A New Keynesian Phillips curve for Tunisia: Estimation and analysis of sensitivity

Samir Ben Ali

10. November 2010
A New Keynesian Phillips curve for Tunisia: Estimation and analysis of sensitivity

Samir BEN ALI
University 7 November of Carthage
November 10, 2010

Abstract

In this paper, we study some empirical issues in the estimation of a New-Keynesian Phillips curve for Tunisia. In this purpose, we compare the performance of the strict and hybrid forms in the validation of data. In addition, we try to establish the sensitivity of the Phillips curve estimation to some empirical specifications. It includes the measures to be used for the output gap variable, as well as the implementation of the generalized method of moments for the estimation of this curve.

Key words: monetary policy, New Keynesian Phillips curve, Inflation, sensitivity, Tunisia

JEL Classification: C22, E31, E52
1 Introduction

Analysis of the dynamics of inflation and its interactions with aggregates of the real economy continues to be an essential task for the implementation of monetary policy, in particular for how central banks should react in order to maintain inflation targets. Within this framework, and following the work of Gali and Gertler (1999), the New-Keynesian Phillips curve (NKPC) is gaining in popularity, due to its robustness to the Lucas critique. Thus, building on the foundations of microeconomic theory, the structural parameters of this curve are insensitive to changes in policy regime. The standard formulation of this curve was supported by assumptions reflecting nominal rigidities in the behaviour of economic agents. The New-Keynesian models used for this purpose led to a strict version explaining the evolution of inflation through a combination joining solely its anticipated level and a measure of marginal cost or output gap. However, this purely prospective "forward looking" version had not met empirical success. Indeed, applications of this curve, especially for the U.S. and European data have shown its limitations in the description of the data generator process. The latter continues to describe some persistence in the inflation dynamics calling for the introduction of lagged values of this variable in the so-called hybrid Phillips curve.

Furthermore, these empirical works provide more evidences about the inflation dynamics. In particular, Gali, Gertler, and Lopez-Salido (2001, 2003) suggest that the forward-looking component in the hybrid Phillips curve is much more important than the backward-looking component.

However, despite the success that could be attributed to this specification of the Phillips curve, the estimation of its parameters continues to be confronted with empirical issues. These are related to the choice of variables to be used for the validation of this equation as well as to the adequacy of the estimation
technique. Indeed, the estimation of this curve requires the specification of the variable reflecting the marginal cost or the output gap. However, the choice of this variable is often constrained by empirical issues. Furthermore, the implementation of the generalized method of moments, widely used in this framework, raised other issues related to the weak identification and the relevance of the instruments. All these empirical issues could have important implications, especially on the sensitivity of the estimated parameters of the Phillips curve.

In this framework, our paper presents estimates of a New-Keynesian Phillips curve for Tunisia. We examine the sensitivity of the estimated parameters to some choices made about the variables included in the model, as well as about the instruments used in the implementation of the generalized method of moments. The relevance of this investigation is related to its attachment to a widely used category of models, especially in analysis and forecasts in inflation targeting implementation frameworks. In Tunisia, the Central Bank has expressed its interest in a possible transition towards targeting inflation. Preliminary work of modelling the transmission mechanisms have been launched for this purpose. In this context, it may be possible to try to estimate the dynamics of inflation in referring to a New-Keynesian Phillips curve. Therefore, knowledge of sources of uncertainties surrounding the estimation of this curve would be desirable.

Until now, few studies have focused on this version of the Phillips curve for Tunisia. The main work of this category is returning to Pierre Richard Agenor and Nidhal Bayraktar (2007) who provided empirical estimates of the Phillips curve for eight middle-income developing countries including Tunisia (for the period 1979-2006). These authors have proposed a modified version of the hybrid NKPC to reflect the role of openness and the impact of borrowing costs on marginal production costs. The main results of this study have shown that the lagged
and lead inflation rate variables are statistically significant and have overall a positive effect on the current inflation. However, in contrast to most studies, they haven’t found a predominant weight of the forward-looking component in the model. Noting that the authors have used as a measure of the output gap the log difference of output to its trend component, where output is the real industrial production index and the trend component is calculated using a generalized version of the Baxter-King filter.

The remainder of this paper is organised as follows. The next section provides the theoretical framework of the NKPC. Section 3 describes the empirical specifications used for our estimation of this curve for the Tunisian case. Section 4 presents our empirical results, including an analysis of the sensitivity of the estimated parameters to several empirical specifications. Section 5 provides concluding remarks.

2 New-Keynesian Phillips curve specification: strict versus hybrid form

The New-Keynesian Phillips curve can be derived from the analysis of intertemporal optimization behavior of economic agents in an economy composed by a representative consumer and a large number of companies in a position of monopolistic competition. The specification of the strict version of this curve is based mainly on the hypothesis of Calvo (1983) assuming that only a fraction \((1 - \alpha)\) of firms are able to vary their prices each period, while the fraction \(\alpha\) of companies keep their prices unchanged. The equation derived from this formulation suggests that the current inflation rate is positively linked to the prospective rate of inflation and to the current real marginal cost or the output gap used as a proxy for this later, ie :
\[ \pi_t = \beta E_t \pi_{t+1} + \kappa y_t + \eta_{it} \]

Where \( \pi_t \) is the inflation rate, \( y_t \) is the output gap, \( \beta \) is a discount factor and \( \kappa \) is a structural parameter reflecting the link between the output gap and the real marginal cost.

However, the empirical limitations of this representation resulted in the identification of other forms of the Phillips curve by allowing some persistence of lagged values of inflation. The deduction of these hybrid forms are based on the consideration of other possibilities, than the Calvo hypothesis, on the conduct of companies in setting prices. It is, in particular, to admit that a fraction \( \alpha \) of firms is unable to adjust prices, while among the \((1-\alpha)\) companies which are able to vary their prices, an under fraction \( \omega \) does not adjust prices to optimal, but to reflect the past inflation rate. This representation seems to be more suitable to the Tunisian case where a fraction of the products included in the consumer prices index are administered, hence the need to reflect a fraction of companies unable to adjust their prices. The remaining companies may, of course, adjust prices to optimal or to reflect the past inflation rate.

The assumption made here is to suppose that a fraction \( \omega \) of companies does not adjust their prices in an optimal way, but chooses a price:

\[ p_t^* = p_{t-1}^* + \pi_{t-1} \] where \( p_{t-1}^* \) is the level of average prices in the previous period.

The \((1-\omega)\) remaining companies continue to adjust their prices in an optimal.

Accordingly, the level of average prices for the period \( t \) can be written as follows:

\[ p_t^* = (1-\alpha) \left[ (1-\omega) p_{t-1}^{1-\omega} + \omega \left( p_{t-1}^* + \pi_{t-1} \right) \right] + \alpha p_{t-1}^* \]

The corresponding Phillips curve admits the following hybrid form:

\[ \pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \lambda y_t + \eta_{it} \] (2)

Note that if \( \omega = 0 \) we have \( \gamma_b = 0 \) and \( \lambda = \kappa \). We find, therefore, the formulation of the strict New-Keynesian Phillips curve.
It is important to clarify in this context that, in general, formulations of the New-Keynesian Phillips curve, including those defined by Gali and Gertler, are expressed in terms of real marginal cost and not in terms of the output gap. Despite the possibility to establish a theoretical link between the marginal cost and the output gap, some authors think that in reality this later can not be a valid proxy for marginal cost. Indeed, Fuhrer and Moore (1995) have criticized the positive relationship between inflation and output gap in the Phillips curve involving the absence of an alternation between these two variables. In fact, with a transformation by iteration of the strict form of the Phillips curve, it is possible to establish that the current inflation depends solely on the present value of future output gap. In this regard, Gali and Gertler, have noted, in contrast, that the observed data show that the current output gap has a positive connection with future inflation and a negative relationship with past inflation.

Despite those empirical findings, works involved in estimating optimal rules continue to use Phillips curves approximating the marginal cost by the output gap. This is, obviously, because of links sought with the IS curve and the objective function of the central bank. In our case, estimates of the Phillips curve for Tunisia were conducted using only the output gap because of the unavailability of statistics on the real marginal cost.

3 Empirical specifications

Constrained by the availability of quarterly data, especially for output, the estimates have been conducted on the period 1991:1 - 2007:2. Regarding the data selected to represent the variables of the model, we chose to use the annual change in the quarterly consumer prices index (CPI) as a measure of the inflation rate, namely:
\[ \pi_t = 400 \times (\ln(CPI_t) - \ln(CPI_{t-4})). \]

Indeed, despite the imperfections that may exist in this index, in particular the problems of administered prices regulation and it’s high exposure to supply shocks, this measure remains attractive because of its extended employability in targeting inflation policy schemes.

![Figure 1: Evolution of the inflation rate in Tunisia](image)

The graph above shows the evolution of inflation rate, measured on the basis of the Tunisian CPI, over the period 1991: 1 - 2007: 2. The examination of this chart shows an evolution consistent with our expectations about the stationarity of the serie, despite the slight downward trend characterizing the beginning of the sample. The Augmented Dickey-Fuller test (ADF) applied on this measure of inflation confirmed this presumption.

With regard to the output gap which is a non observable variable, we used 3 different measures. The First one is based on the HP filter \((\lambda = 1600)\). The second measure is derived from the estimation of the unobserved components model of
Watson (1986). In this model, production is decomposed into two components, a trend component and a stationary cyclical component, measuring the output gap. The estimation of these components is conducted through the use of the iterative Kalman filter. The last measure of the output gap is generated by the estimation of a SVAR model. Indeed, we defined a VAR including three variables, ie the first differences of the logarithm of real output ($\Delta y_t$), inflation ($\Delta \pi_t$) and real interest rate ($\Delta r_t = i_t - \pi_t$). Following the suggestions of Schwarz and Hannan-Quinn key criterias, we selected a VAR with 3 delays. Then we estimated the potential output by imposing the restrictions of Blanchard and Quah (1989) to form the structural VAR.  

We present in the following graph the results of the estimates generated by each of these techniques. We note them, respectively, by $y_{HP}$, $y_{UCM}$ and $y_{SVAR}$. The stationarity of these different measures of the output gap has been well proven.

---

1 Blanchard and Quah assumed that long-term production is not affected by the structural shocks on inflation and that real interest rate and inflation does not depend on structural shock on the real interest rate.
In order to lead the estimation of the Tunisian New-Keynesian Phillips curve, we used a limited information approach based on the method of GMM. Given their impact on the results and the validation of the estimates produced by this method, the choice of instruments is particularly important. They must, indeed, meet two key criteria for selection, namely their ability to predict the prospective variable $\pi_{t+1}$ and the lack of correlation with the error term.

In this context, the values of lagged variables $\pi_t$ and $y_t$ which have a predictive capability on $\pi_{t+1}$, can be used as instruments as they are not correlated with the error term. As such, Gali and Gertler asserted that this condition of no correlation is well satisfied if the errors terms are independent and identically distributed ($iid \ (0, \sigma^2_\epsilon)$). This assertion is based on the hypothesis of rational expectations involving the orthogonality of the forecast errors over all the available information.

Regarding the Tunisian case, we have used as instruments lagged values of inflation, output gap, as well as the short-term interest rate $i_t$. The latter was measured by the quarterly average of the money market interest rate. Note also that given the small size of the sample, we chose to limit the number of instruments in order to avoid the effects of a high order of over-identification on the estimation results. In addition and in order to study sensitivity of the estimated parameters of the Phillips curve to the choice of these instruments, we used two sets of them.

As a first set we chose three lags of inflation and output gap, ie:

$$S^1_t = \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, y_{t-1}, y_{t-2}, y_{t-3}$$

For the second set of instruments, we added two lags of the interest rate, ie:

$$S^2_t = (\pi_{t-1}, \pi_{t-2}, \pi_{t-3}, y_{t-1}, y_{t-2}, y_{t-3}, i_{t-1}, i_{t-2}).$$
4 Empirical results

We begin firstly by presenting the estimation results of the strict New-Keynesian Phillips curve defined above by equation (1):

\[ \pi_t = \beta E_t\pi_{t+1} + \kappa y_t + \eta_{\pi t} \]

This strict form was estimated on the period between the first quarter of 1991 and the second quarter of 2007. As mentioned above, inflation was measured by the quarterly rate of change in the overall consumer prices index (CPI), while the output gap has been expressed through alternative measures generated by the HP filter, the unobserved components model of Watson, and by the SVAR model. Given the similarity of the results on the two sets of instruments, we will only present in the following table those using the set \( S^1 \) of instruments.

Table 1: Estimates of equation (1) using the GMM

\[ S^1_t = (\pi_{t-1}, \pi_{t-2}, \pi_{t-3}, y_{t-1}, y_{t-2}, y_{t-3}) \]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>measure of ( y )</th>
<th>( y_{HP} )</th>
<th>( y_{SVAR} )</th>
<th>( y_{UCM} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>( \beta )</td>
<td>1.01**</td>
<td>1.09**</td>
<td>1.03**</td>
</tr>
<tr>
<td>( \kappa )</td>
<td></td>
<td>-0.08</td>
<td>0.21*</td>
<td>-0.16</td>
</tr>
<tr>
<td>statistic ( J )</td>
<td></td>
<td>7.46</td>
<td>6.11</td>
<td>4.79</td>
</tr>
<tr>
<td>( \rho ) (probability)</td>
<td></td>
<td>0.11</td>
<td>0.19</td>
<td>0.31</td>
</tr>
<tr>
<td>( R^2 )</td>
<td></td>
<td>0.81</td>
<td>0.73</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Note: The terms in parentheses represent standard deviations of estimators.

* * Denotes that the estimate is significant at the 1% level.

* Denotes that the estimate is significant at the 5% level.
It appears clearly that the estimation of the New-Keynesian Phillips curve strict form resulted in parameters with sign or value non-conforming with those recommended by the theory. Indeed, as a discount factor, the parameter $\beta$ should naturally be less than 1. Also, the parameter $\kappa$ related to the output gap variable, which is used as a proxy of the marginal cost, should be positive. Contrary to these theoretical suggestions, estimates of the parameter $\beta$ led to figures slightly above 1 for the 3 output gap measures. Moreover, parameter $\kappa$ appeared with negative sign for estimates using HP filter and unobserved components model measurements of the output gap.

Already at this level, we can note the impact of the output gap measure choice on the results of estimates of the Phillips curve. From theoretical point of view, the results obtained through the use of the measure provided by the SVAR model would be satisfactory if the value of the parameter $\beta$ was less than unity. Also, we note that the appearance of a negative sign on the parameter $\kappa$ should not surprise, since it is a widespread problem in the estimates of Phillips curve referring to the output gap as a proxy for the marginal cost variable.

At this level of analysis, we can not decide definitively on the empirical support for this Phillips curve strict form. It is only through the comparison with the hybrid form estimation results that we can assess the interest of the introduction of a dynamic, reflecting a potential persistence of inflation in improving the estimation quality. As such, the results of the hybrid Phillips curve estimation, as defined above by equation (2), may provide clarification as to the contribution of the introduction of this dynamic in improving the performance of the curve in the description of inflation in Tunisia. In fact, the significance of the parameter related to the inflation lagged variable in equation (2) ie $\gamma_b$, would allow to judge the empirical validity of the correct form of the Phillips curve.
Conducted on the same period from the first quarter of 1991 to the second quarter of 2007, the estimation of the hybrid Phillips curve was based on the reduced form defined by (2), namely:

\[ \pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \lambda y_t + \eta_{nt} \]

The estimation procedure followed the same steps described above, i.e., the use of 3 respective measures of output gap (HP filter, unobserved components model and SVAR model) and the estimation of the models with GMM method using two scenarios of instruments.

The following table summarizes the hybrid Phillips curve estimation outputs using the \( S^1 \) instruments set.

<table>
<thead>
<tr>
<th>( S^1 ) ( = (\pi_{t-1}, \pi_{t-2}, \pi_{t-3}, y_{t-1}, y_{t-2}, y_{t-3}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>( \gamma_f )</td>
</tr>
<tr>
<td>( \gamma_b )</td>
</tr>
<tr>
<td>( \lambda )</td>
</tr>
<tr>
<td>statistic ( J ) (probability)</td>
</tr>
<tr>
<td>( R^2 )</td>
</tr>
</tbody>
</table>

Note: The terms in parentheses represent standard deviations of estimators.

** Denotes that the estimate is significant at the 1% level.

* Denotes that the estimate is significant at the 5% level.

Preliminary analysis of the estimation results carried out on the three measures of the output gap leads to some similarities, especially in the confirmation of the
dominant character of the forward-looking behavior in these models. Indeed, the 
estimated value of the parameter \( \gamma_f \) referring to the proportion of firms that adjust 
their prices prospectively was located in the neighborhood of the two-thirds of the 
inflation dynamic. Also, these results show that the parameter \( \gamma_b \), linked to the 
presence of a fraction of firms adjusting their prices retrospectively, is significantly 
different from zero. This significance attests the need of the introduction of an 
inflation lagged term in the Phillips curve in order to capture the persistence in 
this variable evolution. Being close and slightly below 1, the sum of these two co-
efﬁcients \( \gamma_f \) and \( \gamma_b \) expressing, respectively, the retrospective and the prospective 
inflation dynamic is also consistent with theoretical suggestions.

The dominance of the forward-looking behavior and the signiﬁcance of the re-
rospective term in the hybrid Phillips curve go well with the results of empirical 
tests conducted elsewhere, including the U.S. economy and the Euro zone. Indeed, 
the estimations of the hybrid Phillips curve, carried out on U.S and the Euro zone 
data, especially by Gali and Gertler (1999) and López-Salido (2001), have allowed 
to prove the importance of the forward looking behavior in this curve, as well as 
the statistical signiﬁcance of the retrospective component. In addition, the same 
tests conducted by these authors have shown that the output gap variable is not a 
have written in this context that "the use of real marginal cost as the relevant 
real sector forcing variable in the hybrid NKPC (as the theory suggests) is criti-
cal to the empirical success. Speciﬁcations based instead on ad-hoc “output gap” 
measures (e.g., detrended log GDP) do not perform well: The parameter on the 
output variable is either insigniﬁcant or signiﬁcant but with the wrong sign.”.

The same can be noted from the Phillips curve estimation on tunisian data. 
The results above show that the estimated parameter of the output gap variable,
while significant for the three alternative measures of this variable has appeared with the expected sign only for the estimate using the measure $y_{svar}$. For the two other measures of the output gap, the estimated parameter had a negative sign that does not reflect the theoretical link, positive in its state, between the inflation rate and the marginal cost. This finding helps to confirm the concern that we have made about the influence of the choice of the output gap measure on the estimates results of the Phillips curve.

In addition, and to study the sensitivity of the parameters of this Phillips curve to the choice of the GMM instruments, estimates have been renewed using the other set of instruments, namely $S^2$. The following tables present the results of these estimates.

Table 3: Estimates of equation (2) using the GMM

$$S^2_t = (\pi_{t-1}, \pi_{t-2}, \pi_{t-3}, y_{t-1}, y_{t-2}, y_{t-3}, i_{t-1}, i_{t-2})$$

<table>
<thead>
<tr>
<th>parameters $\gamma_f$</th>
<th>measure of $y$</th>
<th>$y_{HP}$</th>
<th>$y_{SVAR}$</th>
<th>$y_{UCM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma_f$</td>
<td>0.74**</td>
<td>0.56**</td>
<td>0.62**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.061)</td>
<td>(0.046)</td>
<td>(0.042)</td>
</tr>
<tr>
<td>$\gamma_b$</td>
<td></td>
<td>0.26**</td>
<td>0.44**</td>
<td>0.37**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.060)</td>
<td>(0.045)</td>
<td>(0.039)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td></td>
<td>-0.11**</td>
<td>0.07*</td>
<td>-0.09**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.056)</td>
<td>(0.034)</td>
<td>(0.036)</td>
</tr>
<tr>
<td>statistic $J$</td>
<td></td>
<td>8.53</td>
<td>8.13</td>
<td>7.43</td>
</tr>
<tr>
<td>$\rho$ (probability)</td>
<td></td>
<td>0.13</td>
<td>0.15</td>
<td>0.19</td>
</tr>
<tr>
<td>$R^2$</td>
<td></td>
<td>0.91</td>
<td>0.92</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Note: The terms in parentheses represent standard deviations of estimators.

* * Denotes that the estimate is significant at the 1% level.

* Denotes that the estimate is significant at the 5% level.
Overall, the analysis of the results of the Phillips curve estimates conducted using $S^2$ and recorded in table 3 allows to see their similarity to those obtained using $S^1$ (Table 2), testifying to the strength and the validity of the estimation procedure. Indeed, the same empirical conclusions drawn from the estimates conducted under $S^1$ can be confirmed by those using $S^2$. Precisely, these conclusions concern the dominance of the prospective dynamic and the emergence of the expected signs of the output gap parameter, only when using the measure $y_{svar}$ for this variable.

Moreover, comparison of results of estimations shows the sensitivity of parameter estimates to changes in instrument sets. The finding is so important because the two sets of instruments seem to be appropriate. Indeed, it is not possible for the two sets to reject the hypothesis that all the over-identifying restrictions are satisfied. The goodness of fit is satisfactory for both of them.

Nevertheless, and despite the similarities in the estimation results, it is possible to notice the existence of differences, more or less important, in the values of estimated parameters. These differences reflect the sensitivity of estimates to the choice of the instruments introduced for the implementation of the GMM method. Note that the sensitivity of estimated parameters constitute an important factor of uncertainty, particularly when used for the determination of optimal rules of monetary policy.

5 Conclusion

 Concerned in this paper by the validation of the New Keynesian Phillips curve on the Tunisian data, we conducted several estimates of this curve in order to demonstrate sensitivity of the estimated parameters to some choices that should be made about the included variables and the implementation of the estimation procedure.
The results of the estimates on Tunisian data have called for the convenience of a hybrid form of the Phillips curve. Indeed, the weak performance of the strict form and the statistically significant parameter of the inflation retrospective term in the hybrid form suggest the supremacy of the latter in explaining the evolution of inflation. However, even assuming the legitimacy of the use of an hybrid version of the Phillips curve, uncertainty regarding the estimated parameters is well proven.

The presence of these uncertainties has been confirmed through the identification of the impact of the output gap measure on the estimated values of these parameters. In addition, we were able to demonstrate that the parameters estimates of this curve are dependent on the set of instruments introduced in the implementation of the generalized method of moments (GMM). Note also that these sources of uncertainty are not restrictive. Other examples that we have not tried to test empirically can be considered. These include uncertainties related to the length of the retrospective dynamic in the hybrid form of the Phillips curve, or the use of another method than the GMM in the estimation of parameters.
BIBLIOGRAPHY


[38] Rubene Leva and Guarda Paolo : The new keynesian Phillips curve: Empirical results for Luxembourg, Cahier d’études de la Banque Centrale de Lux-
embourg n°11, Juin 2004.

