Endogenous Market Structures and the Business Cycle

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Endogenous Market Structures and Labor Market Dynamics[1]

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Abstract

We propose a flexible prices model where endogenous market structures and search and matching frictions in the labor market interact endogenously. The interplay between firms endogenous entry, strategic interactions among producers and labor market frictions represents a strong amplification channel of technology shocks on labor market variables, and helps addressing the unemployment-volatility puzzle. Consistently with U.S. evidence, new firms create a large fraction of new jobs and grow faster than more mature firms, net firms' entry is procyclical and the price mark up is countercyclical.

JEL classification: E24, E32, L11.

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1 *Introduction*

We present a framework where search and matching frictions in the labor market interact with the dynamics of the number of firms and their strategic behavior. Market structures are said to be endogenous since the number of producers and the price mark ups are determined both in the short and in the long run.

Three stylized facts motivate this paper: the large fraction of job creation (destruction) in the U.S. economy due to the birth (death) of firms, the procyclical variations in the number of market competitors and in aggregate profits, and the countercyclicality of price mark ups.[2]

We consider an economy with distinct sectors, each one characterized by many firms supplying goods that can be imperfectly substitutable to a different degree. As in Colciago and Etro (2010 a and b), we take strategic interactions into account and allow firms within a sector to compete either in prices (Bertrand competition) or in quantities (Cournot competition). Following Ghironi and Melitz (2005) and Bilbiie, Ghironi and Melitz (2007) (BGM 2007 henceforth), entry is subject to sunk entry costs and a time-to-build lag. The free entry condition equates the expected present discounted value of profits to the sunk cost to endogenize the number of firms in each sector. As a result the degree of market power, measured as the mark up that firms can impose over marginal costs, depends endogenously on the form of competition, on the degree of substitutability between goods and on the number of firms in the sector. Firms are large, since they employ multiple-workers and the labor market is characterized by Mortensen and Pissarides (1999)-style search and matching frictions. Workers may separate from a job for two reasons: either because the firm where the job is located exits from the market or because the match is destroyed. As a consequence, we can identify the contribution to job creation coming from the birth of firms and the contribution to job destruction due to the exit of firms from the market. Further, and differently from the standard labor search model, the endogeneity of the number of producers, together with the large firms assumption, allows to realistically distinguish between the dynamics of the number of producers and that of employment.

This set up delivers two sets of results. First, in line with the facts discussed above, new entrants give a sizeable contribution to job creation and grow faster than more mature producers. Further, profits are procyclical and price mark ups are countercyclical. Second, without resorting to rigidities in wages (real or nominal) or prices, and under

[2] As in Jaimovich and Floetotto (2008) we use the terms new firm and new competitors in a broad sense. They refer to both start ups and to new establishments.

a conservative and standard calibration of parameters, our framework substantially outperforms the basic search and matching model at replicating the observed variability of the unemployment rate, vacancies and market tightness.

Both outcomes are the result of a novel propagation mechanism of technology shocks.[3] A rise in technology initially increases profits, which leads to entry of new firms. Since firms' creation requires output, incumbent firms increase their labor demand both at the intensive and at the extensive margin. The need to hire workers boosts vacancy posting on impact. As new entrants start producing, vacancy posting and job creation are further amplified with respect to a model with exogenous market structures. Also, as the goods' market becomes more crowded, the relative price of existing varieties rises. At the same time, stronger competition leads to a lower mark up and thus to a higher demand by consumers. Both effects provide firms with an incentive to create vacancies which is absent in a model with exogenous market structures. The responses of the job finding rate and the vacancy filling rate are similarly amplified.

This dynamics translates into a response of the unemployment rate in the period after the shock which is almost three times larger, and more persistent, than that observed in a search model with exogenous market structures but identical calibration. Importantly, our framework generates countercyclical mark ups together with procyclical profits and matches the time pattern of the correlation between output and the mark up identified in the literature (see e.g. BGM (2007) and Colciago and Etro (2010)). Turning to second moments, we find that endogenous market structures magnify the volatility of unemployment, hours, vacancies and the job market tightness in response to a technology shock. For this reason, we argue that a model where the market structure is endogenous helps alleviating the so called Shimer (2005) puzzle.

In the long run, stronger competition in the goods market leads to lower unemployment and to higher real wages. The endogenous steady state share of gross job creation due to new firms is 25 percent and the share of overall employment due to startups equals 2.5 percent. These figures are in line with U.S. averages. Haltiwanger et al. (2009) consider U.S. annual data between 1992 and 2005. They find that business startups account for roughly 3 percent of U.S. total employment in any given year. While this is a reasonably small share of the stock, it is large relative to net job creation which averages around 2.2 percent of total

[3]With minor differences in the explanation, the propagation mechanism we describe applies to any output-expanding shock.

employment per year.[4] Also, Davis and Haltiwanger (1990) on the basis of U.S. manufacturing data between 1972 and 1986 estimate that 25 percent of annual gross job creation is due to new establishments births. Similarly, Jaimovich and Floetotto (2008) focus on employment data at the establishment level. They estimate that the average fraction of quarterly job-gain (losses) that can be explained by the opening (closing) of establishments is about 20 percent.

Finally, we show that the interaction between endogenous market structures and search and matching frictions generates time varying wedges between the competitive equilibrium allocation and the socially efficient one. The time-varying nature of price mark up distorts both the allocation of the intensive and the extensive margin of labor. Moreover, since new entrants need to build up a workforce at the beginning of their activity, they suffer lower profits and are characterized by a lower value with respect to incumbent producers. We show that this asymmetry distorts the allocation of consumption across different dates. Importantly, the Hosios (1990) condition is necessary but not sufficient to achieve efficiency in the allocation of the extensive margin of labor. In particular, the competitive equilibrium allocation is characterized by under-employment under both Cournot and Bertrand competition even when the Hosios condition is satisfied.

There is convincing macroeconomic evidence in support of the business cycle implications of our approach. Jaimovich and Floetotto (2008) document that around a third of the cyclical volatility of the job-gains (losses) comes from opening (closing) of establishments, which suggests that the dynamics of the number of market competitors is a relevant explanatory variable for the dynamics of job creation (destruction) and thus of unemployment. The strongly procyclical response of the job finding probability delivered by our model is consistent with the evidence in Hall (2005) who, using post-war data, finds that periods of boom, when the unemployment rate decreases, are associated to a high probability of finding a job.[5] In line with our results Davis et al. (2009) find that the vacancy filling probability is strongly countercyclical. An early reference on the procyclicality of firms' entry in the U.S. is Chatterjee and Cooper (1993), while a more recent one is Bergin and Corsetti (2008). Portier (1995) reports a similar pattern for France. Bils (1987), Rotemberg and Woodford (1999) and Galì *et al.* (2007) document price mark ups countercyclicality. Campbell and Hopenayn (2005) and Martins et al. (1996)

[4] Haltiwanger et al (2009) warn that it would be misleading to conclude that new firms account for more than 100% percent of all net new jobs. Other, mature, firms are creating jobs. However the net growth from new firms alone exceeds the average.

[5] Hall (2005) also reports a relatively acyclical job separation rate.

convincingly report evidence suggesting that a variation in the number of competitors affects the degree of competition in the market and through this way the mark up that firms can impose on marginal costs. BGM (2007) and Etro and Colciago (2010) emphasize the procyclicality of real profits.

Our work bridges two apparently uncorrelated recent lines of research. The first one is constituted by theoretical and empirical contributions studying the role of firms' entry and the creation of new products for the business cycle. Recent empirical works on the manufacturing sector by Broda and Weinstein (2009) and Bernard *et al.* (2008) have emphasized the importance of the extensive margin in the process of product creation or innovation. For this reason BGM (2007) design the entry process as a process of creation of new products with limited substitutability which may depend on the number of available products. Jaimovich and Floetotto (2008) in an RBC framework and Colciago and Etro (2010) in a frictional, BGM-like set up consider Bertrand and Cournot competition between and endogenous number of producers. These contributions show that the extensive margin of product creation improves the performance of an otherwise standard flexible prices DSGE model at matching impulse responses and business cycle moments for U.S. data. Recently Bergin and Corsetti (2008) and Lewis (2010) uncovered a correlation between firms' dynamics and monetary policy, suggesting that the extensive margin may constitute a further welfare dimension for monetary policy intervention. The second line of research relevant for our analysis is constituted by those contributions trying to solve the unemployment-volatility puzzle presented by Shimer (2005) and Hall (2005) mainly by modifying the basic Mortensen-Pissarides model. To this group belongs the work by Hall (2005), who argues that the lack of amplification characterizing the basic Mortensen-Pissarides model lies in the wage Nash bargaining assumption. He suggests that real wage rigidity could help solving the puzzle. In a similar vein Gertler and Trigari (2009) extend the RBC labor search model of Andolfatto (1996) and Merz (1995) by introducing staggered multi-period wage contracting and show that their model exhibits a strong amplification. We collocate in this group also the article by Hagedorn and Manovskii (2008) who adopt the same Mortensen-Pissarides model used by Shimer (2005), but show that the model features a stronger propagation mechanism once parametrized using a different calibration strategy.

This paper argues that oligopolistic competition between an endogenous number of producers is a relevant dimension to consider in order to understand the connection between labor markets and the goods markets and to improve the ability of the Mortensen-Pissarides theoretical framework to fit key labor market statistics. Also, to our knowledge, the model we propose is the first one to address in a unified framework the three stylized facts listed at the beginning of this Introduction.

The papers closest related to ours are Blanchard and Giavazzi (2003), and more recently Hebel and Haefke (2009), Shao and Silos (2008) and Kaas and Kircher (2011). With respect to Blanchard and Giavazzi (2003) we provide a fully specified DSGE model where the dynamics of the number of firms is explicitly modeled. Hebel and Haefke (2009) consider a labor search model with firms' entry. In their model entry and exit of firms are exogenous and the number of producers is determined endogenously in the long run by means of a zero profits condition. Their analysis focuses on the long run effects of deregulation in the goods markets for the level of unemployment and the real wage. Shao and Silos (2008) introduce firms' entry in a Mortensen-Pissarides-style model with monopolistic competition in the goods market characterized by small firms. They identify the countercyclical value of vacancies as the main propagation channel of technology shocks and study the business cycle implications of their model for the labor and profit shares of output. Also, the small firms assumption does not allow to address the empirical evidence on job creation by new entrants. Our analysis differs from that in Kaas and Kircher (2011) with respect to both assumptions and focus. For what concerns assumptions we feature strategic interactions among large firms which bargaining the wage on a period-by-period basis with their employees. The aforementioned authors consider an alternative framework to characterize firms' dynamics in a frictional labor market, where large, risk-neutral firms can commit to long-term wage contracts. With respect to the focus, we analyze endogenous market structures both in the long and the short run, and we emphasize their role under different forms of competition for the propagation of exogenous technology shocks on labor market variables. Kaas and Kircher (2011) focus instead on the efficiency of the competitive equilibrium in the presence of multiple-workers firms that can commit to long-term wage contracts. In this environment efficiency obtains on all margins of job creation and destruction, both with idiosyncratic and aggregate shocks. Their model is also consistent with several empirical regularities about firm size, job flows and pay.

The remainder of the paper is organized as follows. Section 1 describes the model and its dynamic properties in the short and long run. Section 2 provides the conditions for efficiency of the decentralized equilibrium. Section 3 displays the analysis of the impulse response functions and the second moments to exogenous technology shocks. Section 4 focus on the efficiency of the competitive equilibrium. Section 5 concludes.

Technical details are left in the Appendix.

2 *The model*

2.1 *Labor and Goods Markets*

There are two main building blocks in the model: oligopolistic competition with endogenous entry in the goods market and search and matching frictions in the labor market. In this paragraph we outlay their main features.

As in Colciago and Etro (2010 a and b), the economy features a continuum of sectors, or industries, on the unit interval. Sectors are indexed with $k \in (0,1)$. Each sector k is characterized by different firms $i = 1, 2, ..., N_{kt}$ producing the same good in different varieties. At the beginning of each period N_{kt}^e new firms enter into sector k, while at the end of the period a fraction $\delta \in (0,1)$ of market participants exits from the market for exogenous reasons.[6] Below we describe the entry process and the mode of competition within in each sector in detail.

The labor market is characterized by search and matching frictions, as in Andolfatto (1996) and Mertz (1995). A fraction u_t of the unit mass population is unemployed at time t and searches for a job. Firms producing at time t need to post vacancies in order to hire new workers. Unemployed workers and vacancies combine according to a CRS matching function and deliver m_t new hires, or matches, in each period. The matching function reads as $m_t = \gamma_m \left(v_t^{tot}\right)^{1-\gamma} u_t^{\gamma}$, where γ_m reflects the efficiency of the matching process, v_t^{tot} is the total number of vacancies created at time t and u_t is the unemployment rate. The probability that a firm fills a vacancy is given by $q_t = \frac{m_t}{v_t^{tot}}$, while the probability to find a job for an unemployed worker reads as $z_t = \frac{m_t}{u_t}$. Firms and individuals take both probabilities as given. Matches become productive in the same period in which they are formed. Each firm separates exogenously from a fraction $1 - \varrho$ of existing workers each period, where ϱ is the probability that a worker stays with a firm until the next period.

As a result a worker may separate from a job for two reasons: either because the firm where the job is located exits from the market or because the match is destroyed. Since these sources of separation are independent, the evolution of aggregate employment, L_t, is given by

$$L_t = (1 - \delta) \varrho L_{t-1} + m_t \tag{1}$$

where the number of unemployed workers searching for a job at time t

[6] As discussed in BGM (2007), if macroeconomic shocks are small enough N_{kt}^e is positive in every period. New entrants finance entry on the stock market.

is $u_t = 1 - L_{t-1}$.[7]

2.2 *Households and Firms*

Using the family construct of Mertz (1995) we can refer to a representative household consisting of a continuum of individuals of mass one. Members of the household insure each other against the risk of being unemployed. The representative family has lifetime utility:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 \ln C_{kt} dk - \chi L_t \frac{h_t^{1+1/\varphi}}{1+1/\varphi} \right\} \quad \chi, \varphi \geq 0 \tag{2}$$

where $\beta \in (0,1)$ is the discount factor and the variable h_t represents individual hours worked. Note that C_{kt} is a consumption index for a set of goods produced in sectors $k \in [0,1]$, defined as

$$C_{kt} = \left[\sum_{i=1}^{N_{kt}} C_{kt}(i)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{3}$$

where $C_{kt}(i)$ is the production of firm i of this sector, and $\varepsilon > 1$ is the elasticity of substitution between the goods produced in each sector. The distinction between different sectors and different goods within a sector allows to realistically separate limited substitutability at the aggregated level, and high substitutability at the disaggregated level. Contrary to many macroeconomic models with imperfect competition, our focus will be on the market structure of disaggregated sectors: intrasectoral substitutability (between goods produced by firms of a same sector) is high, while intersectoral substitutability is low.[8] The family receives real labor income $w_t h_t L_t$ and profits from the ownership of firms. Further, we assume that unemployed individuals receive an unemployment benefit b in real terms, leading to an overall benefit for the household equal to $b(1 - L_t)$. This is financed through lump sum taxation by the government. Notice that the household recognizes that employment is

[7] Given that population is normalized to one, the number of unemployed workers and the unemployment rate are identical.

[8] Our functional form implies unitary elasticity of substitution between goods produced in different sectors. In this case the aggregate consumption bundle enjoyed by the household could be defined as $C_t^A = \exp\left(\int_0^1 \ln C_{kt} dk\right)$ and associated to the aggregate price index $P_t^A = \exp\left(\int_0^1 \ln P_{kt} dk\right)$. The same approach has been proposed by Colciago and Etro (2010 a). Atkeson and Burnstein (2008) consider a trade model with multiple sectors. Even if they allow for general substitutability across sectors, their numerical results are obtained assuming a unitary intersectoral elasticity of substitution.

determined by the flows of its members into and out of employment according to

$$L_t = (1-\delta)\,\varrho L_{t-1} + z_t u_t \tag{4}$$

Households choose how much to save in riskless bonds and in the creation of new firms through the stock market according to standard Euler and asset pricing equations.[9]

The intratemporal optimality conditions for the optimal choices of C_{kt} requires:

$$P_{kt}C_{kt} = EXP_t \;\text{ for any } k \tag{5}$$

where EXP_t is total nominal expenditure allocated to the goods produced in each sector in period t and P_{kt} is the price index for consumption in sector k: due to the unitary elasticity of substitution, total expenditure is identical across sectors.

The marginal value to the household of having one member employed rather than unemployed, Γ_t, which is a determinant of the wage bargaining problem is

$$\Gamma_t = \frac{1}{C_t}\left(w_t h_t - b\right) - \chi \frac{h_t^{1+1/\varphi}}{1+1/\varphi} + \beta E_t\left[(1-\delta)\,\rho - z_{t+1}\right]\Gamma_{t+1} \tag{6}$$

Each firm i in sector k produces a good with a linear production function. We abstract from capital accumulation issues and assume that labor is the only input. Output of firm i in sector k is then:

$$y_{kt}(i) = A_t n_{kt}\,(i)\,h_{kt}(i) \tag{7}$$

where A_t is the, common to all sectors, total factor productivity at time t, $n_{kt}\,(i)$ is firm i's time t workforce and $h_{kt}(i)$ represent hours per employee. Since each sector can be characterized in the same way, in what follows we will drop the index k and refer to the representative sector.[10]

2.3 *Endogenous Market Structures*

Following Ghironi and Melitz (2005) and BGM (2007) we assume that new entrants at time t will only start producing at time $t+1$. Given the exogenous exit probability δ, the average number of firms per sector, N_t, follows the equation of motion:

$$N_{t+1} = (1-\delta)(N_t + N_t^e) \tag{8}$$

[9] We report these conditions in Appendix A.
[10] We provide analytical details in Appendix A.

where N_t^e is the average number of new entrants at time t. We assume that entry requires a fixed cost ψ, which is measured in units of output. In each period, the same nominal expenditure for each sector EXP_t is allocated across the available goods according to the direct demand function:

$$y_t(i) = Y_t \left(\frac{p_t(i)}{P_t}\right)^{-\varepsilon} = \frac{p_t(i)^{-\varepsilon}}{P_t^{1-\varepsilon}} Y_t P_t = \frac{p_t(i)^{-\varepsilon} EXP_t}{P_t^{1-\varepsilon}} \qquad i = 1, 2, ..., N_t \tag{9}$$

where P_t is the price index

$$P_t = \left[\sum_{j=1}^{N_t} p_t(j)^{-(\varepsilon-1)}\right]^{\frac{-1}{\varepsilon-1}} \tag{10}$$

such that total expenditure satisfies $EXP_t = \sum_{j=1}^{N_t} p_t(j) C_t(j) = C_t P_t$.[11] Inverting the direct demand functions, we can derive the system of inverse demand functions:

$$p_t(i) = \frac{y_t(i)^{-\frac{1}{\varepsilon}} EXP_t}{\sum_{j=1}^{N_t} y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}} \qquad i = 1, 2, ..., N_t \tag{11}$$

Period t real profits of an incumbent producer are defined as

$$\pi_t(i) = \rho_t(i)\, y_t(i) - w_t(i)\, n_t(i)\, h_t(i) - \kappa v_t(i) \tag{12}$$

where $\rho_t(i)$ $(= \frac{p_t(i)}{P_t})$ is the real price of firm i's output, $v_t(i)$ represents the number of vacancies posted at time t and κ is the output cost of keeping a vacancy open. The value of a firm is the expected discounted value of its future profits

$$V_t(i) = E_t \sum_{s=t+1}^{\infty} \Lambda_{t,s} \pi_s(i) \tag{13}$$

where $\Lambda_{t,t+1} = (1-\delta)\beta \left(\frac{C_{t+1}}{C_t}\right)^{-1}$ is the households' stochastic discount factor which takes into account that firms' survival probability is $1 - \delta$. Incumbent firms which do not exit from the market have a time t individual workforce given by

$$n_t(i) = \varrho n_{t-1}(i) + v_t(i)\, q_t \tag{14}$$

[11] The demand of the individual good and the price index are the solution to the, usual, consumption expenditure minimization problem.

2.3.1 *Bertrand Competition*

Let us consider competition in prices. Contrary to the traditional Dixit-Stiglitz (1977) approach which neglects strategic interactions between firms, we take these into consideration and derive the exact Bertrand equilibrium. Each firm i chooses $p_t(i)$, $n_t(i)$ and $v_t(i)$ to maximize $\pi_t(i) + V_t(i)$, taking as given the price of the other firms. Maximization is subject to three constraints, namely (7), (9) and (14).

The variable $\phi_t(i)$ is the Lagrange multiplier of the latter constraint, and represents the time-t value of an additional workers to the firm; $mc_t(i)$ is the time t real marginal cost faced by firm i and represents the Lagrange multiplier of constraint (7).

In what follows we distinguish between incumbent firms according to their period of entry. We define as *first period incumbent firms* those producers which entered the market in period t-1 and at time t produce for the first time. The term *mature incumbent firms* refers, instead, to producers which entered the market in period t-2 or prior. The distinction is relevant because first period incumbents have no beginning of period workforce. Nevertheless, Proposition 1 and 2 show that in both the Bertrand and Cournot equilibria incumbent producers, no matter the period of entry, have the same size, impose the same mark up over marginal costs and have the same individual level of production.

Proposition 1 (Bertrand Equilibrium) *In the Bertrand equilibrium, no matter the period of entry: i) the marginal cost and the value of an additional worker are identical across producers: $mc_t(i) = mc_t$ and $\phi_t(i) = \phi_t$; ii) firms set the same mark up over the nominal marginal cost, given by*

$$\mu_t^P(\varepsilon, N_t) = \frac{\varepsilon(N_t - 1) + 1}{(\varepsilon - 1)(N_t - 1)} \tag{15}$$

iii) firms have the same level of production, the same size and demand the same number of hours per employee: $y_t(i) = y_t$, $n_t(i) = n_t$ and $h_t(i) = h_t$.

Proof. See Appendix B1 ■

Since in equilibrium firms set the same prices, it follows from (10) that the relative price is also identical across producers and reads as $\rho_t(\varepsilon, N_t) = \frac{p_t}{P_t} = N_t^{\frac{1}{\varepsilon-1}}$. The mark up $\mu_t^P(\varepsilon, N_t)$ is decreasing in the degree of substitutability between products ε, with an elasticity $\epsilon_\varepsilon^P = \frac{\varepsilon N_t}{(1-\varepsilon+\varepsilon N_t)(\varepsilon-1)}$. Moreover, the mark up vanishes in case of perfect substitutability: $\lim_{\varepsilon\to\infty} \mu^P(\varepsilon, N_t) = 1$. Finally, the mark up is decreasing in the number of firms, with an elasticity $\epsilon_N^P = \frac{N_t}{(1+\varepsilon(N_t-1))(N_t-1)}$. Notice that the elasticity of the mark up to entry under competition in prices is decreasing in the level of substitutability between goods, and it tends to zero when the goods are approximately homogenous. When $N_t \to \infty$ the mark up tends to $\varepsilon/(\varepsilon-1)$, the traditional one under monopolistic competition. As well known, strategic interactions between a finite number of firms lead to a higher mark up than under monopolistic competition.

2.3.2 *Cournot Competition*

In this section we consider competition in quantities, which has been largely neglected in general equilibrium macroeconomic models with imperfect competition. In this case firms maximize $\pi_t + V_t$ choosing their production $y_t(i)$ beside $n_t(i)$ and $v_t(i)$, taking as given the production of the other firms. Maximization is subject to the same constraints as above, taking care to replace the direct demand function (9) with the inverse demand function given by equation (11). Most of the considerations drawn in the Bertrand competition case extend to Cournot competition. Proposition 2 fully characterizes the equilibrium under Cournot competition.

Proposition 2 (Cournot Equilibrium) *Points i),iii) and iii) of Proposition 1 extend to the Cournot case. The symmetric Cournot equilibrium generates the individual output*

$$y_t = \frac{\varepsilon - 1}{\varepsilon} \frac{N_t - 1}{N_t^2} \frac{EXP_t}{MC_t} \tag{16}$$

where MC_t is the nominal marginal cost, the associated equilibrium mark up is:

$$\mu^Q(\varepsilon, N_t) = \frac{\varepsilon N_t}{(\varepsilon - 1)(N_t - 1)} \tag{17}$$

Proof. See Appendix B2. ■

For a given number of firms, the mark up under competition in quantities is always larger than the one obtained under competition in prices, as well known for models of product differentiation (see for instance Vives, 1999). Notice that the mark up is decreasing in the degree of substitutability between products ε, with an elasticity $\epsilon_\varepsilon^Q = 1/(\varepsilon - 1)$, which is always smaller than ϵ_ε^P: higher substitutability reduces mark ups faster under competition in prices. In the Cournot equilibrium, the mark up remains positive for any degree of substitutability, since even in the case of homogenous goods, we have $\lim_{\varepsilon\to\infty} \mu^Q(\varepsilon, N_t) = N_t/(N_t - 1)$. This allow us to consider the effect of strategic interactions in an otherwise standard setup with perfect substitute goods within sectors (as in the standard RBC setting with search and matching frictions of Andolfatto (1996) and Mertz (1995)).

In the general formulation the mark up is decreasing and convex in the number of firms with elasticity $\epsilon_N^Q = 1/(N-1)$, which is decreasing in the number of firms (the mark up decreases with entry at an increasing rate) and independent from the degree of substitutability between goods. Since $\epsilon_N^Q > \epsilon_N^P$ for any number of firms or degree of substitutability, we can conclude that entry decreases mark ups faster under competition in quantities compared to competition in prices, a result that will have an impact on the relative behavior of the economy under the two forms of competition. Only when $N_t \to \infty$ the mark up tends to $\varepsilon/(\varepsilon-1)$, which is the traditional mark up under monopolistic competition.[12]

2.3.3 ***Job Creation and Vacancy Posting***

Combining the first conditions for profits maximization (see Appendix B) we get the Job Creation Condition (JCC), which, under both forms of competition, reads as

$$\frac{\kappa}{q_t} = \left(\frac{\rho_t}{\mu_t}A_t - w_t\right)h_t + \varrho E_t \Lambda_{t,t+1}\frac{\kappa}{q_{t+1}} \tag{18}$$

The JCC equates the real marginal cost of hiring a worker, the left hand side, with the marginal benefit, the right hand side. Note that we assumed that firms take individual wages as given when choosing employment.[13] Importantly, the marginal benefit depends positively on the ratio $\frac{\rho_t}{\mu_t}$, which is a positive function of the number of firms in the market, N_t.[14] As the number of market competitors increases, the relative price of existing varieties rises. At the same time, stronger competition leads to a lower mark up and thus to a higher demand by consumers. Both effects provide firms with an incentive to create vacancies which is absent in a model with exogenous market structures. Moreover, the incentive changes with the extent of competition and, since net entry is procyclical as we show below, it is procyclical.

Let $\pi_t^{FP}$and v_t^{FP} be, respectively, the real profits and the number of vacancies posted by a first period incumbent. Symmetrically, π_t and v_t define, respectively, the individual profits and vacancies posted by mature incumbent firms.

[12]In what follows, to lighten the notation, we suppress the dependance of ρ_t and μ_t from ε and N_t.

[13]A similar assumption can be found, *inter alia*, in chapter 3 of Pissarides (2000) and Krause and Lubik (2007). This assumption rules out the the hiring externality emphasized by Ebell and Haefke (2009) . However, the same authors show that the over-hiring effect on unemployment and wages is quantitatively very small.

[14]Of course, μ_t differs according to the mode of competition.

Proposition 3 (Profits and hiring policy) *Under both Bertrand and Cournot competition it follows that: i)* $v_t^{FP} = \frac{n_t}{q_t} = v_t - \varrho \frac{n_{t-1}}{q_t}$ *and ii)* $\pi_t^{FP} = \pi_t - \kappa \varrho \frac{n_{t-1}}{q_t}$.

Proof. See Appendix B3. ■

Since all incumbent firms are characterized by the same size, the optimal hiring policy of first period incumbent firms, which have no initial workforce, consists in posting at time t as many vacancies as required to reach the size of a mature incumbent producer. Given vacancy posting is costly, they will suffer lower profits.

As a consequence of their hiring policy, first period incumbent producers will grow faster than mature incumbent producers. This is consistent with the U.S. empirical evidence in Haltiwanger et al. (2009), which suggests that a start-up creates on average more new jobs than an incumbent firm.

2.3.4 *Endogenous Entry*

In each period the level of entry is determined endogenously to equate the value of a new entrant, V_t^e, to the entry cost

$$V_t^e = \psi \tag{19}$$

The next Proposition provides a useful relationship between the value of a new entrant and the value of an incumbent firm, denoted by V_t.

Proposition 4 (Value of an Incumbent Firm) *The value of an incumbent firm is larger than that of a new entrant*

$$V_t = V_t^e + \kappa \varrho E_t \Lambda_{t,t+1} \frac{n_t}{q_{t+1}} \tag{20}$$

Proof. See Appendix B4. ■

Perspective new entrants have lower value than incumbent firms because they will have, in case they do not exit from the market before starting production, to set up a workforce in their first period of activity. The difference in the value between an incumbent producer and a new entrant is, in fact, the discounted value of the higher vacancy posting cost that the latter will suffer, with respect to the former, in the first period of activity.

2.4 *Bargaining over Wages and Hours*

We assume Nash wage bargaining, so that the firm and each worker split the joint surplus of their employment relationship. Thus, the real wage is set to maximize the product

$$(\phi_t)^{1-\eta} (\Gamma_t C_t)^{\eta} \tag{21}$$

Recall that the term in the first bracket is the value to the firm of having an additional worker, the second term is the household's surplus expressed in units of consumption. The parameter η reflects the parties' relative bargaining power. The FOC for Nash bargaining is

$$\eta\phi_t = (1-\eta)\,\Gamma_t C_t \tag{22}$$

Using the definitions of ϕ_t and Γ_t gives, after some manipulations, the wage equation

$$w_t = (1-\eta)\,b + \eta mc_t A_t + (1-\eta)\,\chi C_t \frac{h_t^{1/\varphi}}{1+1/\varphi} + \frac{\eta\beta\kappa}{h_t} E_t \frac{z_{t+1}}{q_{t+1}} \frac{C_t}{C_{t+1}} \tag{23}$$

Since $\frac{z_t}{q_t} = \theta_t$, $\Lambda_{t,t+1} = (1-\delta)\,\beta\left(\frac{C_{t+1}}{C_t}\right)^{-1}$ and, importantly, $mc_t = \frac{\rho_t}{\mu_t}$ we obtain

$$w_t = (1-\eta)\,b + \eta\frac{\rho_t}{\mu_t} A_t + (1-\eta)\,\chi C_t \frac{h_t^{1/\varphi}}{1+1/\varphi} + \frac{\eta\kappa}{(1-\delta)} \frac{1}{h_t} E_t \Lambda_{t,t+1}\theta_{t+1} \tag{24}$$

Clearly the mark up function, μ_t, differs according to the form of competition, whether Bertrand or Cournot. In both cases, however, the direct effect of entry on the real wage is captured through the term $\eta\frac{\rho_t}{\mu_t}A_t$. Notice that $\frac{\rho_t}{\mu_t}A_t$ represents the marginal revenue product (MRP) of labor, while η represents the share of the MRP which goes to workers. As described above, entry leads to an increase in the MRP of labor. Thus, ceteris paribus, stronger competition shifts the wage curve up. This result is similar to that in Blanchard and Giavazzi (2003), who find a positive effect of competition on the real wage.

Hours are set to maximize the joint surplus of the match, given by $S_t = \phi_t + \Gamma_t C_t$. The FOC with respect of h_t is

$$\chi C_t h_t^{1/\varphi} = \frac{\rho_t}{\mu_t} A_t \tag{25}$$

where, as above, μ_t depends on the form of competition. Hours worked are such that the the marginal rate of substitution between hours and consumption equals the MRP of labor. Stronger competition leads to an increase in hours bargained between the workers and firms for the same reasons for which competition positively affects the wage schedule.

2.5 *Aggregation and Market Clearing*

Considering that the individual workforce, n_t, is identical across producers leads to

$$L_t = n_t N_t \tag{26}$$

To obtain aggregate output notice that $P_tY_t = \sum_{i=1}^{N_t} p_t y_t = N_t p_t y_t$, further given $\rho_t = \frac{p_t}{P_t}$ and the individual production function it follows that

$$Y_t = \rho_t N_t y_t = \rho_t A_t L_t h_t \tag{27}$$

Aggregating the budget constraints of households we obtain the aggregate resource constraint of the economy

$$C_t + \psi N_t^e = W_t h_t L_t + \Pi_t \tag{28}$$

which states that the sum of consumption and investment in new entrants must equal the sum between labor income and aggregate profits, Π_t, distributed to households at time t. Aggregate profits are defined as

$$\Pi_t = (1-\delta) N_{t-1}\pi_t + [N_t - (1-\delta) N_{t-1}] \pi_t^{NP} \tag{29}$$

where $(1-\delta) N_{t-1}$ is the number of mature incumbent producers, and $N_t - (1-\delta) N_{t-1}^e$ is the number of time-t first period incumbent firms. Goods' market clearing requires

$$Y_t = C_t + N_t^E \psi + \kappa v_t^{tot} \tag{30}$$

Finally, the dynamics of aggregate employment reads as

$$L_t = (1-\delta) \varrho L_{t-1} + q_t v_t^{tot} \tag{31}$$

which shows that workers employed to a firm which exits the market join the mass of unemployed. Appendix C lists the full set of equilibrium conditions for the economy.

2.6 *Steady State and Calibration*

In order to obtain values for the steady state levels of variables and for the deep structural parameters, we need to impose 14 restrictions. Calibration is conducted on a quarterly basis. The discount factor, β, is set to the standard value of 0.99 for quarterly data, while the rate of business destruction, δ, equals 0.025 to match the U.S. empirical level of 10 percent business destruction a year reported by BGM (2007). With no loss of generality, the value of χ is such that steady state labor supply equals one. The Frish elasticity of labor supply reduces to φ, to which we assign a value of one as in Monacelli et al. (2010). We choose $\varepsilon = 6$ as the baseline value for inter-sectoral elasticity of substitution between goods. Notice, however, that our model allows for a large variety of combinations of substitutability between goods (ε) and mark up (μ), which in turn depends on the mode of competition. Steady state TFP equals $A = 1$. The baseline value for the entry cost is $\psi = 1$. Alternative combinations of A and ψ affect the endogenous level of market power because a low entry cost compared to the size of the market leads to a larger number of competitors and thus to lower mark ups, and *viceversa*. However, the impulse response functions below are not qualitatively affected by values of ψ within a reasonable range. The baseline parameterization leads to a price mark up of 23 percent under Bertrand Competition and of 33 percent under Cournot competition, which are broadly in line with the evidence in Scarpetta et al. (1996).[15]

Next we turn to parameters that are specific to the search and matching framework. We adopt a conventional parameterization. The aggregate separation rate is $1 - (1 - \delta)\varrho$. We set ϱ such that the the latter equals 0.1, as suggested by estimates provided by Hall (1995) and Davis (1996). The elasticity of matches to unemployment is $\gamma = \frac{1}{2}$, which is within the range of the plausible values of 0.5 to 0.7 reported by Petrongolo and Pissarides (2001) in their survey of the literature on the estimation of the matching function. We impose symmetry in bargaining and set $\eta = \frac{1}{2}$, as in the bulk of the literature. Importantly, as we show below, the equality between the elasticity of matches to unemployment and the workers' power in wage setting does not guarantee efficiency. We normalize the value of $\theta = 1$.[16] Following Dee Haan et al. (2000) and Shimer (2005) we fix the probability that a vacancy is filled to $q = 0.7$. Finally, we set the unemployment benefit b such that the replacement ratio $\frac{b}{w} = 0.42$, as in Shimer (2005) and Gertler and Trigari (2009). Given these parameters we can recover the cost of posting a vacancy κ by equating the steady state version of the JCC and the steady state wage setting equation.

The steady state rate of unemployment is equal to

$$u = \frac{1 - (1 - \delta)\varrho}{q\theta + (1 - (1 - \delta)\varrho)} = 0.125$$

which is increasing in the rate, δ, of business destruction and in the exogenous, firm-level job separation rate, ϱ. As expected the unemployment rate is decreasing in the job filling probability q. While the

[15] Pissarides (2003) provides an index for entry delay as the average number of business days necessary to set up a new firm. Ebell and Haefke (2009) convert this index in months of lost output to get a value of the entry cost ψ. They find that entry costs amount to 15 percent of quarterly output in the U.S. Under the baseline parameterization, steady state aggregate entry costs ($\frac{N^e\psi}{Y}$) amount to 14 percent of output under Bertrand competition and to 18 percent under Cournot competition.

[16] The value of θ does not affects the dynamic of the model. See the discussion in Shimer (2005) and more recently in Monacelli et al. (2010).

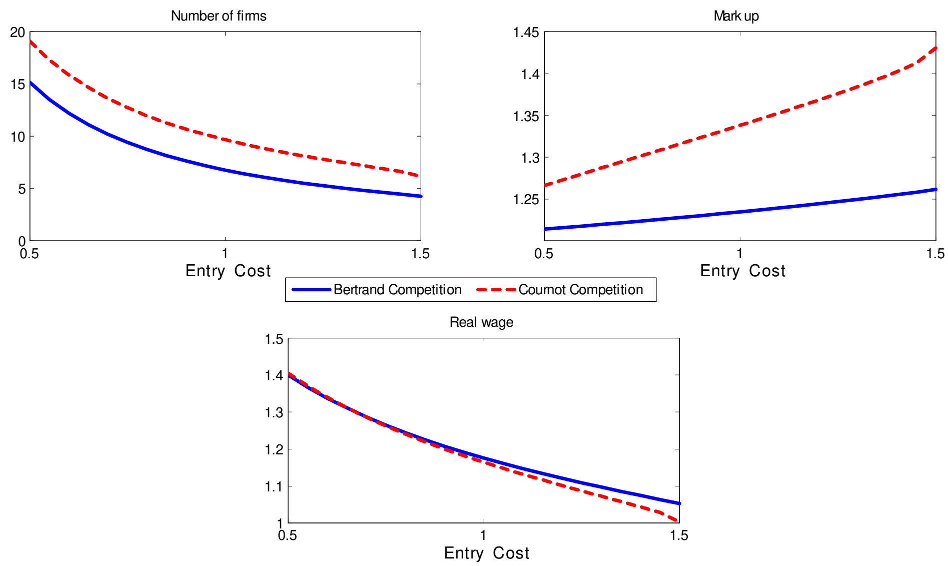

Figure 1: Steady state value of some selected variables as a function of the entry cost ψ.

endogenous steady state rate of unemployment is larger that the average quarterly rate for the U.S., it is in line with the value used by Krause and Lubik (2007) and much lower that those in Andolfatto (1996) and Trigari (2009).[17]

Figure 1 shows the steady state values of some key variables as a function of the entry cost under both Cournot and Bertrand Competition. As well known, when firms compete in prices the equilibrium mark ups are lower, which in turn allows for a lower number of firms to be active in the market. As a consequence, given the entry costs, the Bertrand equilibrium is characterized by a lower number of goods compared to the Cournot equilibrium. Not surprisingly a lower entry cost is, in both frameworks, associated to a higher number of producers and thus to stronger competition and to a lower mark up. As mentioned earlier, the real wage is higher in more competitive environments.

Notice that the steady state ratio between jobs created by first period incumbent firms (JC^{FP}) and total job creation (JC) is given by

$$\frac{JC^{FP}}{JC} = \frac{(1-\delta)\, N^e v^{NP} q}{v^{tot} q} = \frac{\delta}{\theta q}\frac{(1-u)}{u} = 0.25$$

which implies that job creation by new producers account for about 25 per cent of total (gross) job creation, close to the quarterly U.S. average

[17]The computation of the steady state is in Appendix D.

of 20 per cent reported by Jaimovich and Floetotto (2008). Also notice that the ratio between workers employed by first period incumbent firms (L^{FP}) and total employment (L) is

$$\frac{L^{FP}}{L} = \frac{(1-\delta)\, N^e \frac{L}{N}}{L} = \delta = 0.025$$

New producers account for about 2.5 percent of total employment, slightly lower than the 3 percent reported by Haltiwanger et al. (2009) as the average value for the U.S. between 1976 and 2005. Notice that the shares considered are independent of both the entry cost and the competitive framework.

3 *Business Cycle Analysis*

In what follows we will first study the impulse response functions to a technology shock, and finally we will evaluate the second order moments. To assess the role of endogenous market structures, we compare the performance of the Bertrand and Cournot model to that of a standard search model, characterized by monopolistic competition in the goods market and capital accumulation. Monopolistic competition implies that the market structure, i.e. number of producers and mark ups, is exogenous. Specifically, firms do not interact strategically and set a constant mark up $\mu = \frac{\varepsilon}{\varepsilon - 1}$ over marginal costs.[18]

The benchmark search model features a Cobb-Douglas production function of the form $y_t = A_t k_{t-1}^{a} (L_t h_t)^{1-\alpha}$ and a dynamics of physical capital given by $k_t = (1-\delta)\, k_{t-1} + I_t^k$, where I_t^k represents investment. As in the bulk of the literature we set $\delta = 0.025$ and $\alpha = \frac{1}{3}$.[19] The calibration strategy of remaining parameters is identical across the models.

3.1 *IRFs to a technology Shock*

In this section we show the qualitative reactions of the economy to a persistent technology shock. Technology is assumed to follow a first order autoregressive process given by $\hat{A}_t = \rho_A \hat{A}_{t-1} + \varepsilon_{At}$, where $\hat{A}_t = \ln(A_t/A)$ and $\rho_A \in (0,1)$ and ε_{At} is a white noise disturbance, with zero expected value and standard deviation σ_A.

Figure 2 and Figure 3 depict percentage deviations from the steady state of key variables in response to a one percent technology shock with persistency $\rho_A = 0.9$; time on the horizontal axis is in quarters. We

[18] We say that the markup is exogenous in the case of monopolistic competition because its value is fully determined once the elasitcity of substitution between goods is fixed. In other words the two magnitudes cannot be set independently.

[19] We compare our baseline model to the search model with capital so that both are characterized by endogenous investment.

consider alternative market structures. Solid lines refer to the case with competition in prices, dashed line to that with competition in quantities and, finally, dotted line refer to the benchmark search model.

Under Bertrand and Cournot competition the market structure is generated endogenously and the steady state mark ups are respectively 23 per cent and 33 per cent. On the contrary in the case of monopolistic competition the mark up is exogenous and equals 20 per cent. Figures 2 and 3 show that the quantitative reactions of the main aggregate variables to the shock are very similar in both the Bertrand and the Cournot frameworks.

The temporary shock increases output and creates large profit opportunities. This, in turn, leads to entry of new firms and lowers the mark up. Recall that entry is subject to a one period time-to-build lag, which implies that the number of producing firms, N_t, does not change on impact. Given that both the love for variety and the mark up are just functions of N_t they are also initially muted. Output, instead, has its peak in the initial period due to the strong response of investment (in new firms) and consumption. To satisfy the higher demand, existing firms use both the intensive margin and the extensive margin of labor. The need to hire workers boosts vacancy posting on impact. In the periods after the shock vacancy creation is more sustained, with respect to the benchmark model, for two reasons. The first one in due to vacancy creation by new entrants which need to build up their workforce. The second one is due to the rise in the number of producers that leads to an increase in the ratio $\frac{\rho_t}{\mu_t}$, which provides an incentive for vacancy posting to both existing firms and to new entrants. Figure 3 shows that under endogenous market structures the response of the job finding rate and the vacancy filling rate are strongly amplified with respect to those obtained in the benchmark search model. The large number of vacancies posted on impact by incumbent firms, and vacancy posting by entrants in the periods after the shocks, makes it harder to fill a vacancy for a producers. The counterpart of a markedly countercyclical vacancy filling rate is a strongly procyclical job finding rate. Hall (2005) argues that the job finding rate is the key variable in understanding the large fluctuation in unemployment over the past 50 years. The strongly procyclical response of the job finding rate delivered by the Bertrand and Cournot models is at the basis of the large swing in unemployment. The endogeneity of the market structures implies a response of the unemployment rate in the period after the shock which is almost three times larger, and more persistent, than that observed in the benchmark search model.

While the shock vanishes and entry strengthens competition, output and profits of the firms drop and the incentives to enter disappear. At

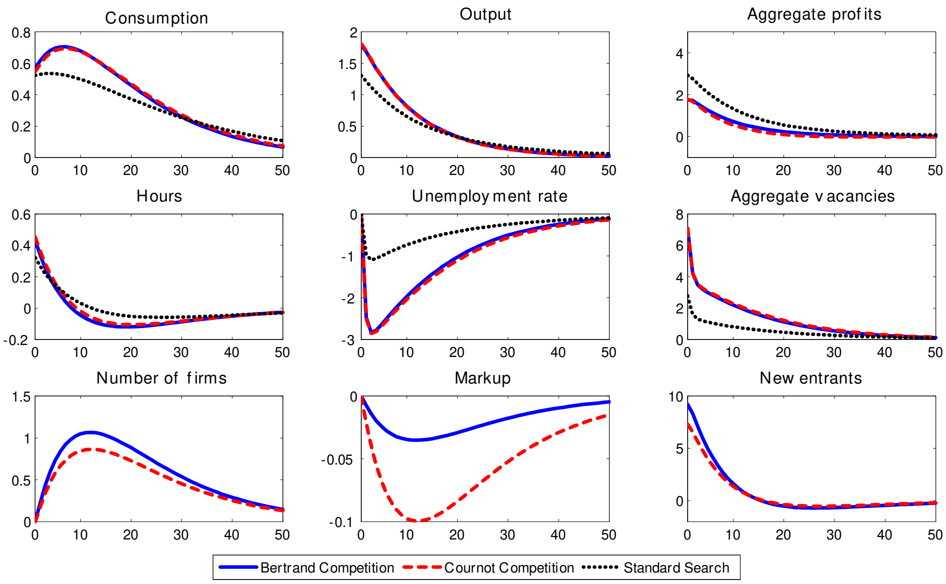

Figure 2: Impulse response function to a temporary technology shock.

some point net exit from the market occurs and the rate of unemployment, the mark up and thus the incentive to create vacancy gradually return to the steady state.

Importantly, notice that our framework delivers procyclical profits together with countercyclical mark ups. Further notice that output jumps on impact in response to a TFP shock, while the mark up does not change on impact and falls more in future periods. This correlation pattern is consistent with the analysis in Rotemberg and Woodford (1999) and with the VAR evidence for the U.S. in Colciago and Etro (2010 a).

To sum up, the impact of a temporary shock on the main macroeconomic variables is magnified in the presence of endogenous market structures. The model displays a much a stronger, with respect to the case of exogenous market structures, response of unemployment, vacancy creation and job market tightness.

3.2 *Second Moments*

To further assess the implications of endogenous market structures for the business cycle, we compute second moments of the key macroeconomic variables. In this exercise we follow the RBC literature and assume that the only source of random fluctuations are temporary exogenous technology shocks. We calibrate the productivity process as in King and Rebelo (2000), with persistence $\rho_A = 0.979$ and standard deviation $\sigma_A = 0.0072$. We use the same process as in King and Rebelo (2000)

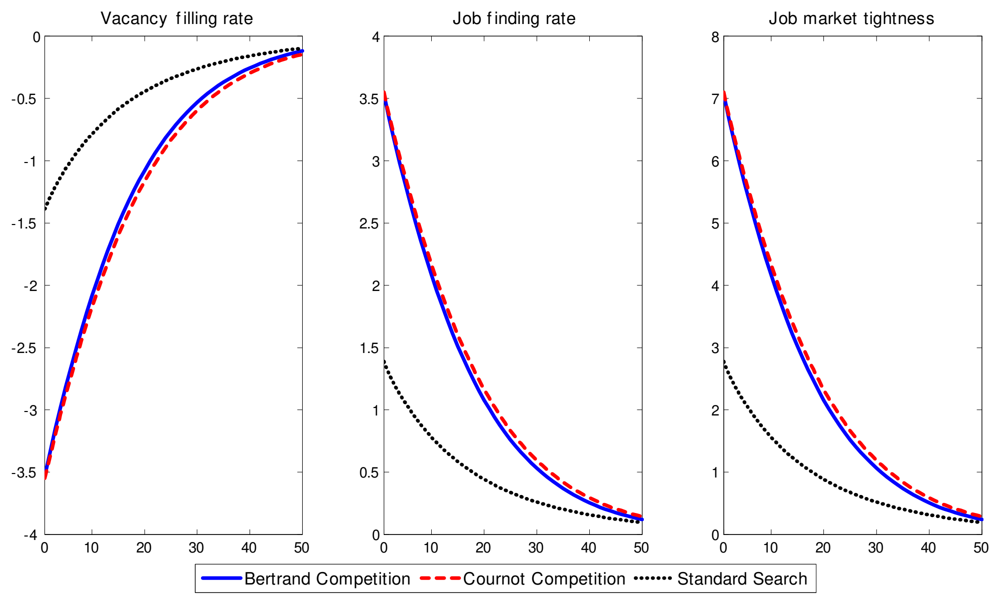

Figure 3: Impulse response function to a temporary technology shock.

for comparison purposes with the bulk of the literature and to verify the additional impact of our propagation channel for a given shock.

We report in Table 1 the statistics on US data (1951:1 / 2009:3) for output Y, consumption C, investment I, aggregate hours, L,, aggregate profits Π, the unemployment rate u, the mark up μ, the job finding rate z and vacancies v.[20] In the same table we report the moments produces by the benchmark search model. As it is well known, the basic search model fails at replicating the high variability of unemployment, vacancies, aggregate hours and the job finding rate. Also, given the monopolistic competitive nature of the market structure, it cannot deliver mark up countercyclicality coupled with profit procyclicality.

Table 2 reports second moments of Y, C, u, $I \equiv N^e V^e$, L, Π, μ, z and v^{tot} for our model with competition in quantities and with competition in prices under the baseline parameterization.

[20] Variables have been logged. We report moments of HP filtered variables with a smoothing parameter equal to 1600. Most of the data derive from FRED, the Federeal Reserve Economic Database of the Federal Reserve Bank of St. Louis. Vacancies are proxied by the Help-Wanted Advertising Index computed by the Conference Board. Monthly job finding probability is constructed in the way suggested by Shimer (2005). For monthly data series, the average of the monthly data in each quarter is used. Profits include both the remuneration of capital and the extra-profits due to market power. The mark-up is computed using a labor-share based measure along the lines suggested by Rotemberg and Woodford (1999) and described in Colciago and Etro (2010 a).

$Variable$	$\sigma(X)$	$\sigma(X)/\sigma(Y)$	$E(X_t, X_{t-1})$	$Corr(X_t, Y_t)$
Y	$1.58, 1.03$	1	$0.84, 0.72$	1
C	$1.22, 0.89$	$0.77, 0.86$	$0.82, 0.73$	$0.86, 0.99$
I	$5.12, 1.54$	$3.24, 1.49$	$0.90, 0.75$	$0.86, 0.99$
u	$13.09, 1.06$	$8.28, 1.03$	$0.88, 0.79$	$-0.65, -0.71$
L	$1.92, 0.15$	$1.21, 0.14$	$0.88, 0.76$	$0.82, 0.99$
z	$8.38, 1.29$	$5.3, 1.25$	$0.80, 0.71$	$0.82, 0.99$
v	$14.10, 1.98$	$8.92, 1.92$	$0.91, 0.43$	$0.75, 0.92$
$\frac{v}{u}$	$26.35, 2.57$	$16.68, 1.53$	$0.90, 0.71$	$0.79, 0.99$
Π	$8.59, 2.56$	$5.43, 2.48$	$0.80, 0.73$	$0.68, 0.99$
μ	$0.96, 0.00$	$0.61, 0.00$	$0.77, 0.00$	$-0.28, 0.00$

Table 1: Second moments. LEFT: U.S. data. RIGHT: benchmark search model

Both models deliver a very similar performance at replicating the U.S. business cycle. The volatility of output under endogenous market structures is almost as large as that of US data. The endogeneity of market structures implies a substantially higher volatility of aggregate hours with respect to the benchmark model. This is due to both a higher volatility of the intensive and extensive margin of labor.[21] Unemployment is far from being as volatile as in the data. However, while it is as volatile as output in the benchmark model, it is twice as volatile as output in our framework. Similar considerations extend to vacancies, which are 3.3 times as volatile as output, the job finding probability, which has a volatility more than double with respect to output. Both frameworks deliver an extremely volatile job market tightness with respect to the basic search model. Similarly, the volatility of investment is about three times higher in our framework, essentially matching that in the data. The volatilities of profits and the mark up are severely underestimated, however the model does a relatively good job at matching the negative contemporaneous correlation between output and the mark up.

Considering that we adopt a very standard and conservative model calibration we see the performance of both the Bertrand and the Cournot frameworks as a success for three main reasons. First, without resorting to nominal rigidities in wages (real or nominal) or prices, it substantially outperforms a standard model of search in the labor market in terms of variability of labor market variables. Second, the model can reproduce the procyclicality of entry and the countercyclicality of the mark up observed in the data. Third, it matches the nonlinear time profile of the

[21] The standard deviation of individual hours is 0.17 under both Bertrand and Cournot competition, while it is 0.02 under the standard search model.

correlation between the mark up and the cycle documented by Rotemberg and Woodford (1999) and emphasized by BGM (2007). For these reasons we claim that endogenous market structures are a relevant amplification channel of technology shocks in an otherwise standard model of search in the labor market.

$Variable$	$\sigma(X)$	$\sigma(X)/\sigma(Y)$	$E(X_t, X_{t-1})$	$Corr(X_t, Y_t)$
Y	$1.46, 1.47$	1	$0.74, 0.74$	1
C	$0.83, 0.83$	$0.56, 0.56$	$0.77, 0.77$	$0.98, 0.97$
I	$5.48, 4.41$	$3.75, 3.0$	$0.74, 0.74$	$0.98, 0.98$
u	$2.71, 2.72$	$1.85, 1.85$	$0.80, 0.80$	$-0.71, -0.70$
L	$0.51, 0.52$	$0.35, 0.35$	$0.76, 0.76$	$0.99, 0.99$
z	$3.26, 3.27$	$2.23, 2.22$	$0.73, 0.73$	$0.99, 0.98$
v	$4.94, 4.95$	$3.38, 3.37$	$0.45, 0.45$	$0.93, 0.93$
$\frac{v}{u}$	$6.52, 6.56$	$4.46, 4.46$	$0.73, 0.74$	$0.99, 0.98$
Π	$1.43, 1.44$	$0.98, 0.98$	$0.76, 0.75$	$0.99, 0.99$
μ	$0.02, 0.04$	$0.01, 0.027$	$0.96, 0.96$	$-0.14, -0.13$

Table 2: Second moments. Left: Bertrand Competition. Right : Cournot Competition

4 Social Efficiency

Before concluding, we study a scenario where a benevolent social planner (SP) maximizes households' lifetime utility by choosing quantity directly. In doing this, the SP is subject to the same technological constraints and the matching frictions described in the previous sections. Since the economy resource constraint implies $C_t = \rho_t(N_t) A_t L_t h_t - N_t^E \psi - k v_t^{tot}$, the SP problem can be written as:

$$\max_{\left\{C_t, N_{t+1}, v_t^{tot}, L_t, h_t, N_t^e\right\}_{t=0}^{\infty}} E_t \begin{pmatrix} \ln\left(\rho_t A_t L_t h_t - N_t^E \psi - k v_t^{tot}\right) + \\ -\chi L_t \frac{h_t^{1+1/\varphi}}{1+1/\varphi} \end{pmatrix}$$

subject to (8) and (31) or, substituting the second constraint into the objective function and considering that $q_t = \frac{m_t}{v_t^{tot}} = \frac{\gamma_m u_t^\gamma \left(v_t^{tot}\right)^{1-\gamma}}{v_t^{tot}}$

$$\max_{\left\{N_{t+1}, L_t, v_t^{tot}, h_t\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \begin{bmatrix} \ln\left(\rho_t A_t L_t h_t - \left(\frac{1}{1-\delta} N_{t+1} - N_t\right)\psi - k v_t^{tot}\right) + \\ -\chi L_t \frac{h_t^{1+1/\varphi}}{1+1/\varphi} \end{bmatrix}$$

such that $L_t = (1-\delta)\varrho L_{t-1} + \gamma_m (1 - L_{t-1})^\gamma \left(v_t^{tot}\right)^{1-\gamma}$. The social planner takes into account the effect of the number of varieties, N_t, on the relative price ρ_t and the effect of vacancy posting on the probability of filling a vacancy q_t. Let Υ_t be the Lagrange multiplier of the unique constraint. FOCs with respect to $N_{t+1}, L_t, v_t^{tot}, h_t$ are, respectively:[22]

$$\psi = (1-\delta)\,\beta E_t \frac{C_t}{C_{t+1}}\left[\rho_{N,t+1} A_{t+1} L_{t+1} h_{t+1} + \psi\right] \tag{32}$$

$$\frac{\rho\,(N_t)\,A_t h_t}{C_t} - \frac{\chi h_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} = \Upsilon_t - \beta E_t \Upsilon_{t+1}\left[(1-\delta)\,\varrho - \gamma_m \gamma \left(\frac{v_{t+1}^{tot}}{u_{t+1}}\right)^{1-\gamma}\right] \tag{33}$$

$$\frac{\kappa}{C_t} = \Upsilon_t\,(1-\gamma)\,\gamma_m \left(\frac{v_t^{tot}}{u_t}\right)^{-\gamma} \tag{34}$$

$$\chi C_t h_t^{\frac{1}{\varphi}} = \rho_t A_t \tag{35}$$

Substituting condition (34) into equation (33) yields

$$\frac{\kappa}{q_t} = (1-\gamma)\left[\rho_t A_t - \chi C_t \frac{h_t^{\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right] h_t - \frac{\gamma\kappa}{(1-\delta)} E_t \Lambda_{t,t+1}\theta_{t+1} + \kappa\varrho E_t \frac{\Lambda_{t,t+1}}{q_{t+1}} \tag{36}$$

Definition (Planning Equilibrium) A planning equilibrium consists of an allocation $\{L_t,\ h_t,\ N_{t+1}, v_t^{tot}\}_{t=0}^{\infty}$ satisfying equations (32), (33), (36) and (35) for given N_0, L_0 and $\{A_t\}_{t=0}^{\infty}$.

Next we evaluate the efficiency of the competitive equilibrium (CE). In particular we are interested to assess whether the Hosios (1990) condition, that is $\gamma = \eta$, is sufficient to achieve social efficiency. For convenience, in this section we assume that $b = 0$. We first reduce the Euler equations for the assets of new entrants and incumbent firms to a single equation. The latter reads as

$$S_t = (1-\delta)\,\beta E_t \frac{C_t}{C_{t+1}}\left[\left(1 - \frac{w_{t+1}}{A_{t+1}}\right) Y_{t+1} - k v_{t+1}^{tot} + S_{t+1}\right] \tag{37}$$

where $S_t = (1-\delta)\,(N_t V_t + N_t^e \psi)$ is the aggregate market value of firms in the economy. Also, we combine the JCC and the wage schedule to obtain the equivalent of equation (36) in decentralized economy, which is

$$\frac{\kappa}{q_t} = (1-\eta)\left[\frac{\rho_t}{\mu_t} A_t - \chi C_t \frac{h_t^{1/\varphi}}{1+1/\varphi}\right] h_t - \frac{\eta\kappa}{(1-\delta)} E_t \Lambda_{t,t+1}\theta_{t+1} + \kappa\varrho E_t \frac{\Lambda_{t,t+1}}{q_{t+1}} \tag{38}$$

[22] We denote with $\rho_{N,t}$ the derivative of the relative price ρ_t with respect to the number of varieties, N_t.

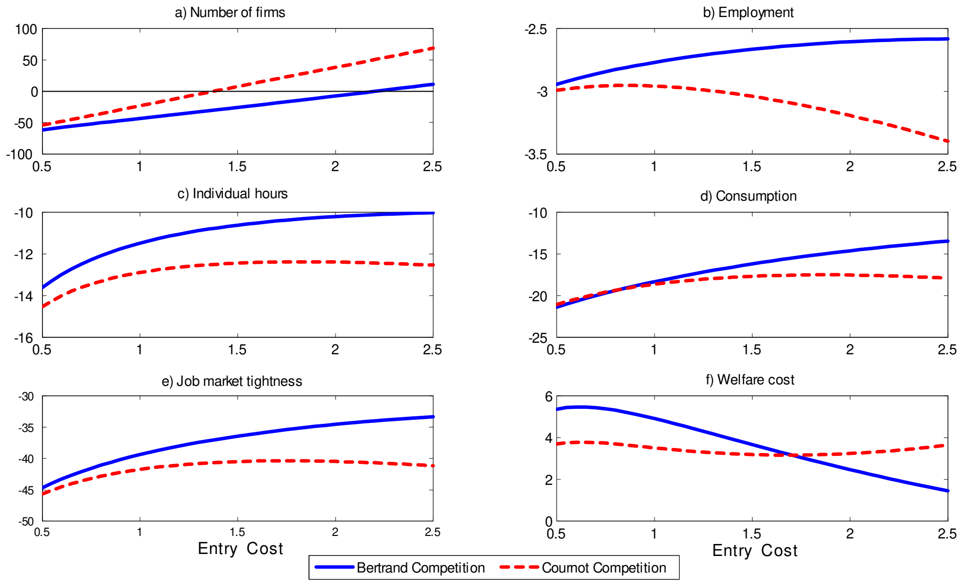

Figure 4: Panels *a)-e):* gap between the CE allocation and the SP allocation as a function of the entry cost. Panel *f):* welfare cost due to an inefficient allocation.

Finally recall that the condition for optimal labor supply in the CE reads as

$$\chi C_t h_t^{1/\varphi} = \frac{\rho_t}{\mu_t} A_t \tag{39}$$

In the remainder we compare the conditions for social efficiency with their counterparts in the CE and highlight a number of distortions.

Equation (38) reduces to its SP counterpart, that is equation (36), under two alternative scenarios. In the first one, $\gamma = \eta = 1$, the Hosios condition holds, workers have all the bargaining power but, as pointed out by Ebell and Haefke (2009), the matching function is degenerate. In the second one $\gamma = \eta < 1$ and $\mu_t = \mu = 1$. Oligopolistic competition affects the value of the job market tightness even if the usual Hosios condition is satisfied. Ruling out the distortion on the extensive margin of labor due to imperfect competition in the goods market requires setting $\mu_t = \mu = 1$. However, this would lead to no entry since, as pointed out by BGM (2006), in the absence of a positive net mark up firms could not recover the entry cost.

The positive mark up also implies a time-varying wedge between the marginal rate of substitution between consumption and the marginal rate of transformation, which leads to an inefficiently low supply of individual hours. Thus, the price mark up creates an intratemporal distortion in both the extensive and the intensive margin of labor. Next, consider equation (37) and compare it the SP correspondent condition (32). In the CE new entrants and incumbent firms are characterized by different values due to the different initial hiring policy. This heterogeneity creates an intertemporal wedge between the CE and the SP equilibrium which distorts the allocation of consumption across different dates.

As a consequence, both fluctuations and the stationary equilibrium are inefficient in the CE. In particular the extensive margin of labor is distorted even if the Hosios condition is satisfied. Inducing efficiency in the decentralized equilibrium would require a combination of fiscal instruments aimed at undoing the intratemporal and the intertemporal wedges described above.[23]

In what follows we compare the steady state allocations of the CE to the SP one. We assume that the Hosios condition holds throughout the analysis. In this case we can evaluate both the magnitude and the sign of the distortion due to oligopolistic competition. Panels *a)-e)* of Figure 4 display the gap between the CE allocation and the SP allocation of some selected variables as a function of the entry cost, under both Bertrand and Cournot competition. The percentage difference is measured in terms of the SP allocation. Panel *f)* displays the welfare cost due to an inefficient allocation as the percentage variation in consumption that a consumer should experience in the CE to be as well off as under the SP equilibrium.[24]

Few remarks are in order. In the case of a relatively high entry cost, the CE could be characterized by an excessive number of varieties. Under Cournot this happens quite quickly since the latter competitive framework is characterized by a higher number of varieties with respect to Bertrand. Recall that the number of varieties in the CE, or equivalently the number of firms, is decreasing in the entry cost. However, the opposite holds for the price-mark up. A high mark up could, in turn, lead to high profits and thus to an excessive number of variety produced. Due to imperfect competition the CE is characterized by under-employment and by an inefficiently low level of hours worked no matter the hosios condition. The same can be said for output. As

[23] To achive first best any fiscal instrument needs to be financed through lump sum taxation.

[24] We report on the vertical axes of panel *a)-e)* the value $\lambda_x = (X^{CE} - X^{SP})/X^{SP} \times 100$. The vertical axe of panel *f)* reports the value $\lambda_c \times 100$, where λ_c is such that

$$\log C^{CE}\left(1+\lambda_c\right) - \chi L^{CE}\frac{\left(h^{CE}\right)^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}} = \log C^{SP} - \chi L^{SP}\frac{\left(h^{SP}\right)^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}}$$

the entry cost increases Bertrand competition implies a lower welfare cost with respect to Cournot competition. However, under our baseline calibration ($\psi = 1$) the Cournot competition provides a better welfare performance. Nevertheless, as we have seen above, the dynamics in response to a technology shock and the second moments are very similar across the models.

5 *Conclusions*

We provided a DSGE model where firms' dynamics and matching frictions in the labor market interact endogenously. We accounted for strategic interactions in both prices and quantities among producers. The interplay between search and matching frictions, endogenous entry and strategic interactions among producers constitutes a strong amplification channel of technology shocks on labor market variables. Without resorting to rigidities in wages (real or nominal) or prices, and under a conservative and standard calibration of parameters, our framework substantially outperforms the standard search and matching model at replicating the observed variability of the unemployment rate, vacancies and the job market tightness. Also, the model explains the procyclicality of profits together with the countercyclicality of price mark ups.

Our analysis could be extended in various dimensions. One aspect we neglect is the asymmetry between market competitors in terms of both size and the probability of exit form the market. Davis et al. (2009) document that the distribution of vacancy creation is strongly biased in favor of small firms; Haltiwanger et al. (2009) show that younger firms are more likely to exit from the market than more mature firms. Another important aspect that we do not discuss is, as documented by Davis et al. (2009), that a large fractions of new hires happens without prior vacancy creation.

In ongoing research we extend our framework to a government sector and analyze the transmission of government spending shocks to the labor market. We believe that the strong propagation embodied in the model with endogenous market structures could help resolving the unemployment fiscal multiplier puzzle emphasized in some recent contributions without departing from a flexible prices approach.

References

Andolfatto, David (1996) "Business Cycles and Labor Market Search", American Economic review 86, pp112-132.

Atkeson, A. and A. Burnstein (2008). Pricing to Market, Trade Costs, and International Relative Prices, The American Economic Review, 98, 5, pp. 1998-2031.

Bergin, P. and G. Corsetti (2008). The Extensive Margin and Monetary Policy, Journal of Monetary Economics, 55, pp. 1222-37.

Bernard, A., S. Redding and P. Schott (2008). Multi-Product Firms and Product Switching, mimeo, Yale University.

Bilbiie, F., F. Ghironi and M. Melitz (2007). Endogenous Entry, Product Variety, and Business Cycles, NBER WP 13646.

Bilbiie, F., F. Ghironi and M. Melitz (2008a). Monetary Policy and Business Cycles with Endogenous Entry and Product Variety, in NBER Macroeconomic Annual 2007, Edited by D. Acemoglu, Kenneth Rogoff and Michael Woodford, pp. 299 - 353.

Bilbiie, F., F. Ghironi and M. Melitz (2008b). Monopoly Power and Endogenous Variety: Distortions and Remedies, NBER WP 14383.

Bils, M., 1987, The Cyclical Behavior of Marginal Cost and Price, The American Economic Review, 77, pp. 838-55.

Blanchard, O. and N. Kiyotaki (1987). Monopolistic Competition and the Effects of Aggregate Demand, The American Economic Review, 77, 4, pp. 647-66.

Blanchard, O. and Giavazzi, F. Macroeconomic Effects of Regulation and Deregulation in Goods and Labor Markets, (2003). The Quarterly Journal of Economics, vol. 118(3), 879-907

Bresnahan, T. and P. Reiss (1987). Do Entry Conditions Vary across Markets?, Brookings Papers on Economic Activity, 3, pp. 833-81.

Broda, C. and D. Weinstein, (2009). Product Creation and Destruction: Evidence and Price Implications, The American Economic Review, in press.

Campbell, J. and H. Hopenhayn (2005). Market Size Matters, Journal of Industrial Economics, 53, 1, pp. 1-25.

Chatterjee, S. and R. Cooper (1993). Entry and Exit, Product Variety and the Business Cycle, NBER WP 4562.

Colciago, A. and F. Etro (2010 a). Real Business Cycles with Cournot Competition and Endogenous Entry, Journal of Macroeconomics, Vol. 32, 1101-1117.

Davis,S.J., Haltiwanger,J.C. and Schuh,S., (1996). Job Creation and Job Destruction. The MIT Press,Cambridge, MA.

Davis, S. J and Haltiwanger, J.C., (1992). Gross Job Creation, Gross Job Destruction, and Employment Reallocation, The Quarterly Journal of Economics, vol. 107(3), 819-63.

Davis, S. J., Faberman, R.J. and Haltiwanger, J.C., (2009). The Establishment-Level Behavior of Vacancies and Hiring, WP 09-14, Federal Reserve Bank of Philadelphia.

Den Haan, W., Ramey, G., Watson, J., (2000). Job destruction and the propagation of shocks. American Economic Review 90, 482–498.

Ebell, M. and Haefke, C. (2009). Product Market Deregulation and the U.S. Employment Miracle, Review of Economic Dynamics, vol. 12(3), 479-504.

Etro, F. and Colciago, A. (2010 b). Endogenous Market structures and the Business Cycle. Economic Journal, Vol. 120, 1201-1233.

Galì, J., M. Gertler and D. López-Salido (2007a). mark ups, Gaps, and the Welfare Costs of Business Fluctuations, Review of Economics and Statistics, 89, 1, pp. 44-59.

Ghironi, F. and M. Melitz (2005). International Trade and Macroeconomic Dynamics with Heterogenous Firms, Quarterly Journal of Economics, pp. 865-915.

Gertler, M. and Trigari, A. (2009). Unemployment Fluctuations with Staggered Nash Wage Bargaining, Journal of Political Economy 117 (1), 38-86.

Hagedorn, M. and Manovski, J. (2008). The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited, American Economic Review, 98(4), pp. 1692-1706

Hall, R.E. , (2005). Job Loss, Job Finding, and Unemployment in the U.S. Economy over the Past Fifty Years, NBER Macroeconomics Annual, 101-137.

Hall,R.E.,(1995). Lost jobs. Brookings Papers on Economic Activity, 1, 221–256.

Haltiwanger,J. C. Jarmin, R. S. and Miranda, J. (2010). Who Creates Jobs? Small vs. Large vs. Young, NBER Working Paper No. 16300.

Hosios, Arthur J., (1990). On the Efficiency of Matching and Related Models of Search and Unemployment, Review of Economic Studies, 57(3), 279-298

Jaimovich, N. and M. Floetotto (2008). Firm Dynamics, mark up Variations, and the Business Cycle, Journal of Monetary Economics, 55, 7, pp. 1238-52.

Kass, L. and P. Kircher (2011). Efficient Firm Dynamics in a Frictional Labor Market. Manuscript available at http://personal.lse.ac.uk/kircher/Papers/Efficient-Firm-Dynamics-in-a-Frictional-Labor-Market.pdf

King, R. and S. Rebelo (2000). Resuscitating Real Business Cycles, Ch. 14 in Handbook of Macroeconomics, J. B. Taylor & M. Woodford Ed., Elsevier, Vol. 1, pp. 927-1007.

Lewis, V. (2009). Business Cycle Evidence on Firm Entry, Macroeconomic Dynamics, in press.

Merz, Monica. (1995). Search in the labor market and the real business cycle, Journal of Monetary Economics 36: 269-300.

Monacelli, T., Trigari A., Perotti, R. (2010). Unemployment Fiscal Multipliers, Journal of Monetary Economics 57 (5), pp. 531-553

Mortensen, D. and C. Pissarides (1999). New Developments in Models of Search in the Labor Market, in: Orley Ashenfelter and David Card (eds.):Handbook of Labor Economics.

Oliveira Martins, J. and S. Scarpetta (1999). The level and Cyclical Behavior of Mark-ups Across Countries and Market Structures. OECD Working Paper No. 213, OECD Publishing.

Petrongolo, B. and C. Pissarides (2001). Looking into the Black Box: A Survey of the Matching Function. Journal of Economic Literature 39(2): 390-431.

Pissarides, Christopher A. (2000). Equilibrium Unemployment Theory, 2nd edition, Cambridge, MIT Press.

Rotemberg, J. and M. Woodford (1992). Oligopolistic Pricing and the Effects of Aggregate Demand on Economic Activity, Journal of Political Economy, 100, 6, pp. 1153-207.

Rotemberg, J. and M. Woodford (2000). The Cyclical Behavior of Prices and Costs, Ch. 16 in Handbook of Macroeconomics, J. B. Taylor & M. Woodford Ed., Elsevier, Vol. 1, pp. 1051-135.

Shao, E. and Silos P. (2008). Firm Entry and Labor Market Dynamics, Atlanta Fed Working Papers

Shimer, R. (2005). The Cyclical Behavior of Equilibrium Unemployment and Vacancies, American Economic Review, 95(1): 25-49.

Shimer, R. (2009). Convergence in Macroeconomics: The Labor Wedge, American Economic Journal: Macroeconomics, 1(1): 280–297.

Trigari, A. (2009). Equilibrium Unemployment, Job flows and Inflation Dynamics, Journal of Money, Credit and Banking 41 (1), pp. 1-33.

Appendix

A. Analytical Details

The representative agent maximizes intertemporal utility (2) choosing how much to invest in bonds and risky stocks out of labor and capital income. We assume that household invest in both incumbent firms and new entrants. Bonds and stocks are denominated in terms of an aggregate price index P_t^A.

The budget constraint expressed in nominal terms is

$$\begin{aligned}
&P_t^A B_{t+1} + \int_0^1 P_{kt} C_{kt} dk + \\
&+ P_t^A \int_0^1 V_{kt} N_{kt} s_{kt+1} dk + P_t^A \int_0^1 V_{kt}^e N_{kt}^e s_{kt+1}^e dk \\
= &W_t L_t h_t + (1 - L_t) P_t^A b + (1 + r_t) P_t^A B_t + \\
&+ (1 - \delta) P_t^A \int_0^1 [\pi_{kt}(\varepsilon, N_{kt}) + V_{kt}] N_{kt-1} s_{kt} dk + \\
&+ (1 - \delta) P_t^A \int_0^1 \left[\pi_{kt}^{FP}(\varepsilon, N_{kt}) + V_{kt}\right] N_{kt-1}^e s_{kt}^e dk - P_t^A T_t
\end{aligned} \tag{40}$$

where B_t is net bond holdings with interest rate r_t, V_{kt} is the value of an incumbent firm in sector k and V_{kt}^e is the value of a new entrant in sector k. The variables N_{kt} and N_{kt}^e represent the number of active firms in sector k and the new firms in this sector at the end of the period. The variable s_{kt} is the share of the stock market value of the incumbent firms of sector k that are owned by the agent while s_{kt}^e is the share of portfolio of new entrants held by the household. The term $(1 - \delta) P_t^A \int_0^1 [\pi_{kt}(\varepsilon, N_{kt}) + V_{kt}] N_{kt-1} s_{kt}$ represents the sum between the value of the portfolio of mature incumbent firms held by the household and the profits distributed by these firms. Notice that in period t there are $(1 - \delta) N_{kt-1}$ mature incumbent firms in each sector. The term $(1 - \delta) P_t^A \int_0^1 \left[\pi_{kt}^{FP}(\varepsilon, N_{kt}) + V_{kt}\right] N_{kt-1}^e s_{kt}^e$ denotes the sum between the value of the portfolio of first period incumbent firms held by the household and the profits distributed by these firms, where $(1 - \delta) N_{kt-1}^e$ is the number of first period producers at time t. Recall that the superscript FP indicates variables relative to first period incumbent firms. In the budget constraint we have imposed the condition that $V_{kt}^{FP} = V_{kt}$ i.e. symmetry between incumbents. Finally $P_t^A T_t$ represent nominal lump sum taxes imposed to finance unemployment benefits. Equations **??** and 4 represents the constraint to the utility maximization problem. We denote with ξ_t the Lagrangian multiplier of the first constrain, while Γ_t is the one of the second constraint.

The intertemporal optimality conditions with respect to s_{kt+1}, s_{kt+1}^e for each sector, and with respect to B_{t+1} are:

$$P_t^A V_{kt} = \beta E_t (1 - \delta) \frac{\xi_{t+1}}{\xi_t} P_{t+1}^A [\pi_{kt+1}(\varepsilon, N_{kt+1}) + V_{kt+1}] \tag{41}$$

$$P_t^A V_{kt}^e = \beta E_t (1 - \delta) \frac{\xi_{t+1}}{\xi_t} P_{t+1}^A \left[\pi_{kt+1}^{FP}(\varepsilon, N_{kt+1}) + V_{kt+1}\right] \tag{42}$$

$$P_t^A \xi_t = \beta E_t (1 + r_{t+1}) P_{t+1}^A \xi_{t+1} \tag{43}$$

The optimal choice of consumption of the bundle of good produced in sector k, C_{kt}, is instead

$$P_{kt}C_{kt} = P_t^A C_t = EXP_t \text{ for any } k \in [0,1] \tag{44}$$

the latter implies that nominal expenditure is identical in each sector and, given sectors are atomistic with aggregate unit mass, that sector nominal expenditure equals aggregate nominal expenditure, defined as EXP_t. Also, it follows that $\xi_t = \frac{1}{P_t^A C_t}$. Notice that Γ_t has the meaning of the marginal value to the household of having a member employed rather than unemployed. The latter affects bargaining over the real wage and individual hours and it is given by

$$\Gamma_t = \frac{1}{C_t}\left(w_t h_t - b\right) - \chi \frac{h_t^{1+1/\varphi}}{1+1/\varphi} + \beta E_t\left[\left(1-\delta\right)\rho - z_{t+1}\right]\Gamma_{t+1} \tag{45}$$

where $w_t = \frac{W_t}{P_t^A}$ is the real wage. Following Ghironi and Melitz (2005), we adopt a probability $\delta \in [0,1]$ with which any firm can exit from the market for exogenous reasons in each period. The dynamic equation determining the number of firms in each sector is then:

$$N_{kt+1} = \left(1-\delta\right)\left(N_{kt} + N_{kt}^e\right) \quad \forall k \tag{46}$$

which provides the dynamic path for the average number of firms:

$$N_{t+1} = \left(1-\delta\right)\int_0^1 \left(N_{kt} + N_{kt}^e\right)dk = \left(1-\delta\right)\left(N_t + N_t^e\right) \tag{47}$$

where, of course, we have $N_t \equiv \int_0^1 N_{kt}dk$ and $N_t^e \equiv \int_0^1 N_{kt}^e dk$.

Market clearing in the asset markets requires $B_t = 0$ for any t in the bond market, and $s_{kt} = s_{kt}^e = 1$ for any sector k in the stock market. In a symmetric equilibrium, the number of firms, the mark up and individual profits are the same in every sector, which leads to the following equilibrium relations:

$$P_{kt} = P_t^A \quad C_{kt} = C_t \ \ \forall k \tag{48}$$

$$V_t = E_t \Lambda_{t,t+1}\left[\pi_{t+1}(\varepsilon, N_{t+1}) + V_{t+1}\right] \tag{49}$$

$$V_t^e = E_t \Lambda_{t,t+1}\left[\pi_{t+1}^{FP}(\varepsilon, N_{t+1}) + V_{t+1}\right] \tag{50}$$

$$C_t^{-1} = \beta(1+r_{t+1})E\left(C_{t+1}^{-1}\right) \tag{51}$$

The variable $\Lambda_{t,t+1} = \beta\left(1-\delta\right)\frac{C_t}{C_{t+1}}$ represents the household's stochastic discount factor, which takes into account that a firm exits from the market with probability δ.

B. Proofs of Propositions

B1. Proposition 1

Proof. Notice that $C_{k,t}(i) = \left(\frac{p_{k,t}(i)}{P_{k,t}}\right)^{-\varepsilon} C_{k,t} = \frac{p_{k,t}^{-\varepsilon}(i)}{\left(P_{k,t}\right)^{1-\varepsilon}} P_{k,t}C_{k,t} = \frac{p_{k,t}^{-\varepsilon}(i)}{\left(P_{k,t}\right)^{1-\varepsilon}} EXP_{k,t} = \frac{p_{k,t}^{-\varepsilon}(i)}{\left(P_{k,t}\right)^{1-\varepsilon}} EXP_t$. Since

$$P_{k,t} = \left[\sum_{j=1}^{N_t} p_{k,t}(j)^{-(\varepsilon-1)}\right]^{\frac{-1}{\varepsilon-1}} \tag{52}$$

we can write the demand faced by firm i as

$$C_{k,t}(i) = \frac{p_{k,t}(i)^{-\varepsilon} EXP_t}{\left[\sum_{j=1}^{N_{k,t}} p_{k,t}(j)^{-(\varepsilon-1)}\right]} \tag{53}$$

Each sector can be similarly described, so we drop the index referring to sectors and consider a representative sector. Substituting the direct demand for the individual good into period t real profits, we obtain

$$\pi_t = \frac{p_t(i)^{1-\varepsilon}}{\left[\sum_{j=1}^{N_t} p_t(j)^{-(\varepsilon-1)}\right]} \frac{EXP_t}{P_t^A} - w_t\left(i\right) n_t\left(i\right) h_t\left(i\right) - kv_t\left(i\right) \tag{54}$$

The profit maximization problem of a mature producer reads as

$$\max_{\{p_t(i), n_t(i), v_t(i)\}_t^\infty} \pi_t + E_t \sum_{s=t+1}^{\infty} \Lambda_{t,s}\pi_s \tag{55}$$

subject to

$$A_t n_t\left(i\right) h_t(i) = \frac{p_t(i)^{-\varepsilon} EXP_t}{\left[\sum_{j=1}^{N_t} p_t(j)^{-(\varepsilon-1)}\right]} \tag{56}$$

$$n_t\left(i\right) = \rho n_{t-1}\left(i\right) + v_t\left(i\right) q_t \tag{57}$$

Lagrangian multipliers on constraints (56), and (57) are respectively $mc_t\left(i\right)$ and $\phi_t\left(i\right)$. Setting up the Lagrangian $\mathbb{L}$, the FOCs for profit maximization are

$$\frac{\partial \mathbb{L}}{\partial n_t\left(i\right)} = 0 : w_t\left(i\right) h_t\left(i\right) + \phi_t\left(i\right) - mc_t\left(i\right) A_t h_t\left(i\right) = \varrho E_t \Lambda_{t,t+1}\phi_{t+1}\left(i\right) \tag{58}$$

$$\frac{\partial \mathbb{L}}{\partial v_t(i)} = 0 : k = \phi_t(i)\, q_t \tag{59}$$

$$\frac{\partial \mathbb{L}}{\partial p_t(i)} = 0 : \frac{(1-\varepsilon)\left[\sum_{j=1}^{N_t} p_t(j)^{-(\varepsilon-1)}\right] - (1-\varepsilon)\, p_t(i)^{1-\varepsilon}}{\left[\sum_{j=1}^{N_t} p_t(j)^{1-\varepsilon}\right]^2} p_t(i)^{-\varepsilon} \frac{EXP_t}{P_t^A} \tag{60}$$

$$mc_t(i) \frac{\varepsilon p_t(i)^{-1}\left[\sum_{j=1}^{N_t} p_t(j)^{-(\varepsilon-1)}\right] + (1-\varepsilon)\, p_t(i)^{-\varepsilon}}{\left[\sum_{j=1}^{N_t} p_t(j)^{1-\varepsilon}\right]^2} p_t(i)^{-\varepsilon} EXP_t$$

$$= 0$$

Note that we assumed that firms take individual wages as given when choosing employment. The second condition shows that $\phi_t(i)$, the surplus created by a match, is identical across mature incumbent firms. Before providing an explicit formula for the individual price level and the price mark up, we turn to the profit maximization problem of a first period incumbent producer which sets the price for the first time. The relevant difference with respect to the previous case is represented by the form of constraint (57) which reads as $v_t(i)\, q_t = n_t(i)$, since producers in their first period of activity have no stock initial workforce. However, FOCs with respect to $p_t(i)$, $n_t(i)$ and $v_t(i)$ are identical to those reported above. Since the surplus ϕ_t created by a match is identical across incumbent firms, they will face the same wage bargaining problem, thus will face the same wage, $w_t(i) = w_t$, the same marginal cost, $mc_t(i) = mc_t$, and will demand the same amount of hours, $h_t(i) = h_t$. As a result the third condition can be written as

$$(1-\varepsilon)\, P_t^{1-\varepsilon} - (1-\varepsilon)\, p_t(i)^{1-\varepsilon} = MC_t \left[\varepsilon p_t(i)^{-1} P_t^{1-\varepsilon} + (1-\varepsilon)\, p_t(i)^{-\varepsilon}\right] \tag{61}$$

where $MC_t \left(= P_t^A mc_t\right)$ is the nominal marginal cost, which shows that $p_t(i)$ does not depend on any firm specific variable. In other words all incumbent firms, no matter the period of entry, choose the same price. Since firms face the same demand function and adopt the same technology, it follows that $y_t(i) = y_t$ and $n_t(i) = n_t$. We are now ready to provide an expression for the common price chosen by firms. Given firms choose the same price level, it follows that $P_t^A = P_t = \left[\sum_{j=1}^{N_t} p_t(j)^{-(\varepsilon-1)}\right]^{\frac{-1}{\varepsilon-1}} = N_t^{\frac{1}{1-\varepsilon}} p_t$. Imposing symmetry and rearranging, condition 3 can be rewritten as

$$p_t = \mu_t^P MC_t \tag{62}$$

where

$$\mu_t^P = \frac{\varepsilon (N_t - 1) + 1}{(\varepsilon - 1)(N_t - 1)} \tag{63}$$

■

B2. Proposition 2

Proof. The main difference with the proof of proposition 1 is that profit maximization must take the inverse demand function as a constraint. The latter is

$$p_t(i) = \frac{y_t(i)^{-\frac{1}{\varepsilon}} EXP_t}{\sum_{j=1}^{N_t} y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}} \tag{64}$$

which implies that period profits can be written as

$$\pi_t = \frac{y_t(i)^{1-\frac{1}{\varepsilon}}}{\sum_{j=1}^{N_t} y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}} \frac{EXP_t}{P_t^A} - w_t(i)\, n_t(i)\, h_t(i) - kv_t(i) \tag{65}$$

and constraint 56 is replaced by $A_t n_t(i)\, h_t(i) = y_t(i)$. We proceed as above and initially consider the problem of a mature incumbent. Setting up a Lagrangian function as in the proof of Proposition 1 and differencing with respect to $y_t(i), n_t(i), v_t(i)$, it can be easily verified that the FOCs with respect to $n_t(i), v_t(i)$ are unchanged with respect to the Bertrand case. Turning to the problem of a first period incumbent firm, it can be verified that the consideration made under Bertrand competition extend to this case. Incumbent firms, independently of the period of entry, face the same marginal cost and assign the same value to the marginal worker. In particular, notice that the FOC with respect to $y_t(i)$ reads as

$$\frac{\frac{\varepsilon-1}{\varepsilon} y_t(i)^{-\frac{1}{\varepsilon}} \sum_{j=1}^{N_t} y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} - \frac{\varepsilon-1}{\varepsilon} y_t(i)^{\frac{\varepsilon-2}{\varepsilon}}}{\left[\sum_{j=1}^{N_t} y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\right]^2} \frac{EXP_t}{P_t^A} = mc_t \tag{66}$$

which shows that individual production is not firms specific. Imposing symmetry and rearranging leads to the individual output

$$y_t = \frac{\varepsilon - 1}{\varepsilon} \frac{N_t - 1}{N_t^2} \frac{EXP_t}{MC_t} \tag{67}$$

Substituting the latter into the inverse demand function, after imposing symmetry, we get

$$p_t = \frac{EXP_t}{N_t} {y_t}^{-1} = \frac{EXP_t}{N_t} \frac{\varepsilon N_t^2 MC_t}{(\varepsilon - 1)(N_t - 1) EXP_t} = \mu_t^Q MC_t \tag{68}$$

where

$$\mu_t^Q = \frac{\varepsilon}{(\varepsilon - 1)} \frac{N_t}{(N_t - 1)} \tag{69}$$

■

B3. Proposition 3

Proof. Since all incumbent firms are, under both forms of competition, characterized by the same size, first period incumbent firms, which have no initial workforce, must post at time t as many vacancies as required to reach the size of a mature incumbent producer. Given the time-t workforce of a first period incumbent is $v_t^{FP} q_t = n_t$, i) follows. To prove ii) notice that

$$\pi_t^{FP} = \frac{p_t}{P_t} y_t - w_t h_t n_t - k v_t^{FP} = \frac{p_t}{P_t} y_t - w_t h_t n_t - k \frac{n_t}{q_t} \tag{70}$$

Since it also holds that $n_t = \rho n_{t-1} + v_t q_t$ the latter can be written as

$$\begin{aligned} \pi_t^{FP} &= \frac{p_t}{P_t} y_t - w_t h_t n_t - k \frac{\varrho n_{t-1} + v_t q_t}{q_t} \\ &= \underbrace{\frac{p_t}{P_t} y_t - w_t h_t n_t - k v_t}_{\pi_t} - k \varrho \frac{n_{t-1}}{q_t} \\ &= \pi_t - k \frac{\varrho n_{t-1}}{q_t} \end{aligned} \tag{71}$$

■

B4. Proposition 4

Proof. The value of a new entrant reads as

$$V_t^e = E_t \Lambda_{t,t+1} \pi_{t+1}^{NP} + E_t \sum_{s=t+2}^{\infty} \Lambda_{t,s} \pi_s = E_t \Lambda_{t,t+1} \left(\pi_{t+1}^{NP} + V_{t+1} \right) \tag{72}$$

Proposition 3 implies that

$$\pi_{t+1}^{NP} = \pi_{t+1} - k \frac{\varrho n_t}{q_{t+1}} \tag{73}$$

Using the latter into (72) it follows

$$V_t^e = E_t \Lambda_{t,t+1} \left(\pi_{t+1} - k \frac{\varrho n_t}{q_{t+1}} \right) + E_t \Lambda_{t,t+1} V_{t+1} \tag{74}$$

Notice that the value of an incumbent firm must satisfy the recursive equation

$$V_t = E_t \Lambda_{t,t+1} \left(\pi_{t+1} + V_{t+1} \right) \tag{75}$$

Substituting the latter into (74) we obtain equation (20). A similar result can be obtained combining equations (50) and (49) and using the result in Proposition 3. ■

C. Equilibrium Conditions

In what follows we list the equilibrium conditions of the model. The definition of aggregate employment is

$$L_t = N_t n_t \tag{76}$$

Since $P_t Y_t = N_t p_t y_t$ and $\rho_t = \frac{P_t}{p_t}$ it follows that aggregate output reads as

$$Y_t = \rho_t N_t y_t = \rho_t A_t L_t h_t \tag{77}$$

In equilibrium $B_t = B_{t-1} = 0$ and $s_t = s_{t+1} = s^e_{t+1} = 1$. Further since the Government runs a balanced budget it follows that $G_t = b\left(1 - L_t\right) = T_t$ and the aggregate resource constraint reads as

$$C_t + V^e_t N^e_t = W_t L_t h_t + (1-\delta) N_{t-1} \pi_t + (1-\delta) N^e_{t-1} \pi^{NP}_t \tag{78}$$

Good's market clearing requires

$$Y_t = C_t + N^E_t \psi + k v^{tot}_t \tag{79}$$

where

$$v^{tot}_t = (1-\delta) N_{t-1} v_t + (1-\delta) N^e_{t-1} v^{NP}_t \tag{80}$$

and

$$v^{NP}_t = \frac{n_t\left(i\right)}{q_t} \tag{81}$$

The motion of the number of firms reads as

$$N_t = (1-\delta)\left(N_{t-1} + N^E_{t-1}\right) \tag{82}$$

while the dynamic of aggregate employment

$$L_t = (1-\delta)\varrho L_{t-1} + q_t v^{tot}_t \tag{83}$$

The JCC

$$\frac{k}{q_t} = \left(mc_t A_t - w_t\right) h_t + \varrho E_t \Lambda_{t,t+1} \frac{k}{q_{t+1}} \tag{84}$$

where

$$q_t = \frac{m_t}{v_t^{tot}} \tag{85}$$

The definition of the household's stochastic discount factor is

$$\Lambda_{t,t+1} = (1-\delta)\,\beta\left(\frac{C_{t+1}}{C_t}\right)^{-1} \tag{86}$$

The wage schedule reads

$$w_t = (1-\eta)\,b + \eta mc_t A_t + (1-\eta)\,\chi\frac{C_t h_t^{1/\varphi}}{1+1/\varphi} + \eta k E_t \frac{C_t/h_t}{C_{t+1}}\theta_{t+1} \tag{87}$$

where job market tightness is defined as

$$\theta_t = \frac{v_t^{tot}}{u_t} \tag{88}$$

Hours worked satisfy

$$h_t = \left(\frac{1}{\chi}\frac{\rho_t}{\mu_t}\frac{A_t}{C_t}\right)^{\varphi} \tag{89}$$

The mark up function depends of the form of competition; for Bertrand competition we have

$$\mu_t^P = \frac{\varepsilon\,(N_t-1)+1}{(\varepsilon-1)\,(N_t-1)} \tag{90}$$

while under Cournot Competition

$$\mu_t^Q = \frac{\varepsilon N_t}{(\varepsilon-1)\,(N_t-1)} \tag{91}$$

Next we have to consider three Euler equations; the one for bonds

$$\frac{1}{C_t} = \beta(1+r_t)E_t\left(\frac{1}{C_{t+1}}\right) \tag{92}$$

that for shares of incumbent firms

$$V_t = E_t\Lambda_{t,t+1}\left(\pi_{t+1} + V_{t+1}\right) \tag{93}$$

and finally the Euler equation for shares in new entrants

$$V_t^e = E_t\Lambda_{t,t+1}\left(\pi_{t+1} - k\frac{\varrho n_{t+1}}{q_{t+1}} + V_{t+1}\right) \tag{94}$$

Next we consider the pricing equation

$$mc_t = \frac{\rho_t}{\mu_t} = \frac{N^{\frac{1}{\varepsilon-1}}}{\mu_t} \tag{95}$$

and the definition of profits of incumbent firms which have been in the market for more than a period

$$\pi_t = \frac{p_t}{P_t} y_t(i) - w_t n_t L_t - k v_t(i) \tag{96}$$

The total number of matches is

$$m_t = \gamma_m (u_t)^{\gamma} \left(v_t^{tot}\right)^{1-\gamma}, \tag{97}$$

where the definition of the unemployment rate is

$$u_t = 1 - L_{t-1} \tag{98}$$

Finally we have to take into account the entry condition

$$V_t^e = \psi \tag{99}$$

and the definition of the job finding rate

$$z_t = \frac{m_t}{u_t} \tag{100}$$

The equilibrium contains 24 equations for 25 variables: 24 endogenous variables Y_t, L_t, n_t, h_t, C_t, m_t, q_t, z_t, θ_t, N_t^e, N_t, v_t^{tot}, v_t^{NP}, v_t^e, mc_t, w_t, $\Lambda_{t,t+1}$, μ_t, r_t, V_t, π_t, u_t, V_t^e, ρ_t and 1 exogenous variable, A_t. In addition the equilibrium features 13 parameters: γ, γ_m, κ, δ, ϱ, β, φ, χ, η, ε, b, A and ψ.

D. Steady State

Given the restrictions reported in the text, the steady state can be obtained as follows. By definition $q = \frac{m}{v^{tot}} = \gamma_m \theta^{-\gamma}$, thus $\gamma_m = q\theta^{\gamma}$ and $z = \frac{m}{u} = \gamma_m \theta^{1-\gamma}$. To pin down the steady state rate of unemployment notice that $v^{tot} = \theta u = \theta(1-L)$. Substituting for total vacancies into the steady state counterpart of equation (83) leads to

$$\begin{aligned} L &= (1-\delta)\varrho L + q v^{tot} = (1-\delta)\varrho L + q\theta(1-L) \\ &= \frac{q\theta}{1-(1-\delta)\varrho + q\theta} \end{aligned} \tag{101}$$

As a consequence we can determine

$$v^{FP} = (1-\delta)\frac{N^e}{N}\frac{L}{q} = \delta\frac{L}{q} \tag{102}$$

$$u = \frac{1-(1-\delta)\varrho}{1-(1-\delta)\varrho + q\theta}. \tag{103}$$

and

$$v = v^{tot} - v^{FP} \tag{104}$$

Notice that $b = \frac{b}{w}w$, where we calibrate the ratio $\frac{b}{w}$. Evaluating the wage schedule and the JJC at the steady state leads respectively to

$$wh = \left[1-(1-\eta)\frac{b}{w}\right]^{-1}\left[\frac{\eta}{\mu}\rho Ah + (1-\eta)\chi C\frac{h^{1+1/\varphi}}{1+1/\varphi} + \eta\kappa\beta\theta\right] \tag{105}$$

and

$$wh = mcAh - (1-\varrho(1-\delta)\beta)\frac{\kappa}{q} \tag{106}$$

Combining the latter two equations, after substituting for $\chi = \frac{\varrho}{\mu}\frac{A}{C}h^{-1/\varphi}$, delivers the cost of posting a vacancy, k, as a function of the number of firms

$$\kappa = \frac{1-\left(1-(1-\eta)\frac{b}{w}\right)^{-1}\left(\frac{\eta+\varphi}{1+\varphi}\right)}{\frac{(1-\varrho(1-\delta)\beta)}{q} + \left(1-(1-\eta)\frac{b}{w}\right)^{-1}\eta\beta\theta}\frac{\rho}{\mu}Ah \tag{107}$$

The value of k increasing with the extent of competition since $\frac{\rho}{\mu}$ is an increasing function of N. The same holds for the steady state wage, given by

$$w = mcA - (1-\varrho(1-\delta)\beta)\frac{\kappa}{hq}$$

Combining the steady state counterparts of equations (78) and (79) delivers

$$Y = wLh + (1-\delta)N_{t-1}\pi_t + (1-\delta)N_{t-1}^{e}\pi_t^{NP} + kv^{tot} \tag{108}$$

where $\pi = \left(\rho(N) - \frac{w}{A}\right)\frac{AL}{N} - (1-\rho)\frac{k}{q}\frac{L}{N}$, $V = \frac{1-\Lambda}{\Lambda}\pi$ and $\pi^{NP} = \frac{\psi}{\Lambda} - V = \frac{\psi}{\Lambda} - \frac{\Lambda}{1-\Lambda}\pi$. Substituting the definitions of π, V and π^{e} into (108) delivers and equation which can be solved for N. Our numerical analysis shows that the latter has a unique solution for N>1.