated equilibrium" - the return on the shadow market is going to be lower than the one in the "unregulated" equilibrium, because I showed in the previous discussion that $R^*(s) < \hat{R}(s) = R(s)$ in any state of the world. This means that at such regulated return the short asset is dominated by the long asset, and no intermediary will hold liquidity. This cannot be an equilibrium, because market clearing in the shadow bond market is violated, i.e. early consumers would like to borrow, but no one would lend to them. Thus, the only way the system can support an equilibrium in which the regulated return on the hidden market is lower than the return on the long technology is via the introduction of a minimum liquidity requirement, so that banks are forced to hold enough resources to finance the consumption of the early consumers. By picking the right floor, the regulator can then manipulate banks' portfolio allocation, and let them implement the constrained efficient allocation.

More formally, the date 0 banking problem in country i is modified with the additional constraint in (31). The following proposition characterizes the regulatory intervention:

Proposition 3. *The Tier-1 capital requirement F^i that implements the efficient allocation is:*

$$F^i = \frac{1}{\mu^i} \sum_s q_1^R(s)\pi^i(0,s)\mathcal{I}^*(s) \tag{32}$$

where the equilibrium prices of the Arrow-Debreu securities $\tau_1^i(s)$ in equilibrium are:

$$q_1^R(s) = \frac{u'(I^*(s))\frac{\hat{R}(s)}{R^*(s)}\left[\pi^i(0,s) + \pi^i(1,s)\frac{R^*(s)}{\hat{R}(s)}\right]}{\sum_{k=1}^S u'(I^*(k))\frac{\hat{R}(k)}{R^*(k)}\left[\pi^i(0,k) + \pi^i(1,k)\frac{R^*(k)}{\hat{R}(k)}\right]} \tag{33}$$

and $\{\mathcal{I}^*(s), R^*(s)\}$ come from the solution to the planner problem.

Proof. In Appendix C. ■

The tier-1 capital requirement that implements the efficient allocation is therefore a weighted average of all the possible expenses that banks will face at date 1 if early consumers are entitled with the efficient amount of consumption ($\pi^i(0,s)\mathcal{I}^*(s)$). The weights are taken from the market, as are the prices of the Arrow-Debreu securities in the regulated equilibrium itself. Notice that from the comparison between (33) and (14) it is clear that the inefficiency in the intermediated equilibrium comes from the suboptimal spread between the official return and the hidden return.

As a last point, it is interesting to briefly compare the optimal regulation derived here with the one proposed in the Basel III Accord. In particular, the former features two of the main characteristics of capital buffers proposed by the Bank for International Settlements (2010): it dampens capital requirement cyclicality, by creating an ex-ante uncontingent rule; it promotes forward-looking provisions, by taking into account (and weighing) all the possible future states of the world. Notice that the other two mentioned objectives (build buffers to be used in period of financial distress and protect bank from excessive credit growth) are instead related to the role of regulation in preventing default, and as I showed in the previous sections that never emerges in equilibrium if markets are complete.

6 Conclusion

In the present paper I addressed the role of unobservable markets in igniting bankruptcy in the financial system. In a tractable model of intermediation inspired by Diamond and Dybvig (1983), I proved that hidden borrowing and lending impose a burden on financial intermediaries in competitive equilibrium: banks are not able to balance the two objectives of providing incentives and insurance, and do not improve over the autarkic allocation. Moreover, such burden does influence bankruptcy decisions, but only as long as markets are not complete: were banks able to trade in full state-contingent claims, they would perfectly insure against default risk. To put it differently, market incompleteness is the actual reason for banks' default. I guess the origin of default as an equilibrium phenomenon might then be found in environments with endogenous market incompleteness, that is where the pricing in the market is inefficient due to the presence of informational asymmetries or transaction costs. In fact, the mispricing of over-the-counter assets (especially those connected to the real estate market) is generally acknowledged as the main reason for the 2007-2009 financial crisis. As a general point I can then draw the conclusion that the presence of hidden markets is not a theoretical rationale for government intervention on bank's default decisions through resolution regimes.

In the last section I also showed that a tier-1 capital ratio, similar to the one proposed in the Basel III Accord, is indeed necessary to overcome the distortion imposed by unobservability on the consumption allocation. This has something to say about the interaction between macroprudential policy and monetary policy, both recently put in the hands of central banks. As it is clear from the analysis, the friction imposed by hidden savings on the financial system is a *real* phenomenon. Thus, a *real* instrument is enough to implement the efficient allocation. I guess that in more complete models of financial intermediation, in which there is a fundamental role for money as a meaning of transaction and eventually for monetary policy, the same result would hold. Monetary policy and macroprudential policy might then be distinct solutions to distinct problems. More work is needed in that direction.

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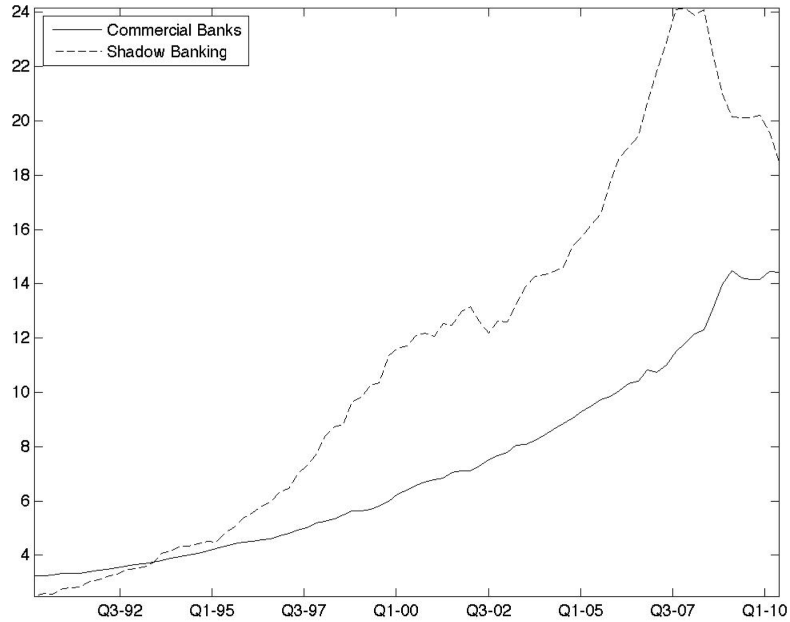

Figure 1: Total Financial Assets of US commercial banks vs. the shadow banking system, in US$ Billions. The shadow banking system is defined as the sum of: security brokers and dealers, funding corporations, real estate investment trusts, finance companies, issuers of asset-backed securities, agency and GSE-backed mortgage pools, government-sponsored enterprises, closed-end funds, mutual funds, money market mutual funds. *Source*: Federal Reserve, Flow of Funds of the United States, 2nd quarter 2010.

A Why only bonds in the shadow market?

In the proof, I follow Golosov and Tsyvinski (2007). Remember that, when borrowing and lending, individual types are still private information. In order to complete the set of traded securities, we may then add claims paying 1 unit of consumption conditional on reporting type θ in state s. Define the price of such securities as $Q(\theta, s)$. I can prove the following:

Lemma 6. $Q(\theta, s) \geq \frac{1}{R(s)}$ *for every type* $\theta \in \{0, 1\}$ *and* $s = 1, \ldots, S$.

Proof. Notice that $1/R(s)$ is the price of a risk-free bond delivering one unit of consumption in the following period for each unit invested, in state s. I prove the lemma by contradiction. Assume $Q(\theta, s) < \frac{1}{R(s)}$ for some θ and s. That would give rise to arbitrage opportunities: agents would buy infinitely many of such securities, sell uncontingent bonds in the same amount, then report exactly type θ, and enjoys infinite utility. That cannot be an equilibrium. ■

Given that $Q(\theta, s) \geq \frac{1}{R(s)}$, it is easy to argue that no type-contingent claim will be traded: individuals will never exchange securities which yield one unit of consumption if a specific type is reported, when they have the opportunity to trade a cheaper bond which yields one unit of consumption whatever type is reported.

B Constrained Efficient Allocation without Hidden Savings

The social planner chooses the optimal contract and the efficient portfolio allocation in order to maximize the total ex-ante welfare of the economy. In doing so, she is subject to the constraint that the portfolio allocation must provide enough resources to pay consumption in both periods to any agent of any type. In addition, I assume that agents still have private information about their individual types. I can then apply the Revelation Principle and restrict the social planner problem to truth-telling mechanisms in which every agent correctly reports her type. Formally, planner's problem is:

$$\max_{\substack{X,Y,\{C^i(\theta,s)\}_{s=1,\ldots,S} \\ \theta\in\{0,1\} \\ i=1,\ldots,n}} \sum_i \sum_s \nu(s) \sum_\theta \pi^i(\theta, s) U(C^i(\theta, s), \theta)$$

subject to:

$$X + Y \leq 1 \tag{34}$$

$$\sum_i \sum_\theta \pi^i(\theta, s) c_1^i(\theta, s) \leq X \tag{35}$$

$$\sum_i \sum_\theta \pi^i(\theta, s) c_2^i(\theta, s) \leq \hat{R}(s) Y \tag{36}$$

for each $s = 1, \ldots, S$, and:

$$U(C^i(0, s), 0) \geq U(C^i(1, s), 0) \tag{37}$$

$$U(C^i(1, s), 1) \geq U(C^i(0, s), 1) \tag{38}$$

for each $i = 1, \ldots, n$ and $s = 1, \ldots, S$. I report the solution to this problem in the next lemmas, which fully characterizes the equilibrium:

Lemma 7. *The planner chooses the optimal allocation such that in every state $s = 1, \ldots, S$, and for every $i, j = 1, \ldots, n$:*

$$\begin{aligned} c_1^i(1,s) = c_2^i(0,s) &= 0 \\ \frac{u'(c_1^i(0,s))}{u'(c_1^j(0,s))} = 1 &= \frac{u'(c_2^i(1,s))}{u'(c_2^j(1,s))} \\ \beta\hat{R}(s)u'(c_2^i(1,s)) &= u'(c_1^i(0,s)) \end{aligned} \tag{39}$$

and

$$\sum_i \left[\pi^i(0,s)c_1^i(0,s) + \pi^i(1,s)\frac{c_2^i(1,s)}{\hat{R}(s)}\right] = 1 \tag{40}$$

The constrained efficient allocation is equivalent to the unconstrained optimum.

Proof. Guess (37) and (38) are slack. Re-write (34), (35) and (36) as:

$$\sum_i \sum_\theta \pi^i(\theta,s)\left[c_1^i(\theta,s) + \frac{c_2^i(\theta,s)}{\hat{R}(s)}\right] \leq 1 \qquad \forall s = 1, \ldots, S \tag{41}$$

Assign multipliers $\lambda(s)$ to each constraint. Clearly, $c_1^i(1,s)$ and $c_2^i(0,s)$ are optimally set to zero, since they would be only costs. The first order conditions with respect to $c_1^i(0,s)$ and $c_2^i(1,s)$ read:

$$u'(c_1^i(0,s)) = \lambda(s) \tag{42}$$

$$\beta u'(c_2^i(1,s)) = \lambda(s)\frac{1}{\hat{R}(s)} \tag{43}$$

for each $s = 1, \ldots, S$. Then we easily derive (39). Simplification of (41) leads to (40).

Finally, I need to verify that the incentive compatibility constraints are actually slack. The expressions in (37) and (38) now become:

$$\nu(s)u(c_1^i(0,s)) \geq u(c_1^i(1,s)) = 0 \tag{44}$$

$$u(c_2^i(1,s)) \geq u(c_2^i(0,s)) = 0 \tag{45}$$

We need to prove that in equilibrium $c_1^i(0,s) > 0$ and $c_2^i(1,s) > 0$ for each i and s.

More specifically, given that the Euler equation holds in each country, we just need to prove that one of the two is true. Assume $c_1^i(0,s) = 0$ for some i and s. Then, by Inada conditions $u'(c_1^i(0,s)) = +\infty$ and by (39) also $u'(c_1^i(0,s)) = +\infty$ for any i and s. This implies $c_1^i(0,s) = 0$ for any i, and by the Euler equation $c_2^i(1,s) = 0$ for any i. But then (40) clearly gives $0 = 1$, which is a contradiction. ■

C Proofs

Proof of lemma 1. Rewrite the problem of the agent in the shadow market as:

$$\begin{aligned} V(C^i(\theta,s), R(s), \theta, s) = & \max_{x_1^i, x_2^i, b^i, \theta'} U(x_1^i, x_2^i, \theta) \\ s.t. \qquad & x_1^i + \frac{x_2}{R(s)} = c_1^i(\theta', s) + \frac{c_2^i(\theta', s)}{R(s)} \end{aligned}$$

For type 1 and 2, the incentive compatibility then respectively reads:

$$\begin{aligned} V(C^i(0,s), R(s), 0, s) \geq V(C^i(1,s), R(s), 0, s) \\ V(C^i(1,s), R(s), 1, s) \geq V(C^i(0,s), R(s), 1, s) \end{aligned}$$

which can be rewritten as:

$$\begin{aligned} u\left(c_1^i(0,s) + \frac{c_2^i(0,s)}{R(s)}\right) \geq u\left(c_1^i(1,s) + \frac{c_2^i(1,s)}{R(s)}\right) \\ u(R(s)c_1^i(1,s) + c_2^i(1,s)) \geq u(R(s)c_1^i(0,s) + c_2^i(0,s)) \end{aligned}$$

The result then follows. ■

Proof of lemma 3. The proof is in different steps by backward induction. First, I solve the ex post investment problem in (4). Attach multipliers $\xi^i(s)$ and $\chi^i(s)$ to (5) and (6), respectively. The first order conditions are:

$$\begin{aligned} L^i(s): \qquad & r\xi^i(s) \geq \hat{R}(s)\chi^i(s) \qquad & [= \text{if } L^i(s) \neq 0] \\ M^i(s): \qquad & \xi^i(s) \geq \chi^i(s) \qquad & [= \text{if } M^i(s) \neq 0] \end{aligned}$$

Clearly, these cannot hold together, because either $L^i(s) \neq 0$ and $M^i(s) = 0$ or vice versa, or $L^i(s) = M^i(s) = 0$. Hence the solution must be the following. Either:

$$\begin{aligned} M^i(s) &= X^i - \sum_\theta \pi^i(\theta, s) c_1^i(\theta, s) \\ L^i(s) &= 0 \end{aligned}$$

or:

$$\begin{aligned} L^i(s) &= \frac{1}{r}\left[-X^i + \sum_\theta \pi^i(\theta, s) c_1^i(\theta, s)\right] \\ M^i(s) &= 0 \end{aligned}$$

or $L^i(s) = M^i(s) = 0$. Notice that in any of the first two cases $-rL^i(s) + M^i(s) = X^i - \sum_\theta \pi^i(\theta, s) c_1^i(\theta, s)$. The intuition for this result is the following: Either the amount of short assets stored from date 0 is exactly equal to the needed liquidity (hence there is no need for further storage or liquidation), or the bank is forced to adjust ex post its investment strategy in a costly way.

Second, I solve for the problem at date 0 in (7). Argue that $c_2^i(0, s) = c_1^i(1, s) = 0$ (see the text for the intuition). Then it must be always the case that $c_2^i(1, s) = R(s) c_1^i(0, s) = \hat{R}(s) c_1^i(0, s)$, where $R(s)$ comes from the solution of lemma 2. Guess the equilibrium ex ante investment strategy is the one proposed. Then if ex post $\pi^i(0, s) c^i(0, s) = X^i$ there is no need to store or liquidate, hence the resource constraint in (10) gives $c_1^i(0, s) = 1$. If $\pi^i(0, s) c^i(0, s) < X^i$, then clearly $M^i(s) = X^i - \pi^i(0, s) c^i(0, s) > 0$ and $L^i(s) = 0$, and simple algebra gives the result from (10). Similarly, if $\pi^i(0, s) c^i(0, s) > X^i$, ex post budget clears if $M^i(s) = 0$ and $L^i(s) = 1/r\left[-X^i + \pi^i(0, s) c_1^i(0, s)\right]$, and again the resource constraint gives the result.

It is easy to verify that the proposed ex ante portfolio allocation clears the date 0 budget constraint, since $\pi^i(0, s) + \pi^i(1, s) = \mu^i$ in any state by hypothesis. This ends the proof. ■

Proof of proposition 1. We start from the banking problem at date 1. Attach multipliers $\xi^i(s)$ and $\chi^i(s)$ to (5) and (6), respectively. The first order conditions are:

$$\begin{array}{lll} Z^i(s): & \xi^i(s) \geq \hat{R}(s)\chi^i(s) & [= \text{if } Z^i(s) \neq 0] \\ L^i(s): & r\xi^i(s) \geq \hat{R}(s)\chi^i(s) & [= \text{if } L^i(s) \neq 0] \\ M^i(s): & \xi^i(s) \geq \chi^i(s) & [= \text{if } M^i(s) \neq 0] \end{array}$$

Clearly, these three cannot hold together, hence the solution must be the following. If $Z^i(s) = -\sum_{j\neq i} Z^j(s)$:

$$Z^i(s) = X^i + \tau^i(s) - \sum_\theta \pi^i(\theta, s) c_1^i(\theta, s)$$
$$L^i(s) = M^i(s) = 0$$

If $Z^i(s) < -\sum_{j\neq i} Z^j(s)$:

$$M^i(s) = X^i + \tau^i(s) - \sum_\theta \pi^i(\theta, s) c_1^i(\theta, s)$$
$$L^i(s) = Z^i(s) = 0$$

If $Z^i(s) > \sum_{j\neq i} Z^j(s)$:

$$L^i(s) = \frac{1}{r}\left[-(X^i + \tau^i(s)) + \sum_\theta \pi^i(\theta, s) c_1^i(\theta, s)\right]$$
$$Z^i(s) = M^i(s) = 0$$

Notice that in any case $Z^i(s) - rL^i(s) + M^i(s) = X^i + \tau^i(s) - \sum_\theta \pi^i(\theta, s) c_1^i(\theta, s)$.

The problem at date 0 is the following. Again, guess $c_2^i(0, s) = c_1^i(1, s) = 0$, and that in the equilibrium market completeness will ensure perfect cross-group risk sharing. Moreover, guess $L^i(s) = M^i(s) = 0$. Then, it is easy to see how the proposed consumption allocation comes from the resource constraint and incentive compatibility.

Now I solve for the investment strategy. Rewrite the budget constraint at date 2 as:

$$Y^i + (Z^i(s) - rL^i(s) + M^i(s)) - \left(1 - \frac{1}{\hat{R}(s)}\right) M^i(s) - (1-r)L^i(s) + \frac{\tau_2^i(s)}{\hat{R}(s)} = \sum_\theta \pi^i(\theta, s) \frac{c_2^i(\theta, s)}{\hat{R}(s)}$$

and making use of the ex post investment strategy and of incentive compatibility simplify into:

$$Y^i + X^i + \tau_1^i(s) + \frac{\tau_2^i(s)}{\hat{R}(s)} - \left(1 - \frac{1}{\hat{R}(s)}\right) M^i(s) - (1-r)L^i(s) = \mu^i c_1^i(0, s) \qquad (46)$$

Plug the equilibrium consumption into (46). Clearly, for (12) and (46) to hold together, it must be the case that:

$$\tau^i(s) + \frac{\tau_2^i(s)}{\hat{R}(s)} - \left(1 - \frac{1}{\hat{R}(s)}\right) M^i(s) - (1-r)L^i(s) = \sum_k q_1(k)\tau_1^i(k) + q_2(k)\tau_2^i(k) \tag{47}$$

Before going on, I need to solve for prices. Attach multipliers λ^i and $\zeta^i(s)$ to (12) and (46), respectively. The first order conditions with respect to $c^i(0,s)$, $\tau_1^i(s)$ and $\tau_2^i(s)$ are:

$$\nu(s)\pi^i(0,s)u'(c_1^i(0,s)) = \zeta^i(s)\mu^i \tag{48}$$

$$\lambda^i q_1(s) = \zeta^i(s) \tag{49}$$

$$\lambda^i q_2(s) = \frac{\zeta^i(s)}{\hat{R}(s)} \tag{50}$$

Sum (49) across states of the world, and normalize $\sum_s q(s) = 1$, so that $\lambda^i = \sum_s \zeta^i(s)$. Given that $c_1^i(0,s) = 1$, then $\lambda^i = u'(1)\sum_s \frac{\pi^i(0,s)}{\mu^i}\nu(s)$ and we can derive $q_1(s)$ as in (14). Moreover (49) and (50) imply $q_2(s) = q_1(s)/\hat{R}(s)$. Use this result into (47): multiply both sides by $q_1(s)$ and sum across states. Then:

$$\sum_s q_1(s)\left[\left(1 - \frac{1}{\hat{R}(s)}\right) M^i(s) + (1-r)L^i(s)\right] = 0$$

But given that both $M^i(s)$ and $L^i(s)$ are non-negative, then the solution must be $M^i(s) = L^i(s) = 0$, and as a consequence also $Z^i(s) = 0$. Using this and the solution to the date-1 problem, I can solve for $\tau_1^i(s)$ and $\tau_2^i(s)$.

With the solution to $\tau_t^i(s)$ and $q_t(s)$, it is easy to check that the proposed allocation clears the budget in every period, and that perfect cross-group risk sharing is verified. Given the equilibrium allocation, no agent will trade in the shadow market, hence $\sum_i b^i = 0$ is also satisfied. ■

Proof of lemma 5. We want to prove that the allocation that solves (20) can be implemented for some $\{R(s), \mathcal{I}^i(s)\}$ satisfying (22), and that given $\{R(s), \mathcal{I}^i(s)\}$ solution to (22) we can set $c_t^i(\theta,s) = x_t^i(\mathcal{I}^i(s), R(s), \theta, s)$ and check that it is feasible in (20).

Start from the first part. Take any allocaton $c_t^i(\theta,s)$ solution to (20). We know it satisfies incentive compatibility, hence:

$$c_1^i(0,s) + \frac{c_2^i(0,s)}{R(s)} = c_1^i(1,s) + \frac{c_2^i(1,s)}{R(s)} \tag{51}$$

Call this present discounted value $\mathcal{I}^i(s)$. Notice that $c_t^i(\theta, s)$ from (20) is solution to the "hidden problem" delivering $V(C^i(\theta,s), R(s), \theta, s)$ with equilibrium return $R(s)$, hence it also solves (22) provided $\theta' = \theta$, i.e. the true type is reported. That means that any solution to (20) can be implemented with the right choice of $\mathcal{I}^i(s)$ and $R(s)$. This ends the first part of the proof.

Now for the second part. Assume $\{R^*(s), \mathcal{I}^{i*}(s)\}$ is solution to (22). Pick $c_t^i(\theta, s) = x_t^i(\mathcal{I}^i(s), R(s), \theta, s)$. We want to see whether this allocation is feasible in (20). It clearly satisfies the resource constraint. Notice that:

$$\mathcal{I}^{i*}(s) = c_1^i(0,s) + \frac{c_2^i(0,s)}{R^*(s)} = c_1^i(1,s) + \frac{c_2^i(1,s)}{R^*(s)} \tag{52}$$

meaning:

$$\tilde{V}(\mathcal{I}^{i*}(s), R^*(s), \theta, s) = V(C^i(\theta,s), R^*(s), \theta, s) \tag{53}$$

because with income $\mathcal{I}^{i*}(s)$ every agent does as well by reporting the true type or the other. Given that x_t^i solves (7), it must be the case that $U(x_1^i, x_2^i, \theta) \geq V(C^i(\theta,s), R^*(s), \theta, s)$. This ends the proof. ■

Proof of proposition 2. Attach multiplier $\lambda(s)$ to the resource constraint. First order conditions are:

$$\begin{aligned}
\mathcal{I}^i(s): \quad & \nu(s)[\pi^i(0,s)u'(\mathcal{I}^i(s)) + \beta R(s)\pi^i(1,s)u'(R(s)\mathcal{I}^i(s))] = \\
& = \lambda(s)\left[\pi^i(0,s) + \pi^i(1,s)\frac{R(s)}{\hat{R}(s)}\right] && (54)\\
R(s): \quad & \nu(s)\beta\sum_i \pi^i(1,s)u'(R(s)\mathcal{I}^i(s))\mathcal{I}^i(s) = \frac{\lambda(s)}{\hat{R}(s)}\sum_i \pi^i(1,s)\mathcal{I}^i(s) && (55)
\end{aligned}$$

Multiply both sides of (54) by $\mathcal{I}^i(s)$ and sum across i. Making use of (55), we can then express $\lambda(s)$ as:

$$\lambda(s) = \frac{\nu(s)\sum_i \pi^i(0,s)u'(\mathcal{I}^i(s))\mathcal{I}^i(s)}{\sum_i \pi^i(0,s)\mathcal{I}^i(s)} \tag{56}$$

Use (56) back into (55) to derive the following condition:

$$\frac{\sum_i \pi^i(1,s)R(s)\mathcal{I}^i(s)}{\sum_i \pi^i(0,s)\mathcal{I}^i(s)} = \frac{\beta\hat{R}(s)\sum_i \pi^i(1,s)u'(R(s)\mathcal{I}^i(s))R(s)\mathcal{I}^i(s)}{\sum_i \pi^i(0,s)u'(\mathcal{I}^i(s))\mathcal{I}^i(s)} \tag{57}$$

The unconstrained optimum in (39)-(40) is solution to the constrained efficient problem. To see that, plug (39) into (57) and see that it is satisfied. Resources are exhausted, too. ■

Proof of proposition 3. Given that I am looking for the capital requirement that implements the efficient allocation, I can impose $Z^i(s) = L^i(s) = M^i(s) = 0$ right away. The date-0 banking problem reads:

$$\max_{\substack{c_1^i(\theta,s),c_2^i(\theta,s)\\X^i,Y^i,\tau^i(s)}} \sum_s \nu(s)\left[\pi^i(0,s)u\left(c_1^i(0,s)+\frac{c_2^i(0,s)}{R(s)}\right)+\beta\pi^i(1,s)u(R(s)c_1^i(1,s)+c_2^i(1,s))\right]$$

subject to:

$$X^i + Y^i + \sum_{s=1}^{S} q_1(s)\tau_1^i(s) + q_2(s)\tau_2^i(s) \leq \mu^i \tag{58}$$

$$X^i + \tau^i(s) \leq \sum_\theta \pi^i(\theta,s)c_1^i(\theta,s) \tag{59}$$

$$\hat{R}(s)Y^i \leq \sum_\theta \pi^i(\theta,s)c_2^i(\theta,s) \tag{60}$$

$$\frac{X^i + \sum_s q_1(s)\tau_1^i(s)}{\mu^i} \geq F^i \tag{61}$$

and the incentive compatibility constraint in (3). Remember that we proved that in competitive equilibrium $q_2(s) = q_1(s)/\hat{R}(s)$. Multiply both sides of (59) by $q_1(s)$, and of (60) by $q_2(s)$. Then sum across states and use (58) to obtain the intertemporal budget constraint:

$$\sum_s q_1(s)\sum_\theta\left[c_1^i(\theta,s)+\frac{c_2^i(\theta,s)}{\hat{R}(s)}\right] = \mu^i \tag{62}$$

Similarly, constraint (61) becomes:

$$\sum_s q_1(s)\sum_\theta \pi^i(\theta,s)c_1^i(\theta,s) \geq F^i\mu^i \tag{63}$$

Apply the following change of variables:

$$\mathbf{I}^i(s) = c_1^i(0,s) + \frac{c_2^i(0,s)}{R(s)}$$

$$\mathbf{H}^i(s) = \frac{1}{\mu^i}\sum_\theta \pi^i(\theta,s)c_2^i(\theta,s)$$

First, I express the constraints of the program in terms of $\mathbf{I}^i(s)$ and $\mathbf{H}^i(s)$. The capital requirement in (63) becomes:

$$
\begin{aligned}
&\sum_s q_1(s)[\pi^i(0,s)c_1^i(0,s)+\pi^i(1,s)c_1^i(1,s)+\mu^i c_1^i(0,s)-\mu^i c_1^i(0,s)] = \\
&=\sum_s q_1(s)[\mu^i c_1^i(0,s)+\pi^i(1,s)(c_1^i(1,s)-c_1^i(0,s))] = \\
&=\sum_s q_1(s)\left[\mu^i c_1^i(0,s)+\pi^i(1,s)\frac{c_2^i(0,s)-c_2^i(1,s)}{R(s)}\right] = \\
&=\sum_s q_1(s)\left[\mu^i\left(c_1^i(0,s)+\frac{c_2^i(0,s)}{R(s)}\right)-\frac{\pi^i(0,s)c_2^i(0,s)+\pi^i(1,s)c_2^i(1,s)}{R(s)}\right] \\
&=\sum_s q_1(s)\left[\mathbf{I}^i(s)-\frac{\mathbf{H}^i(s)}{R(s)}\right]\geq F^i \qquad (64)
\end{aligned}
$$

Similarly, the intertemporal budget constraint now reads:

$$\sum_s q_1(s)\left[\mathbf{I}^i(s)-\mathbf{H}^i(s)\left(\frac{1}{R(s)}-\frac{1}{\hat{R}(s)}\right)\right]=1 \qquad (65)$$

The problem then is to choose $\{\mathbf{I}^i(s),\mathbf{H}^i(s)\}$ to maximize the objective function, subject to (64) and (65). Attach multipliers η^i and ξ^i, respectively. Then the first order conditions with respect to $\mathbb{I}^i(s)$ and $\mathbf{H}^i(s)$ are:

$$
\begin{aligned}
q_1(s)(\xi^i-\eta^i) &= \nu(s)[\pi^i(0,s)u'(\mathbf{I}^i(s))+\beta R(s)\pi^i(1,s)u'(R(s)\mathbf{I}^i(s))] \qquad (66)\\
\frac{\eta^i}{R(s)} &= \xi^i\left[\frac{1}{R(s)}-\frac{1}{\hat{R}(s)}\right] \qquad (67)
\end{aligned}
$$

Plug the constrained efficient allocation into the program:

$$
\begin{aligned}
\mathbf{I}^i(s) &= \mathcal{I}^{i*}(s)\\
\mathbf{H}^i(s) &= \frac{1}{\mu^i}\pi^i(1,s)R^*(s)\mathcal{I}^{i*}(s)\\
R(s) &= R^*(s)
\end{aligned}
$$

I need to prove that at the constrained efficient allocation the multipliers are positive, FOCs are satisfied and markets clear for some positive prices. Notice that at the constrained efficient allocation the RHS of (66) can be simplified using the Euler equation. Merge the

FOCs into:

$$q_1(s)\xi^i \frac{R^*(s)}{\hat{R}(s)} = u'(I^*(s)) \left[\pi^i(0,s) + \pi^i(1,s) \frac{R^*(s)}{\hat{R}(s)} \right]$$

Sum the latter across states of the world, so as to derive:

$$\xi^i = \sum_s u'(\mathcal{I}^*(s)) \frac{\hat{R}(s)}{R^*(s)} \left[\pi^i(0,s) + \pi^i(1,s) \frac{R^*(s)}{\hat{R}(s)} \right]$$

which is positive by concavity of the utility function. Then I can easily derive the equilibrium prices of the Arrow securities in (33).

Notice that also η^i is positive by (67), since ξ^i is positive and $R^*(s) < \hat{R}(s)$. Therefore the minimum capital requirement is a binding constraint. By plugging the efficient allocation into (63) I derive (32).

■