Does pension debt mean anything after all?

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DOES THE IMPLICIT PENSION DEBT MEAN ANYTHING AFTER ALL?

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Abstract
We discuss the meaning of the concept of implicit pension debt (unfunded pension liabilities) from a public finance perspective and contrast different definitions such a variable with the notion of public debt. We conclude that the implicit pension is deeply different from public debt but nevertheless is meaningful for economic policy. We compute the implicit pension debt associated to retired workers for several countries for different years adopting a homogeneous algorithm. Our results show that the major countries have implicit pension debt of very different size with different trends in the last few years.

JEL: H55, H63

Keywords: Pension liabilities, implicit debt, sovereign debt.

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1. **Introduction: what are we talking about?**

The “implicit pension debt” (“unfunded pension liabilities”) consists in the present value of pension promises, net of future pension contributions, that are implicit in the current legislation. The literature on the topic was started by Feldstein (1974) who looked at such a variable from the perspective of individuals and introduced a wealth variable (“social security wealth”) arguing that it affects consumption. Such a variable might also affect fertility (Cigno, 1986) and/or human capital accumulation (Werding 2006) and can be used in order to evaluate the distributive effects of pension reforms (Beltrametti, 1996; Blanchet and Le Minez, 2008).

If we look at the same variable from the perspective of a public, pay-as-you-go (PAYG) pension scheme, we get a measure that might be meaningful for public finances both under a traditional public budget approach (Castellino 1985, Dornbusch 1999, Holzmann et al. 2001) and under more radical approaches such as “net worth” (Boskin 1982, Buiter 1983 and 1985, Eisner and Pieper 1984, Blejer and Cheasty 1991 and Bohn 1992) and “generational accounting” (Auerbach et al 1991, 1992). The main analogy between the implicit pension debt associated with the operating of a PAYG pension scheme and public debt refers to the fact that in both cases we have a promise to pay in the future made by the State.

More recently, the relevance of implicit pension debt for economic policy has been put forwards under several points of view.

First, it has been argued (Boeri and Tabellini 2004) that traditional public deficit measures embodied - e.g. in the European Stability and Growth Pact - are inadequate when structural pension reforms (aimed at reducing the weight of a public PAYG while expanding private fully funded schemes) are implemented. The Council of the European Union (2005) received this point of view by asserting that “the Council agrees that an excess close to the reference value which reflects the implementation of pension reforms introducing a multi-pillar system that includes a mandatory, fully funded pillar should be considered carefully. Although the implementation of these reforms leads to a short-term deterioration of the budgetary position, the long-term sustainability of public finances clearly improves.”

Second, the significance of implicit pension debt for the sustainability of sovereign debt has been directly stressed by the highest European authorities. In particular, the European Commission in its Proposal for a Council Regulation (EU) amending Regulation (EC) No 1467/97 on speeding up and clarifying the implementation of the excessive deficit procedure (29 September 2010, p. 5) states that the decision of starting an excessive deficit procedure “would need to take into account all the factors that are relevant, in particular for the assessment of debt developments, such as whether very low nominal growth is hampering debt reduction, together with risk factors linked to the debt structure, private sector indebtedness and implicit liabilities related to ageing” (emphasis added).
Third, leading market players (e.g. Credit Suisse, Market Focus May 24th 2010, p. 6) have included “pension and health liabilities” among the critical long term factors affecting sovereign debt sustainability. Moody’s (Hampton et al., 2011) has used debt and pension metrics in order to better rate U.S. state bonds and asserts that “treating pension liabilities as a form of debt, and combining the unfunded amount with outstanding indebtedness, improves transparency by providing a more complete comparison of states based on their total long-term obligations as a portion of available revenue and taxing capacity” (p. 2). Much on the same line, Munnell et al. (2011) find that unfunded pension liabilities have a significant, albeit modest, effect on the spread of bond yields of U.S states.

In this paper we discuss the possible meanings of pension debt from a public finance perspective and provide estimates for a few industrial countries by using a homogeneous algorithm. We restrict to the pension debt that considers only pension promises made to current retirees (see Appendix for a discussion of possible definitions of pension debt). This latter choice has five main motivations: i) future payments are totally unconditional (no social security contributions are due) and thus we deal with a variable that is closer to public debt; ii) the political cost of reforms that modify already-going pension payments are much higher than those of reforms affecting retirement age and/or rules for determining the amount of pension benefits1; iii) measurement is more accurate since we do not have to make assumptions about future labour incomes, employment rates, future contribution rates, age at retirement2, accrual rate for future benefits; furthermore, a shorter time horizon implies that hypotheses about discount rate are less decisive; iv) by limiting our analysis to retired workers we make our data more comparable cross country since active workers in different countries may be subject to different probabilities of pension reform before they retire; v) by limiting our estimates to the pension debt held by retirees, we shortcut all the controversies about the pension debt definition to be used (see Appendix) since all definitions coincide as far as retirees are concerned.

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1 The rationale for this could be that retirees have very limited room for adjusting their work supply and for smoothing their consumptions inter-temporally; furthermore, they have a full perception of the effects of pension reform cutting their benefits and therefore make a much stronger opposition to it than active workers do to pension reforms that affect their future entitlements. It is no surprise that among dozen of pension reforms that occurred in the last decades around the world only very few affected retired people. By analyzing the key features of major pension reforms that occurred since 1990 in 21 countries, Whiteford and Whitehouse (2006) show that only in five cases countries have cut ongoing pension benefits through a change in the indexation of the pensions in payment (Finland, Germany, Hungary, Italy and Poland); to our knowledge, only Greece was forced to cut running pension benefits (above a given amount) under the pressure of 2009 sovereign debt crisis. Also from the juridical point of view, once the payment of pension has been started, the retirees have a fully accrued individual right to it: under many constitutional frameworks this right receives a much stronger protection than the accrued to date rights held by workers. Also “verbal” attitudes are consistent with our approach: in the case of public debt and of the pension debt of retirees any, even minor, unilateral change in rules is labelled as “default”; on the contrary, changes in rules affecting future benefits are currently labelled as “pension reform”.

2 The measurement of pension debt of the active workers implies therefore the use of (dynamic) micro-simulation models (see e.g. Mazzaferrro and Morciano, 2008; Werding 2006; Blanchet and Le Minez 2008).
In the case of retirees, the amount of future benefits to be paid and the time of payment are well defined, unconditional and more robust from both the legal and the political point of view: this makes the notion of pension debt closest to that of public debt. Indeed, what makes the public debt “special” from the public finance perspective (in contrast with other commitments to future expenditures) is the fact that it involves sovereign promises that are jointly unconditional and precisely defined.

In sum, we will deal only with the pension debt held by retirees for both conceptual reasons (it’s a variable that is more relevant for public finances) and for practical reasons (we get more precise and uncontroversial measures).

The size of both the public debt and of pension debt offer a measure of the size of resources the State has to withdraw from the private sector (in form of taxes and/or pension contributions and/or new debt issues) in order to comply with promises made in the past; for any given level of future income growth, the implicit pension debt to GDP ratio gives a measure of the size of the tax wedge that has to be imposed on young generations in order to comply with promises already made to the elderly.

However, even if the similarities between pension debt and public debt are significant, there are fundamental differences between the two:

a) while the subscription of public debt voluntarily occurs on markets, “subscription” of pension debt (i.e. the payment of pension contribution that will lead to the pension benefit) is compulsory. Nevertheless (Castellino, 1985) in both cases as the payments of debt come due, the state has the option to meet the cash requirements by issuing new debt or by extinguishing it by means of taxation.

b) The pension debt is not tradable; thus, we cannot use prices in order to learn about agent’s expectations. The impossibility of arbitrage implies that the State can give different rates of return to different groups of agents; there is also the possibility for the State of selectively repudiating the outstanding liabilities (see Bohn, 1992).

c) The pension debt is affected by life expectancy while of course public debt is not.

d) While the stock of public debt necessarily is the result of past deficits, this is not the case of pension debt: the latter exists even in a hypothetical country in which a public PAYG pension scheme has always been balanced since its inception.

Points a) and b) jointly imply that the pension debt is totally held by residents and that the conditions for its renewal do not depend on market conditions.

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3 For Switzerland and Sweden only old-age and disability pensions (for the former) and old-age (for the latter) are considered.

4 Of course, the size of the tax wedge has implications that go beyond public finance and affect firms’ location decisions and workers’ choices about regular/irregular labor market participation.
As first noted by Samuelson (1958), a PAYG pension scheme plays an economic role that exhibits important analogies with the role of fiat money: both money and a PAYG allow agents to transfer consumption intertemporally and both institutions work as long as each generation conforms to them on the expectation that also the following will. While both pension liabilities and fiat money rest to some extent on the State enforcing them with its power, on the contrary the public debt arises when the State decides not to use its very own power of collecting taxes and printing money. In the case of both pension liabilities and fiat money there is a form of seignorage that is exploited by the State: in the case of pensions the Government gains the gratitude of the first generation of retirees (who did not pay social security taxes during their working lives), in the case of fiat money the story is well known. While the dynamics of both pension liabilities and money is pro-cyclical (see Fact 1 in the Appendix), on the contrary the dynamics of the public debt tends to be anti-cyclical.

In sum, even in a perfectly balanced PAYG scheme the pension debt is positive for all parameter values (see Appendix); its size is not a direct measure of the financial sustainability: it simply offers a measure of how costly is going to be to meet past pension promises. In other words, the size of the pension debt gives a measure of the perspective distortionary effects of the tax wedge associated with the working of a PAYG pension scheme. A deficit (surplus) in a PAYG system does not imply that pension liabilities are rising (decreasing) and vice versa (see Fact 3 Appendix).

2. Measurement issues

In general, from the point of view of its measurement, the pension debt can be divided into the component associated with active workers and the one associated with retirees.

As we have said above, when we limit our calculation to the pension debt held by already retired persons, the measurement is simple since the current benefit is known and the only uncertainty refers to indexation and expected life duration.

The computation of pension debt consists in calculating the present value at year t of pension benefits whose right can be claimed by those who already receive a pension:

\[
PD_t = \sum_{j=1}^{v_{s,a,t}} \sum_{s=1}^{2} \sum_{a=1}^{n} \left( \frac{B_{s,t}}{(1+r)^j} \right) \cdot P_{s,a,t}
\]

where B is the average amount of benefits for individuals of gender s and age a in year t; P is the number of such benefits, v_{s,a,t} is the (country specific) expected residual life of the individuals of gender s, at age a, at time t and r is the discount factor. We take five years age cohorts (except the
last one that includes all the individuals with more than 95 years) and attribute to individuals in each cohort the corresponding median age\(^5\) (for the last cohort we attribute a median age of 97).

We refer to all kind of pensions (old age, disability, ...) currently being paid under the national PAYG scheme(s).

We consider before tax benefits: of course, this assumption implies a substantial over-estimation of financial burden for the State but makes our measures independent from assumptions about the existence of other revenues or about family composition; note that even in traditional public debt literature the fact that the State taxes interests is neglected. Our measures are overestimated also because, for lack of data, the same expected residual life is applied for those individuals that receive a disability pension even if it is known that disabled people have shorter life expectations. However, these two sources overestimation are partially balanced by the fact that we do not consider future pensions payable to surviving spouses. We are not able to give a measure of the overall, net effect of these assumptions.

3. Results and conclusions

We consider nine countries and we are able to consider changes across time for Italy (1999-2007), USA (1990-2008), Germany (2003-2009) and United Kingdom (2003-2010).

Our computations show a wide range of values for the pension debt /GDP ratio. In the scenario with no discounting (Table 1A), it varies from 41% (USA, 1990) to 250% (Italy 1999); as we said above, such figures alone do not imply any judgment on the financial sustainability of a PAYG pension scheme: they simply reflect different sizes of public PAYG schemes. Values of pension debt to public debt ratios range from 74% (USA, 1990) to 447% (Finland, 2008). The share of pension debt held by women ranges from 42.4% (USA, 2003) to 60.2% (New Zealand, 2006); this latter ratio depends jointly on female labour force participation rates, gender differentials in wages, age at retirement and life expectations. Duration has a minimum of 7.9 years in Sweden (2008) and a maximum of 14.8 years in New Zealand (2006).

For Italy and USA we have observation over a relatively long time spell. In Italy, over the 1999-2007 period the pension debt to GDP ratio has decreased (from 250% to 242%) while duration has slightly increased. In the USA over the period 1990-2008 the pension debt to GDP has increased from 41% to 67% while duration increased from 8.6 to 10.2.

For 3 countries (USA, Germany, UK) we have observations for the period 2003-2008. We note that the pension debt to GDP ratio decreased in Germany by 32 percentage points; on the contrary, it increased in the USA and the UK by, respectively 11 and 7 percentage points. Duration shortens in

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\(^5\) This implies a (minor) over-estimation when a discount rate is used.
all countries but the USA and the share of pension debt held by women decreases in Germany while increases in the USA and the UK.

Table 2 gives an insight of the determinants of changes in value of implicit pension debt: life expectancies, the number of beneficiaries (both in absolute and in % of population) increases in all the countries considered; the increase in the number of beneficiaries in the USA is noticeable. On the contrary, other variables move in different direction in different countries: the generosity of the pension schemes (average pension benefit/per capita GDP) increases in the USA and UK while decreases in Italy and Germany; the share of beneficiaries over both 65 and 75 increases in all countries but the USA; similarly, the average pension benefit of retirees over 65 grows (with respect to average benefit) in USA and declines in UK and Germany; the symmetric occurs when retirees 75+ are considered.

The choice of a *discount factor* for pension liabilities is very controversial\(^6\); in Table 1B a factor equal to the yield of the 10 years government bonds is introduced: the size of the fall in the pension debt depends on national interest rates, the age and gender distribution of beneficiaries and the relative amount of benefits by age and gender. According to these factors, two facts are noticeable and both are due to a dramatic fall in interest rates in the national bond market. The duration of UK pension debt increases in the period 2003-2010 (it shortens with no discounting). Since women have longer life expectation than men, discounting tends to reduce their share of pension debt; similarly discounting has stronger effect where the benefits of the older beneficiaries are relatively high. This occurs in all countries but the UK because of the particular structure of benefits by age and gender. In general, it is not possible to compare our results with the ones obtained in previous studies\(^7\). The only possibility is the comparison of our results with those by Van den Noord and Herd (1993) for the pension debt of retirees for USA 1990. Such a comparison is very encouraging since the pension debt/GDP ratio with no discounting is almost the same (42% in Van den Noord and Herd work and 41% in ours).

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\(^6\) Brown and Wilcox (2009) argue that an ideal discount rate for pension liabilities should be derived from securities that deliver cash flows that are fully taxable, with high degree of assurance, that are traded in markets without extraordinary liquidity characteristics and that are free of flight-to quality effects. It is not obvious however to identify a security with all those characteristics and scholars often use values taken from the yield of medium or long term government bonds: e.g. Gokhale and Smetters (2003, p. 23) use a discount rate of 3.6% “corresponding to the average yield on thirty-year Treasury bonds during the past several years”. Since in this paper we deal only with the pension debt of retired persons, it seems appropriate to refer to 10 year government bonds; furthermore, this maturity closely matches the average durations of retirees pension debt (see Tables 1A and 1B).

\(^7\) There are several reasons for that: first, methodology is different (e.g. Van den Noord and Herd, 1993, use the generational accounting computation or Brugiavini, 1987, considers a sample survey on family budgets); secondly, the disaggregation between retirees and active workers is often not detailed. Moreover, some author takes into account only particular schemes (for example, Castellino 1985 and Beltrametti 1996 consider only private sector pension schemes). Finally, previous estimations are quite old and our results are instead referred to the last available year for data with the needed detail.
Finally, note that the level of data aggregation has a substantial effect on results. For instance, we use specific data on pension benefit for each age and gender: when we use (as it is done in most of the previous literature) aggregated data that do not take into account differences in benefit for different cohorts then the differences in results are sizeable (see Table 3). In particular such differences are in the order of 10% in the case of Italy and UK.

The important differences in the size of the pension debt that we have found suggest that the tax wedge associated with the working of the PAYG pension schemes are going to be substantially different across the considered countries. Huge distributive and efficiency implications follow.
Table 1A: Implicit pension debt (only retirees) with no discounting

<table>
<thead>
<tr>
<th>COUNTRY</th>
<th>Year</th>
<th>PENSION DEBT (national currency, bn)</th>
<th>PENSION DEBT/GDP %</th>
<th>PENSION DEBT/PUBLIC DEBT %</th>
<th>DURATION (years)</th>
<th>PENSION DEBT HELD BY FEMALE POPULATION %</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITALY</td>
<td>2007</td>
<td>3741</td>
<td>242</td>
<td>234</td>
<td>10.3</td>
<td>44.9%</td>
</tr>
<tr>
<td></td>
<td>2003</td>
<td>3108</td>
<td>233</td>
<td>223</td>
<td>10.5</td>
<td>45.1%</td>
</tr>
<tr>
<td></td>
<td>1999</td>
<td>2819</td>
<td>250</td>
<td>220</td>
<td>10.2</td>
<td>45.7%</td>
</tr>
<tr>
<td>USA</td>
<td>2008</td>
<td>8888</td>
<td>67</td>
<td>102</td>
<td>10.2</td>
<td>44.7%</td>
</tr>
<tr>
<td></td>
<td>2003</td>
<td>6298</td>
<td>56</td>
<td>93</td>
<td>10.2</td>
<td>42.4%</td>
</tr>
<tr>
<td></td>
<td>1990</td>
<td>2387</td>
<td>41</td>
<td>74</td>
<td>8.6</td>
<td>42.5%</td>
</tr>
<tr>
<td>UK</td>
<td>2010</td>
<td>1918</td>
<td>142</td>
<td>169</td>
<td>13.1</td>
<td>55.3%</td>
</tr>
<tr>
<td></td>
<td>2008</td>
<td>2154</td>
<td>149</td>
<td>286</td>
<td>13.4</td>
<td>54.7%</td>
</tr>
<tr>
<td></td>
<td>2003</td>
<td>1614</td>
<td>142</td>
<td>368</td>
<td>13.7</td>
<td>52.7%</td>
</tr>
<tr>
<td>GERMANY</td>
<td>2009</td>
<td>2829</td>
<td>118</td>
<td>161</td>
<td>8.4</td>
<td>51.3%</td>
</tr>
<tr>
<td></td>
<td>2008</td>
<td>2781</td>
<td>111</td>
<td>169</td>
<td>8.5</td>
<td>51.5%</td>
</tr>
<tr>
<td></td>
<td>2003</td>
<td>2548</td>
<td>143</td>
<td>175</td>
<td>8.7</td>
<td>51.7%</td>
</tr>
<tr>
<td>FINLAND</td>
<td>2008</td>
<td>282</td>
<td>152</td>
<td>447</td>
<td>9.8</td>
<td>51.3%</td>
</tr>
<tr>
<td>SWEDEN</td>
<td>2008</td>
<td>4176</td>
<td>153</td>
<td>395</td>
<td>7.9</td>
<td>46.1%</td>
</tr>
<tr>
<td>SWITZERLAND*</td>
<td>2005</td>
<td>196</td>
<td>44</td>
<td>115</td>
<td>10.6</td>
<td>54.1%</td>
</tr>
<tr>
<td>DENMARK*</td>
<td>2010</td>
<td>284</td>
<td>124</td>
<td>305</td>
<td>10.7</td>
<td>56.9%</td>
</tr>
<tr>
<td>NEW ZEALAND*</td>
<td>2006</td>
<td>82</td>
<td>78</td>
<td>267</td>
<td>14.8</td>
<td>60.2%</td>
</tr>
</tbody>
</table>

Source: our computations on national data.

* Average pension benefit by age not available: we use the average benefit by gender

Table 1B: Implicit pension debt (only retirees) with 10 year national bonds discounting

<table>
<thead>
<tr>
<th>COUNTRY</th>
<th>Year</th>
<th>PENSION DEBT (national currency, bn)</th>
<th>PENSION DEBT/GDP %</th>
<th>PENSION DEBT/PUBLIC DEBT %</th>
<th>DURATION (years)</th>
<th>PENSION DEBT HELD BY FEMALE POPULATION %</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITALY</td>
<td>2007</td>
<td>3046</td>
<td>197</td>
<td>191</td>
<td>9.2</td>
<td>44.4%</td>
</tr>
<tr>
<td></td>
<td>2003</td>
<td>2574</td>
<td>193</td>
<td>185</td>
<td>9.5</td>
<td>44.6%</td>
</tr>
<tr>
<td></td>
<td>1999</td>
<td>2032</td>
<td>180</td>
<td>158</td>
<td>8.8</td>
<td>45.0%</td>
</tr>
<tr>
<td>USA</td>
<td>2008</td>
<td>7309</td>
<td>55</td>
<td>84</td>
<td>9.3</td>
<td>44.2%</td>
</tr>
<tr>
<td></td>
<td>2003</td>
<td>4935</td>
<td>44</td>
<td>73</td>
<td>9.1</td>
<td>41.9%</td>
</tr>
<tr>
<td></td>
<td>1990</td>
<td>2056</td>
<td>35</td>
<td>64</td>
<td>8</td>
<td>42.2%</td>
</tr>
<tr>
<td>UK</td>
<td>2010</td>
<td>1837</td>
<td>136</td>
<td>162</td>
<td>12.7</td>
<td>55.4%</td>
</tr>
<tr>
<td></td>
<td>2008</td>
<td>2125</td>
<td>147</td>
<td>282</td>
<td>13.4</td>
<td>54.7%</td>
</tr>
<tr>
<td></td>
<td>2003</td>
<td>1082</td>
<td>95</td>
<td>246</td>
<td>10.2</td>
<td>53.1%</td>
</tr>
<tr>
<td>GERMANY</td>
<td>2009</td>
<td>2289</td>
<td>95</td>
<td>130</td>
<td>7.7</td>
<td>50.7%</td>
</tr>
<tr>
<td></td>
<td>2008</td>
<td>2362</td>
<td>95</td>
<td>144</td>
<td>7.9</td>
<td>51.0%</td>
</tr>
<tr>
<td></td>
<td>2003</td>
<td>1938</td>
<td>109</td>
<td>133</td>
<td>7.7</td>
<td>50.9%</td>
</tr>
<tr>
<td>FINLAND</td>
<td>2008</td>
<td>277</td>
<td>150</td>
<td>440</td>
<td>9.7</td>
<td>51.2%</td>
</tr>
<tr>
<td>SWEDEN</td>
<td>2008</td>
<td>3691</td>
<td>135</td>
<td>349</td>
<td>7.6</td>
<td>45.8%</td>
</tr>
<tr>
<td>SWITZERLAND*</td>
<td>2005</td>
<td>175</td>
<td>39</td>
<td>102</td>
<td>10.2</td>
<td>53.8%</td>
</tr>
<tr>
<td>DENMARK*</td>
<td>2010</td>
<td>273</td>
<td>119</td>
<td>294</td>
<td>10.4</td>
<td>56.9%</td>
</tr>
<tr>
<td>NEW ZEALAND*</td>
<td>2006</td>
<td>54</td>
<td>51</td>
<td>176</td>
<td>10.9</td>
<td>58.2%</td>
</tr>
</tbody>
</table>

Source: our computations on national data.

* Average pension benefit by age not available: we use the average benefit by gender
<table>
<thead>
<tr>
<th>COUNTRY</th>
<th>Year</th>
<th>Life expectations at 67 (males)</th>
<th>Life expectations at 67 (females)</th>
<th>Average pension benefit/per capita gdp</th>
<th>Beneficiaries per 100 inhabitants</th>
<th>% beneficiaries 65+</th>
<th>% beneficiaries 75+</th>
<th>Average pension benefit 75+/average pension benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITALY</td>
<td>2007</td>
<td>16.34</td>
<td>19.85</td>
<td>0.53</td>
<td>28.4</td>
<td>52.7%</td>
<td>6.4%</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td>2003</td>
<td>15.32</td>
<td>18.90</td>
<td>0.52</td>
<td>28.5</td>
<td>48.91%</td>
<td>16.08%</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>1999</td>
<td>14.75</td>
<td>18.36</td>
<td>0.57</td>
<td>27.4</td>
<td>44.50%</td>
<td>13.80%</td>
<td>1.03</td>
</tr>
<tr>
<td>USA</td>
<td>2008</td>
<td>17</td>
<td>19.70</td>
<td>0.31</td>
<td>13</td>
<td>60.40%</td>
<td>18.40%</td>
<td>1.03</td>
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<td>16.4</td>
<td>19.20</td>
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<td>12.2</td>
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<td>19.97%</td>
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<td>1990</td>
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<td>17.10</td>
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<td>10.9</td>
<td>66.10%</td>
<td>18.60%</td>
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<td>2010</td>
<td>16.03</td>
<td>18.58</td>
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<td>30.9</td>
<td>50.50%</td>
<td>17.20%</td>
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<td>16.21</td>
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<td>2003</td>
<td>14.85</td>
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<td>0.27</td>
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<td>GERMANY</td>
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<td>16.1</td>
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<td>24.30%</td>
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<td>19.00</td>
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<td>15.7</td>
<td>18.50</td>
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<td>62.70%</td>
<td>17.80%</td>
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<td>19.65</td>
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<td>14.60%</td>
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<td>2008</td>
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<td>19.30</td>
<td>0.52</td>
<td>21.6</td>
<td>82.70%</td>
<td>22.90%</td>
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<td>2005</td>
<td>16.8</td>
<td>17.50</td>
<td>0.3</td>
<td>8.1</td>
<td>49.70%</td>
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<td>1.00</td>
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<tr>
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<td>2010</td>
<td>15.51</td>
<td>17.84</td>
<td>0.37</td>
<td>15.7</td>
<td>59.70%</td>
<td>16.80%</td>
<td>1.00</td>
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<tr>
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<td>2006</td>
<td>16.42</td>
<td>18.98</td>
<td>0.24</td>
<td>16.8</td>
<td>46.30%</td>
<td>15.10%</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Source: our computations on national data.
* Average pension benefit by age not available: we use the average benefit by gender
Table 3: Differences between measures of the pension debt when using specific benefits for each gender and cohort (as in equation 1) and when using average benefit for all cohorts.

<table>
<thead>
<tr>
<th>COUNTRY</th>
<th>DIFFERENCE BY PENSION DEBT§ (no discounting)</th>
<th>ABSOLUTE DIFFERENCE BETWEEN DURATION§ (no discounting)</th>
<th>PENSION DEBT WITH REAL DISCOUNT RATE=10 YEARS COUNTRY BOND (PD with zero discounting=100)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>9%</td>
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<td>81.4</td>
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<tr>
<td>USA 2008</td>
<td>3%</td>
<td>0.4</td>
<td>82.2</td>
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<td>UNITED KINGDOM 2010</td>
<td>11%</td>
<td>1.8</td>
<td>95.9</td>
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<tr>
<td>GERMANY 2009</td>
<td>4%</td>
<td>0.4</td>
<td>80.9</td>
</tr>
<tr>
<td>FINLAND 2008</td>
<td>4%</td>
<td>1.3</td>
<td>98.2</td>
</tr>
<tr>
<td>SWEDEN 2008</td>
<td>4%</td>
<td>0.4</td>
<td>88.4</td>
</tr>
<tr>
<td>SWITZERLAND 2005</td>
<td>-</td>
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</tr>
<tr>
<td>DENMARK 2010</td>
<td>-</td>
<td>-</td>
<td>96.1</td>
</tr>
<tr>
<td>NEW ZEALAND 2006</td>
<td>-</td>
<td>-</td>
<td>65.8</td>
</tr>
</tbody>
</table>

Source: our computations on national data

§ Measure of the size of the overestimation (+) or underestimation (-) that occurs when the computation is done using average pension benefit by gender with no differentiation by age group.
References


Data sources:
Danmarks Statistik (2009): www.dst.dk,
Istituto Nazionale di Statistica (ISTAT) : www.istat.it .
APPENDIX: The simple arithmetic of pension liabilities

Three definitions of Social Security Wealth (SSW) and, symmetrically, implicit pension liabilities are most commonly used in the literature: a) the net SSW for the closed group of those who are currently retired or in the labour force; b) the net SSW for the open group of all born and unborn generations; c) the accrued to date SSW.

Definition "a". This definition was suggested by Feldstein (1974: pp. 911): “Gross social security wealth... is the present value in year t of the retirement benefits which could eventually be claimed by all who are either in the labour force or already retired in year t... Net social security wealth... equals gross social security wealth minus the present value of the social security taxes to be paid by those who are currently in the labour force”.

Definition "b". The net SSW for the open group of all born and unborn generations is equal to the net SSW for the closed group of those who are currently retired or in the labour force (definition "a") plus the present value of the social security benefits to which those who will join the labour force in the future (even if not yet born) will be entitled, net of the social security taxes they will have to pay.

Definition "c". The accrued to date SSW restricts the calculation to the present value of the part of future pension benefits the right to whom has already been fully acquired.

The differences among these definitions can be traced back to different underlying interpretations of the concept of vested pension right (see Beltrametti 1996); the choice of the "best" definition obviously depends on the purpose of the analysis.

A very simple model

Let us consider a deterministic economy with three overlapping generations. In each period $t$, agents belonging to generation 1, 2 and 3, respectively, live. Agents work in period 1 and 2 of their lives receiving a wage $W$ and being subject to a social security tax rate $\tau$; during period 3 they are retired and receive a pension benefit $B$. Population and wage grow, respectively, at rate $n$ and $\omega$: by assuming that $\omega$ is exogenously given we assume that SSW does not affect capital accumulation and growth.

Net and gross pension liabilities (definition “a”)

\[ PL_a = p_1^3 B_t + p_2^2 \left( \frac{1}{1 + r} \right) B_{t+1} + p_3^1 \left( \frac{1}{1 + r} \right)^2 B_{t+2} - \left[ \tau_t W_t (p_1^1 + p_2^2) + \frac{1}{1 + r} \tau_{t+1} W_{t+1} p_3^2 \right] \]

where $r$ is the discount rate that we take to be equal to the real interest rate. The first three terms correspond to the Feldstein's Gross SSW and the term in square bracket is the present value of the social security taxes to be paid by those who are currently in the labour force.

Pension liabilities according to definition "b"

It is useful to rewrite (2) as follows:

\[ PL_a = D_t + X_t \]

where:

\[ D_t = \tau_t W_t (p_1^1 + p_2^2) + \frac{1}{1 + r} \tau_{t+1} W_{t+1} p_3^2 \]

\[ X_t = p_1^3 B_t + p_2^2 \left( \frac{1}{1 + r} \right) B_{t+1} + p_3^1 \left( \frac{1}{1 + r} \right)^2 B_{t+2} \]

---

9 The discussion that follows will make it clear in this case it is irrelevant whether you refer to the closed group of those who are currently retired or in the labour force or to the open group of all born and unborn generations.
10 It is the sum of the benefits to which those who are currently retired are entitled and the present value of future benefits which could eventually be claimed by each worker taken in the proportion in which she has already paid the total social security taxes that will entitle her to an old age pension; no deduction of future social security taxes is made.
\[ D_t = P_t^3B_t - (P_t^1 + P_t^2)\tau W_t \]

is the current deficit of the pension scheme in time \( t \) and

\[ X_t = \left[ P_{t+1}^3 \left( \frac{1}{1+r} \right) B_{t+1} - P_{t+1}^2 \left( \frac{1}{1+r} \right) \tau W_{t+1} \right] + P_{t+2}^3 \left( \frac{1}{1+r} \right)^2 B_{t+2} \]

is a term that contains the present values of payments that concern generations alive in period \( t \) but that will occur in \( t+1 \) and \( t+2 \). Similarly, if we consider not only the generations living in \( t \) but also those living in all periods from \( t \) to \( t+m \) we have\(^\text{11}\):

\[ PLa_{t,t+m} = \sum_{i=0}^{m} D_{t+i} \left( \frac{1}{1+r} \right)^i + \left( \frac{1}{1+r} \right)^m X_{t+m} \cdot \]

By definition, we have:

\[ PLb_t = \lim_{m \to \infty} (PLa_{t,t+m}) = \sum_{i=0}^{\infty} D_{t+i} \left( \frac{1}{1+r} \right)^i + \lim_{m \to \infty} \left( \frac{1}{1+r} \right)^m X_{t+m} \]  

(3)

Accrued to date pension liabilities (definition "c")

\[ PLC_t = P_t^3B_t + \frac{1}{2} P_t^2 \left( \frac{1}{1+r} \right) B_{t+1} \].

(4)

That is, we compute the right to receive a pension benefit that has been already fully acquired by the old generation and we compute half the benefit to be received at the end of the following period by the middle age generation.

A perfectly balanced scheme

Let us assume that the pension scheme is perfectly balanced, i.e. that in each period the social security taxes paid by the workers equal the pension benefits paid to the retirees; we take \( \tau \) as an exogenously given constant and thus have to assume that \( B_t \) is endogenously determined so that the budget constraint is met:

\[ B_t = \frac{\tau W_t (p_t^1 + p_t^2)}{P_t^3} \]

(5).

Under the assumption that the system is perfectly balanced and noticing that \( p_t^1 = p_{t+1}^2 = p_{t+2}^3 \) and \( p_t^3 = \frac{p_t^2}{1+n} = \frac{p_t^1}{(1+n)^2} \) we get:

\[ \text{Of course, in case people know that the system will be terminated in some future time, we have to assume that some working generations are forced to pay contributions even if they are otherwise not required to. In our framework this would be equivalent to assuming that such generations do not support the consumption of old generations, i.e. all the resources are produced by the working generations while the extension of social security does not affect the size of current and future output.} \]
\[ PLa = \tau W_i p_i^1 \left( k + k \frac{2 + n}{1 + n} \right) \] 
\[ (2') \]
\[ PLb_i = \tau W_i p_i^1 k \left( 1 + \frac{2 + n}{1 + n} k \right) \lim_{m \to \infty} k^m \] 
\[ (3') \]
\[ PLC_i = \tau W_i p_i^1 \frac{2 + n}{1 + n} \frac{2 + k}{2} \] 
\[ (4') \]

where \( k = \frac{(1 + \omega)(1 + n)}{1 + r} \).

A trivial non-negativity condition for population, wage and the present value of wage and pension benefits implies that \( \omega > -1, n > -1, r > -1 \) and that therefore \( k > 0 \). Of course, since \( k \) is the ratio between the growth rate of the economy and the interest rate, its value being lower than 1 characterizes an economy being dynamically efficient (see below).

**Fact 1**: in a perfectly balanced PAYG system, both net and accrued to date pension liabilities are positive for all admissible parameter values.

Thus, the very existence and the size of pension liabilities have nothing to do with the sustainability of the scheme\(^{12}\). Note that net pension liabilities consider the payments that the same agents make and receive in different periods of time while the possible net burden to public finances has to do with the payments that different generations make and receive in the same periods of time. The gross definition is a useful measure of the cost of future pension benefits payments.

Also note (equations 2, 3, 4) that, since under all the definitions unfunded pension liabilities are increasing with \( W, \tau, \omega \) and \( n \), pension liabilities are procyclical, and decrease as population growth slows down. International comparison of the size of unfunded pension liabilities (see, e.g. World Bank, 1994, p. 139; Disney, 2000, Table 1 and in Tables 1A and 1B in this paper) have no value in signalling different levels of current or perspective pressure on public finances. It would be so only if all the national schemes shared the same parameters values. For instance large differences in contribution rates imply large differences in pension liabilities even among perfectly balanced schemes. It seems also misleading to argue (Hemming, 1998, p15) that “the size of this unfunded pension liabilities would indicate the extent to which future contribution rates have to be increased to meet future pension payments”. The size of pension debt simply signals the size of the tax wedge that will occur in the future because of past pension promises.

**Fact 2**: In a perfectly balanced PAYG net pension liabilities for all born and unborn generations are positive only if \( k > 1 \), that is, if the economy is dynamically inefficient; indeed:

\[ PLb_i = +\infty \quad \text{if } k > 1 \]
\[ PLb_i = \tau W_i p_i^1 \left( 1 + \frac{2 + n}{1 + n} k \right) = PLa_i \quad \text{if } k = 1 \]
\[ PLb_i = 0 \quad \text{if } k < 1. \] 
\[ (3'') \]

\(^{12}\) This fact is discussed by Franco (1995).
Note from eq. 3 that if \( k < 1 \) then pension liabilities according to the definition "b" are the sum of the present values of all the future deficits implicit in the current design of the pension scheme: by definition, in a balanced scheme this sum is zero.

An unbalanced scheme

Let us consider an unbalanced scheme: from equations 2 and 3 we get the expression of changes in value of pension liabilities (definitions “a” and “b”) from time \( t \) and \( t+1 \):

\[
P_{La_{t+1}} - P_{La_t} = \left[ B_{t+1} P_t^2 \left( \frac{r}{1 + r} \right) + B_{t+2} P_t^1 \left( \frac{r}{(1 + r)^2} \right) + B_{t+3} \frac{P^1_{t+1}}{(1 + r)^3} \right] + \tau[A] - B_t P_t^3
\]

where

\[
A = \left[ W_t (P_t^1 + P_t^2) + \frac{W_{t+1}}{1 + r} - W_{t+1} (P_t^1 + P_t^1) + \frac{W_{t+2}}{1 + r} P^1_{t+1} \right]
\]

that is, the change in the value of pension liabilities under definition “a” from time \( t \) to time \( t+1 \) depends on the “issuing” of new debt (see the first squared bracket) plus a term (see the second squared bracket) that says that if population and wages grow, than liabilities decrease since the State expects greater contributions in the future minus the part of debt that is reimbursed (see last term).

For definition “c” we get

\[
P_{Lc_{t+1}} - P_{Lc_t} = B_{t+1} \left[ P_t^2 - \frac{1}{2} P_t^2 \left( \frac{1}{1 + r} \right) \right] + \frac{1}{2} B_{t+2} P_t^2 \frac{1}{1 + r} - B_t P_t^3
\]

That is, the change in the value of pension liabilities under definition “c” from time \( t \) to time \( t+1 \) is given by the old generation acquiring the right to the “second half” of the pension benefit (first squared bracket) plus the middle age generation acquiring the right to the first half of the pension benefit (second squared bracket) minus the reimbursement of the pension debt towards the previous old generation.

Since the pension scheme runs a current deficit if:

\[
\tau W_t (P_t^1 + P_t^2) < B_t P_t^3
\]

we see that:

**Fact 3:** A deficit (surplus) in a PAYG system does not imply that pension liabilities are rising (decreasing) and vice versa.

Indeed this is a very intuitive result: think at a PAYG that is being closed down: a current deficit is associated with declining pension liabilities; on the contrary, in early stages of a new PAYG current surpluses are associated with increasing pension liabilities. Nevertheless, it is often argued that changes in pension liabilities have to do with sustainability: for instance the European Central Bank (ECB 2000, p. 63) states that “the improvements in current primary balances required to prevent an unsustainable build up of public pension liabilities have been estimated to be substantial”. The very fact a PAYG is unbalanced does not imply that pension liabilities increase.
Few other facts in a balanced PAYG

One could like to take into account the social security taxes paid by the living generation before time \( t \) and could ask what is the net worth of the participation in the PAYG scheme for all the living and future generations as a whole (NSSWL&F). Let us start by assuming that the pension system will be terminated with certainty in some future period \( t+m \). In this case we have:

\[
NSSWL & F_{t,t+m} = PLa_{t,t+m} - \left( \frac{1}{1+r} \right)^m X_{t+m} - \left[ P^1_t \tau W_t \left( k^{-2} + k^{-1} \left( 1 + n \right)^{-1} + k^{-1} \right) \right]
\]  

(6)

that is, we have to subtract from \( PLa_{t,t+m} \) the present values of both the payments that would have been made and received hadn't the system been closed in time \( t+m \) and the payments that the living generations have made before time \( t \).

**Fact 4:** Under the assumption that the system is perfectly balanced the net worth of the participation in the PAYG scheme for all the living and future generations as a whole is negative for all the admissible values of \( k \).

Indeed the sum of first two terms on the right end side of (6) gives zero while the term in squared brackets is positive. The same result occurs if we assume that the pension scheme's duration is infinite. In other words, once the first generation that receives the benefits with out having paid contributions at the start of the system has gone, the participation in the system for the remaining generations has always negative sum\(^1\). As is well known, the very existence of a PAYG may therefore appear puzzling\(^2\).

**Fact 5:** if our PAYG scheme is perfectly balanced, then:

\[ PLa_t > PLC_t \quad \text{iff} \quad k > 1 - \frac{1}{8} n + \frac{15}{32} n^2 \]

\[ PLa_t < PLC_t \quad \text{iff} \quad k < 1 - \frac{1}{8} n + \frac{15}{32} n^2. \]

**Pension liabilities and dynamic efficiency**

Since the deterministic economy we are considering grows at a rate \( \gamma = \omega + n + \omega n \), we note that \( k \) is greater (lower) than 1 only if the growth rate of the economy is greater (lower) than the rate of interest. In other terms, \( k < 1 \) implies that the economy is dynamically efficient; in other terms, \( k < 1 \) implies that the PAYG offers a rate of return on contributions that is lower than market (riskless by definition) investments. Of course in the real world uncertainty about returns and administrative costs might question this fact.

The pension liabilities associated with the generation that is entering the labour force (generation 1) are:

\[
PLa^1_t = P^1_t \left[ \left( \frac{1}{1+r} \right)^2 B_x t^2 - \tau W_t - \left( \frac{1}{1+r} \right) \tau W_{t+1} \right]
\]

\(^{13}\) It is well known that in such a case rational backwards induction will prevent the PAYG scheme from surviving.

\(^{14}\) This result is well known: see e.g. Bohn (1992, p. 44); in the case the economy has an infinite duration, it can been noted that (obviously) a negative but finite figure divided among an infinite number of generations still gives a zero burden to each of them.

\(^{15}\) See, e.g. Tabellini (1991, 2000) and Boldrin and Rustichini (2000).
plugging (5) we get

\[ PLa^1_i = \tau W_i P^1_i \left( \frac{2 + n}{1 + n} - \frac{1}{1 + n} k - 1 \right) \]

or

\[ PLa^1_i = \tau W_i P^1_i (k - 1) \left( \frac{2 + n}{1 + n} k + 1 \right) \]  

(7).

Since both \( k \) and \( \frac{2 + n}{1 + n} \) are positive, for a balanced PAYG we have the following:

**Fact 6**: if a PAYG is perfectly balanced and the economy is dynamically efficient then new generations get a negative value from the participation to the scheme (when the discount factor is the market interest rate).

**Fact 7**: if you know that the economy is dynamically efficient, and then \( PLa^1 > 0 \) implies that the system is not sustainable.